UNIVERSITÄT KARLSRUHE

INSTITUT FÜR STATISTIK
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GAME THEORY AND EXPERIMENTAL ECONOMICS N° 10

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March 1999

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Resolving Commons Dilemmas by Cooperative Games

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April 6, 1999

Abstract

Usually common pool games are analysed without taking into account the cooperative features of the game even when communication and (non-binding) agreements are involved. Using the fact that the corresponding cooperative TU-common pool games are clear and convex, negotiators may agree on attractive solutions in the grand coalition. In symmetric situations the solution is trivial. Assuming asymmetry, especially the (in this case) single valued kernel has the appealing property to balance the losses/gains among the players. For an experimental test the data set reported by Hackett, Schlager and Walker 1994 is reanalysed from a cooperative point of view. In order to represent to what extend the subjects obey efficiency and fairness we present the patterns of the corresponding excess vectors. Moreover it is analysed up to what degree intermediate coalitions can be content with the agreements realised.

1 INTRODUCTION

In commons situations agents jointly manage a resource where the exploitation by one user restricts the consumption by or production opportunities of other agents. Rivalry in use exposes externalities to the other agents and one would expect by arguments like those of Hardin in 1968 [4] that the commons are endangered through the overuse by the appropriators of the jointly managed resource.

"The authors are thankful to Hackett, Schlager and Walker for placing the data set of their paper [3j at the authors disposal. The data set was a valuable tool to discuss the cooperative approach to heterogeneous commons dilemmas.

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Judgements like those are strongly supported by non-cooperative game-theoretical analysis, because it is well known that the incentive structure induces an inefficient Nash equilibrium. Therefore, under the assumption of individual rationality the corresponding model prescribes an overuse or even a destruction of the jointly used resource. These results are widely challenged by field and experimental studies, if allowance is made for costless communication among the agents in the bargaining process although the communication process had no effect on the incentive structure of the underlying game. In the corresponding studies researchers have found evidence that appropriators of the commons exploit the resource much more efficiently than the non-cooperative game-theoretical approach would suggest (cf. with Ostrom et. al. [15]). In general appropriators rely on different institutions to treat the resource with care. Communication facilities, control, and sanctioning regimes are institutions which allow for a successful self-management of the commons (cf. [15]). Agents can use the communication as a strategy coordination device for enhancing the efficiency in the commons. In an experiment Hackett, Schlager and Walker [3] have shown that subjects in doing so increase the efficiency considerably.

In face-to-face communication the agents invest a considerable amount of time and effort to reach an agreement upon a joint optimal strategy. But if we allow for such communication among the agents, so that they can use yield enhancing strategies, then we also must consider which arguments can be presented in the bargaining process, that is to say, we have to rely on cooperative solution concepts. The purpose of the paper is to rely on some well established cooperative solution concepts for reanalysing the data ascertain by Hackett, Schlager and Walker [3] (HSW) from the cooperative game-theoretical viewpoint. In the paper we will consider both efficiency and fairness. Because in contrast to the non-cooperative game theory the cooperative game theory can capture fairness reflection. The kernel and nucleolus capture very well fairness and having the appealing property to balance the losses/gains among the agents.

We will argue in the sequel of the paper that it doesn't matter which fairness concept we will consider here, because both solution concepts are closely related to each other. Ostmann [12] has shown that cooperative common pool games are clear and due to a result worked out by Meinhardt [9, 10] we know that such games are as well convex. Note that convexity implies that the kernel is a singleton and agrees with the nucleolus ([16, p. 319]). According to these results we are able to represent the data set given by Hackett, Schlager and Walker [3] through a pattern of corresponding excess vectors.

To answer the question to what extent cooperative game theory enables us to reinterpret the HSW-data we introduce three measurement concepts for efficiency, contentment and fairness. By considering the corresponding excesses for the grand coalition we are able to quantify the sacrifice with respect to the Pareto-optimal outcome. Alternatively, measuring efficiency as the ratio between the aggregate payoffs and the welfare optimum does not quantify the sacrifice of the agents. The later measure gives only an indication of how efficient agents are in self-managing the common. Our second measurement concept refers to the issue whether the agents feel the proposed payoff allocation as an fair compromise while looking at the excesses for intermediate coalitions. Consequently, we quantify of how content agents are with agreement points. The last measurement concept determines the Euclidian distance with respect to the nucleolus being our point of fairness. Thus, a growing distance to the nucleolus means a decrease in fairness.

The rest of the paper is organised as follows. The next section contains the formal game-theoretical model from which the HSW experimental design was drawn. Whereas the authors consider only non-cooperative concepts we also rely on on the cooperative game induced. Section 3 reconsiders the HSW experimental design and the explicit cooperative game in characteristic function form is given. In section 4 the HSW-data set is reanalysed by the cooperative game-theoretical point of view. Some concluding remarks can be found in Section 5.

2 COMMON POOL GAMES

In a common pool problem individuals have access to a resource, where it is difficult to implement exclusion from the use and where we observe rivalry in the consumption or yield. That is to say, that individuals cannot be excluded from using the resource or for cases where it is feasible to exclude individuals it is not economically meaningful to do so, because they can only be excluded with prohibitive costs. Rivalry in the consumption or yield means for these kinds of economic problems that a unit which has been consumed (extracted) by one individual, cannot be consumed (extracted) by others. Thus, the consumption of the resource imposes negative externalities to the other individuals. Notice that the rivalry in use is the main difference to the so-called public good problems, where there is no rivalry in use. Formally, we can represent the underlying incentive structure of a common pool problem by a normal form game from which we are able to derive the corresponding cooperative game.

