

**Searching for Keynes:  
with Application to Canada, 1870-2000**

**by**

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## Abstract

Keynes' idea (1936) that governments can stabilize aggregate activity is one of the most important innovations in public policy thinking in the twentieth century. However, the extent to which Keynesian theory has actually influenced policy actions remains an open question. We reconsider the elements required to address that question and then conduct our own 'search for Keynes'. To do so, we develop an intertemporal probabilistic voting model of fiscal structure in order to distinguish empirically between transitory and permanent components of fiscal structure - where Keynesianism concerns the transitory elements - and to construct a counterfactual showing what would have happened to transitory policy 'after Keynes', had pre-Keynesian stabilization continued. Democratic governments have always been concerned with alleviating the hardship of voters in bad times and this must be accounted for in assessing the impact, if any, of the Keynesian revolution. Together, these and other steps take us on a fascinating tour through the methodology and substance of fiscal history.

While the model could be adapted to study the fiscal structure of any competitive political system, we look for evidence of *attempts* at Keynesian stabilization in the policy actions of the Canadian government after 1945. The study uses consistent budgetary data for 1870 to 2000 constructed by Irwin Gillespie (1991) and updated by the authors. The Canadian case is of particular interest for a number of reasons. The *White Paper on Employment and Income* in 1945 signalled the acceptance of Keynesian ideas in senior policy circles and R.B. Bryce, one of Keynes' early students (who considers 1939 as the date of the first Keynesian budget in Canada), played an important role for many years in the Department of Finance.

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*This [revolutionary] extension of the role of the state was, implicitly, of a general and over-all nature, involving all departments of government; but in accordance with Keynesian economic theory, the greater part of this new responsibility came to focus on particular aspects of governmental operations—its fiscal and monetary policies; and what was called for in fiscal policy especially was an even more radical break with the traditions of the past.*

*H. Scott Gordon, 1965*

## **1. Introduction**

The idea that national governments can stabilize aggregate economic activity, introduced by J. M. Keynes in his *General Theory of Employment, Interest and Money* (1936), is one of the most important innovations in economic policy thinking in the twentieth century. The *General Theory* has profoundly influenced both macroeconomic theory and popular opinion about what governments can and should do with respect to the business cycle.<sup>1</sup> However, the extent to which Keynesian theory has actually influenced the course of public policy remains an open question. We address that question here.

In addition to his influence on macroeconomics, Keynes' arguments about the instability of the economy and the role for state intervention serve as an important ideological pillar for the widely held view favoring a managerial state and a mixed economy (Hall 1989, 365). In this paper however, we focus on the evolution of fiscal policy (non-interest public expenditure, current taxation and the deficit net of debt interest) which, as Gordon notes, is the more radical element of the Keynesian prescription.<sup>2</sup>

Since stabilization involves the transitory or cyclical components of public policy, any search for Keynes requires that policy actions be separated into permanent and transitory components. There is little work on the challenging issue of separating trend and cycle in public finance, and this aspect of the paper effectively constitutes a second theme, one that is of interest in its own right. Our decomposition of fiscal policy into permanent and transitory components is based on an intertemporal extension of the static probabilistic voting model of Coughlin and Nitzan 1981 (used by Hettich and Winer 1999, Persson and Tabellini 2000 and others) in which all policies are considered as part of an evolving political equilibrium. To maximize the probability of reelection, the governing party balances the interests of two types of citizens, those who are liquidity constrained against those who are not.<sup>3</sup> In such a setting, we show that public policies always contain both a long run component that depends on the fundamental structure of the economy and society, and a transitory component that depends on the incumbent party's reaction to transitory shocks.

We look for evidence of *attempts* at Keynesian stabilization in the transitory components of the fiscal policy actions of the Government of Canada after 1945, using consistent budgetary data for 1870 to 2000 constructed by Irwin Gillespie (1991) and updated by the authors.<sup>4</sup> The long time series is required in order to construct a counterfactual that shows what governments would have planned to do 'after Keynes', if Keynesianism had not in fact been present. Only by comparing planned transitory policy 'after Keynes'

**For an international comparison of the role of Keynesianism in policy making, see Hall (1989). An interesting recent study of Keynesianism in action (in the United States) is Prachowny (2000). The political context of Keynesianism is discussed in Hall (1989), Buchanan and Wagner (1977), and Brenner (1994) among others. See also Skidelsky (1995, 2000).**

<sup>1</sup> Thus we leave the investigation of Keynesianism in monetary policy for future research. It may be noted here that the budget constraints and data that are used in this study are specified in a way that is consistent with the relationship between the government and the central bank. It should also be noted that we do not attempt to estimate the consequences of policy for macroeconomic outcomes.

This distinction between types of voters builds on work by Campbell and Mankiw (1990).

We do not attempt to estimate the consequences of policy for macroeconomic outcomes.

with such a counterfactual is it possible to uncover the incremental impact of Keynes' ideas on the course of public policy. Democratic governments have always been concerned with alleviating the hardship of voters in bad times, and this fact must be accounted for in assessing the impact, if any, of the Keynesian revolution.

While the methodology and model we developed can be used to study the evolution of policy in any competitive political system, the Canadian case is of particular interest for a number of reasons. The *White Paper on Employment and Income* in 1945 signalled the acceptance of Keynesian ideas in senior Canadian policy circles, and allows us to date a potential shift in policy regimes. Moreover, Robert Bryce, one of Keynes' early students, played an important role in implementing Canadian fiscal policy from the Department of Finance. Bryce (1986) himself considers 1939 as the date of the first Keynesian budget in Canada.<sup>5</sup>

The paper proceeds as follows. In section two we identify the elements that are essential for assessing the role of Keynesianism in policy choices, and we briefly review previous statistical research in this context. A basic model of public policy in a competitive political system appropriate for modelling policy both 'before and 'after' Keynes' is set out in section three, and a system of reduced form estimating equations of the permanent and transitory elements in policy are derived. The model restricts the signs of coefficients in the transitory components and so allows a test of the short run model implicit in our overall approach. At this point we also clarify the distinction between our model and the traditional approach to the study of Keynesianism that relies on an 'automatic' versus 'discretionary' decomposition of fiscal policies. In section four, the long run and transitory components are estimated, and a test for the presence of Keynesianism that identifies the transitory components both 'before Keynes' (in the counterfactual) and 'after Keynes' is implemented. Brief conclusions complete the paper.

## 2. Essential Elements

The elements that are essential to any investigation of whether Keynesianism actually influenced public policy must include the following:

**First**, any search for Keynes must consider how policy evolved 'after Keynes' *relative to* what would have happened if the General Theory had not in fact influenced policy choices. A counterfactual is then required to indicate what would have happened 'after Keynes' if the pre-Keynesian policy regime had continued in place. Otherwise we risk attributing to Keynesianism what is due to pre-Keynesian or non-Keynesian responses to transitory economic shocks. In our case, the counterfactual recognizes that governments in the nineteenth and first half of the twentieth centuries have always had an incentive to respond to economic hardship, if only because of the concern expressed by voters and pressure groups. Our approach to the counterfactual is to estimate an model (developed below) over the period from 1870-1938, before Keynesian ideas could have been relied upon in the making of policy. We then use this model to forecast into the period 'after Keynes'.

