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ON THE EXPLOITATION OF AN UNMANAGED LOCAL COMMON.

HARDIN'S ANALYSIS RECONSIDERED.

by

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Abstract
A system of communally owned grazeland and privately owned livestock is considered. The local common is unmanaged in the sence that none of the fixed number of herdsmen owning and using the common pay any attention to their impact on the common when deciding the size of their livestock and offtake. The exploitation of the common is studied under different social and economic constraints and four types of harvesting strategies are analysed. Three of the strategies reproduce Hardin's verbal notion of a 'tragedy of the common' while a fourth one can result in a relatively robust and stable ecological system.

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1. Introduction

Common property resources are resources in which there exist property rights, but property rights that are exercised collectively by members of a group. There must also be rivalry in consumption of the resource within the group; that is, an increase in the amount consumed by one invidual reduces the amount remaining for others to consume (Seabright 1993). A common resource can be defined as a local common resource if the number of members in the specified group is 'small'. Thus, irrigation, grazing on pastures, in-shore fisheries, wildlife habitats and so on in most developing countries are local commons where the access to the resources usually are restricted to small local commmunities. Some of these resources are common resources for practical and economic reasons because of difficulty of exclusion. For example it does not pay to put a fence around the arid and sesonal Sahelian savannas where the resources are charactrized by low renewal rates. On the other hand, some of these resources are common resources for cultural and institutional reasons (Stenseth 1989). An local common can be said to be a managed common if the exploitation of the common is executed in some cooperative manner among its owners (see, e.g. Bardhan 1993 and Seabright 1993). On the other hand, a local common can be said to be unmanaged if no cooperation is present. Under such a scheme, each owner follows his self interests and the effects on the common resource base are not taken into account when exploiting the common.

Part of our understanding of the problems of unmanaged local commons comes from Garret Hardin's(1968) famous article 'The tragedy of the common'. Following Hardin the tragedy goes like this. Consider a herdsman grazing his herd in some common property land. He is free to choose how many animals he wants in his herd. He asks himself: Should I add more animals to my herd? If he does so, he will obtain almost all the profit from this extra animal when selling it on the market. This is so, because by letting his animals graze the common pastures, he in effect transfers a piece of common property (the grazeland) to some private property (the meat of his animals). The cost of letting an additional animal graze on the pasture will, however, be shared by all herdsmen using the grazeland. It follows, as an obvious consequence, that the herdsman finds it economically rewarding and rational at the moment of taking the decision, to add one more animal to his herd. By the same logic it also follows that it is rational to add yet another one, and another one... But this is the conclusion reached by every herdsman sharing the common. From this Hardin concludes that: "Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit -in a world that is limited... Freedom in a common brings ruin to all" (Hardin 1968, p. 1244).

It is well known that the above reasoning of Hardin is obscure. And according to Dasgupta(1982), it seems difficult to locate another passage of comparable fame containing as many errors. Dasgupta's main critisism is, of course, that it would be wrong to suppose that each herdsman will add cattle without limit because animals are not costless, even to the herdsmen who own them. And such private costs set limits on the number of animals each herdsman finds profitable to introduce into the common pasture. What Hardin analysis obviously lacks, is therefore a well founded analysis of the behaviour of the herdsmen in the absence of a binding mutual agreement, i.e. in an unmanaged environment.

In what follows we will take a closer look at that type of ecological and economic system discussed by Hardin(1968). More specific, the objective of the paper is to study, from a theoretical point of view, the effects on an unmanaged ecological system being exploited by herdsmen under different social and economic constraints. Differences in social and economic constraints will be represented by various types of behaviour among the herdsmen, and four types of harvesting rules will be considered. The main point then is to compare the outcome for the ecological system under the given harvesting rules. The outcomes are evaluated in terms of degree of stability and types of oscillations, and particulary any possibility of a 'tragedy of the common' scenario will be considered. In line with Hardin, it will all throughout be supposed that the livestock is privately owned and the grazeland is communally owned. Moreover, the number of herdsmen is assumed to be fixed. The study draws on the recent analysis by Brekke and Stenseth(1994) which is one of the few papers analysing Harding's scheme in a proper ecological context[1]. Their ecological model is the starting point, but the present analysis is extended to study man-made influences under different social and economic conditions, not only utility maximization as they do.

Just to fix ideas and to reveal some important characteristics of the ecological system, we start in section 2 to analyse the plant-herbivore ecological model in the absence of man. In section 3 the exploitation of the common is analysed under the different harvesting schemes. As will be shown, it is to some extent possible to characterize the different models analytically. This typically includes the (local) stability conditions. However, it is not possible to characterize the different trajectories in great details. In section 4 we therefore proceed by illustrating the models numerically. The paper is closed by some concluding remarks in section 5.

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[1]Numerous papers have taken Hardins analysis as a starting point when discussing management problems of local commons. One of the earlist formulation of the problem in a well-defined economic framework, explicitely capturing the problem as the presence of negative external effects, was Dasgupta and Heal(1979). Their framework was a static one-stock fishery model. Several attemps have also been undertaken to construct dynamic models of local commons. To a large extent, these models have focused on 'fish-wars' (see, e.g. Munro 1979, Levhari and Mirman 1980 and Plourde and Yeung 1989). However, all these papers on fish wars and static analysis à la Dasgupta and Heal depart from Hardin's(1968) analysis in the sence that Hardin was considering a bioecological system, i.e. two types of resources interacting under influence of man.

2. The bioecological model

As already noted, we consider a system of plants and livestock. All plants are assumed belonging to one species. This is also the case for the domestic animals. Plants are assumed to be communally owned while each herdsman has his own livestock animals. It is restricted access to the local common so the number of herdsmen is fixed. Moreover, all the N herdsmen ($i = 1,..,N$) are assumed to be identical.

