IASC P

A TWO-AGENT MODEL FOR THE ARCTO-NORWEGIAN COD

by

Ussif Rashid Sumaila

Abstract. A two-agent model for the exploitation of the Arcto-Norwegian cod stock is developed to investigate a possible social loss from a non-unified management. These two agents are identified as a trawl fishery versus a coastal fishery. When only the trawl or coastal fishery exploits the resource, the well-known sole-ownership regime results. On the other hand, when both fisheries harvest the cod, we are more likely to end up with a non-cooperative game. Standard economic theory predicts that, *cet. par.*, the sole ownership regime should give the optimal solution whereas non-cooperative exploitation will generally entail inefficiencies. However, due to the differences in fishing gear and grounds, the question of which case gives the optimal solution is not obvious. Using a non-cooperative game framework, we show that given current Norwegian prices and costs, the highest social benefit is achieved when only the trawl fishery exploits the resource. This conclusion is, however, rather sensitive to perturbations in costs and prices.

Acknowledgements. I seize the opportunity here to thank the Norwegian Fisheries Research Council and the Chr. Michelsens Institute for their support. Mention must also be made of the invaluable professional advice received in the course of this work from Professors S.D. Flåm and R. Hannesson.

A TWO-AGENT MODEL FOR THE ARCTO-NORWEGIAN COD

1. Introduction. This study is an applied analysis of the economic benefits realisable from the exploitation of the Arcto-Norwegian cod stock (*Gadus morhua*). The focus is on the joint versus divided management of the stock and the effect of this on the economic rent accruable.

1.1 The Arcto-Norwegian cod fishery. The Arcto-Norwegian cod is a member of the Atlantic cod family, arguably the world's most important fish species. It is a shared resource jointly managed by Norway and Russia. One of the problems we face in this analysis is the lack of information on costs and prices in the Russian fishing industry. To bypass this problem, we ignore the ownership question and apply Norwegian prices and costs in the study. While this is not a true representation of reality, it still will help to illuminate issues of international resource management by using a quasi empirical approach. A second simplification is necessary due to the fact that several different types of gear/vessels are employed in the overall fishery (see Lønnsomhetsundersøkelser, 1979 - 1990). If we were to consider all these the analysis would become too complex and nontractable. We thus group the gears into two broad categories, namely, the coastal and the trawl fisheries, respectively. Furthermore, we select from each of these categories, the *most cost effective* vessels[1] and assume that only these are employed. This simplification can be justified as follows: Since our interest is in calculating optimal solutions, it is natural to select cost effective vessels from the outset. The selected vessels are then placed under the management of two separate and distinct management authorities, henceforth to be known as Coastal Fisheries Management (CF), and Trawl Fisheries Management (TF).

The assignment of two separate and distinct fleets to the two management authorities captures, to some extent, the division of the stock between Norway and Russia, but even in Norway a division is usually made between the coastal fleet and the trawlers, and the Norwegian quota is divided between these.

Now, we have designed a situation where the stock is managed by two separate and distinct management authorities, each employing only the most cost effective vessel

[1] Cost effectiveness is defined here in terms of least cost per kilogram of fish landed.

available to it[2]. The questions we attempt to answer are, what is the maximum social benefit that can be realised from the resource in this situation? Assuming that the stock is managed by only one of the authorities, instead of both, what will the maximum social benefits be in these cases? Which of these gives the optimal solution? The main issue is whether the optimal solution involves CF or TF when they operate alone, and how the optimal solution compares with the game solution with two players. Other questions we address include: How do the costs and prices faced by the players; their discount factors; their selectivity patterns; their time horizon; and the survival rate of the stock; affect the results of the study?

1.2 Methodology. A non-cooperative game theoretic framework is used for the analysis. The reason for this choice is two-fold. First, when both TF and CF exploit the resource, a conflict emerges, where each management seeks to maximise its net discounted profit from the resource, given that its rival does the same[3]. Second, when exploitation is limited to only one player, the so-called sole ownership regime results, whose optimum can easily be analysed by slightly modifying the game theoretic framework.

The economic theory of fishery exploitation is well endowed with models of open access or sole ownership (Andersen and Lee, 1986). The equilibrium solutions for these types of models have also been well predicted and analysed (Gordon (1954), Scott (1955), and Clark (1976)). However, the same cannot be said of models belonging to game theory proper, that is, models that lie "in-between" the open access and sole (exclusive) ownership models, where there are more than one and less than infinite participants. Efforts at game theoretic modelling of these situations have been receiving the attention of some researchers in recent times (see Levhari and Mirman (1980), Munro (1990) and Armstrong and Flaaten (1991)). It is important to note here that very few (if any) of these works, put *computation* at the centre stage as done in this study.

Typically, in games of the sort constructed herein, there are problems related to both the existence and uniqueness of equilibrium solutions. However, it is shown in Cavazutti and Flåm (1992) that, under certain conditions, equilibria for the class of games under consideration here do exist. In addition, the authors show that if along

---

[2] The consequence of this simplification on the results of the study is discussed in section 5.

[3] Due to the difficulties that are likely to arise in enforcing cooperation agreement, the possibility for cooperation between the two managements are considered here to be quite slim: There is a general lack of credible threats. This is even more so when it comes to countries.

the equilibrium profile all players impute the same shadow prices (Lagrange multipliers) to the resource constraint, then the equilibrium tends to be unique.

1.3 Summary of the results. The results of the analysis indicate that, using current prices, costs and discount factors, a maximum present value (PV) of economic rent of Nkr 47.59 billion is obtained (over a 15 year time horizon) when only TF exploits the cod stock. When only CF is allowed to exploit the resource, the corresponding amount is Nkr 44.51 billion. A PV of economic rent of Nkr 42.29 billion is realised when both companies exploit the resource. Clearly, it is economically suboptimal to allow both fleets to exploit the resource simultaneously in a noncooperative environment. Sensitivity analysis also shows that CF needs a reduction in costs relative to those of TF of just over 18% or an increase in price relative to that of TF of under 5% to produce the optimal solution.