For preparation to reanalyse the data set by Hackett, Schlager and Walker [3] we reconsider the formal underlying game-theoretical model from which they draw their experimental design and from which we will deduce the transferable utility characteristic function game. The normal form game is described by the player set N of cardinality n with the players representing the appropriators who will jointly manage the common pool resource (CPR). In addition, each appropriator i is endowed with input factors Wi which he can invest partly or completely in production activities for harvesting the common pool resource. In sake of simplicity, we will abstract from the investment decision into a private good in contrast to the Hackett, Schlager and Walker (HSW) setting, where the investment decision into a private good is explicity taken into account (cf. [3, p. 105]). Observe, that the endowment Wi > 0 represents an upper bound of the feasible strategy set for player i denoted

by

$$F_i := \{x_i \in \mathbb{R}_+ \,|\, 0 \leq x_i \leq \omega_i\} \tag{2.1}$$

The term $x_i \geq 0$ denotes player i's investment decision concerning the common pool resource. To capture heterogeneous agents, it is supposed that the individual endowments are different and this implies that the feasible strategy sets do not coincide. Because of the jointly managed resource, the individual payoff depends not only on the investment decision taken by the appropriator i but in addition to that it depends on the aggregate investment taken in the resource, i.e. $x(N) = \sum_{i \in N} x_i$. Let c describe the private marginal costs which are the same for all appropriators, so that the term $c\,x_i$ represents the total private costs from investing in the CPR for appropriator i. The whole return is expressed by the **joint production function** $f(x(N))$, where the function f is concave, such that $f(0) = 0$ and $f'(0) \geq c$. The payoff function u can now be given by

$$u(x) = (u_i(x))_{i \in N} = (q_i\, f(x(N)) \; - c\,x_i)_{i \in N} \qquad \text{with} \quad q_i = \frac{x_i}{x(N)} \tag{2.2}$$

where the function q_i describes of how the whole group return of the CPR is distributed between the agents. Observe, that the function q_i represents a **proportional sharing rule** for distributing the whole group return. For this cases a public good can be characterised by an equal sharing rule, i.e. $q_i = \frac{1}{n}$. That is to say, that both types of games belong to a more generalised class of games.

Usually, the game theoretical analysis, which is based on the assumption of selfish and rational individuals, let us predict that the unique Nash-equilibrium for commons dilemma games is inefficient compared to the Pareto-optimum. More precisely, the game theoretical analysis expects that the appropriators will overuse the commons. And indeed, for symmetric CPR games the unique interior Nash-equilibrium is expressed by the following formula

$$c - f' = \frac{n-1}{n}\,(\frac{f}{x(N)} - f'), \tag{2.3}$$

that expresses formally the overuse. According to the concavity property of the joint production function, we can conclude that equation (2.3) implies an overuse in the commons. Observe, that the right hand side of equation (2.3) is positive, since the average return exceeds the marginal product ($\frac{f}{x(N)} > f'$), whereby Pareto efficiency claims that the marginal costs equal the marginal product, i.e. $f' = c$. Therefore, there is no incentive for rational agents to choose strategies which lead to a mild extraction in the jointly used resource. But if we assume that individuals can communicate, then rational agents can use communication as a strategy coordination device for reaching better outcomes in the CPR situation. Hence, if communication is allowed between the agents than we can expect that coalitional arguments will be used in the bargaining process and cooperative solution concepts have to be considered.

Let us now turn to the **transferable utility game** which we can derive from the above normal form game. Notice that cooperative transferable utility games are defined by the player set N and a real-valued set function v on the power set 2^N, whereby v is called the characteristic function of the game. An element of the power set is called coalition and it is denoted by S. The real number $v(S)$ represents the worth or the value that will be produced to the members of coalition S by mutual cooperation.

From the normal form game which was described above we can model different types of arguing in the bargaining process if we allow for communication among the agents. In our cooperative game-theoretical investigation we will focus on the so-called α and β TU-games (or characteristic function form) introduced by von Neumann and Morgenstern [17] and have been further developed for TU and Non-TU games by Aumann [1]. These two types of characteristic function specifications reflect two different kinds of believes by the members of a particular coalition S about their possibilities to react actively or passively against a chosen strategy combination $x_{N\setminus S}$ of the opposition (or complement coalition) $N\setminus S$. We will write the joint strategy combination of a coalition S as $x_S = (x_i)_{i\in S}$. The corresponding feasible strategy set for a coalition S is restricted by the sum of the individual endowments of its members, hence it is given by

$$F_S := \left\{ x_S \in \mathbb{R}^s_+ \mid \sum_{i\in S} x_i \leq \sum_{i\in S} \omega_i \right\} \tag{2.4}$$

Now, we are able to introduce the definition of the characteristic α-function which is denoted by

$$v_\alpha(S) := \max_{x_S\in F_S} \min_{x_{N\setminus S}\in F_{N\setminus S}} \sum_{i\in S} u_i(x_S, x_{N\setminus S}) \tag{2.5}$$

A natural interpretation of the characteristic α-function is that the α-value describes a **prudent perception** by the members of coalition S about their capabilities to guarantee to themselves the payoff $v_\alpha(S)$. Regardless of the joint strategy of the complement coalition, the coalition S can ensure no less than the value $v_\alpha(S)$ if its members choose a joint strategy x_S that produce exactly this value.

The characteristic β-function is denoted by

$$v_\beta(S) := \min_{x_{N\setminus S}\in F_{N\setminus S}} \max_{x_S\in F_S} \sum_{i\in S} u_i(x_S, x_{N\setminus S}) \tag{2.6}$$

For the characteristic β-function we get a similar interpretation, where the β-value describes an **optimistic perception** by the members of coalition S about their capabilities not to be prevented from the worth $v_\beta(S)$. Regardless of the strategy choice of the complement coalition the coalition S cannot be prevented from getting less than $v_\beta(S)$ by finding a joint strategy x_S, that brings about exactly this value to the members of S. In general, the β-characteristic function is greater or equal than the α-characteristic function, that is that we have a weak incentive to react passively by awaiting the joint strategies of the outsiders.