The population dynamics of the livestock is given by (1) where herdsman i has Y_i animals at time t (the time index is omitted) and $Z_i \geq 0$ is the offtake. The specific growth rate is given by the amounts of plants X, the natural mortality of the livestock $m > 0$, the corresponding maximum reproductivity rate $h > 0$, and the self-saturation coeffecient $\varepsilon > 0$. For convenience, the reproductivity rate is denoted as $a(X)$. It is seen that $a(X)$ is a strictly concave function with $a(0) = 0$ and $a' > 0$. Moreover, $a(X)$ approaches the maximum specific reproductivity rate h when X becomes large. It is also seen that the reproductivity rate shifts down when the self-saturation constant ε increases.

(1') gives the population dynamics of the total stock of grazing animals and is just the summation over the livestocks of the N identical herdsmen exploiting the commmon. We therefore have $\Sigma Y_i = NY_i = Y$ as the total livestock and $\Sigma Z_i = NZ_i$ as the total offtake.

$$(1)\quad dY_i/dt = [hX/(X+\varepsilon) - m]Y_i - Z_i = [a(X) - m]Y_i - Z_i$$

$$(1')\quad dY/dt = [a(X) - m]Y - NZ_i$$

In the absence of grazing animals, the plant population dynamics is supposed to follow a logistic function as the first term in (2). A >0 and B >0 are the fixed parameters in the logistic growth model with A/B as the carrying capacity and A as the maximum specific growth rate. $h' > 0$ is a parameter determining the maximum grazing rate. h′ and h [in (1)] are related to each other by a constant determined by the effeciency of transforming plant-biomass to animal-biomass so that $h/h' < 1$ should hold (see, e.g. May 1981).

$$(2)\quad dX/dt = (A - BX)X - [h'X/(X+\varepsilon)]Y$$

(1) and (2) corrspond to the bioecological model used by Brekke and Stenseth(1994) and the model is of the Rosenzweig-MacArthur type predator-prey models as discussed in Maynard Smith(1974), May(1974) and May(1981). In the present framework, the predator, the animals, is harvested through a selective harvesting scheme, while there is no direct harvesting of the prey, the grazeland. Notice that the system is closed in two respects. First of all, there is no migration of herdsmen (N is fixed) so there is no inflow of grazing animals due to more herdsmen. Secondly, all recruiting of animals comes from the existing stock. Z_1 and NZ_i are therefore gross =net offtake.

In the absence of man and assumed there will be ecologically equivalents to the livestock animals, the system is defined with $NZ_1 = 0$. $X = 0$ and $Y = 0$ will be isoclines of the system, but these isoclines are of no interest. From an ecological and economic point of view, the interesting isoclines are shown in Figure 1. The X-isocline (when $dX/dt = 0$ in addition to $X > 0$) will be a humped curve and will always have a peak value between $X = 0$ and $X = A/B$. dX/dt will be negative above the isocline while it will be positive below. The Y-isocline of (1′) (when $dY/dt = 0$ and $Y > 0$, in addition to $NZ_1 = 0$) will be a straight line paralell to the Y-axis as given by $[a(X) - m] = 0$. This yields the bioecological steady-state stock of grazeland directly as $m\varepsilon/(h-m)$. dY/dt will be positive on the right hand side of the isocline and negative on the left hand side.

Figure 1 about here

It is well known (see, e.g. Maynard Smith 1974 and May 1974) that such a plant-herbivore system will exhibit damped oscillations if the Y-isocline intersects to the right of the peak value of the X-isocline (panel b, Figure 1). In such a case, the animals are said to be ineffective grazers. On the other hand, the animals will be effective grazers and the system will exhibit limit cycles if the Y-isocline intersects to the left of the peak value (panel a).

As shown by May(1974, Ch. 4), the amplitude of the limit cycle; that is to say, the maximum and minimum values the invidual populations of animals and plants reach during the cycle, will decrease the closer the Y-isocline will be to the peak of the X-isocline. So also is the period, the time to complete one cycle. Thus, initially located in the regime of limit cycles, a larger self-saturation constant, ε, as well as a higher natural mortality rate of the animals, m, will shift the Y-isocline to the right and therefore tend to stabilize the system. On the other hand, a higher maximum specific reproductivity rate, h, tends to destabilize the system. It is also seen that a higher carrying capacity of the plants, A/B, will shift the X-isocline to the right and hence, tends to destabilize the system.

3. Mans exploitation of the common

We are now ready to investigate how the herdsmen's exploitation of the common affects the ecological system under different economic and social constraints. Four types of harvesting rules will be considered. First of all, we study a situation where the herdsmen slaughter a fixed proportion of their privately owned livestock every year. Secondly, it will be assumed that the harvest follows as a result of utility maximization of consumption. Thirdly, the case two situation of utility maximization of consumption is extended with a 'wealth' effect reflecting the fact that a large herd gives status. Finally, the effect on the ecosystem is studied when a large herd as a measure of status is included in a more direct way following by some kind of routinized behaviour.

The hypothesis reflect different assumptions on how institutional and cultural conditions work. Moreover, they reflect various types of 'rationality' among the herdsmen. When a herdsman maximizes present value utility he will therefore be (privately) rational, irrespective of the fact that such a behaviour, as will be shown, will give some adverse effects. By the same token, when the herdsman follows some kind a routinized behaviour like harvesting a fixed ratio of the livestock every period of time, he is not rational in a strict sence. The main point then is to compare the consequences for the ecosystem due to the various types of behaviour and harvesting strategies.

For all harvesting strategies to be considered, it will be assumed that each herdsman make a decision on harvesting and herd size independent of the fact that the other herdsmen are exploiting the common at the same time. Thus, since every herdsman feels he has none or only negligible effect on the vegetation of the common and regards the stock of plants, X, as fixed when deciding his offtake Z_i, he pays no attention to his negative external effect upon the common. In technical terms, it means that the herdsman does not impose any shadow price on the grazeland when he transfers a piece of common property into private property (the meat of his animals). This is in essence, the <u>unmanaged</u> common case and corresponds to the type of exploitation discussed by Garret Hardin.