**Second**, Keynesianism concerns only the transitory response of governments to transitory macroeconomic shocks, making it necessary to distinguish between the permanent and transitory or cyclical parts of public policy. Keynesianism does not involve policy responses to such factors as war or the expectation of permanent increases in income. In their study of macroeconomic policy in Canada after 1962, Kneebone and McKenzie (1999) recognize the need to remove the permanent or longer run part of policy choices, and do so using a simple Hodrick-Prescott trend. We model the long run, 'permanent'

<sup>5</sup> For histories of Keynesian ideas and their influence in Canada, see for example Gordon (1965) and Campbell (1987, 1991) as well as Bryce (1986).

component of fiscal policy and its transitory component explicitly, generating a system of equations that impose testable restrictions on the signs of the coefficients of the transitory or short run elements of policy.

Operationalizing the distinction between trend and cycle is a challenging task in any context (see, for example, Canova 1998). We think that not enough attention has been paid to this issue in the positive political economy of public finance (but see Goff and Tollison 2002, Goff 1998 and Barro 1986).<sup>6</sup> It cannot be avoided here.

**Third,** Keynesianism is about *planned* responses to expected aggregate economic shocks. Hence we must model ex ante stabilization policy actions. It is not about mistakes, though of course mistakes are made. A positive model of planned policy choices by political representatives is therefore required, one that contains both permanent and transitory elements.

The traditional approach to the study of Keynesian stabilization distinguishes discretionary from automatic aspects of fiscal policy. Discretionary changes are typically defined as changes in the budget that result from the explicit adjustment of policy parameters (like a tax rate), while automatic components are budgetary changes that result from fluctuations in economic activity at unchanged policy parameters. Work of this sort on Canada includes Will (1967), Gillespie (1979), Boothe and Davidson (1993), Wilson and Dungan (1993) and Boothe and Petchey (1995). These studies conclude generally that the discretionary component of policy is small compared to the total and often awkwardly timed from the perspective of Keynesian stabilization theory. (But see McKenzie and Kneebone 1999, who find evidence for the existence of discretionary changes in policy especially in the 1990's) Thus, on our reading, this literature on balance suggests that the evidence for the adoption of Keynesianism in Canada is not strong, unless one argues that Keynesian thinking led to the deliberate use of, and/or increased reliance on, fiscal policies that act as automatic stabilizers of aggregate demand, a conclusion that calls into question the initial distinction between automatic and discretionary policy.

Closely associated with the automatic/discretionary decomposition is one that distinguishes cyclical changes in budgetary policy (due to changes in the automatic component of fiscal aggregates) from structural changes (due to changes in the discretionary components). Typically the discretionary component or structural change in a fiscal instrument is estimated by predicting what policy would have been chosen at this year had unemployment remained unchanged, using an autoregressive model that includes unemployment rates, with the difference between this hypothetical policy and last year's actual policy serving as the measure of discretionary change.

In neither case is the political context of policy choice given explicit consideration. Hence in our view the automatic/discretionary or structural/cyclical approaches are incomplete and potentially misleading. A model of political behavior is needed on which to base an estimate of equilibrium planned policy actions in a competitive political system, in the face of expected shocks. One should also note that this political economy setting, the distinction between discretionary and automatic policy is problematic, since all

**Barro (1986) derives an estimating equation based on his tax-smoothing model of the public deficit to explain the behaviour of federal deficits for the period from 1916 to 1985. In his view, deficits should rise when economic activity temporarily falls or when government expenditures rise temporarily relative to some longer run value, in order to smooth tax rates and spread excess burdens over time. Barro finds that the behaviour of the deficit is generally in accordance with this view, though some results indicate that increases associated with an economic recession or with a temporary increase in government spending are larger than can be accounted for by the pure tax-smoothing argument. More importantly from our point of view, he finds no change in the data generating process governing public deficits between the interwar period, 1920-40, and the post World War II sample, 1948-82. These results for the U.S. are, as a whole, consistent with policy 'after Keynes' that looks Keynesian but is not.**

dimensions of policy are continually adjusted in the face of evolving economic activity. A decision not to change a policy parameter in the face of an expected change in aggregate activity is then just as active a policy as the opposite.<sup>7</sup>

**Fourth**, the model used to investigate a possible change in the stabilization regime should allow for the possibility of a Keynesian-like policy and for Keynesian policy to coexist in a political equilibrium. This implies that in our case the model constructing the counterfactual should give competitive political parties some reason for engaging in what might look like countercyclical policy 'before Keynes'. Otherwise, the counterfactual is likely to be biased towards finding evidence of Keynesian policy 'after'. The model must also be precise in allowing us to predict that Keynesian stabilization was actually attempted after Keynes for particular reasons, so that the model of planned policy after Keynes can be properly specified.

The model we develop contains both liquidity-constrained and unconstrained citizens, in contrast to that of Barro (1986, 1981, 1979) and others in which liquidity constraints have no role. Tax smoothing on behalf of both types of voters is then but one feature of a politically profitable fiscal platform. Consumption smoothing for liquidity-constrained voters, the interaction between liquidity-constrained voters and the political demands of voters not so constrained, and redistributions between these two types of voters also play important roles in motivating the choices made by governments both before and after Keynes in the model outlined in section three. In this framework, we interpret the introduction of Keynesianism as stemming from changed views about the benefits from government action to deal with the externalities inherent in an economy with liquidity constrained citizens, an interpretation of Keynesian theory that follows Leijonhufvud (1968).

**Fifth**, it is necessary to allow for structural changes in the public sector that may have nothing to do with Keynesianism, but which affect the responsiveness of the fiscal system to transitory shocks. The evolution of the social welfare system, which may have nothing to do with Keynesianism, affects the responsiveness of planned actions to transitory shocks. Under the assumption - which may not be correct - that the welfare state developed in a way that was completely independent of Keynesian ideas, the effect of this structural change on the responsiveness of the public sector to shocks needs to be controlled for. While it is (in our view, seriously) flawed in other ways, the good sense in the automatic/discretionary decomposition is its allowance for the (assumed to be) unrelated influence of the welfare state on the cyclical response of fiscal policies. We also control for the role of the welfare state, using a different method that allows for the gradual maturation of the welfare state over the post-war period.

**Sixth** and finally, we must construct time series representing economic shocks to which the government responds, and do so in a manner that allows historical work over the long period of time required for the construction of a counterfactual. We do not attempt to break new ground here, and our approach to this task is outlined in section four.

The preceding discussion and general method employed to study fiscal policy can be summarized usefully in Figure 1. Here we illustrate the model of planned policy before Keynes (before 1938) and the forecast based on this model after Keynes that constitutes the counterfactual, for some policy instrument such as public expenditure. In each case, policy consists of a long run (shown here to be constant for simplicity) and a transitory part that varies with transitory economic activity. The figure also illustrates the systematic part of the model of policy after Keynes, and what we call the policy differential - the difference between our model of planned policy after Keynes and that in the counterfactual. In the hypothetical case illustrated, given the nature of economic shocks illustrated in the lower part of the figure, policy after Keynes is substantially more countercyclical than in the counterfactual, indicating the introduction of

<sup>7</sup> The end of the next section distinguishes analytically the difference between our approach and the traditional approach based on the automatic/discretionary distinction.

Keynesian stabilization. The tests for the existence of Keynesian policy we implement in what follows involves regressions explaining changes in estimated policy differentials for each policy instrument.

[Figure 1 here]

### 3. Competitive Political Equilibrium

We regard all policies as equilibrium outcomes in a competitive political system. Our model of policy actions extends work on probabilistic voting (as in Coughlin and Nitzan 1981, Coughlin 1992, Hettich and Winer 1999, and Persson and Tabellini 2000) to an intertemporal setting in which in which some voters are liquidity constrained, and in which there is a distinction between longer run or 'permanent' and shorter run or 'transitory' components of public policy. In this framework, two expected vote maximizing political parties compete for support from citizens whose voting behavior is known only probabilistically. In the next section, we explicitly solve a linearized version of the model in order to derive estimating equations that can be applied both 'before' and 'after' Keynes. Here we consider in turn, government budget constraints, the private economy, individual optimization and political behavior, political parties and the equilibrium outcome of political competition.