In Ostmann [12] it was established that for cooperative common pool games both characteristic function forms coincide and that the core as a solution concept for this class of games is nonempty. Note that the core as a solution set in cooperative transferable utility games describes all allocation vectors that are coalitional rational and Pareto-efficient. No coalition can raise an objection against an allocation that belongs to the core. As a consequence of the former result given in Ostmann [12], there is no advantage in common pool games to react passively by awaiting the joint strategy of the opposition. Notice that these kinds of games are called **clear games**. This game theoretical concept was introduced by Jentzsch [5], for the first time. The second result, shown by Ostmann [12] on the non emptiness of the core, says that the claims that will be presented by the coalitions are relatively modest with respect to the welfare optimum. Hence, that is an incentive for cooperative behaviour in such games, for exhausting the gains that are feasible through mutual cooperation. In Meinhardt [9, 10] a much stronger result has been established. There, it was shown that these kinds of games are **convex**. Convexity of the characteristic function expresses nondecreasing marginal returns with respect to coalition size. Thus convexity of games can be interpreted as a strong incentive for large-scale cooperation.Cooperation within intermediate coalitions is relatively weak compared with the profits available in the grand coalition. The relative weakness for intermediate coalitions yields an over-proportional surplus by joining larger coalitions, which makes large-scale cooperation for rational agents interesting.

In order to analyse to what extend the subjects obey fairness consideration in the experimental design, we have to introduce the **kernel** as a second solution concept that compares the "best arguments" two agents can present against each other in a bargaining process. For this purposes, let us first address to the **excess** denoted by $e(S, u) := v(S) - u(S)$. The excess represents the losses/gains for the members of a coalition S with respect to the value $v(S)$ if its members accept the payoff vector u instead of $v(S)$. Observe, that a positive excess value $e(S, u)$ means that the members have to bear a loss by accepting the payoff vector u, because they can guarantee to themselves the value $v(S)$ instead of $u(S)$ by mutual cooperation. In this sense the excess is a measure of content or discontent by the members of coalition S with the payoff vector u. Note that for core allocations such losses are not acceptable. Now let us introduce the set of coalitions containing agent i but not j denoted by $\mathcal{T}_{ij}$ for all $i, j \in N$ with $i \neq j$, hence

(2.7) $$\mathcal{T}_{ij} := \{S \in 2^N \mid i \in S, j \notin S\}$$

and the maximum gain of agent i over j is given by

(2.8) $$s_{ij}(u) := \max_{S \in \mathcal{T}_{ij}} e(S, u).$$

The kernel now has the appealing property to balance the losses/gains between agents as a fair compromise by using "best arguments" against each other. Note that the condition of excess maximising in equation (2.8) is interpreted as best argument. The agents i and j are in equilibrium or have balanced their losses/gains at u if $s_{ij}(u) = s_{ji}(u)$. In case where $s_{ij}(u) > s_{ji}(u)$ then player i expresses discontentment with the proposed payoff vector u by claiming that player j should render him a share from his claim to balance their losses. Player i can justify his claim by presenting best argument that he can leave the coalition with player j and joins a coalition $S \in \mathcal{T}_{ij}$ where he can get his minimal loss or maximal gain. Certainly, one can imagine situations where a balancing in the losses/gains produce to an agent less than his value. But the kernel concepts exclude this situation, by claiming that every agent should get at least his value. That is, if agent i outweighs agent j at u, i.e. $s_{ij}(u) > s_{ji}(u)$, then $u_j = v(\{j\})$. Moreover, it was proven by Maschler and Peleg [6, p.315] that the kernel preserves the desirability relation of players. A player i is called more desirable than a player j, written as $i \succeq j$, iff

(2.9) $$v(S \cup \{i\}) \geq v(S \cup \{j\})$$

for all $S \in 2^N, S \cap \{i,j\} = \emptyset$. A payoff vector u preserves the desirability relation with respect to the game (N, v), if $i \succeq j$ implies $u_i \geq u_j$, that is, a player i who increases the worth of a coalition S more than a player j gets an individual payoff at least as high as player j.

For convex cooperative transferable-utility games the kernel solution concept agrees with a well-known other solution concept related to the kernel, called **nucleolus**. Since, for convex games the kernel is a single-valued solution (cf. [8, pp.88-91]), whereas in general this is not the case and the nucleolus as a single-valued cooperative solution concept is contained in the kernel, this implies that both solution concepts coincide for convex games. For symmetric cooperative transferable-utility games the solution of the kernel is trivial, because it proposes a distribution in equal proportions to the members of the grand coalition. For the asymmetric case the solution can be more complex, but agents of equal strength get the same payoff. For a more detailed discussion of cooperative solution concepts we refer the reader to Rosenmüller [16, chap. 3].

3 COOPERATIVE SOLUTIONS FOR THE HSW SETTING

In the previous section we covered the game theoretical models which reflect the conflict situation behind the joint management of a common pool resource in both the non-cooperative and the cooperative view. We argued that in respective situations in which the actors involved can communicate and can negotiate for an agreement how to coordinate actions and/or how to allocate payoffs it is necessary to consider cooperative solution concepts, even if the agreements can not be guaranteed to be binding. From the foregoing discussion, we know that the equilibrium is not efficient for a CPR and that the underlying cooperative game is convex, so that there is a strong incentive for cooperation. It is now an empirical question to what extent people dealing with such situations can establish cooperation and can enhance their results by arranging and implementing favourable agreements.

In **experiments** with communication (and without binding agreements) referring to symmetric situations Ostrom et. al. [15] found evidence that results are far from the inefficient (non-cooperative) equilibrium and communication often leads to near-optimal results. Moreover most agreements found during the communication phase are stable in the sense that in the (private) decision after the agreement nobody deviates from it. Nevertheless it can be argued that in such symmetric situation it is easy to establish cooperative solutions because according to all normative standards single valued solutions result in equal shares, even without referring to any strategic considerations. This changes in case of asymmetry. In such a case actors respectively subjects favoured by the conditions have to find convincing arguments that can support the larger share they like to get. An experimental study with an asymmetric common is reported in Hackett, Schlager, and Walker [3]. Results show in an impressive way that even in the asymmetric case just the introduction of communication enhances considerably the efficiency. In most cases subjects find stable agreements realising the gains of cooperation. In the HSW-experiment subjects reached agreements in 72 of 80 rounds, cf [3, p. 118].