<u>Harvesting rule 1: A fixed proportion of offtake</u>

We start to look at the rather simple case where the herdsman follows the routinized type of behaviour and slaughter a fixed proportion α of his livestock every period of time as in (3). (3) combined with (1) yields $dY_i/dt = [a(X)-m-\alpha]Y_i$. It is therefore seen that the only difference compared to the ecological system unexploited by man is that the mortality rate of the livestock increases. The mortality rate will now be the sum of the natural mortality and the man-made mortality as given by the offtake ratio, $(m+\alpha)$.

(3) $Z_i = \alpha Y_i$

The livestock dynamics will be the same for every herdsman, (4) therefore holds for all the fixed number of herdsmen N taken together. Hence, (4) together with (2) give the dynamics of the system of plants and livestock when the offtake ratio is fixed. As seen, this is in principle the same type of dynamic system as the system unexploited by man. Because the mortality rate of the animals increases, however, the Y-isocline shifts to the right compared to that system (cf. Figure 1). Following May(1974), the present harvesting scheme will therefore tend to stabilize the ecological system. That is to say, the amplitudes of the limit cycles will decrease compared to the system unexploited by man. Alternatively, a regime of limit cycles in the unexploited case changes to a regime of damped oscillations when harvesting takes place. Obviously, this can particulary be so if the offtake ratio α is high.

(4) $dY/dt = [a(X) - m - \alpha]Y$

Under the given harvesting scheme, the number of herdsmen has no influence on the outcome of the ecological system, i.e. the number N is not included in the equations (2) and (4)[2]. What counts is just the number of grazing animals $Y = NY_i$. This is an important result and differs from the traditional (static) analysis of unmanaged local commons where more owners generally means a more intensive exploitation of the common resource base (see, e.g. Dasgupta and Heal 1979). The present result hings on the fact that the offtake is homogeneous of degree one in the livestock.

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[2]However, the number of owners can influence the ecological system in a transitional phase through changing initial conditions (cf. section 4 below).

Harvesting rule 2: Utility maximization of consumption
We now proceed to study the strict optimalization case where each herdsman decide the size of his herd and his offtake based on present value utility maximization of consumption. All the offtake is supposed to be consumed, so this type of exploitation of the common resource base reflects an institutional setting where there is no market present for slaughtered animals. When the instantaneous utility function is specified as the constant elasticity of substitution function $U(Z_i) = Z_i^{(1-v)}/(1-v)$, the planning problem of the herdsman is therefore to chose a trajectory of Z_i that maximizes (5) subject to constraint (1). $\delta > 0$ is the rate of discount.

(5) $W = {}_0\int^{\infty}[Z_i^{(1-v)}/(1-v)]e^{-\delta t}dt$

The current-value Hamiltonian of this problem is $H = Z_i^{(1-v)}/(1-v) + \mu[(a(X)-m))Y_i - Z_i]$ with Z_i as the control variable, Y_i as the state variable and μ as the adjoint variable.
As noted, because the herdsman feels that the grazing of his livestock has none effect on the common pasture, the grazeland X is threated as a parameter. (6) yields the necessary condition for maximum when an interior solution is supposed to be present. It captures both the maximum principle condition $Z_i^{-v} = \mu$ and the portifolio balance condition giving the time path of the adjoint variable, $d\mu/dt = \mu(m+\delta-a(X))$. It is seen that the offtake (consumption) will increase in periods when the grazing conditions are well so that $a(X) > (m+\delta)$. On the contrary, the offtake will decrease in periods when the pasture is overgrazed.

(6) $dZ_i/dt = (1/v)[a(X) - m - \delta]Z_i$

All the herdsmen follow the same harvesting rule, so the total offtake is given by (6'). Thus, (6') together with (1') (with NZ_i as Z) and (2) give the dynamics of the system under this harvesting rule. Again it is seen that the number of herdsmen has no influence on the outcome. What counts is, just as above, the total number of animals and the total offtake. (1′), (2) and (6′) represent therefore a system of three interconnected differential equations in the three variables, X, Y and Z.

(6′) $dZ/dt = (1/v)[a(X) - m - \delta]Z$

It seems difficult to characterize the above system analytically. It is possible, however, to give an interpretation under special circumstances and that is when the elasticity of the marginal consumption coeffecient is equal to one. When $v = 1$ and the instaneous utility function takes the logaritmic form $U(Z_i) = \ln Z_i$, (6) reduces to $dZ_i/dt = [a(X)-m-\delta]Z_i$. Comparing with (1), it turns out that these two equations will be jointly satisfied when $Z_i = \delta Y_i$. This is seen by substitution of Z_i into (1) which yields $dY_i/dt = [a(X)-m-\delta]Y_i$. Thus, Z_i as well as Y_i will grow at the rate $[a(X)-m-\delta]$ and this will be in accordance with $Z_i = \delta Y_i$. A consumption trajectory represented by the fixed consumption-livestock ratio δ is therefore a possible solution of the model. Moreover, as shown by Brekke and Stenseth(1994), that will be the consumption trajectory maximizing present value consumption.

Substitution of $Z_i = \delta Y_i$ into (1) and summing over all the herdsmen yields (7). Thus, when the instaneous utility function is assumed to be of the logaritmic type, the dynamics of the ecological system is given by (7) and (2). In addition, the total offtake is fixed to the livestock trajectory by $Z = \delta Y$.

(7) $dY/dt = [a(X) - m - \delta]Y$

The similarities with the system considered above is striking. Indeed, it is just the same type of dynamic system except for differences in the mortality rate of the animals. The mortality rate of the livestock is now $(m+\delta)$ compared to $(m+\alpha)$ as under Harvesting rule 1. Also when the exploitation of the common is directed by utility maximizing, the system therefore tends to be stabilized compared to the system unexploited by man. Again, this will happen because the Y-isocline shifts to the right (cf. Figure 1). However, because δ is likely to be small compared to the fixed offtake ratio α, the ecological system tends to be less stabilized under the present harvesting scheme.