#### 3.1 Government budget constraints and the private economy

We begin with the budget constraints applicable to the vote-maximizing fiscal platform offered by any party. A platform in this context is a set of fiscal policies for the current and for all future periods the parties think voters will care about. The relevant flow government budget constraints for the current and all such future periods are

$$g_{t+s} = t_{t+s} + [(b_{t+s} - b_{t+s-1}) - r_{t+s} \cdot b_{t+s-1}] ; \quad s = 0, 1, \dots, \infty \quad (1)$$

where the set of fiscal policy variables to be chosen by each party at time  $t$  is  $\{g, t, b\} = \{g_t, \dots, g_{t+s}, \dots, g_\infty ; t_t, \dots, t_{t+s}, \dots, t_\infty ; b_t, \dots, b_{t+s}, \dots, b_\infty\}$ .<sup>8</sup> Here we assume the relevant horizon is infinite for convenience. The estimating equations remain agnostic about the politically relevant horizon.

The general equilibrium structure of the private economy is represented simply by a function  $H(\cdot)$  that incorporates the way in which public policies impinge on the private choices of voters. The function below represents both technological and market transformation possibilities facing society together with the reaction of the private sector to the changing incentives that arise from given government expenditure, tax and debt policies. Because liquidity constraints may affect the choices of some voters, and, especially if Keynes is right, can to some extent be affected by government policies, the general equilibrium consequences of liquidity constraints are also included in the  $H$  function:

$$H(g, t, b, x) = 0 \quad (2)$$

<sup>8</sup> By successive substitution for  $b_t$  and imposition of the no-Ponzi game condition that neither households nor governments can be net borrowers at infinity  $\lim_{s \rightarrow \infty} \prod^s R^s b_{t+s} = 0$ , where  $R^s = [1/(1+r_{t+s})]$  and  $\prod^s R^s = [1/(1+r_{t+1})][1/(1+r_{t+2})] \dots [1/(1+r_{t+s})]$  and  $\prod^0 R^0 = 1$ , the constraints in (1) could be combined into one intertemporal constraint (through period  $s$ ) of the form  $(1+r_{t+s})b_{t+s-1} - \sum_{s=0}^{\infty} \prod^s R^s (t_{t+s} - g_{t+s}) = 0$ . Doing so, with  $b_{t+s-1}$  given, would eliminate public debt as a policy variable from the model and for this reason we continue to use the set of flow constraints in (1) rather than the combined intertemporal constraint. Conceptually we do not wish to prejudge which particular policy instrument, if any, provides better evidence of Keynesianism, nor do we wish to prejudge whether individuals or the strategy of political parties is fully cognizant of the existence of a long-run budget constraint. Ultimately all that is required for stability is that government debt grow less rapidly than the interest rate.

where the vector  $\mathbf{x}$  represents the set of factors that determine the long run evolution of public policy. Note that  $H(\cdot)$  allows for the effects of public debt on economic activity. Later it will prove useful to separate the vector  $\mathbf{x}$  into two parts; permanent income, and all other factors (denoted as 's' below) affecting the long run such as the degree of urbanization, age structure of the population, and so on.

### 3.2 Individual voting behaviour

Individuals are assumed to have time separable direct utility functions such that the expected utility of individuals has the following general form:

$$E_t u_t = \sum \beta^s E_t \mu [g_{t+s}, c_{t+s}, \ell_{t+s}]; \quad s = 0, 1, \dots, \infty \quad (3)$$

where utility is assumed to increase with increases in government spending,  $g_{t+s}$ , private consumption,  $c_{t+s}$ , and leisure,  $\ell_{t+s}$ ; where  $\beta$  is the subjective discount factor (i.e.,  $\beta = 1/(1+\rho)$ , where  $\rho$  is the rate of time preference); and  $\beta^s = [1/(1+\rho)]^s$ . Individuals choose the values of  $c_{t+s}$  and  $\ell_{t+s}$  that maximize the discounted value of utility subject to the general equilibrium structure of the economy  $H(\cdot)$  and the set of  $\mathbf{g}$ 's,  $\mathbf{t}$ 's and  $\mathbf{b}$ 's proposed by political parties.

There are two types of voters: type 1 voters are liquidity constrained and type 2 care only about permanent income in their decision making. Within types, individuals are assumed to be homogeneous.

Using  $E_t U_2$  to represent the indirect expected utility of voter type 2 who is *not* liquidity constrained,

$$E_t U_2 = \sum \beta^s E_t \mu [c_{t+s}^*(H(\cdot)), \ell_{t+s}^*(H(\cdot)); \{\mathbf{g}, \mathbf{t}, \mathbf{b}\}_{t+s}]; \quad s = 0, 1, \dots, \infty \quad (4)$$

where the starred values reflect the optimal private choices of individuals and the levels of  $\mathbf{g}$ ,  $\mathbf{t}$  and  $\mathbf{b}$  are parameters for each individual. Using  $U_1$  to represent the indirect utility of voter type 1 who is liquidity constrained,

$$E_t U_1 = \sum \beta^s E_t \mu [c_{t+s}^*(H(\cdot); \mathbf{y}_t), \ell_{t+s}^*(H(\cdot); \mathbf{y}_t); \{\mathbf{g}, \mathbf{t}, \mathbf{b}\}_{t+s}], \quad s = 0, 1, \dots, \infty. \quad (5)$$

$E_t U_1$  differs from  $E_t U_2$  only because the liquidity constrained individual's evaluation of government actions depends upon the realization of a vector of stochastic current period variables  $\mathbf{y}_t$ . In what follows in this section, we define  $\mathbf{y}$  as a deviation from a long run value that is *welfare improving* for liquidity constrained voters, such as the increase in actual output above its long run equilibrium level.

Individuals vote for a political party based on two factors: first, the level of utility offered by the proposed party platform compared to that offered by the alternative; and, second, the non-policy characteristics of the party which are assumed to be stochastic from the parties' perspective. The latter include such things as candidate personalities, perceived competency, and reputation for carrying out promises. These non-policy characteristics are assumed to be exogenous and independent of any particular policy instrument setting.

More formally, the level of utility received by a representative voter of type  $j$  ( $= 1, 2$ ) from the program platform offered by party,  $k$ , is described by

$$V_j^k = E_t U_j [\mathbf{g}(k), \mathbf{t}(k), \mathbf{b}(k)] + \xi_j^k = E_t U_j(k) + \xi_j^k \quad (6)$$

where  $E_t U_j[\cdot]$  is the indirect expected utility function as described in (4) and (5) and the  $\xi_j^k$  describes the level of utility generated by the non-policy related characteristics of party  $k$  and its candidates.



With the political process consisting of an incumbent party (i) and an opposition (o), we may define the non-policy *bias* of a representative voter of type j in favor of party o as

$$\phi_j^o = \xi_j^o - \xi_j^i. \quad (7)$$

By definition, this non-policy bias is independent of  $g$ ,  $t$ ,  $b$ ,  $H(\cdot)$  and  $y_t$ .