1.4 Outline of paper. The rest of the paper is organised in the following manner: In the next section, we present the model, a special feature of which is the explicit modelling of the biologically and economically important age groups of cod. The algorithm for the computation of the (Nash non-cooperative) equilibrium solutions predicted by our model is briefly mentioned in section 3: The detailed algorithm is given in an appendix. In section 4, the results of the study are stated. Finally, section 5 concludes the paper.

2. The model.
We consider a two-agent fishery comprising CF and TF, both of whom are assumed to seek to maximise their net discounted profit from their fishing activity. The two companies are independent decision-makers in the sense that they are free to choose their decision variable (that is, fishing effort) as they deem fit. This freedom is, however, not absolute: It is a partial freedom because of the fact that the players are jointly constrained by the stock dynamics. In this way, the choice of decision variable by one player affects the outcome of the other player. We characterise this type of situation as a non-cooperative game, and proceed to construct a game theoretic model of natural resource exploitation that captures the situation to be studied.

Our model is deterministic, with the assumption that all parameters in the model are known with 100% certainty. For instance, in the part of the model describing the stock, all the parameters associated with the population dynamics are assumed to be known exactly. Extension of the work herein to a stochastic model is on our agenda. It will then be possible to compare the results of this work with that from the stochastic model.

In the presentation of the mathematical equations of the model, three subscripts ($p=1,2$, $a=0,...,A$, and $t=1,...,T$) are used to denote players in the game, age groups of fish, and fishing periods, respectively. We refer to the TF management as player 1 and the CF management as player 2. Based on the life expectancy of cod, the last age group A, is set equal to 15. The finite time horizon of the game, T, is set equal to 15 due to computational limitations.

By a game we mean a normal (strategic) form game involving the two "players" CF and TF. Each player p seeks to maximise his net discounted profit function (π_p) with respect to his effort profiles, e_p, subject to the constraints he faces. The game is dynamic, evolving upto a finite horizon in discrete time, with individual profits depending on the entire time path of the effort profiles.

The value (price) per kilogram of fish faced by player p, denoted by v_p, is assumed to be constant. The harvesting cost of a given player p in period t, $c_{p,t}$, is modelled as an "almost" linear function of his fishing effort:

$$C_{p,t} = \frac{k_p e_{p,t}^{1+b}}{1+b}$$

where $b = 0.01$, and k_p = the marginal cost of engaging one fishing fleet for one year. This formulation of the cost function has two advantages. First, it is a convex cost function, which together with the linear harvest function in the model gives a strictly concave objective function. This is important because strict concavity is a necessary condition for convergence in our model (see Flåm, 1993). Second, by choosing a value for $b = 0.01$, we end up with a marginal cost of fishing effort that can be considered constant for all practical purposes. This makes the definition of fishing effort in terms of, say, number of vessels, or number of fishing fleets of a given vessel type, non-problematic.

Now, the problem of player p is to find a sequence of effort, $e_{p,t}$ ($t=1,2,...,T$) to maximise his present value of profits

$$M_p(n, e_p) = \sum_{t=1}^{T} \delta_p^t \pi_p\left(n_t, e_{p,t}\right)$$

subject to the stock dynamics and nonnegativity constraints (see later). Where

$\delta_p = \dfrac{1}{1 + r_p}$ is the discount factor and $\pi_p = \sum_{a=0}^{A} v_p w_a q_{p,a} n_{a,t} e_{p,t} - \left(\dfrac{1}{1+b}\right) k_p e_{p,t}^{1+b}$ is

the profit in period t. Further, n_t and $n_{a,t}$ are the post catch stock number of fish (both age independence and dependence), respectively; w_a is the weight of fish of age *a*; r_p denotes the discount rate of player *p*, and $q_{p,a}$ is the age and player dependent catchability coefficient, that is, the share of age group *a* cod being caught by one unit of effort. The parameter $q_{p,a}$, plays a central role in this model: It is the device used to account for the special features of our two fisheries.

We want here to isolate and focus attention on interactions between the players at the level of the resource. Therefore the profit function above is formulated so as to exclude the possibility for interactions between the players in the marketplace (such interactions could, however, easily be incorporated): First, a constant price means a competitive market for fish, where the quantity put on the market by any single player does not affect the price. Second, the profit function of player *p* is assumed to depend only on his own effort.

An important component of this game is that players are jointly constrained by the population dynamics of the fish stock. Nature is introduced (as a player) in the game with the sole purpose of ensuring that the joint constraints are enforced. The decision variable of nature is thus the stock level - its objective being to ensure the feasibility of the stock dynamics. Formally, nature's objective is expressed as 0 if the stock dynamics is feasible, and $-\infty$ otherwise.

In addition to the joint constraints mentioned above, the players are faced with nonnegativity restrictions such as $e_{p,t} \geq 0$, $\forall$ p,t; $n_{a,t} \geq 0$, $\forall$ a,t and $n_{a,T+1} \geq 0$, $\forall$ a. We remark here that, unless players enjoy bequest, they will typically drive the fishable age groups of the stock to extinction at the end of the game, if the terminal restriction is simply $n_{a,T+1} \geq 0$, $\forall$ a. To check this tendency, one can exogeneously impose the more restrictive constraint, $n_{a,T+1} \geq \tilde{n}_a$, where $\tilde{n}_a$ is a certain minimum level of the stock of age group *a* that must be in the habitat at the end of period T+1. Alternatively, this restriction can be imposed endogeneously by obliging the players to enter into a stationary regime maintaining constant catches and keeping escapement fixed from T onwards.