In the **asymmetric HSW-experiment** a group of eight subjects participates in two consecutive 10-round sequences. In the first 10 rounds (not to consider here) it is not allowed for the subjects to communicate, while in the last 10 rounds subjects have the opportunity to communicate among themselves. Asymmetry is captured by two different values for the endowment corresponding to two different roles - say the "rich" and the "poor". The half of the group gets high endowments, the other half gets low ones. The HSW trials differ in the assignment of endowments. In the "random" condition subjects are randomly assigned to roles. For the second condition the authors install an auction, where subjects can bid for the right to become rich (cf. [3, pp. 110-112]). There is also a third "more complex" condition excluded from our reanalysis. In the following we refer to the 8 trials with 10 rounds with each communication included. This data set contains 4 random trials and 4 auction trials.

Let us now reconsider **the non-cooperative game** used in the HSW [3] experimental setup. The authors decided to choose the following parameters for the formula (2.2): $c = 5$, $f(x(N)) = x(N)(33 - .25\,x(N))$, $n = 8$ with four players of the"poor" type with an endowment of 8 tokens per round and player, and four players of the "rich" type with 24 tokens per round and player. The unique **Nash equilibrium** of the game prescribes the rich players to choose $x = 16$ an amount smaller than their endowment while the poor players have to allocate their total endowment, that means $x = 8$. The Nash equilibrium produces the following payoff vector to the agents $u^* = (64, 64, 64, 64, 32, 32, 32, 32)$. The aggregate payoff for the "rich" corresponds to 256 and the respective aggregate payoff for the "poor" amounts to 128. In contrast the welfare optimum with equal shares prescribes individually to allocate 7 tokens resulting in a 98 units payoff for every player.

From the specified normal form game we derive the corresponding cooperative transferable utility game. Recall, that TU-CPR games are clear games, so the α- and β-values agree, so we can choose one of the formulas (2.5) and (2.6) to calculate the characteristic function. The respective values are given in Table 1.

Coalitional values for the different profiles					
(# rich,# poor)	0	1	2	3	4
0	0	0	0	4	16
1	4	16	36	64	100
2	64	100	144	196	256
3	196	256	324	400	484
4	400	484	576	676	784

Table 1:

Figure 1 reveals the convexity property of the cooperative game. Returns are non-decreasing in coalition size. If the coalition, a player *i* may join in, increases, the marginal contribution to player *i* will increase too. For example, consider a second poor player, who joins a coalition with one rich player and one poor player, then his marginal contribution will be smaller than in the case that he joins the coalition with one poor player and two rich players. In addition, what should be obvious here, the claims of the smaller coalitions are relatively moderate with respect to the exhaustive potential of the grand coalition (the upper right corner with the value 784). In the experimental situation we can expect that subjects will show a strong preference for an agreement in the grand coalition.

The core of the considered game is relatively large convex polytope. Precisely, the core volume occupies 5.5 % of the set of possible outcomes that are both individual rational and Pareto efficient. Calculations show that we get 36000 extreme points (vertices). These vertices represent the vector of marginal contributions of the players for 40320 different orders of entry into the grand coalition to form. For

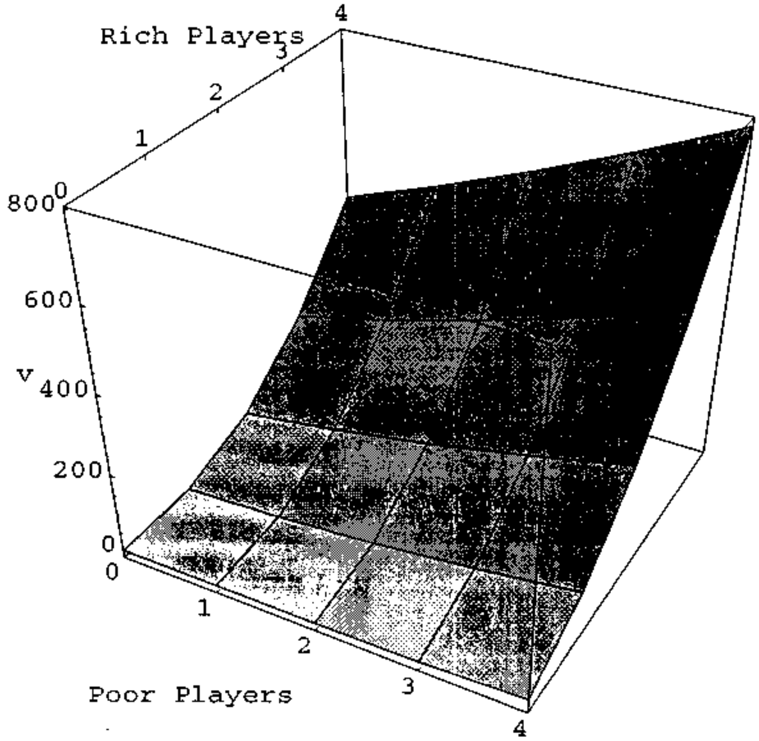

Figure 1: The asymmetric cooperative CPR game for the HSW-setting

the following it is helpful to project the core into the two-dimensional space by aggregating the values for the rich into one component and the values for the poor into the other one. In such a way we can reduce this number of vertices to 32 by considering their aggregated two-dimensional image. In the two dimensional space the image of the core reduces to an interval on the Pareto-frontier with the boundaries $(400, 384)$ and $(768, 16)$.

The reference standard for proposals that are consistent with coalitional fairness considerations is the kernel. As noted before the kernel is single valued and coincides with the **nucleolus** given by the following payoff vector $\nu = (142, 142, 142, 142, 54, 54, 54, 54)$. It should be obvious here that the payoff assigned to the players by the nucleolus preserves the desirability relation. Note that the poor players are strong enough to get positive payoffs besides the fact that the individual value for a poor player is zero. The aggregate $(568, 216)$ corresponds to a 72% share for the rich. Convex games that satisfy the condition $v(\{i\}) \geq 0, \forall i \in N$ are monotonic. For monotonic games Maschler and Peleg [7, p.589] have proven that it is possible to balance the maximal excesses. Moreover, in our case we get for the nucleolus $s_{ij}(\nu) = s_{ji}(\nu) = 0$ for all $i, j \in N$ and $i \neq j$.