Harvesting rule 3; Large herd gives status I

In general, other things than consumption matters for the well-being of the herdsmen and the preferences are closely related to the institutional and social context. It can for example be argued that the stock of animals, as a measure of 'status', will count in some instances. This will probably be so among the pastoralist herdsmen in the Sahal sone in Africa where having a large herd size to meet houshold consumption need for emergency means a rise in social status (see, e.g. Perrings 1993).

One way to include this type of status effect is to extend the utility function with a term reflecting the size of the livestock. $U = \ln Z_i + b\ln Y_i$ represents such an extension where the instaneous utility function is specified as logaritmic additative. This means that a 'wealth effect' à la Kurz(1968) is included in the preferences where $b > 0$ gives the magnitude of the wealth effect.

(8) $W = {}_0\int^{\infty}[\ln Z_i + b\ln Y_i]e^{-\delta t}dt$

The planning problem of the invidual herdsman is now to maximize (8) subject to constraint (1). The current-value Hamiltonian of this problem is $H = \ln Z_i + b\ln Y_i + \mu[(a(X)-m))Y_i - Z_i]$. The necessary condition for maximum follows by (9) when an interior solution is supposed to be present. Comparing with (1), it is seen that these two equations will be jointly satisfied when $Z_i = [\delta/(1+b)]Y_1$ holds. This is verified by first substitution for Z_1 into (1) which yields $dY_1/dt = [a(X)-m-\delta/(1+b)]Y_1$ and next substitution for Y_1 into (9) giving $dZ_1/dt = [a(X)-m-\delta/(1+b)]Z_1$. (1) and (9) will therefore be satisfied when the trajectory $Z_1 = [\delta/(1+b)]Y_1$ holds. Moreover, as above, this trajectory will maximize present value utility.

(9) $dZ_1/dt = [a(X) - m - \delta]Z_1 + bZ_1^2/Y_1$

The necessary condition for maximum in the present problem can therefore be characterized by $dY_i/dt = [a(X)-m-\delta/(1+b)]Y_i$ together with the fixed harvesting ratio as given by $Z_i = [\delta/(1+b)]Y_1$. So the dynamics of the ecological system when a large herd give status through the above type of wealth effect in the utility function is determined by (10) and (2).

(10) $dY/dt = [a(X)-m-\delta/(1+b)]Y$

As seen, the structure of this system is just as under Harvesting rule 2 (and Harvesting rule 1) but with a mortality rate of the animals like $[m+\delta/(1+b)]$. The system is therefore less stable compared to the system where only the consumption stream was included in the utility function. To the extent that cultural changes or modernization reduces the weight of the wealth effect b, this tends to stabilize the ecological system.

Harvesting rule 4: Large herd gives status II

Above the effects on the ecological system was studied when a large herd as a measure of status was included in the utility function. We will now look at this type of wealth argument more directly when assuming that every herdsman wants to keep 'as large livestock as possible'. The herdsman is therefore no longer (privately) rational in a strict sence. Instead, the herdsmen are assumed to follow some kind of routinized behaviour as under Harvesting rule 1 (a fixed proportion of offtake).

The behaviour is supposed to be as follows and is more or less in line with some recent studies of Sahelian pastoralism (see, e.g. Haaland 1991). At every point of time, the rate of harvesting regulates the size of the livestock. From an isolated point of view, a small offtake therefore contributes to keep a large herd. Thus, for every herdsman, keeping as large herd as possible will be synonymus with slaughter as few animals as possible. However, some offtake must take place to meet the basic needs of the herdsman and his family either the offtake is consumed or sold for a market. Keeping as large stock as possible following this type of routinized behaviour, is therefore formulated by setting a lower binding constraint ('target income') on the offtake as in (11) where Z_i^* is fixed through time and independent of the size of the herd and the grazing conditions.

(11) $Z_i = Z_i^*$

(12) $dY/dt = [a(X) - m]Y - NZ_i^*$

Combining (11) and (1) yield (12). (12) together with (2) therefore give the dynamics of the ecological system under the present type of harvesting rule. The isoclines are depicted in Figure 2. The Y-isocline will no longer be paralell to the Y-axis. It will be downward sloping starting from an infinity high value of Y when $a(X)-m = hX/(X+\varepsilon) - m = 0$ and approaches $Y = NZ_i^*/(h-m)$ when X approaches the carrying capacity of the grazeland A/B. The isocline will shift out when the offtake NZ_i^* increases. The shift can either takes place as a result of an increasing value of the fixed offtake Z_i^* or as a result of an increasing number of herdsmen N. In contrast to the other types of harvesting schemes analysed above, it should therefore be clear that the number of herdsmen influences the ecology permanently under the present harvesting rule (the offtake is no longer homogeneous of degree one in the livestock).

Figure 2 about here

In general, the Y-isocline will intersect with the X-isocline at two points and an unique steady-state of the system will no longer be present. If the system is perturbed away from point A when the pont is located to the left of the peak value of the X-isocline (as in Figure 2), there will be limit cycles. However, if A is located to the right of the peak value it can be shown that there will be explosive oscillations. Point B can easily be verified as a saddle point. If the system is 'attracted' by point B the outcome will therefore be extinction of the animals. Extinction of the animals coexisting with a modest pressure on the grazeland will also be the result if the system initially is attracted by point A when A is located to the right of the peak value of the X-isocline. In light of the present problems of degradation of communally owned grazeland, these types of outcomes are, however, of no interest. The steady-state point A located to the left of the peak value of the X-isocline is therefore the relevant attraction point of the system. The initial conditions and the parameters must therefore support limit cycles under this type of harvesting scheme.

4. Numerical illustrations

The main consequences for the ecological system of the four types of harvesting rules considered above can be summarized as follows. The reduced form dynamic equations under Harvesting rule 1 (a fixed proportion of offtake), Harvesting rule 2 (utility maximization of consumption) and Harvesting rule 3 (large herds give status I) will be the same. The only difference is the constant coeffecient yielding the (total) mortality rate of the animals in the equation determining the livestock dynamics. The mortality rate under Harvesting rule 2 will be above that of Harvesting rule 3 on theoretical grounds, while the mortality rate under Harvesting rule 1 will be above that of Harvesting rule 2 on empirical grounds. The ecological system will therefore be more stable when

Harvesting rule 1 is present compared to Harvesting rule 2, which again tends to give a more stable system than Harvesting rule 3. Again, more stability means a smaller amplitude of the limit cycles and/or that a regime of damped oscillations replaces a regime of limit cycles. In none of these three models, the number of herdsmen is included in the reduced form dynamic equations.