The probabilistic voting behaviour of individual can now be described, following Coughlin, Mueller and Murrell (1990). The probability that an individual of type j votes for the opposition, o, rather than the incumbent, i, is

$$P_j^o = \begin{cases} 1 & \text{if } \{E_t U_j(o) - E_t U_j(i)\} > \phi_j^o \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where  $E_t[U_j(o)]$  and  $E_t[U_j(i)]$  represent the levels of expected utility associated with the policy platforms of the opposition and incumbent parties. To a get voter of type j on side, the opposition must deliver enough economic welfare to overcome the non-policy bias of this voter in favour of the incumbent.

We assume that from each party's perspective, the non-policy bias of a representative voter of either type of voter is known to be uniformly distributed over the interval  $(\phi_{\min}^j, \phi_{\max}^j)$ . Representing the cumulative distribution function of  $\phi^j$  as  $F_j$ , both parties view the probability at time t that a representative individual of type j will vote for, say, the opposition, as being equal to the cumulative probability  $F_{jt}\{E_t U_j(o) - E_t U_j(i)\}$  that  $\phi^j$  is less than the utility differential generated in period t by the opposition party:

$$F_{jt}\{E_t U_j(o) - E_t U_j(i)\} = \alpha_j \cdot \{E_t U_j(o) - E_t U_j(i) - \phi_{\min}^j\}, \quad (9)$$

where

$$\alpha_j = \partial F_{jt} / \partial E_t U_j(o) = 1 / (\phi_{\max}^j - \phi_{\min}^j) \quad (10)$$

is the sensitivity of the individual voting probability to a change in welfare.

Assuming further, as is customary in the probabilistic voting approach, that  $[E_t U_j(o) - E_t U_j(i)]$  lies everywhere within the interval on which  $\phi_j$  is defined ensures that parties believe that every voter has some positive probability of voting for it, even if that probability may be small. Consequently, no party will completely ignore any voter of either type, even though one group of voters will be more sensitive to changes in the policies of each party and will therefore receive better treatment by that party's strategists.<sup>9</sup> This assumption plus the probabilistic nature of voting makes the objective function of each party continuous in its policy instruments, rather than being discontinuous at various points as would be the case if voting were strictly deterministic. This continuity of electoral objectives allows a Nash equilibrium in the electoral game to exist despite the multi-dimensionality of the policy space.

### 3.3 Party objectives and Nash equilibrium

We assume that parties choose policies to continually maximize expected support or votes. Given a population of size  $N_t$ , let both parties think that there are  $N_{1t}$  type 1 voters, and a fraction of the population,  $\lambda_t = N_{1t}/N_t$ , that is liquidity constrained. The remaining fraction of type 2 voters,

<sup>9</sup> If this were not true, and the probability that some voters will support one of the parties falls to zero, the expected vote function may not be sufficiently concave for a Nash equilibrium to exist. See for example, Usher (1994).

$1-\lambda_t = (N_t - N_{1t})/N_t$ , respond only to changes in permanent income. Then, the expected number of votes that the opposition party maximizes at time  $t$ ,  $EV_t(o)$ , may be written as

$$EV_t(o) = N_t \lambda_t F_{1t} + N_t (1-\lambda_t) F_{2t}, \quad (11)$$

and the expected vote for the incumbent is then  $EV_t(i) = N_t - EV_t(o)$ .

The continuity of the expected vote functions is insured by the probabilistic structure of voting behaviour. If the set of feasible platforms among which the parties choose is also compact and convex, a pure strategy Nash equilibrium in the electoral game will exist if, after substitution of all relevant constraints on policy choices, each expected vote function is also strictly concave in each policy instrument for each platform chosen by the opposition (see, for example, Owen 1995, Theorem IV.6.2). The concavity of expected vote functions is discussed at length in Enelow and Hinich (1989) and Hinich and Munger (1994). Here we simply note that this amounts to saying that each party can choose an optimal political platform, and assume that all conditions required for the existence of a (possibly non-unique, pure strategy) Nash equilibrium are satisfied.

In this Nash equilibrium, policy platforms converge so that the identity of the party in power is incidental to the determination of the policies actually adopted. Strict concavity of the expected vote functions and the fact that voting depends only on utility differences means that no party can gain a lasting advantage by adopting a platform that differs from that of its opposition (Enelow and Hinich, 1989). Since party platforms converge in an equilibrium, we may drop subscripts identifying the party when it is convenient to do so, and refer only to the governing party or government.

### 3.4 Using a representation theorem to characterize policy equilibrium

A convenient feature of the Nash equilibrium in the probabilistic voting framework is that the equilibrium values of the policy variables chosen by the successful party can be characterized, or represented, by maximizing a particular weighted sum of individual expected utilities (Coughlin and Nitzan, 1981). To see this in the present context, we write the first order conditions defining the vote maximizing choices for one typical policy instrument that must be satisfied at the Nash equilibrium, government non-interest spending,  $g$ . Assuming that all relevant constraints (i.e., (1) and (2)) have been substituted into the expected vote functions (11), and assuming here and below that  $\lambda$  is constant and non-zero, the first order condition that  $g$  must satisfy for both parties at the Nash equilibrium will have the following general form:

$$\begin{aligned} \partial EV / \partial g_t &= N_t \lambda \alpha_1 [\partial(E_t U_1) / \partial g_t] + N_t (1-\lambda) \alpha_2 [\partial(E_t U_2) / \partial g_t] \\ &= \theta_1 [\partial(E_t U_1) / \partial g_t] + \theta_2 [\partial(E_t U_2) / \partial g_t] = 0, \end{aligned} \quad (12)$$

where party and time subscripts have been omitted, and where  $\theta_1 = N_t \lambda \alpha_1$  and  $\theta_2 = N_t (1-\lambda) \alpha_2$ . Analogous conditions hold for the other policy instruments. For later use, we note that the  $\theta$ 's are the derivatives with respect to expected utility of the probability that a representative individual of a given type will vote for the indicated party, times the number of voters of that type.

Next, consider maximization of a political support function,  $S$ , when the same constraints as in (11) have been substituted into the objective function:

$$S = \theta_1 \cdot E_t U_1 + \theta_2 \cdot E_t U_2 \quad (13)$$

and where the  $\theta$ 's are defined in (12). It is straightforward to see that the first order conditions for the maximization of  $S$  with respect to the choice of  $g$  will be exactly the same as those given in (12). The strict concavity of the expected vote functions insures that a solution to the first order conditions for maximizing  $S$  is a global maximiser of expected votes. Hence a policy platform that solves the problem of maximizing  $S$ , subject to both the government budget constraints and the equilibrium structure of the economy, is identical to the common policy platform adopted by the parties in a Nash equilibrium of the electoral game.

This representation theorem shows that the policy outcomes of a competitive political system balance opposing and heterogeneous interests within the electorate, where the  $\theta$ 's in effect measure the relative political influence that each voter type has on the equilibrium outcome.<sup>10</sup>

#### 4. Derivation of Reduced Form Estimating Equations

Using the representation theorem introduced above in its full form for the model we have outlined, the equilibrium policy platform can be found as the solution to the following optimization problem:

$$\begin{aligned} \text{Max } \mathcal{L}_t = & \theta_{1t} \cdot \sum^{\infty} \beta^s E_t \mu^1 [g_{t+s}(H(.)), c^*_{t+s}(H(.)), \ell^*_{t+s}(H(.)); y_{t+s}] \\ & \{g_0, \dots, g_{t+s}, g_{t+s+1}, \dots, t_0, \dots, t_{t+s}, t_{t+s+1}, \dots, b_0, \dots, b_{t+s}, b_{t+s+1}, \dots\} \\ & + \theta_{2t} \cdot \sum^{\infty} \beta^s E_t \mu^2 [g_{t+s}(H(.)), c^*_{t+s}(H(.)), \ell^*_{t+s}(H(.))] \\ & + \sum^{\infty} \psi_{t+s} \{ [\prod^s R^s] [t_{t+s} + (b_{t+s} - b_{t+s-1}) - r_{t+s} b_{t+s-1} - g_{t+s}] \}, \end{aligned} \quad (14)$$

where the expectation  $E_t$  is understood to be conditional on information available at time  $t-1$  and the summation runs from  $s = 0$  to  $s = +\infty$ . In the Nash equilibrium, the incumbent party (or government) will choose levels for current and future policy instruments, given past values and in the light of voters' views about the future, including their beliefs concerning the feasibility and effect of proposed plans.