Now, let the stock dynamics of the biomass of fish in numbers $n_{a,t}$, (that is, the joint constraint mentioned above) be described by

$$n_{0,t} \le f(B_{t-1}),$$
$$n_{a,t} + h_{a,t} \le s_{a-1} n_{a-1,t-1}, \qquad \text{for} \qquad 0 < a < A$$
$$n_{A,t} + h_{A,t} \le s_A n_{A,t} + s_{A-1} n_{A-1,t-1},$$

where

$$f(B_{t-1}) = \frac{\alpha B_{t-1}}{1 + \gamma B_{t-1}}$$
$$B_{t-1} = \sum_a p_a w_{sa} n_{a,t-1}$$
$$h_{a,t} = \sum_p q_{p,a} e_{p,t} n_{a,t}$$

Recall that the variable $n_{a,t}$ is the post-catch stock in numbers of fish; $f(B_{t-1})$ is the Beverton-Holt recruitment[4] function; B_{t-1} represents the post-catch biomass in numbers; p_a is the proportion of mature fish of age a; w_{sa} is the weight at spawning of fish of age a; α[5] and γ are constant parameters chosen to give a maximum stock size of about 6 million tonnes - a number considered to be the approximate carrying capacity of the habitat[6]; s_a is the natural survival rate of fish of age a; $h_{a,t}$ denotes the combined harvest of fish of age a, in fishing season t, by all agents.

The above equations incorporates the fact that at the beginning/end of any fishing period, a certain number of fish are recruited into the habitat (first equation). The recruited fish are then transferred from one age group to the next, after accounting for natural and fishing mortality (second and third equations). The first and last age groups are given special treatment. This is because in the case of the first age group, fish enter this group through recruitment, and not because they have grown too old to be in a younger age group. With regard to the last age group, cod does not grow much after age 15 and for computational reasons we have to cut off somewhere along the line.

---

[4] In this model, recruitment refers to the number of age zero fish that enter the habitat in each fishing period.

[5] $\alpha = f'(0)$ is the number of recruits per unit weight of biomass "at zero".

[6] Researchers at the Institute of Marine Research, Bergen, estimate the maximum sustainable yield (MSY) stock level to be about 3 million tonnes: With an assumption that the MSY stock level is one half of the pristine stock level we get the figure of 6 million tonnes.

Estimating the catchability coefficient. We estimate here the player and age dependent catchability coefficients, $q_{p,a}$. Recall that $q_{p,a}$ is the share of age group *a* cod being caught by one unit of effort. To fix ideas we define two closely related terms. The *catchability* of a fishing gear, cat_p, can be defined as the share of the total stock being caught by one unit of fishing effort. On the other hand, the *selectivity parameter*, $f_{p,a}$, of a fishing gear is the probability of the gear to hit fish of a particular age group.

There is a relationship between cat_p and $f_{p,a}$ which we exploit in order to identify $q_{p,a}$. To reduce unnecessary notational burden, we drop the subscript *t* here and use only the *p* and *a* subscripts in what follows. Let the quantity of cod in weight that can be landed by the gear employed by player *p* in a year, $harv_p$[7], be defined as $harv_p = (cat_p)(stk)e_p$, where cat_p is the catchability of player *p*, stk is the stock size (in weight) in that year, and e_p is proportional to the number of boats of a given size employed by player *p* in a year.

For e_p equal to one, we have $cat_p = \dfrac{harv_p}{stk}$. Thus, given $harv_p$ and stk, catchability can easily be calculated. We approximate these variables using historical data on harvesting capacities and stock levels. Now, player *p's* harvest in weight of age group *a* cod, denoted by $harv_{p,a}$, can be expressed as $harv_{p,a} = (q_{p,a})(stk_a)e_p$. The preceding equation implies that the harvest of player *p* over all age groups can be written as

$$harv_p = \left(\sum_a (q_{p,a})(stk_a)\right)e_p = \left(\sum_a (q_{p,a})\frac{(stk_a)}{(stk)}\right)(stk)e_p$$

From the foregoing equations it can be deduced that cat_p is equal to $\left(\sum_a (q_{p,a})\dfrac{(stk_a)}{(stk)}\right)$.

If we let $q_{p,a} = z_p f_{p,a}$, where z_p is a player-dependent constant of proportionality and $f_{p,a}$ is the selectivity, then

$$z_p\left(\sum_a f_{p,a}\frac{stk_a}{stk}\right) = cat_p.$$

Here the selectivity patterns of TF and CF, that is, $(f_{1,a})_{a=0}^{A}$ and $(f_{2,a})_{a=0}^{A}$, are set equal to (0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1) and (0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1), respectively. These patterns are assumed because TF operates mainly on the feeding grounds of

[7] The variable $harv_p$ is also defined as the "harvesting capacity" of player p's fishing gear.

young age groups, while CF is based principally on the spawning migrations of year classes that have attained the age of maximum biomass (Hannesson, 1993).

With $f_{p,a}$ specified and stk and stk_a obtainable, z_p is easily calculated. This is then multiplied by each element of $f_{p,a}$ to obtain the sought after catchability coefficient, $q_{p,a}$.

Using data on the Arcto-Norwegian cod stock in Kjelby (1993), we calculate z_p for p=1,2 to be equal to (0.074 and 0.089).

3. The algorithm. In this section we outline the conceptual algorithm for the computation of the outcomes predicted by our model. For detailed discussions of the theoretical bases for the algorithm see in particular Flåm (1993), and also Sumaila (1993). The detailed problem-specific algorithm is presented in an appendix.

Suppose for illustrative purposes that all constraints (except nonnegativity ones) are incorporated into one concave restriction of the form $C(n, e_p, e_{-p}) \geq 0$, where e_{-p} is the effort profile of p's rival and n, and e_p are the stock and effort profiles of player *p*, respectively (note that the *a* and *t* subscripts are ignored here). Then we can form the Lagrangian

$$L_p\left(n, e_p, e_{-p}, y\right) = M_p\left(n, e_p\right) + yC^-\left(n, e_p, e_{-p}\right)$$

Where $M_p(n,e_p)$ is the present value of profits to player *p*; y is a Lagrange multiplier, and C^- is given by *min*(0,C). The adjustment rules in the algorithm are then given as follows

$$\dot{e}_p = \frac{\partial L_p(n, e_p, e_{-p}, y)}{\partial e_p} \quad \forall\ p = 1,2$$

$$\dot{y} = -\frac{\partial L_p(n, e_p, e_{-p}, y)}{\partial y} = -C^-$$

where $\frac{\partial L_p(n, e_p, e_{-p}, y)}{\partial e_p}$ and $\frac{\partial L_p(n, e_p, e_{-p}, y)}{\partial y}$ are the partial derivatives of $L_p(n,e_p,e_{-p},y)$ with respect to e_p and y respectively. In similar fashion, the adjustment rules for the stock variable *n* can be derived by setting up the appropriate Lagrangian for the player "nature".