The structure of the system under Harvesting rule 4 (large herds give status II) is somewhat different. First of all, the (total) mortality rate of the animals will no longer be fixed through time. Secondly, the number of herdsmen influences the dynamics of the ecological system directly. Because the <u>number</u> of slaughtered animals will be constant through time, this type of harvesting rule lacks the man-made stabilizer effect as under Harvesting rule 1 (a fixed proportion of offtake) where a large number of animals will be slaughtered at times when there are many animals and few animals will be slaughtered when the herdmen's livestock is small. As argued, limit cycles will be the relevant dynamics when this harvesting rule is present.

Consequently, we know a great deal about how the various harvesting rules are affecting the ecology. However, to get a fuller understanding, they will now be illustrated numerically. In the illustrations, we have chosen the ecological parameters of the model that corresponds reasonably well with the intented interpretation of Sahelian pastoralism. There has been no attempt to estimate the parameters, so the results should thus be taken as a mere illustration. Table 1 gives the details[3]. Regarding the other parameters, the fixed harvesting ratio is set to 50% (α =0.5), the rate of discount is assumed to be 10% (δ =0.1) and the weight of the wealth effect is set to 1 (b =1). As a consequence, the mortality

[3]Nils Chr. Stenseth at the Department of Biology, University of Oslo supported the parameter values (personal communication).

rate of the animals given Harvesting rule 1 (a fixed proportion of offtake) will be $(m+\alpha)$ =1.5, while it will be $(m+\delta)$ =1.1 and $[m +\delta/(1+b)]$ =1.05 under Harvesting rule 2 (utility maximization of consumption) and Harvesting rule 3 (large herd give status I), respectively. Finally, the fixed offtake under Harvesting rule 4 (large herd give status II) is assumed to be 0.2. As a consequence, the initial offtake ratio will be 20% if the initial number of animals is 1.00 (see below).

Table 1
Parameter values numerical illustrations.

Harvesting rule	1	2	3	4
Ecological parameters				
h	5.0	5.0	5.0	5.0
h'	7.5	7.5	7.5	7.5
m	1.0	1.0	1.0	1.0
ε	0.5	0.5	0.5	0.5
A	8.0	8.0	8.0	8.0
B	8.0	8.0	8.0	8.0
Other parameters				
α	0.5	-	-	-
δ	-	0.1	0.1	-
b	-	-	1.0	
NZ_i^*	-	-	-	0.2 (0.1)

What is left is to choose the initial state of the system. But first some points of reference. For the given ecological parameters, it is seen that the carrying capacity of the grazeland will be 1.00. Moreover, it can easily be verified that the peak value of the X-isocline will be 0.60, corresponding to a stock of grazeland as 0.25. The Y-isocline in the absence of man is fixed by a X-value of 0.13 so it intersects to the left of the peak value of the X-isocline. The animals are therefore supposed to be <u>effective grazers</u> (cf. section 2). When choosing the stock of grazeland at t =0 to be $X(0)$ =0.25, it is therefore assumed that the pasture is relatively heavy exploited initially. Moreover, initially it is assumed a heavy grazing pressure as well, the number of animals is therefore set to $Y(0)$ =1.00.

The results are shown in Figures 3-5 as phase portraits. For the given parameters, limit cycles appears under all four harvesting schemes (the outcome of Harvesting rule 3 will be more or less the same as Harvesting rule 2). Comparing Figures 3 and 4 (please notice that the scale of the figures are different), it is seen that the amplitude of the limit cycle following Harvesting rule 2 will be above that of Harvesting rule 1. As noted, this is due to the lower mortality rate of the animals. When comparing Figure 3 and 4 with Figure 5, it should also be clear that Harvesting rule 4, not surprisingly, give rise to the largest oscillations and the less stable system.

In sum, for the given ecological and economic parameters, the pasture will be seriously overgrazed in subsequent periods both when Harvesting rule 2 (utility maximization), Harvesting rule 3 (large herd gives status I) and Harvesting rule 4 (large herd gives status II) are in effect. These scenarios can therefore said to be in accordance with Hardin's(1968) notion of a 'tragedy of the common' in spite of the fact that the common resource base never brakes down. Overgrazing can take place also when Harvesting rule 1 (a fixed proportion of offtake) is present, but never at the extent as the other harvesting schemes suggested that the fixed harvesting ratio is not unrealistic low.

Figures 3-5 about here

As noted, the number of herdsmen has no effect on the dynamic equations except for Harvesting rule 4. However, because $Y = NY_1$ holds, N can influence the dynamics in an <u>indirect</u> way when considering Harvesting rules 1-3 as well. This type of effect comes through a new inital state of the system as the fixed number of herdsmen changes. May(1974) has demonstrated that there will be only one limit cycle for the same dynamic system of this type with an unchanged set of parameters[4]. A changing initial state through a changing number of herdsmen will therefore only affect the dynamics until the limit cycle has been reached. Hence, a changing initial state affects the dynamics only through a transitional phase.

Above it was tacitly assumed that $Y_i(0) = 0.01$ with $N = 100$ so that $Y(0) = 1.00$, and $Z_i^* = 0.002$ so that $NZ_i^* = 0.20$. In what follows, the number of herdsmen is reduced to $N = 50$. If the initial stock of animals per herdsman is kept fixed so that $Y_i(0) = 0.01$ still holds, the total number of animals at $t = 0$ reduces to $Y(0) = 0.50$. Moreover, if Z_i^* is kept fixed, the total fixed offtake reduces to $NZ_i^* = 0.10$. With the initial state given by $X(0) = 0.25$ and $Y(0) = 0.50$ and the same parameters as above except for $NZ_i^* = 0.10$, the consequences for the ecological system are shown in Figure 6 and 7.