To simplify notation in solving the above problem for a political equilibrium, we temporarily ignore the distinction between expected and known quantities and treat the  $x$ 's and  $y$ 's as single variables, even though they would generally refer to vectors of long-run factors and transitory shocks respectively. Accordingly, the first order conditions of an internal optimum for  $g_{t+s}$ ,  $t_{t+s}$  and  $b_{t+s}$  at time  $t+s$  are

$$\partial \mathcal{L}_t / \partial g_{t+s} = \beta^s \{ \theta_1 E_t [\mu^1_g (H(x_{t+s}), y_{t+s})] + \theta_2 E_t [\mu^2_g (H(x_{t+s}))] \} - [\prod^s R^s] \psi_{t+s} = 0, \quad (15)$$

$$\partial \mathcal{L}_t / \partial t_{t+s} = \beta^s \{ \theta_1 E_t [\mu^1_t (H(x_{t+s}), y_{t+s})] + \theta_2 E_t [\mu^2_t (H(x_{t+s}))] \} + [\prod^s R^s] \psi_{t+s} = 0, \quad (16)$$

$$\begin{aligned} \partial \mathcal{L}_t / \partial b_{t+s} = & \beta^s \{ \theta_1 E_t [\mu^1_b (H(x_{t+s}), y_{t+s})] - \theta_2 E_t [\mu^2_b (H(x_{t+s}))] \} - \beta^{s+1} \{ \theta_1 E_t [\mu^1_t (H(x_{t+s+1}), y_{t+s+1})] \\ & + \theta_2 E_t [\mu^2_t (H(x_{t+s+1}))] \} + [\prod^s R^s] [\psi_{t+s} - \psi_{t+s+1}] = 0. \end{aligned} \quad (17)$$

From the first order conditions, two things are immediately apparent. First, optimal vote maximizing behaviour means that government spending, taxes and borrowing will be varied until each becomes equally productive (on the margin) in winning votes. This is reflected in the symmetric form of the equations and the equality of their marginal benefit to the same Lagrangian multiplier and hence to each

<sup>10</sup> If some  $\theta_j$  increases, the expected utility of that voter type will be given greater weight in the Nash equilibrium.

other<sup>11</sup>. Second, from (17) we see that that  $\psi_{t+s+1} \neq \psi_{t+s}$ . Even when we follow the traditional literature by assuming that  $\partial(E_t \mu^2)/\partial b_t = 0$  everywhere for type 2 voters (who are not liquidity constrained and so who are indifferent to the level of government debt), type 1 voters remain liquidity constrained with  $\partial(E_t \mu^1)/\partial b_t \neq 0$ . A government borrowing strategy can then increase type 1 utilities by loosening period specific individual budget constraints and thereby gain additional support. Such action is costly in terms of support from type 2 voters, however, so that the liquidity constraint will not be completely eliminated in political equilibrium.

We can use these first order conditions to determine planned changes in the levels of policy variables over time in political equilibrium. Beginning with government spending, predictions for changes in the level of government spending over time can be derived by comparing the first order conditions across adjacent time periods  $t+s-1$  and  $t+s$ , using the Lagrangian for a given period, say time  $t$ .<sup>12</sup> In this manner we find, from (15), that the marginal rate of substitution across time in the use of public expenditure when there are no constraints on the government's borrowing is

$$\left[ \frac{E_t \beta^s [N_t \lambda \alpha_1 \mu_g^1(H(x_{t+s})) + N_t (1 - \lambda) \alpha_2 \mu_g^2(H(x_{t+s}, y_{t+s}))]}{E_t \beta^{s-1} [N_t \lambda \alpha_1 \mu_g^1(H(x_{t+s-1})) + N_t (1 - \lambda) \alpha_2 \mu_g^2(H(x_{t+s-1}, y_{t+s-1}))]} \right] = \left[ \frac{R^s \psi_{t+s}}{\psi_{t+s-1}} \right] \quad (18)$$

which simplifies to

$$\left[ \frac{E_t [\lambda \alpha_1 \mu_g^1(x_{t+s}; y_{t+s}) + (1 - \lambda) \alpha_2 \mu_g^2(x_{t+s})]}{E_t [\lambda \alpha_1 \mu_g^1(x_{t+s-1}; y_{t+s-1}) + (1 - \lambda) \alpha_2 \mu_g^2(x_{t+s-1})]} \right] = \left[ \frac{(1 + \rho) \psi_{t+s}}{(1 + r_{t+s}) \psi_{t+s-1}} \right] \quad (19)$$

where we assume that  $r_{t+s}$  is observable,  $\beta = (1 + \rho)^{-1}$  is a constant and continue to assume that  $\lambda$  is constant. Similarly for taxes under the same conditions,

$$\left[ \frac{E_t [\lambda \alpha_1 \mu_g^1(x_{t+s}; y_{t+s}) + (1 - \lambda) \alpha_2 \mu_g^2(x_{t+s})]}{E_t [\lambda \alpha_1 \mu_g^1(x_{t+s-1}; y_{t+s-1}) + (1 - \lambda) \alpha_2 \mu_g^2(x_{t+s-1})]} \right] = \left[ \frac{(1 + \rho) \psi_{t+s}}{(1 + r_{t+s}) \psi_{t+s-1}} \right] \quad (20)$$

An analogous equation for public debt applies but is not explicitly solved for since the equations for non-interest public expenditure and taxes together with the government budget constraint in (1) imply the equation for the change in the deficit net of interest payments to the private sector.

To progress to estimating equations, we need to add specificity to the underlying indirect utility functions. If we assume that type 1 utility functions are separable in the current and transitory income variables, then because individuals in the model are distinguished only by the liquidity constraint,

$$\mu^1(x, y) = \mu^1(x) + \mu^1(y) = \mu^2(x) + \mu^1(y). \quad (21)$$

The left hand side of equations (19) and (20) can then be represented as

<sup>11</sup> In interpreting the equation for debt (17), allowance should be made for the optimal spreading of interest payments over time, reflected in the presence of the Lagrange multiplier with subscript  $t+s+1$ . As Kenny and Toma (1997) show, this result generalizes across multiple tax and spending alternatives.

<sup>12</sup> This keeps the decision maker the same across adjacent time periods (so that the Euler equation represents a consistent set of decision makers) and means that cohort changes are reflected in the  $x_{t+s}$ 's.

$$\left[ \frac{E_t [\lambda \alpha_1 \mu_z^1(y_{t+s}) + (\lambda \alpha_1 + (1-\lambda) \alpha_2) \mu_z^2(x_{t+s})]}{E_t [\lambda \alpha_1 \mu_z^1(y_{t+s-1}) + (\lambda \alpha_1 + (1-\lambda) \alpha_2) \mu_z^2(x_{t+s-1})]} \right] = \left[ \frac{E_t [\mu_z^2(y_{t+s}) + \gamma \mu_z^1(x_{t+s})]}{E_t [\mu_z^2(y_{t+s-1}) + \gamma \mu_z^1(x_{t+s-1})]} \right]; z = \{g, t\} \quad (22)$$

where the right hand side is formed by dividing the top and bottom by  $(\lambda \alpha_1 + (1-\lambda) \alpha_2)$  and by using  $\gamma = [N_1 \lambda \alpha_1 / (N_1 \lambda \alpha_1 + N_1 (1-\lambda) \alpha_2)] = N_1 \alpha_1 / (N_1 \alpha_1 + N_2 \alpha_2) < 1$ . Note that  $\gamma$  is interpretable as the relative political weight of type 1 (liquidity constrained) voters in the political process.