The algorithm then comes in differential form: Starting at arbitrary initial points (e_p, y, n), the dynamics represented by the adjustment rules are pursued all the way to the stationary points (e^*_p, y*, n*). Such points satisfy, by definition, the steady-state generalized equation system:

$$0 = C^{-}(n, e_p, e_{-p}),$$

$$0 \in \partial_p\left[M_p\left(n, e_p\right) + yC^{-}\left(n, e_p, e_{-p}\right)\right] \quad \forall\, p = 1,2,$$

with $y^* \geq 0$.

4. The results. Results of the computations are given in the tables and figures below. To obtain these results, the newly developed dynamic simulation software package, POWERSIM[8], is used as computational support. The parameter values listed in table 4.0 are used for the computations. In addition, α and γ are set equal to 1.01 and 1.5 respectively, to give a maximum biomass of 6 million tonnes for a pristine stock. Based on the survival rate of cod, s_a is given a value of 0.81. The price parameter v_p, is set equal to Nkr 6.78. The cost parameter k_p, which denotes the marginal cost of engaging a fleet of vessels (10 and 150 for TF and CF, respectively) for one year, is calculated to be Nkr 210 and 230 million for TF and CF, respectively[9]. The discount rate, r_p, is set equal to 7% as recommended by the Ministry of Finance of Norway. The initial number of cod of each age group is calibrated using the 1992 estimate of the stock size of cod in tonnes[10].

Using the data in table 4.0, we arrive at the equilibrium fishing efforts given in tables 4.1 and 4.2. Table 4.1 gives the solutions in the case where both players are active, while table 4.2 does the same for the cases where only one player is active. Appended to these tables are the corresponding PV of economic rents earned by each player, and

8 Powersim is a dynamic simulation software package developed by ModellData AS in Bergen, Norway. The model has many powerful features, including the ability to process array variables.

9 The price per kilogram of Nkr 6.78 is taken from table 22 in NOS Fiskenstatistikk (1989-1990). The costs per fleet employed in a year is calculated using cost data in Kjelby (1993)

10 The 1992 stock size is estimated at 1.8 million tonnes (Ressursoversikt, 1993).

the total PV of economic rent from the resource. Figures 4.1 and 4.2 illustrate the harvest and stock profiles graphically.

We see from table 4.1 that when both players exploit the resource, the PV of economic rent accruable is Nkr 42.29 billion, that is, the sum of the net discounted profits of the two players. Of this amount TF contributes Nkr 23.6 billion, while CF contributes Nkr 18.7 billion.

From table 4.2 we observe that not only do any player improves his individual net discounted profits when he has monopoly over the exploitation of the resource, but also their individual discounted profits are higher than the sum of discounted profits when both are active. TF obtains Nkr 47.59 billion, while CF makes Nkr 44.51 billion. Clearly, given a noncooperative environment, there is a social gain in limiting the exploitation of the resource to only one of the management authorities, with the optimal solution obtained when only TF is allowed to exploit. Indeed, by allowing only TF (instead of both) to exploit the resource, an improvement in PV of economic rent of well over 12% is achievable. This result is, however, sensitive to changes in the discount rate and the initial age composition of the stock, cf. section 4.1 and 4.5 below. The superiority of the unified management strategy seems to stem partly from the reduction in excess capacity when we change from divided to unified management, cf. table 4.1 with table 4.2.

From figure 4.2 we see that, as expected, the stock profiles are higher when only one player is active, with the highest profile obtained when only CF exploits the resource. Also, it is seen in fig. 1 that harvest levels are higher in the early periods of the game under divided management than under unified management. The reverse is, however, the case in the last periods of the game.

Discussion. Intuitively, the above results can be explained: Both players, knowing that if they let fish escape now, they will be the only ones to harvest it tomorrow, have a better incentive to do so when they have sole fishing rights over the resource. The positive effects of better conservation, or the gains due to the elimination of the "tragedy of the commons" is expected to have a positive effect on the net discounted profit accruable to both players, which by extension leads to higher PV of economic rent to the society as a whole.

Next, we investigate how, *cet. par.*, changes in important parameters affect the outcome of the study.

4.1 Effect of the discount factor. Table 4.3 gives the PV of economic rent accruable for different values of the discount factor. We see that an 8% increase in the discount factor (lesser degree of impatience) of the players, from 0.91 to 0.99, results in a 99% increase in the PV of economic rent when only CF exploits; 89% when only TF is active: and 59% when both fleets exploit the resource. CF seems to benefit most with decreasing impatience. Indeed, it turns out that at $\delta = 1$, the optimal solution is also obtained when only CF is active. This results should not be too surprising: Given that CF exploits older age groups, it becomes more affordable to wait for the stock to build up with high enough discount factors.

4.2 Effect of costs and prices. Assume a 25% decrease in the cost parameter k_P, due to say, the introduction of new and more efficient technology in the fishing industry. The PV of economic rent obtainable under such an assumption are listed in table 4.4. From this table, we observe that such a change in costs results in a 14%, 9%, and 8% increase in PV of economic rent respectively, when both players are active, only CF active, and only TF. This means that a 1% decrease in costs results in about 0.5% improvement in the PV of economic rent, at best, and about 0.3%, at worst. On the other hand, a 25% increase in the price parameter, results in a 37% increase in economic rent when both are active, 39% when only TF is active, and 40% when only TF is active (see table 4.5). This means a 1% increase in price results in about 1.5% increase in PV of economic rent.

Further sensitivity analysis reveals that the general results that TF produces the optimal solution is sensitive to both costs and prices: CF needs to reduce its cost relative to those of TF by just over 18%, or increase the price it faces relative to that faced by TF by under 5%, in order to take over the position of TF as the producer of the optimal solution.