Figures 6-7 about here

Considering Harvesting rule 1 under these two scenarios; that is to say, comparing Figures 3 with Figure 6, it is therefore confirmed that this model settles down to the same limit cycle after an initial phase (again, please notice that the scale of the figures differs). The same happens under Harvesting rule 2 and 3 as well. Consequently, except for a period of transition, the number of herdsmen exploiting the common have no influence the stability of the ecological system and the exploitation of the common grazeland. When Harvesting rule 4 is in effect, it is, however, evident that the number of herdsmen have a permanent effect on the cycles of the ecological system. Comparing Figures 7 and 5 it is seen that the oscillations decreases and the system becomes more stable as N decreases . Thus, when the Y-isocline shifts inwards as a result of a smaller number of herdsmen N (cf. Figure 2), the system becomes more stable.

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[4]For a far more general discussion, see e.g. Lorenz (1993).

5. Concluding remarks

The present analysis has, from a theoretical point of view, studied Garret Hardin's plant-herbivore system of a local common under different social and economic constraints which are represented by various types of hypothetical behaviour among the herdsmen. Four types of harvesting strategies are considered. Two of them are based on strict invidual rationality where present value utility is maximized, while the other two are based on some kind of routinized behaviour. Since each herdsman feels he has none or only negiglible effect upon the vegetation, it is all throughout supposed that none of the herdsmen pay any attention to their impact upon the communally owned pasture when deciding their offtake. In line with Hardin, the common is therefore exploited in an unmanaged way.

The main point has been to compare the different harvesting strategies effect upon the ecological system and particulary study whether any 'tragedy of the common' scenario à la Hardin can be identified. When the ecological model is specified as a type of Rosenzweig-MacArthur model, it is shown that all four harvesting rules will either result in limit cycles or damped oscillations of the plant-herbivore system. It is also demonstrated that the number of owners do not influence the ecological system in a permant way except in the case where every herdsmen slaughter a fixed number of his livestock every period of time. Thus, when the offtake is directed by invidual rationality of the type of maximizing (private) present value utility, the number of herdsmen have no long-term impact on the cycles of the ecological system for the specified functional form of the utility function.

In general, the harvesting rule where each herdsman slaughter a fixed proportion of the livestock every period of time (Harvesting rule 1), gives the less unstable system. If the constant offtake ratio is high in a closer specified way, there will be damped oscillations so there will be no 'tragedy of the common'[5]. On the other hand, the outcome of the other harvesting rules can reproduce Hardin's verbal notion of a 'tragedy of the common' characterized by unstability and serious overgrazing in successive periods. But in absence of uncertainty and ecological shocks, as in the present analysis, the systems never break down. With a more realistic bioecological model, however, such a breakdown can very well take place. This can particulary be so when every invidual herdsman follow the routinized type of harvesting rule where a fixed number of animals are slaughtered every period of time (Harvesting rule 4).

There has been a lot of tragedies, but as pointed out by Ostrom(1990) and Bardhan(1993), managing local commons have also a long history of balanced resource management under highly informal local community institutions. In the present context, such a cooperation has been absent and overall optimality has not been considered[6]. However, to omit large oscillations of the ecological system and any 'tragedy of the common' scenario in absence of cooperation, the main normative conclusion from the above exercise seems to be that every invidual herdsmen should be enforced to or encouraged by an agency ('the state') to slaughter a high and fixed ratio of his livestock every year. How to implement such a policy in practice is, however, another question.

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[5]For the present ecological parameters, the system settles down to damped oscillations if the fixed offtake ratio exceeds 0.67 so that the total mortality rate of the livestock exceeds $(m + \alpha) = 1.67$.

[6]Formally, if present-value consumption is the objective, the program of overall optimality can be formulated as maximization of $W = \int_0^\infty N \ln(Z_1) e^{-\delta t} dt$ s.t. (1) and (2) with $Z = NZ_1$. Because the shadow price of the grazeland no longer is zero, the resulting dynamic system will be quite demanding to analyse.

Figure 1
Dynamics of the ecological system in absence of man. The phase diagramme when the grazer is a effective grazer, panel a. The phase diagramme when the grazer is an ineffective grazer, panel b.

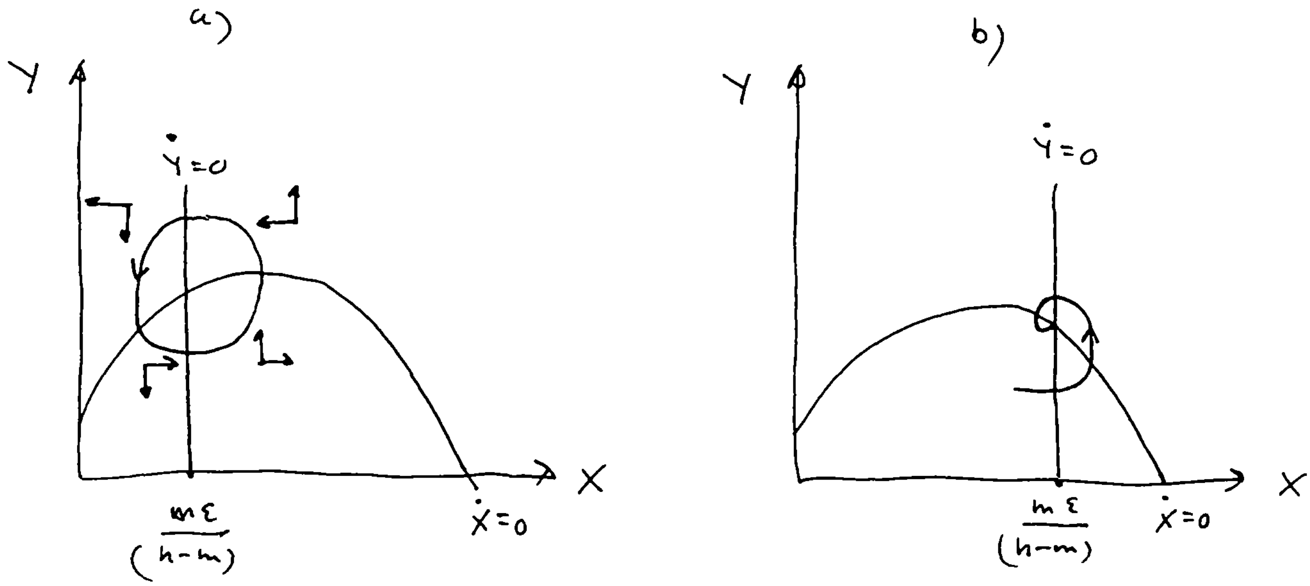


Figure 2
The phase diagrame, Harvesting rule 4 (large herd gives status II).