Taking the logarithm of either (19) or (20) and using (22), we find the general form of the first order conditions for  $g$  and  $t$  as

$$\begin{aligned} \ln [\gamma E_t \mu_z^1(y_{t+s}) + E_t \mu_z^2(x_{t+s})] - \ln [\gamma E_t \mu_z^1(y_{t+s-1}) + E_t \mu_z^2(x_{t+s-1})] \\ = \rho - r_{t+s}^e + D [\ln \psi_{t+s}]; \quad z = \{g, t\} \end{aligned} \quad (23)$$

where, to simplify, we have used the approximation,  $\ln(1+q) \approx q$  for  $q = \{\rho, r\}$  and the  $D$  operator to denote first differences, i.e.,  $D[\ln \psi_{t+s}] = \ln(\psi_{t+s}) - \ln(\psi_{t+s-1})$ .<sup>13</sup>

Further progress can be made by taking the Taylor series approximation of equation (23) for  $g$  and  $t$ . Here we discuss the case of government spending, with details left for the Appendix. We linearize about the long run path of  $g$ ,  $g_{t+s}$ ,  $s = 0, 1, 2, \dots$ , defined as the path of government spending that would occur, given expectations of  $x$ , if all expected transitory deviations from the long run equilibrium path of the economy, the  $y$ 's for different periods, are equal to zero. In the present context this is natural since Keynesian stabilization emphasizes short run responses to transitory shocks.

At each point in time, the permanent path of government spending can be determined by solving the Lagrangian in (14) for current and future periods policies under the condition that all expected  $y$ 's are zero. This long run component of government spending incorporates consumption smoothing on behalf of voters in response to expected movements in long run factors such as the trend in growth of real income and urbanization. To more clearly distinguish between the role of liquidity constraints and of other factors, the permanent component of  $g$  and other policy variables is defined to include the effects of the world wars.<sup>14</sup> The permanent or long run component of each policy instrument has government policy variables moving optimally about the long run path defined by expected values of underlying trends in the economy and society. In the absence of war, this permanent component would depend on the expected full-employment path of the economy.<sup>15</sup>

Because our results concern what the government plans to do in a political equilibrium given their

<sup>13</sup> In a rational expectations-permanent income model of the sort used by Hall,  $\mu^1$  is absent and  $\psi_{t+s} = \psi_{t+s-1}$  since there are no impediments to liquidity constrained voters shifting resources through time. If then  $\rho$  and  $r$  were also constants, equation (23) would reduce to one analogous to that used by Hall and others, namely

$$\ln [E_t \mu_z^2(x_{t+s})] - \ln [E_t \mu_z^2(x_{t+s-1})] = \rho - r; \quad z = \{g, t\}.$$

i.e., (the log of) expected marginal utility with respect to each policy instrument is a martingale. Following Hall, this equation could be converted to an estimating equation by assuming that  $x_{t+s-1}$  is observable, by removing the expectation operator on  $\mu(x_{t+s})$ , and adding to the right side an error term with zero mean and that is uncorrelated with any information available at time  $t+s-1$ .

<sup>14</sup> Our permanent or long run component of policy differs from that of Barro (1990, part III). His permanent component is defined such the effects of wars are excluded.

<sup>15</sup> An early model of government embodying this idea of a long-run or 'permanent' component to current policy outcomes is found in Alt and Chrystal (1983).

expectation of future events we add the superscript "e" to the policy instruments, their partial derivatives where appropriate, and to both the long run factors and transitory shocks on which government plans are based. To provide enough structure so that we may sign some of the coefficients in the estimating equations, we also assume that the expected marginal utility generated by a change in the policy variable is linearly related to the expected level of the state variables  $y_{t+s}^e$  and  $x_{t+s}^e$ , as in  $E_t \mu_g^1 = a + b y_{t+s}^e$  and  $E_t \mu_g^2 = c + d x_{t+s}^e$ , where the scalars  $a$  and  $c$  are positive and both  $b$  and  $d$  are negative, and in addition we assume that  $(\partial y / \partial g_{t+s-1})^e \approx (\partial y / \partial g_{t+s})^e$ . Further simplifications use the assumption that the indirect utility functions are separable in the policy instruments so that cross partial derivatives in the policy instruments are all zero.

When the left side of (23) is linearized around the path defined by

$$\hat{g} = g^e(x_{t+s}^e, x_{t+s-1}^e, y_{t+s}^e=0, y_{t+s-1}^e=0, \dots),$$

that is, around the value of  $g^e$  when transitory shocks are zero, and the result is equated to the right side of (23) and rearranged, we have<sup>16</sup>:

$$\Delta g_{t+s}^e = \Delta \hat{g}_{t+s} + \frac{[\rho - r_{t+s}^e + D \ln(\psi_{t+s})]}{(X_{t+s}^e)^{-1} \gamma b (\partial y / \partial g_{t+s})^e} - \frac{(y_{t+s}^e - y_{t+s-1}^e)}{(\partial y / \partial g_{t+s})^e} - \frac{d (x_{t+s}^e - x_{t+s-1}^e)}{\gamma b (\partial y / \partial g_{t+s})^e} \quad (24)$$

where  $\Delta g^e$  denotes that the equation is an approximation for the **planned** change in government spending,  $\Delta g_{t+s}^e = (g_{t+s}^e - g_{t+s-1}^e)$ ,  $X_{t+s}^e = \gamma a + c + d x_{t+s}^e > 0$ , and where we have assumed that  $X_{t+s-1}^e (X_{t+s}^e)^{-1} \approx 1$ .

Finally, (24) can be given one further useful simplification. Inspection of (17) indicates that  $D \ln(\psi_{t+s})$  is a function of  $y_{t+s}$  through  $E_t \mu_b^1$ .<sup>17</sup> Assuming that the relationship is linear allows us to write  $D \ln(\psi_{t+s}) = \varphi (y_{t+s} - y_{t+s-1}) > 0$  with  $\varphi > 0$ . Hence (24) becomes

$$\Delta g_{t+s}^e = \Delta \hat{g}_{t+s} + \frac{(\rho - r_{t+s}^e)}{Z_{t+s}} + \frac{(\varphi - (X_{t+s}^e)^{-1} \gamma b) (y_{t+s}^e - y_{t+s-1}^e)}{Z_{t+s}} - \frac{d (x_{t+s}^e - x_{t+s-1}^e)}{\gamma b (\partial y / \partial g_{t+s})^e} \quad (25)$$

where  $Z_{t+s} = (X_{t+s}^e)^{-1} \gamma b (\partial y / \partial g_{t+s})^e < 0$ . In interpreting the sign of  $Z_{t+s}$ , recall that  $y^e$  is defined so that positive values represent levels of aggregate output above their long run expected level. Thus if an increase in  $g^e$  increases  $y^e$ ,  $(\partial y / \partial g_{t+s})^e > 0$ . Our discussion of (25) relies on this assumption (although the estimating equations allow for either sign). Since  $X_{t+s}^e > 0$  and  $b < 0$ ,  $Z_{t+s} < 0$ . This implies that the coefficient on  $(y_{t+s}^e - y_{t+s-1}^e)$  is negative.<sup>18</sup> Finally, since  $b < 0$ ,  $d < 0$  and  $\gamma > 0$ , the effect of an increase in  $(x_{t+s}^e - x_{t+s-1}^e)$  on the difference between the current and long run values of  $\Delta g$  is negative.<sup>19</sup>

Equation (25) and its counterparts for current taxation and deficit financing become the basis for empirically implementing our model. To interpret each component in equation (25), it is useful to begin by recalling that  $\Delta \hat{g}$  incorporates all politically optimal intertemporal responses to expected changes in

<sup>16</sup> Our linearization of (23) uses only the first order derivatives in the Taylor series expansion.