In addition to the solution concepts core and nucleolus we calculated the **Shapley value** which also can be interpreted as a measure of power or as another fair allocation of the value of the grand coalition. Note that the Shapley value assigns to the players a payoff according to their contributions to different coalitions of the game. Due to this assignment procedure of the payoffs the Shapley value can be interpreted as fair. Furthermore, the Shapley value is not an element of the core for every balanced game. Therefore, the Shapley value is not coalitionally rational for every game with an non-empty core and for some extreme cases the Shapley value violates individual rationality. In contrast the nucleolus is a core selection for every game with non-empty core and it compares the contentment of the coalitions by optimising the excesses of various coalitions, i.e. it balances the losses/gains among the coalitions. Fairness can be considered by the nucleolus in a much broader sense as by the Shapley value while focusing on internal and outside fairness. These properties make it more preferable to consider the nucleolus as the reference point for fairness. For definitions and interpretations of both solution concepts, see for example Moulin [11, pp 116-129]. In case of convex games the Shapley value is the barycentre of the core. The following payoff vector $\Psi = (\frac{1024}{7}, \frac{1024}{7}, \frac{1024}{7}, \frac{1024}{7}, \frac{348}{7}, \frac{348}{7}, \frac{348}{7}, \frac{348}{7})$ represents the Shapley value of the cooperative CPR game. Clearly, according to the symmetry axiom the Shapley value assigns the payoff to the players in view of the desirability property. For the Shapley value the aggregate $(4096, 1392)/7$ amounts to about 75 % share for the rich.

4 The HSW data reconsidered

In 72 of the 80 rounds observed in the 8 trials the groups reached stable agreements. It was only one trial (namely the random trial 144), in which the group in 8 periods was not able to establish stable agreements. In our statistics we refer to "agreement data" in case we exclude the data from group 144, and we refer to "result data" if included.

In Figure 2 we consider the agreement data and relate the (two-dimensional) aggregated payoff values to the core, the nucleolus, the Shapley value, and the Nash-equilibrium. The bold box nearest to the origin represents the Nash-equilibrium. The declining line corresponds to the Welfare optimum, otherwise called Pareto-frontier or region of efficiency. The triangles at the lower right on the Pareto-frontier represent the vertices of the core. The nucleolus and for completeness the Shapley value are represented by boxes and the payoff data points by stars. The two straight lines starting at the origin represent the equal payoff sharing and the payoff sharing according to the endowment distribution.

Most of the payoff data points lie in between of the two straight lines from the origin and close or on the Pareto-frontier, but no data point lies within the core. Such payoffs that are efficient are not coalitional rational; there are some intermediate coalitions which can block if the proposal is rationally evaluated.

The figure shows that the subjects tend to make both equality and efficiency considerations, since the payoff data points are lying relatively close to the equal payoff sharing and to the welfare optimum. But from the first impression it is not

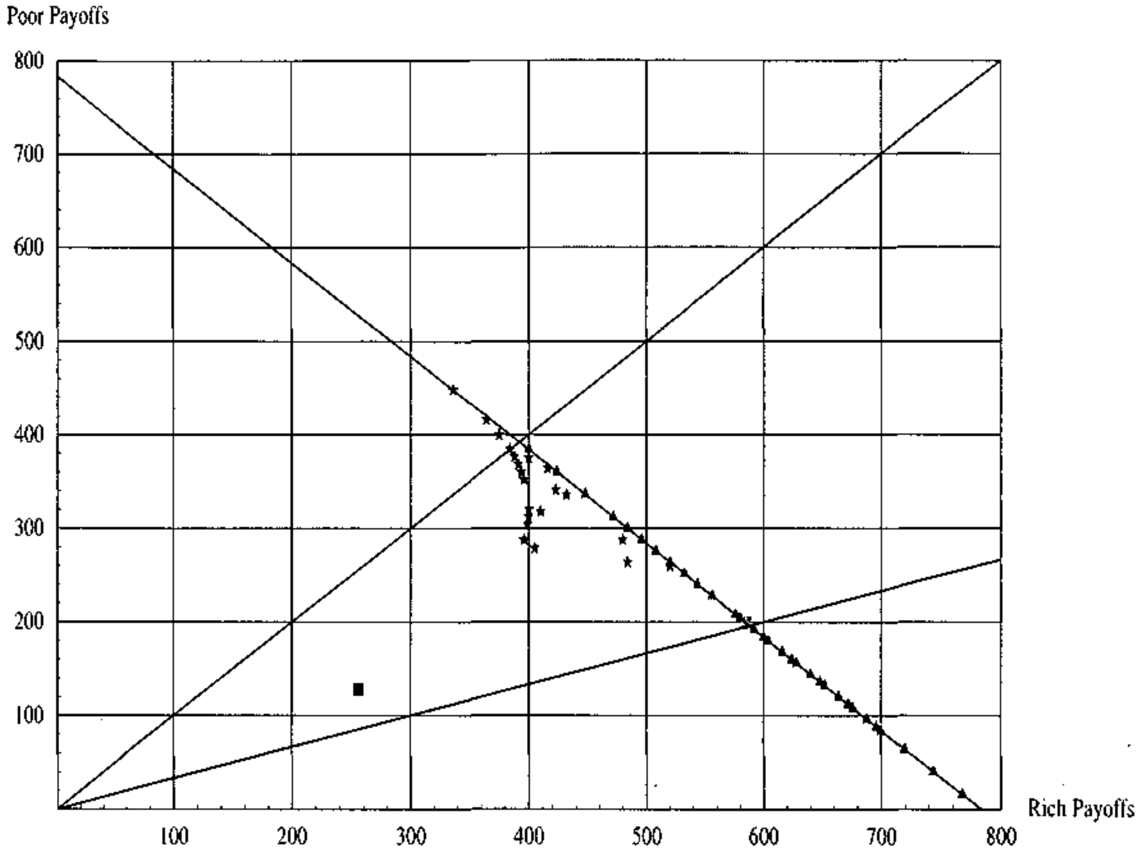

Figure 2: Payoff data set without group 144

clear to what extent they follow fairness considerations as well. For coming up with a respective judgement we now directly rely on the excesses of the result data set.