4.3 Effect of the survival rate. An improvement in the survival rate is quite conceivable. For instance, by reducing the number of predators of cod such as seal and whales. The question we investigate here is, to what extent will a given percentage increase in the survival rate of the cod stock enhance the social benefit from the resource? To answer this question, we allow a 10% increase in the survival rate in the model. Results from such an assumption are given in table 4.6. The results indicate that the PV of economic rent to TF improves by up to 37%; that to CF by 31%: and that when both are active by 35%. So if by some means the survival rate of the cod stock were to be improved by 1%, we should expect an increase in the PV of economic rent of 3.7%, at best, and 3.1%, at worst.

4.4 Effect of different selectivity patterns. We also examined the effect of different selectivity patterns. Table 4.7 states a pair of selectivity patterns, together with the corresponding PV of economic rents. The "Patterns" in table 4.7 denote: Pattern 1: both management authorities assumed to use trawl fishery vessels; and Pattern 2: both management authorities assumed to employ coastal fishery vessels. In effect Patterns 1 and 2 imply a split of the trawl and coastal fisheries into two. It is seen that the social benefit from the resource improves to a good extent when we move from a "split" fishery to a "non-split" one: A gain in PV of economic rent of 13% is realised in the case of the trawl fishery, and 8.5% in the case of the coastal fishery.

4.5 Effect of time horizon. Due to limitations in computational capacity, we had to contend with a short time horizon of 15 fishing periods. It is therefore important to investigate the effect of time horizon on the results of the study. Results of sensitivity analysis using time horizons of 5, 10, and 15 are presented in table 4.8. We observe from this table that the relative profitability of TF and CF is seen to be affected by the length of the time horizon. Somewhat surprisingly the relative profitability of CF is seen to increase with decreasing time horizon. This may be an indication that the relative profitabilty of fleets with different selectivity patterns is sensitive to the initial age composition of the stock. The situation with both active, however, turns out to be inferior in all cases.

5. Concluding remarks.

We have shown in this study that, under the assumptions of our model. current prices and costs, maximum PV of economic rent from the Arcto-Norwegian cod stock is achieved under a unified management. In most of the cases shown, this happens when only TF is permitted to exploit the resource. CF can, however, produce the best results if it is able to reduce its cost relative to those of TF by up to over 18%, or raise the price it receives per kilogram of fish relative to that received by TF by under 5%. This can also happen as a result of changing the discount factor or the time horizon.

To put the results of this study in the right perspective, we now discuss the realism of our assumptions. We assumed (i) that all the vessels used to exploit the Arcto-Norwegian cod stock can be classified as either trawlers or coastal fishery vessels; (ii) that there are only two participants in the fishery; (iii) that only the most cost effective vessels among the trawlers and the coastal fishery vessels are employed in the exploitation. All these are, to some extent. violated in practice.

The consequence of these violations on the results of this study is that the calculated social loss (due to the participation of both players) is a lot lower that the actual loss.

There are two reasons for this claim. First, standard economic theory predicts that the higher the number of players or participants in an open access fishery, the greater the social loss through rent dissipation. Since we have only two players in our model the social loss computed herein must be the lowest possible. Second, by using only the most cost effective vessels in our analysis, we leave out the social loss due to the cost ineffectiveness of the many other vessels used in the exploitation of the resource.

Further work. We seize the opportunity here to discuss the different possible directions for the extension of the work in this report.

One possible extension is to introduce some form of interaction at the market place[11] in the present model, this would facilitate comparison between the solutions obtained therefrom with those obtained by focusing only on interactions at the stock level. Another extension of the model that can be of interest is to introduce "stock shares" or quotas, that is, some kind of property rights to the resource. It is possible to envisage a situation where the different players in the game have certain *à priori* property rights to the stock. For example, for historical reasons one participant may have (say) 60% rights to the stock size, and the other the remaining 40%. It is not difficult to see that such property rights can influence the results of the analysis: Consider, in particular, the case where the economically inefficient player has the larger share. In this situation, inefficiency would result if the players are allowed to fish only up to the percentage to which they have rights.

So far the model is deterministic, which is surely not realistic enough. It is common knowledge that there are many elements of uncertainty in all fisheries. For instance, both variability in recruitment and uncertainty in stock size are common. It will therefore be a fruitful exercise to modify the current model into a stochastic one.

Finally, extension of the model to handle multispecies analysis is quite feasible and important. This is clearly a fruitful area for further work, especially, with respect to the cod stock, where the relationship between cod and capelin are close, and of economic significance. Just as in the case of the single species model, here too both deterministic and stochastic models can be developed.

[11] This can be done by introducing, say, oligopolistic markets, instead of the competitive markets assumed in the current version of the model.

APPENDIX

In this appendix we work out and present the algorithm for the computations of the outcomes predicted by our model.

The algorithm. With $n_{a,0}$, that is, the initial number of fish of each age group given, the following non-standard Lagrangian function for player p follows from our model:

$$
Lp(n, e, y) = \sum_{t=1}^{T} \begin{bmatrix} \delta_p^t \pi_p(n_t, e_{p,t}) + y_{0,t}\left(f(B_{t-1}) - n_{0,t}\right)^- \\ + y_{A,t}(s_A n_{A,t-1} + s_{A-1} n_{A-1,t-1} - n_{A,t} - h_{A,t})^- \\ + \sum_{a=1}^{A-1} y_{a,t}(s_{a-1} n_{a-1,t-1} - n_{a,t} - h_{a,t})^- \end{bmatrix}
$$

where $y := y_{a,t}$ is the player-invariant, but age and season-variant multipliers, and all other variables are as defined earlier.

The negative superscript on the constraints above is a kind of device introduced to increase the efficiency of the algorithm by focussing attention on the situations where there are constraint violations. Such a device results in multipliers that are different from those that would result from the classical Lagrangians. There is a relationship between the two kinds of multipliers, however, the exact relationships are not so easy to retrieve. It is therefore necessary, at this juncture, to call for caution when interpreting the computed equilibrium multiplier levels.