<sup>17</sup> From (17), using  $E_t \mu_b^2 = 0$ ,  $D(\psi_{t+s+1}) = [\theta, \beta^s / \Gamma^s R^s] [E_t \mu_b^1(y_{t+s}) - \beta E_t \mu_b^1(y_{t+s+1})] > 0$  if and only if the liquidity constraint is expected to be increasingly binding. That is, as  $y_{t+s}$  increases relative to  $y_{t+s+1}$ ,  $E_t \mu_b^1(y_{t+s})$  and  $D(\psi_{t+s+1})$  falls. The resulting positive relationship between  $\Delta y$  and  $D(\psi)$  is represented by  $D \ln(\psi_{t+s}) = \varphi (y_{t+s} - y_{t+s-1})$  with  $\varphi > 0$ .

<sup>18</sup> That is, with  $\varphi > 0$ ,  $(X_{t+s}^e)^{-1} \gamma b < 0$  so that  $[\varphi - (X_{t+s}^e)^{-1} \gamma b] > 0$ .

<sup>19</sup> In the special case of the steady state defined by  $y_{t+s}^e = y_{t+s-1}^e = 0$ ,  $x_{t+s}^e = x_{t+s-1}^e = x^*$ ,  $r_{t+s} = \rho$ , and  $\psi_{t+s} = \psi_{t+s-1}$ ,  $\Delta g_{t+s}^e = \Delta \hat{g}_{t+s} = 0$ .

the  $x$ 's, except those arising because of the existence of liquidity constraints and transitory shocks to aggregate economic activity. It includes consumption smoothing on behalf of taxpayers in response to expected changes in the  $x$ 's that would occur over a longer horizon when the  $y$ 's are expected to be zero. Under electoral pressure, an optimizing government will however moderate its attempt to achieve optimal consumption smoothing on behalf of its non-liquidity constrained taxpayers as it attempts to deal with the economic and political consequences of liquidity constraints. This aspect of government fiscal policy is captured by the last three terms in equation (25), which constitutes the transitory part of spending policy.

The nature of the transitory part of  $\Delta g^e$  in any period can be understood by inquiring into the three reasons why  $\Delta g^e$  and  $\Delta \hat{g}$  may differ. This logic will prove useful in understanding the signs on coefficients in the estimating equations that are imposed by the theory:

(i) The first reason for  $\Delta g^e$  depart from  $\Delta \hat{g}$  is if  $\rho \neq r_{t+s}$ . If the interest rate falls below the rate of time preference, for example, individuals will prefer more consumption earlier rather than later. For any given value of  $y^e$ , the resulting equilibrium increase in spending reduces the contemporaneous need for spending in relation to liquidity problems. This is the intuition behind the negative sign on the coefficient on  $\rho - r_{t+s}$  in (25).

(ii) The second reason, represented by the third term in (25), incorporates reactions to expected transitory shocks. Because an expected increase in transitory income this year diminishes the scale of the liquidity constraint relative to last year, it reduces political support for government spending and results in a smaller change in  $g$  than otherwise.<sup>20</sup> Consequently, the coefficient on  $(y_{t+s}^e - y_{t+s-1}^e)$  is negative.

(iii) The third and final reason for the change in planned spending to deviate from its long run path involves the rate at which government spending adjusts to changes in the underlying permanent variables. This is represented by the last term in (25), the effect of which is to reduce the increase in  $g$  relative to the long run. Because the government responds to the intertemporal wishes of their voters, expected increases in  $x$ 's lead individuals to want to spread additional lifetime consumption smoothly over time. As a result - given  $(\partial y / \partial g_{t+s})^e > 0$  - such actions reduce the magnitude of liquidity problems and moderate the need for public spending in the short run. The coefficient on  $(x_{t+s}^e - x_{t+s-1}^e)$  is therefore negative.

The equation for  $\Delta t_{t+s}^e$  corresponding to (25) can now be presented as

$$\Delta t_{t+s}^e = \Delta \hat{t}_{t+s} + \frac{(\rho - r_{t+s}^e)}{W_{t+s}} + \frac{(\phi - (V_{t+s}^1) \gamma f) (y_{t+s}^e - y_{t+s-1}^e)}{W_{t+s}} - \frac{k (x_{t+s}^e - x_{t+s-1}^e)}{\gamma f (\partial y / \partial t_{t+s})^e} \quad (26)$$

where  $E\mu_t^1 = e + f \cdot y_{t+s}^e < 0$  and  $E\mu_t^e = h + k \cdot x_{t+s}^e < 0$  so that higher taxation reduces utility. With  $f$  and  $k$  both positive, the negative effect of taxation is reduced as  $y$  and  $x$  increase. We assume that  $(\partial y / \partial t_{t+s})^e$  is negative, and that the negative effect of taxation on welfare increases at the margin.<sup>21</sup>

While  $V_{t+s}$  and  $W_{t+s}$  have the same general form as  $X$  and  $Z$ , they must be interpreted in terms of the effect that taxes have on income and utility, i.e.,  $W_{t+s} = (V_{t+s}^1)^{-1} \gamma \cdot f \cdot (\partial y / \partial t_{t+s})^e > 0$  with  $V_{t+s} = \gamma e + h + kx_{t+s}^e < 0$ . This implies that the coefficient on  $(y_{t+s}^e - y_{t+s-1}^e)$  is positive. Moreover, the coefficient on  $(x_{t+s}^e - x_{t+s-1}^e)$  is negative, so that an expected increase in  $x$  leads to an increase in taxation. It follows that the equation for  $\Delta t_{t+s}^e$  is symmetric in form to that of  $\Delta g_{t+s}^e$ , but with the negative effect of taxes on utility

<sup>20</sup> Because the change in government spending itself affects transitory income, a second order effect will moderate the scale of the two primary effects. This is captured in  $Z_{t+s}$  in the third term of (25) and  $(\partial y / \partial g_{t+s})^e$  in the fourth.

<sup>21</sup> When  $\partial (E\mu_t^1) / \partial t = f (\partial y / \partial t_{t+s})^e < 0$ , the decline in welfare following an tax increase increases with the tax burden.

resulting in a reversal of the sign of the individual coefficients.

Equation (26) incorporates tax smoothing in two ways. First, tax smoothing in response to expected changes in the permanent factors is incorporated into  $\Delta \hat{t}$ . Long run planned changes in taxes also incorporate all the optimal responses to expected changes in the  $x$ 's, except for those arising because of liquidity constraints. The remaining three terms involve tax smoothing as part of the government's plan to deal with the changes in liquidity occurring in the adjustment both to the evolution of its long run desired size and in response to the emergence of transitory shocks. The latter shocks first impact voters who are liquidity constrained and only later, through changes in marginal tax rates, on those who are not liquidity constrained.<sup>22</sup> The last term in (26) emerges as the government's optimal response to expected changes in the  $x$ 's. In this case, tax adjustments arising from changes in the permanent factors spill over on liquidity, altering the way that taxes would have responded in the absence of these considerations.