The variable **excess for the grand coalition** *(gcex)* can be interpreted as an efficiency measure in addition to the efficiency measure relating the aggregate payoffs to the welfare optimum (cf. HSW [3, p. 116]). According to this measurement concept we are able to quantify the sacrifice with respect to the welfare optimum resulting from the respective agreement or result. Observe that due to convexity the grand coalition is the coalition that normally will show the largest excess or say loss, since the exhaustive potential of the grand coalition is large in comparison with the intermediate coalitions.

Nevertheless there are extreme cases in which the variable *gcex* of an agreement does not coincide with the **maximum** excess *mex*. Consider for example the equally distributed welfare optimum with an individual share of 98. For such an agreement we get as maximiser the coalition of the rich *(mex* = 8, *gcex* = 0). It is true for superadditive games (and convex games are superadditive) that in case that the grand coalition N is not a maximiser of excess a maximising coalition S subsidises outsiders. Starting with the inequality

$$mex = v(S) - x(S) > v(N) - x(N), \tag{4.1}$$

and inserting $v(N - S) + v(S) \leq v(N)$ rising from convexity (respectively superadditivity) we get

$$v(S) - x(S) > v(N) - x(N) \geq v(N - S) + v(S) - x(S) - x(N - S), \tag{4.2}$$

and finally

$$0 > v(N - S) - x(N - S). \tag{4.3}$$

Hence the opposition of the coalition that has to bear the largest losses/gains from such an agreement *x*. Compared with the former measurement concept, the maximum excess gives us an indication of the maximal discontent with regard to the respective payoff allocation. In case that the grand coalition excess is not maximal we can state, that the corresponding proposal, agreement, or result is significantly unbalanced or unfair. In our setting a typical case for such a payoff vector is when some payoff of a poor player is larger than the payoff of some rich player. In such a case the desirability relation of players is obviously violated.

In our result data set in 66 of the 80 cases maximum excess and grand coalition excess coincide. In the auction subset there is only one exception from coincidence (*gcex* = 56.25, *mex* = 64). For the pooled set the average difference for the 14 deviations is 70 (*s.d.* = 38.6). A closer look reveals that in the random subset there are also eight (and without trial 144 four) extremely unfair results in which the aggregate income of the poor surpasses the aggregate income of the rich. In the random trials fairness considerations seem to play only a minor role.

The values for the Kolmogorov-Smirnov test in Table 2 indicate that the difference between the cumulative distributions for the random and for the auction groups show the weakest significance for the efficiency measure *gcex* (0.0072). The missing significant effect on *gcex* showed by the Kruskal-Wallis test indicates that a location-shift is probably not the cause of the difference between the distributions even when looking just at the mean (*mean* : 31 to 50.7) suggests it otherwise. The larger mean for the random setting is caused by only a few data points with very large excesses. Considering the standard deviation we correspondingly observe that the standard deviation is larger for the random sample (*s.d.* : 29.1 to 79.9). Figure 3 reveals the source of the difference in the mean and the standard deviation, namely the three data points close to 300. We may ask for other causes that may account for the occurrence of these unusual data points. In summary we reject the usual presumption that market entitlements induce more efficient results in general.

Due to the foregoing note that no payoff allocation belongs to the core it should be clear that all excesses for the grand coalition and the maximum excesses are non-

Comparison of "auction" and "random"				
		gcex	mex	dnuc
mean	auction	31.0	31.2	175.5
	random	50.7	75.1	242.8
	pooled	40.8	53.1	209.2
s.d.	auction	29.1	29.3	49.8
	random	79.9	76.8	43.5
	pooled	60.1	58.1	46.8
median	auction	16.0	16.0	189.6
	random	18.1	48.0	242.2
	pooled	16.0	22.6	212.2
K-Sm (DN, sign.)		0.375, 0.0072	0.6, E-6	0.7, 6 E-9
mean rank	auction	39.6	31.1	26.2
	random	41.4	49.9	54.9
K-W (Z,sign.)		0.13, 0.72	13.5, 2.4 E-4	30.8, 3 E-8

Table 2: **Legend** *dnuc:* euclidian distance to the nucleolus; *gcex:* excess for the grand coalition; *mex:* maximum excess; K-W: Kruskal-Wallis; K-Sm: Kolmogorov-Smirnov

negative. In the sequel, the lines with the bold points indicate the auction sample while the random sample is indicated by the lines with the crosses.

In contrast, by looking on our contentment measure *mex* (= maximum excess), we get between the auction and the random group a significant difference as indicated by the Kolmogorov-Smirnov test and shown in Figure 4. The discontent in the auction sample is stochastically dominated by that one in the random sample. This result suggests that in the random sample subjects are not so much concerned about the arguments of the intermediate size coalitions. Means and medians for *mex* show a smaller value for the auction sample. Despite the remarkably smaller standard deviation for the auction group the Kruskal-Wallis test indicates a significant difference in location. It may be that the lower discontent in the auction sample is based on the fact that subjects have had the opportunity to dissipated rents through the bidding process for the high entitlement. Or let us say they have had the better opportunity to learn what a role is worth, when they have to solve the additional task of bidding.