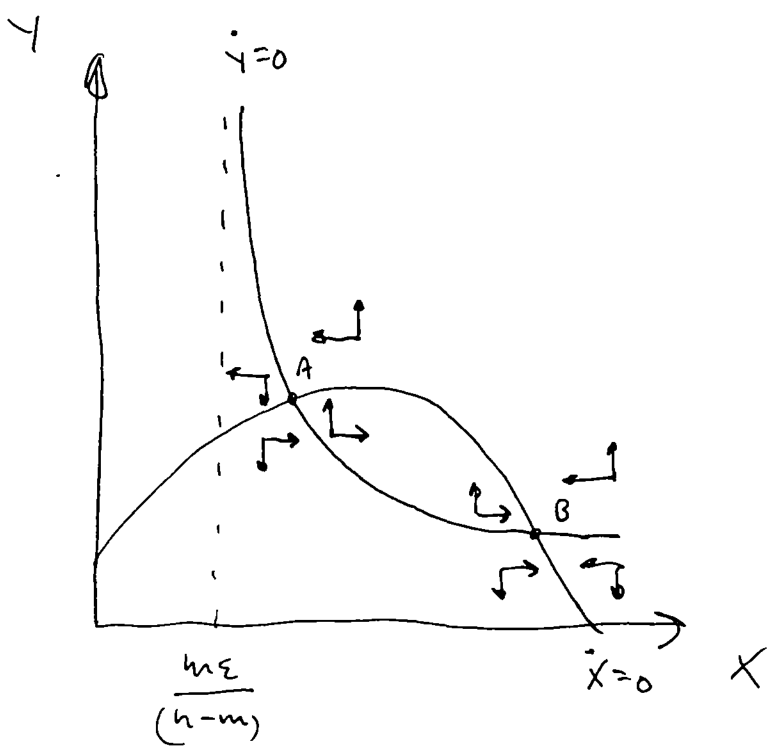


Figure 3
Phase portrait of grazeland (X) and livestock (Y), Harvesting rule 1 (a fixed proportion of offtake). Initial stock of animals Y(0) =1.00.

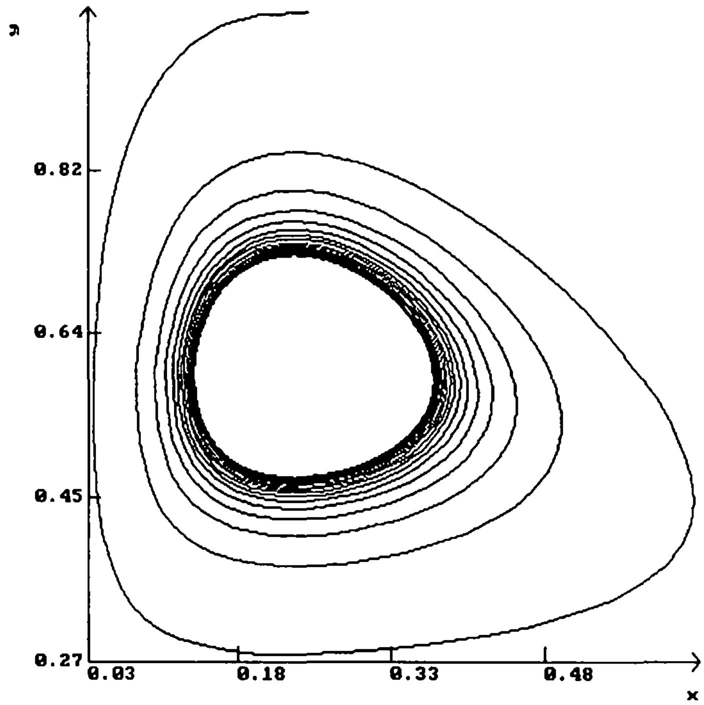


Figure 4
Phase portrait of grazeland (X) and livestock (Y), Harvesting rule 2 (utility maximization of consumption). Initial stock of animals Y(0) =1.00.

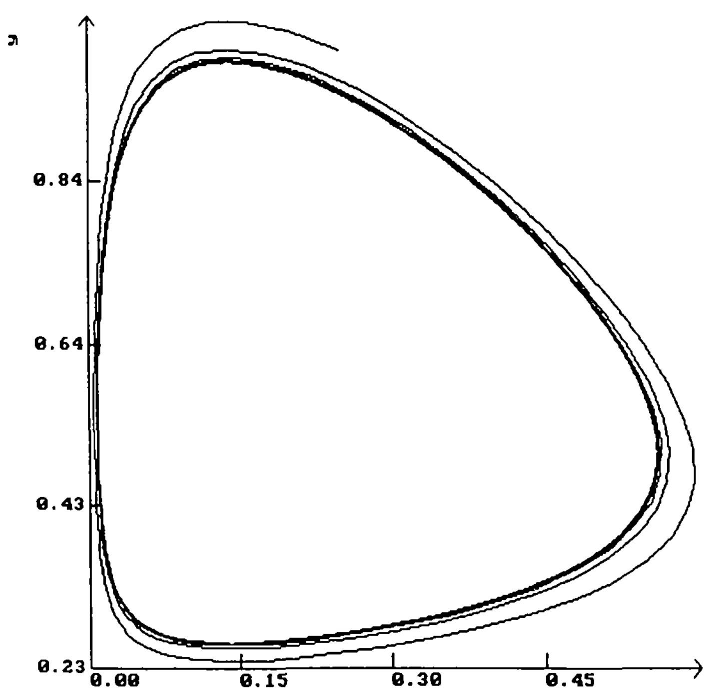


Figure 5
Phase portrait of grazeland (X) and livestock (Y), Harvesting rule 4 (large herd gives status II). Initial stock of animals Y(0) =1.00, fixed offtake NZ_i^* =0.20.