The gradient information obtainable from $L_p(n,e,y)$, gives the adjustment equations for effort levels, stock levels and multipliers, respectively. For the sake of simplicity, we first introduce a special (switch) function before we state the adjustment equations. Let the function $H(r) = 1$ if $r<0$, and $H(r) = 0$ otherwise. If $r \geq 0$ were a constraint equation, then H(r) will attain a value of 1 if the constraint is violated, otherwise it attains a value of 0. In writing the adjustment equations below, this switch function is used.

Starting at arbitrary initial guesses of y_{at}, n_{at}, and e_{pt}, we pursue the dynamics given by the adjustment equations below, all the way to the equilibrium solutions.

Effort adjustment. The adjustment equation for effort given by $\frac{\partial L_p}{\partial e_p}$ is

$$
\begin{aligned}
e_{p,t} = {} & \delta_p^t\Big(\sum_a v_p w_a q_{p,a} n_{a,t} - k_p e_{p,t}^b\Big) \\
& + \sum_{a=1}^{A-1} y_{a,t} H(s_{a-1} n_{a-1,t-1} - n_{a,t} - h_{a,t})(-q_{p,a} n_{a,t}) \\
& + y_{A,t} H(s_A n_{A,t-1} + s_{A-1} n_{A-1,t-1} - n_{A,t} - h_{A,t})(-q_{p,A} n_{A,t})
\end{aligned}
$$

The message from this equation to player p is, in any given period compute the present value of marginal profit and adjust this for deductions due to constraint violation, if the results so obtained is positive, then increase your effort by the magnitude of this positive quantity, if the results is negative, decrease effort accordingly, else maintain your effort at the same level as before.

Nature's adjustment of the stock level. Natures objective can be expressed as

$$
\begin{aligned}
L_N = {} & y_{0,t}\big(f(B_{t-1}) - n_{0,t}\big)^- \\
& + \sum_{a=1}^{A-1} y_{a,t}(s_{a-1} n_{a-1,t-1} - n_{a,t} - h_{a,t})^- \\
& + y_{A,t}(s_A n_{A,t-1} + s_{A-1} n_{A-1,t-1} - n_{A,t} - h_{A,t})^-
\end{aligned}
$$

This equation is derived from the fact that once the stock dynamics is obeyed, nature's net benefit is 0. hence, L_N consist of only the constraint equations.

The updating rules for age groups. a=0, a=1,...,A-2, a=A-1, and a=A are different and are given below separately. These are obtained by partially differentiating L_N, with respect to the corresponding stock level. That is, they come from $\frac{\partial L_N}{\partial n_{a,t}}$.

(1) The stock level of age zero fish is adjusted sequentially in accordance with the equation,

$$
\begin{aligned}
\dot{n}_{0,t} = {} & y_{0,t+1} H(f(B_t) - n_{0,t+1}) f'(B_t) \frac{\partial B_t}{\partial n_{0,t}} \\
& - y_{0,t} H(f(B_{t-1}) - n_{0,t}) \\
& + y_{1,t+1} H(s_0 n_{0,t} - n_{1,t+1} - h_{1,t+1}) s_0
\end{aligned}
$$

(2) Fish of age groups between 1 and A-2 are updated as follows,

$$\dot{n}_{a,t} = y_{0,t+1}H(f(B_t) - n_{0,t+1})f'(B_t)\frac{\partial B_t}{\partial n_{a,t}} + y_{a,t}H(s_{a-1}n_{a-1,t-1} - n_{a,t} - h_{a,t})(-1 - \sum_p q_{p,a}e_{p,t}) + y_{a+1,t+1}H(s_a n_{a,t} - n_{a+1,t+1} - h_{a+1,t+1})s_a$$

(3) The last but one age group of fish (i.e., the A-1 age group) is adjusted in accordance to the equation,

$$\dot{n}_{A-1,t} = y_{0,t+1}H(f(B_t) - n_{0,t+1})f'(B_t)\frac{\partial B_t}{\partial n_{A-1,t}} + y_{A-1,t}H(s_{A-2}n_{A-2,t-1} - n_{A-1,t} - h_{A-1,t})(-1 - \sum_p q_{p,A-1}e_{p,t}) + y_{A,t+1}H(s_A n_{A,t} + s_{A-1}n_{A-1,t} - n_{A,t+1} - h_{A,t+1})s_{A-1}$$

(4) Finally, the last age group, is updated using the following equation,

$$\dot{n}_{A,t} = y_{0,t+1}H(f(B_t) - n_{0,t+1})f'(B_t)\frac{\partial B_t}{\partial n_{A,t}} + y_{A,t+1}H(s_A n_{A,t} + s_{A-1}n_{A-1,t} - n_{A,t+1} - h_{A,t+1})s_A + y_{A,t}H(s_A n_{A,t-1} + s_{A-1}n_{A-1,t-1} - n_{A,t} - h_{A,t})(-1 - \sum_p q_{p,A}e_{p,t})$$

Thus at any point in time in the computations, the right hand side (RHS) of the equation is calculated and then the corresponding stock level adjusted according to the magnitude and direction of the calculated result.

Multiplier adjustment. The equations for the sequential adjustment of the multipliers are obtained by partially differentiating the objective function of player p, with respect to the appropriate multiplier, that is, they come from $-\frac{\partial L_p}{\partial y_{a,t}}$.

For age group zero fish, the multiplier is adjusted according to the equation

$$\dot{y}_{0,t} = -H(f(B_{t-1}) - n_{0,t})(f(B_{t-1}) - n_{0,t})$$

Multipliers for fish of age groups between 1 and A-1 are adjusted as follows

$$\dot{y}_{a,t} = -H(s_{a-1}n_{a-1,t-1} - n_{a,t} - h_{a,t})(s_{a-1}n_{a-1,t-1} - n_{a,t} - h_{a,t})$$

For the last age group, multiplier adjustment is according to

$$\dot{y}_{A,t} = -H(s_A n_{A,t-1} s_{A-1} n_{A-1,t-1} - n_{A,t} - h_{A,t})(s_A n_{A,t-1} s_{A-1} n_{A-1,t-1} - n_{A,t} - h_{A,t})$$

Here too, the RHS of the equations are calculated and then the corresponding multipliers adjusted accordingly.