Let us now turn to our third measurement concept not already mentioned yet, that captures a special kind of fairness consideration. The reader will find the respective statistics in the last column of Table 2. For relying on fairness consideration, we have determined the Euclidian distances of the payoff vectors with respect to the nucleolus. Recall, that the kernel (coinciding with the nucleolus) equalises and minimises simultaneously all losses for the players, that players can present by best arguments against each other. Fairness decreases with growing distance to the

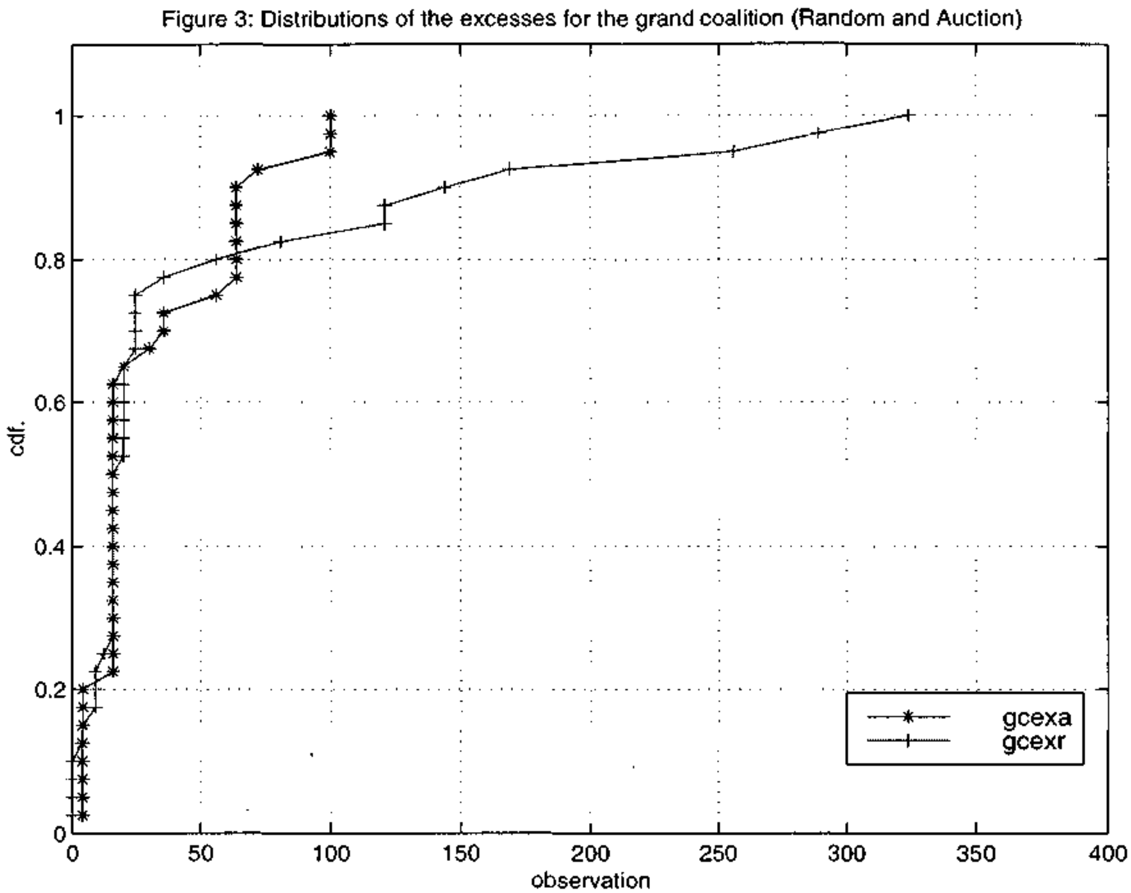

Figure 3: Distributions of the excesses for the grand coalition (Random and Auction)

nucleolus.

With respect to distance to fairness the random sample stochastically dominates the auction sample. This fact is visualised in Figure 5. The corresponding statistics are given in Table 2; as for the maximum excesses, subjects of the random group are significantly less concerned about fairness reflections than subjects of the auction group.

For the pooled data set we can argue that the kind of fairness consideration provided by kernel or nucleolus arguments are not the main aim for the subjects, since a mean of 209.2 in the pooled sample indicates a large distance from the nucleolus. We would rather say that subjects behave in such a way that payoffs