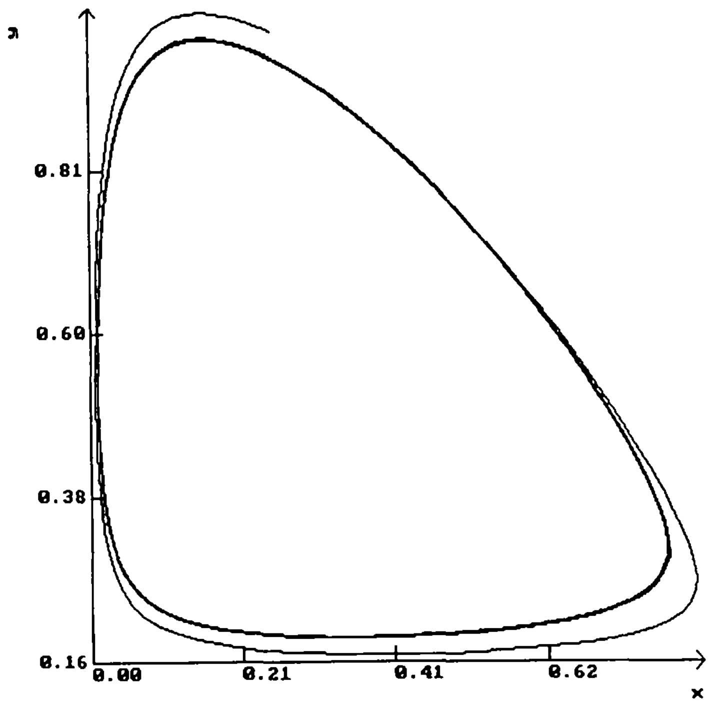


Figure 6
Phase portrait of grazeland (X) and livestock (Y), Harvesting rule 1 (a fixed proportion of offtake). Initial stock of animals Y(0) =0.50.

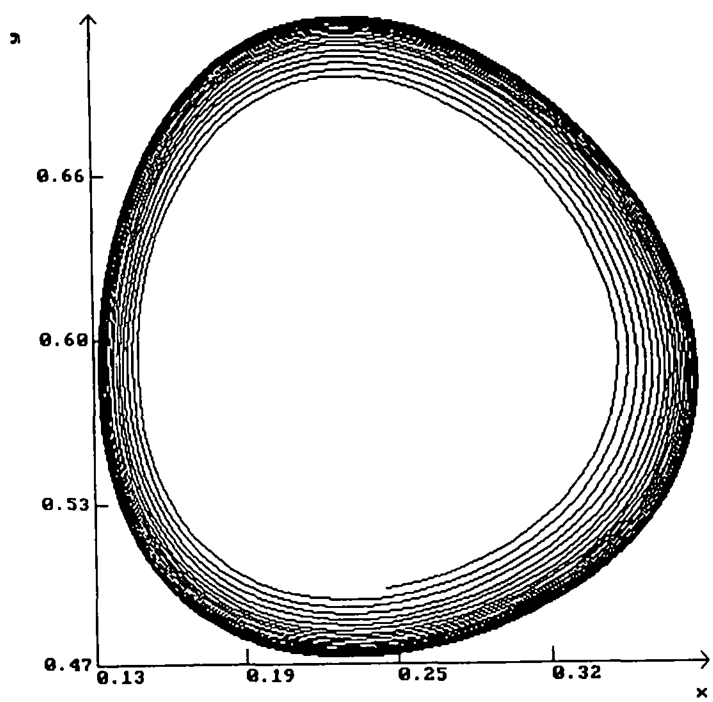
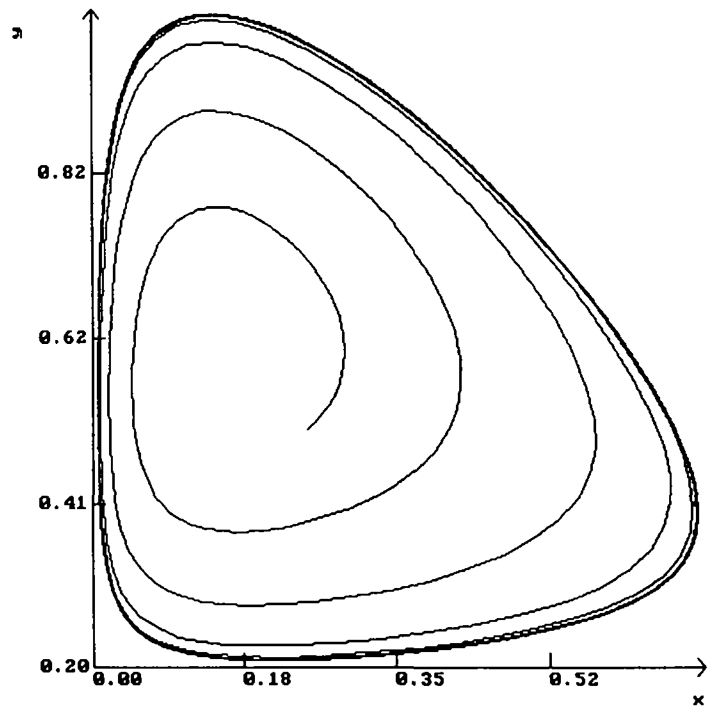


Figure 7
Phase portrait of grazeland (X) and livestock (Y), Harvesting rule 4 (large herd gives status II). Initial stock of animals Y(0) =0.50, fixed offtake $NZ_1 = 0.10$.



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