REFERENCE LIST

Andersen. L. G. and D.W. Lee (1986): Optimal governing instrument, operation level, and enforcement in natural resource regulation: The case of the fishery, *American Journal of Agricultural Economics 68*, 678-770.

Armstrong, C. and Flaaten, O. (1991). Non-cooperative and cooperative Management of Transboundary. Interdependent Natural Resources. Dep. of Econ. and Admin., Univ. of Tromsø Working Paper.

Bertsekas D.P. and J.N. Tsitsiklis (1989): *Parallel and distributed Computations*, Prentice-Hall, New York.

Cavazutti and Flåm (1992): Evolution to selected Nash-equilibria. In E. Giannessi (ed.): *Nonsmooth Optimization Methods and Application*. London: Gordon and Beach.

Central Bureau of Statistic of Norway (1989-1990): Fishery Statistics. Oslo, Norway.

Clark. C.W. (1976): *Mathematical bioeconomics: the optimal management of renewal resources*. New York: Wily.

Lønnsomhetsundersøkelser for fiskefartøyer 13 meter lengste lengde og over. Fiskeridirektoratet. Bergen (1979 - 1990).

Flåm. S.D. (1993): Path to constrained Nash equilibria. *Applied Mathematics and Optimisation.*

Gordon, H.S. (1954): Economic theory of a common property-resource: The fishery, *Journal of Political Economy* 62, 124-142.

Hannesson, R. (1993): *Bioeconomic analysis of fisheries*. Fishing News Books, London.

Kjelby, T. (1993): Det norske torskefiskeriet som nasjonalformue. SEFOS Notat 90.

Levhari, D. and L.J. Mirman (1980). The Great Fish War: An Example Using a dynamic Cournot-Nash Solution. *Bell Journal of Economics* 11:322-334.

Munro, G. R. (1990). The optimal Management of transboundary fisheries: Game Theoretic Considerations. *Natural Resource Modelling* 4: 403-426.

Ressursoversikt (1993): Havforskningsinstituttet, Senter for Marine Ressurser. Særnummer 1.

Scott, A.D. (1955): The fishery: The objectives of sole ownership. *Journal of Political Economy* 63. 116-124.

Sumaila, U.R. (1993): Simulation and computation of Nash equilibria in fisheries games. SEFOS Notat nr.79.

Table 4.0: Values of parameters used in the model

Age a (years)	Selectivity q(p,a) p=1	p=2	Weight at spawning w(s,a) (kg)	Weight in catch w(a) (kg)	Initial numbers (millions)
0	0	0	0.090	0.10	167.0
1	0	0	0.270	0.30	135.0
2	0	0	0.540	0.6	108.0
3	0	0	0.900	1.00	88.3
4	0.074	0	1.260	1.40	71.7
5	0.074	0	1.647	1.83	58.3
6	0.074	0	2.034	2.26	46.7
7	0.074	0.089	2.943	3.27	38.3
8	0.074	0.089	3.843	4.27	30.8
9	0.074	0.089	5.202	5.78	0.25
10	0.074	0.089	7.164	7.96	20.3
11	0.074	0.089	8.811	9.79	16.7
12	0.074	0.089	1.0377	11.53	13.3
13	0.074	0.089	12.456	13.84	10.8
14	0.074	0.089	13.716	15.24	8.67
15	0.074	0.089	14.706	16.34	7.0

Table 4.1: Effort levels (e(p,t)) in number of vessels: (Both players active)

e(1,1)	57.0	e(2,1)	810.0
e(1,2)	65.0	e(2,2)	920.0
e(1,3)	68.0	e(2,3)	938.0
e(1,4)	67.0	e(2,4)	918.0
e(1,5)	66.0	e(2,5)	893.0
e(1,6)	64.0	e(2,6)	864.0
e(1,7)	62.0	e(2,7)	839.0
e(1,8)	60.0	e(2,8)	813.0
e(1,9)	58.0	e(2,9)	788.0
e(1,10)	56.0	e(2,10)	764.0
e(1,11)	55.0	e(2,11)	741.0
e(1,12)	53.0	e(2,12)	714.0
e(1,13)	50.0	e(2,13)	680.0
e(1,14)	47.0	e(2,14)	633.0
e(1,15)	42.0	e(2,15)	564.0

TF and CF achieve net discounted profits of Nkr 23.6 and 18.7 billion, respectively. Hence, total net discounted profit from the fishery as a whole is Nkr 42.29 billion.

Table 4.2: Effort levels (e(p,t)) in number of vessels: (Only one player active)

e(1,1)	63.0	e(2,1)	910.0
e(1,2)	76.0	e(2,2)	1,086.0
e(1,3)	81.0	e(2,3)	1,154.0
e(1,4)	82.0	e(2,4)	1,151.0
e(1,5)	80.0	e(2,5)	1,124.0
e(1,6)	78.0	e(2,6)	1,091.0
e(1,7)	75.0	e(2,7)	1,059.0
e(1,8)	72.0	e(2,8)	1,026.0
e(1,9)	70.0	e(2,9)	993
e(1,10)	67.0	e(2,10)	962.0
e(1,11)	65.0	e(2,11)	929.0
e(1,12)	62.0	e(2,12)	897.0
e(1,13)	59.0	e(2,13)	864.0
e(1,14)	55.0	e(2,14)	821.0
e(1,15)	50.0	e(2,15)	761.0

TF and CF achieve net discounted profits of Nkr 47.59 and 44.51 billion respectively.
Here, the economic rent is equal to the individual net discounted profits.

Fig. 4.1: Harvest profiles (in million tonnes). The figure illustrates graphically the total harvests in each period for the 3 scenarios.