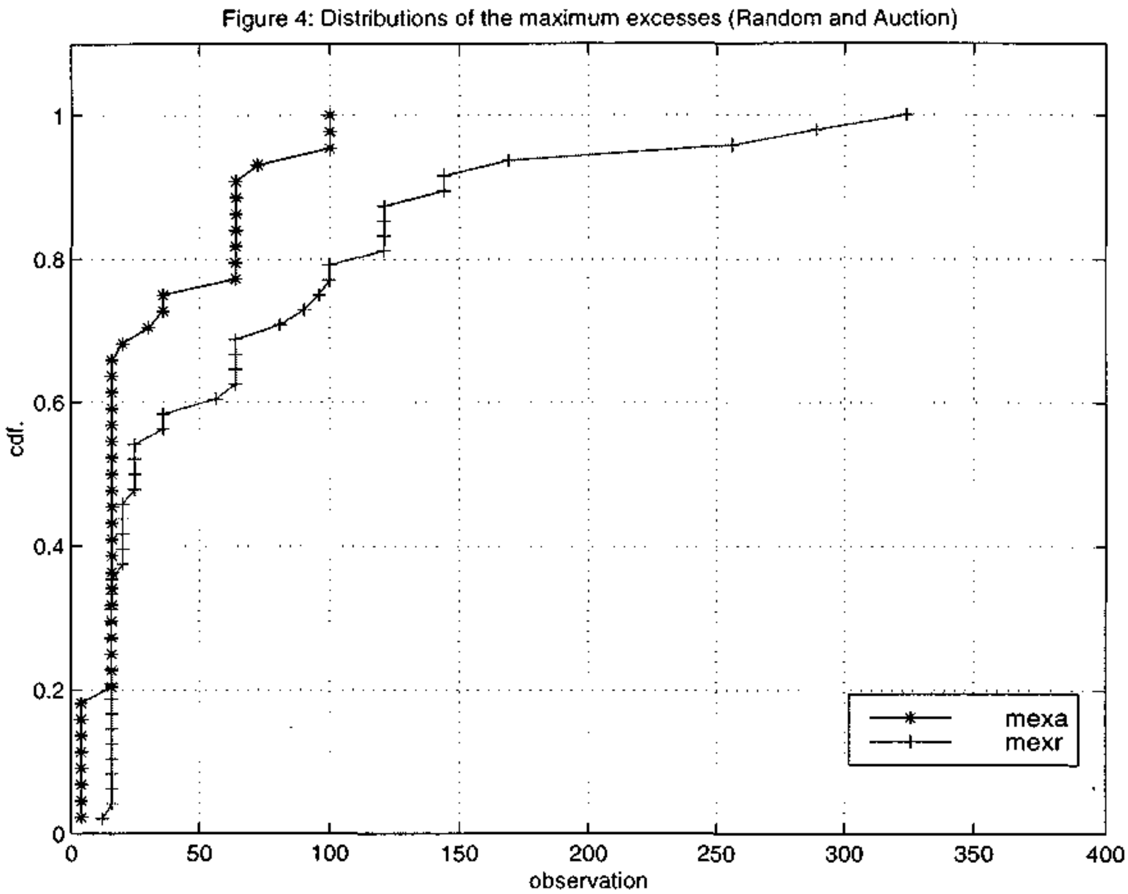

Figure 4: Distributions of the maximum excesses (Random and Auction)

compromise between the fairness and the egalitarian solution.

5 CONCLUDING REMARKS

Through a cooperative game-theoretical reanalysis of the HSW-data set we get for the whole data set also the result that subjects obey efficiency considerations very well. Additionally, we can show through our alternative efficiency measurement concept that the difference in the efficiency between the auction and the random samples vanish. The Kolmogorov-Smirnov test indicates the weakest significance in difference in the cumulative distribution functions between the auction and the random group. Both cumulative distribution functions coincide except for some extreme data points. With our cooperative measurement concept for efficiency we can reject the hypothesis that market entitlements by an auction induce more efficiency than a random assignment of endowments.

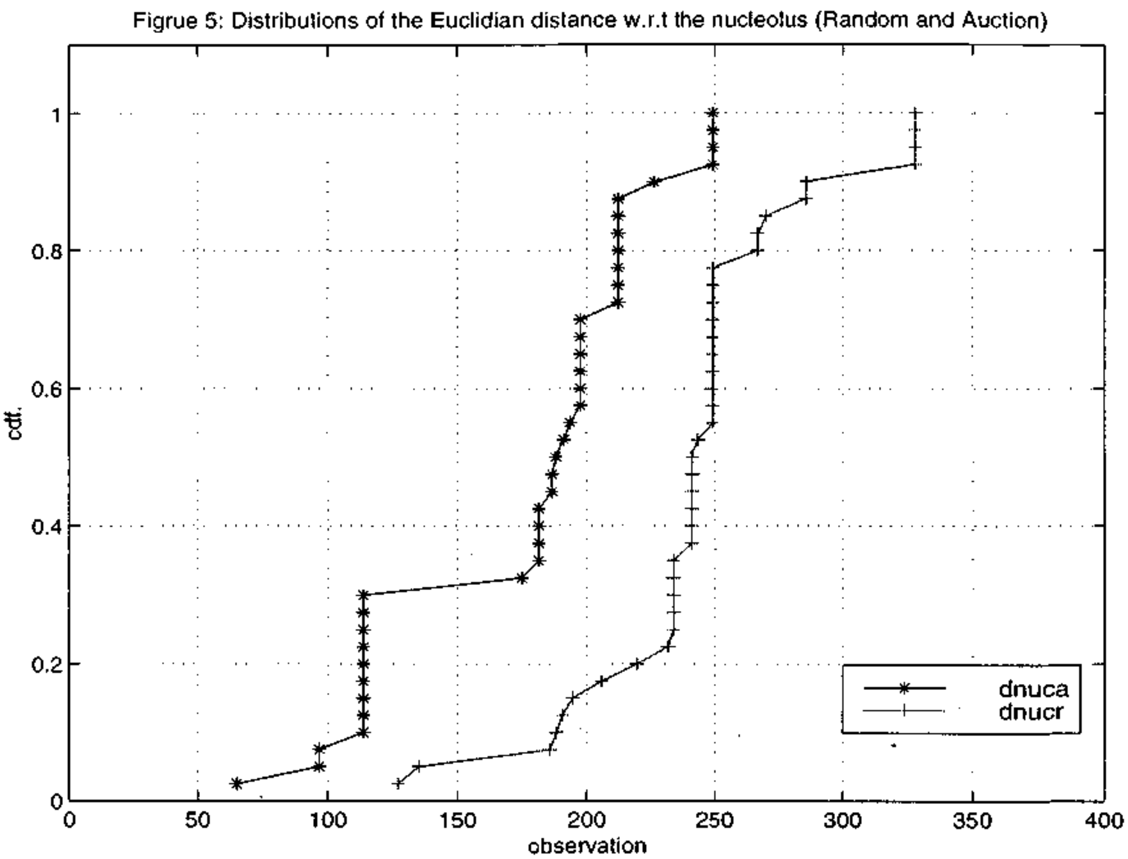

Figure 5: Distributions of the Euclidian distance w.r.t. the nucleolus (Random and Auction)

The solution and value concepts of the cooperative game theory consider not only the outcomes that are feasible in the grand coalition, these concepts takes also into account the feasible outcomes in intermediate coalitions. A measure that takes the claims of intermediate coalitions into account was given with our measure *mex*. The results of the Kolmogorov-Smirnov test on our contentment measure *mex* suggest on a first glance that in the auction sample subjects are more concerned about the arguments presented by intermediate coalitions. Nevertheless, by relating the whole data set to our fairness reference point the nucleolus the statistics test exhibits a large distance to the nucleolus. Instead of balancing and minimising simultaneously their losses subjects compromise in direction to a more egalitarian solution. That is, subjects are not able to reach an agreement which results to a fair outcome between all of them. This result was also revealed by the violation of the desirability property for some extreme result data in which the aggregate income of the poor surpasses the aggregate income of the rich. For summing up we conclude that subjects try to compromise between fairness and the egalitarian solution.

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