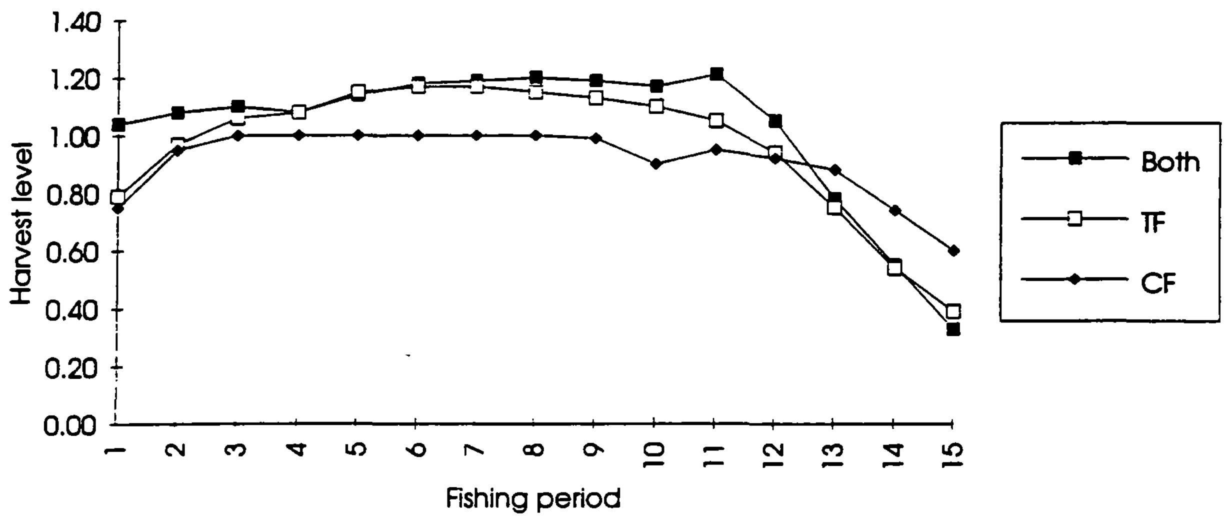

Fig. 4.2: Stock profiles (in million tonnes). The figure illustrates graphically the post catch stock levels in each period for the 3 scenarios.

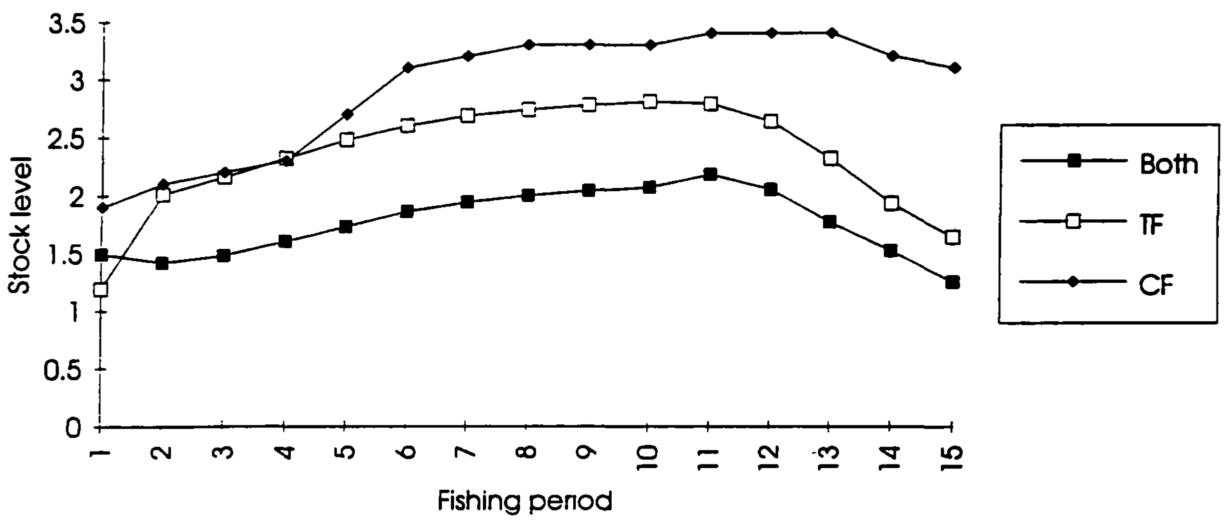

Table 4.3

Effect of discount factor on economic rent (in billion Nkr.). It gives the PV of economic rent accruable for different values of the discount factor.

	$\boxed{\delta_p = 0.91}$	$\boxed{\delta_p = 0.935}$	$\boxed{\delta_p = 0.99}$	%age increase change from 0.91 to 0.99
Both active	36.01	42.29	57.22	59
TF active	38.31	47.59	72.59	89
CF active	35.95	44.51	71.52	99

Table 4.4

Effect of costs on economic rent (in billion Nkr.). It gives the PV of economic rent accruable for different values of the cost parameter.

	kp=(0.21,0.23)	kp=(0.158,0.173)	%age increase (25% decrease in kp)
Both active	42.29	48.13	14
TF active	47.59	51.55	8
CF active	44.51	48.59	9

Table 4.5

Effect of price on economic rent (in billion Nkr.). It gives the PV of economic rent accruable for different values of the price parameter.

	vp=(6.78,6.78)	vp=(8.48,8.48)	%age increase (25% increase in vp)
Both active	42.29	48.13	37
TF active	47.59	66.12	39
CF active	44.51	62.15	40

Table 4.6

Effect of survival rate on economic rent (in billion Nkr.). It gives the PV of economic rent for different values of the survival rate.

	sa=0.81	sa=0.90	%age increase (10% decrease in sa)
Both active	42.29	57.24	35
TF active	47.59	64.97	37
CF active	44.51	58.18	31

Table 4.7

Effect of selectivity pattern on economic rent (in billion Nkr.). It gives the PV of economic rent for different selectivity patterns.

	Pattern 1	Pattern 2
"Split" fishery	42.23	41.02
"Non-split"	47.59	44.51
%age increase	13	8.5

Table 4.8

Effect of time horison on economic rent (in billion Nkr.). It gives the PV of economic rent for different time horizons.

	T=5	T=10	T=15
Both active	5.60	27.78	42.29
TF active	9.05	32.10	47.59
CF active	11.12	33.68	44.51