Finally, we note that since  $\Delta b_{t+s}^e = \Delta g_{t+s}^e - \Delta t_{t+s}^e$ , the signs on the resulting variables in the implied equation for the net deficit can be derived in a straightforward manner. In view of the preceding discussion, the signs on the last three terms in the implied equation for the net deficit will be the same as in the public spending equation.

#### 4.1 Estimating equations

The estimating equations for planned fiscal policy that are implied by the preceding model can now be stated. For estimation purposes, the original model is reinterpreted in real per capita terms and the deficit is defined net of interest payments to the private sector.

The system defined by (25), (26) and the implied equation for the net deficit is:

$$\Delta g_{t+s} = \Delta \hat{g}_{t+s} + \gamma_1 + \gamma_2 (r_{t+s}^e) + \gamma_3 (\Delta y_{t+s}^e) + \gamma_4 (\Delta x_{t+s}^e) + \varepsilon_{t+s}^g, \quad (28)$$

$$\Delta t_{t+s} = \Delta \hat{t}_{t+s} + \tau_1 + \tau_2 (r_{t+s}^e) + \tau_3 (\Delta y_{t+s}^e) + \tau_4 (\Delta x_{t+s}^e) + \varepsilon_{t+s}^t \quad (29)$$

$$\Delta(\Delta b_{t+s} - r_{t+s} b_{t+s-1}) = \Delta(\Delta \hat{b}_{t+s} - r_{t+s}^e \hat{b}_{t+s-1}) + \beta_1 + \beta_2 (r_{t+s}^e) + \beta_3 (\Delta y_{t+s}^e) + \beta_4 (\Delta x_{t+s}^e) + \varepsilon_{t+s}^b \quad (30)$$

where  $g$  and  $t$  are real non-interest spending and tax revenue per capita;  $\Delta(\Delta b_{t+s} - r_{t+s} b_{t+s-1})$  is the change in the real per capita net deficit; a 'hat' denotes a permanent or long run value,  $\gamma_2 > 0$ ,  $\gamma_3 < 0$ ,  $\gamma_4 < 0$ ;  $\tau_2 < 0$ ,  $\tau_3 > 0$ ,  $\tau_4 > 0$ ;  $\beta_i = (\gamma_i - \tau_i)$  for all  $i$  as a result of the government budget constraint; and where  $\varepsilon_{t+s}^g = (\Delta g_{t+s} - \Delta g_{t+s}^e)$  is the difference between the actual and the predicted (permanent plus transitory) values, with  $\varepsilon_{t+s}^t$  and  $\varepsilon_{t+s}^b$  being defined analogously.<sup>23</sup> The budget restraint also implies that  $\varepsilon_{t+s}^b = \varepsilon_{t+s}^g - \varepsilon_{t+s}^t$ .

In the system above, the  $\varepsilon$ 's are error terms that reflect mistakes made by governments in forecasting relevant information and which lead to deviations of planned and actual changes in policy instruments.

<sup>22</sup> For example, as  $y$  falls and type 1 voters become more liquidity constrained, an easing of their tax burden allows the government to win votes by permitting type 1 voters greater flexibility in smoothing both consumption and labor supply decisions. As government responds to type 1 voters, the change in the current tax burden impinges on the plans of type 2 voters (previously optimized in  $\Delta \hat{t}$ ) and this begins to lose the government votes.

<sup>23</sup> While no constant term appears in equations (25) and (26), one is added here by separating the first term on the right side of each equation ( $p - r_{t+s}^e$ ) into a part that depends on  $p$ , assumed constant, and a part that depends on  $r_{t+s}^e$ . The expected real interest rate  $r^e$  is estimated by including current and past real interest rates in each equation (see Holtz-Eakin et al 1994). Allowing the constant term to vary freely provides further flexibility in the construction of a proxy for  $r^e$ , as well as for expectations of the  $y$ 's and the  $x$ 's.



Note that the errors may be correlated over time if aggregate shocks are but should be stationary since we are modelling an equilibrium system. The errors will also be uncorrelated with all explanatory variables if the forecasting of the government is rational in the sense that no information available at time  $t+s-1$  could be used to improve forecasts of activity in the next period.

An important assumption that we shall make concerning the estimating equations is that the *impacts* of government policies, Keynesian or otherwise, occur with at least a one period lag. This implies that the data we shall use, including interest rates, changes in economic shocks and changes in permanent income, are predetermined with respect to contemporaneous policy actions.

## 4.2 The meaning of Keynesianism and testing for Keynes.

The preceding model incorporates normal political responses by government to liquidity and non-liquidity constrained voters. The question now arises: what exactly is Keynesianism in this framework, and how would the strength of Keynesian elements in public policy be assessed? Because our competitive political environment means that a government will not undertake a new set of policies unless that innovation had popular support, *we must address the way in which individuals believe that Keynesian policy could be effective.*

Textbook Keynesian in the above framework would arise from a new understanding by voters that a co-ordinated reduction in the impact of liquidity constraints would allow constrained agents to realize more of their "notional" trading plans, thereby opening new market opportunities for both liquidity and non-liquidity constrained individuals, and increasing individual expected welfare. In other words, greater attempts at stabilization could arise through individuals' enhanced appreciation of the potential role of government as an agent that is able to internalize externalities inherent in relaxing individual liquidity constraints through fiscal and other policies. Keynesianism in this sense follows Leijonhufvud's (1968) seminal interpretation of Keynes.<sup>24, 25</sup>

Greater recognition by the community of the desirability of using government in this way increases the expected utility generated by government action and, hence, the expected political benefit of using fiscal policy to depart from the desired equilibrium path. Specifically, with  $E\mu_g^1 = a + b y_t^e$  [with  $a > 0$  and  $b < 0$ ], a rise in the value to the community of using  $g$  in relation to the liquidity constraint will increase the reason for actual government spending to depart from its equilibrium path in response to a given  $\Delta y_{t+s}^e$ . A rise in the value of either  $a$  or  $b$  then increases the absolute value of the coefficient on the third right-

<sup>24</sup> A rise in the expected utility that would be generated by government actions is to be distinguished from an increase in the effectiveness of fiscal intervention on the economy, involving a rise in  $\partial y / \partial g_{t+s}$ . Here a productivity shock increasing the effectiveness of government policy actually reduces government intervention in the model, as can be confirmed from equation (25). While it may seem surprising that greater fiscal effectiveness reduces the scale of government intervention, this result follows naturally from the incentives political parties have to continually maximize expected votes. On the margin, no more resources will be devoted to dealing with liquidity constraints than are necessary. Because the channelling of resources to type 1 voters reduces support from voters who are not liquidity constrained, and because resources used are now more effective, the government finds it can increase overall support by easing back on the extent of its intervention if  $\partial y / \partial g_{t+s}$  increases. In assessing this argument, it should be recalled that what is demanded by voters is welfare, not intervention per se. Thus, by analogy, when the marginal cost of an hour of 'light' falls because 'light bulbs' (fiscal interventions) now 'burn' twice as long, more hours of 'light' are demanded but fewer 'light bulbs' are used, assuming that the demand for 'light' is downward sloping. Textbook Keynesianism then cannot be due to this sort of productivity shock in the present model. Rather it is Leijonhufvud's interpretation of Keynes' revolution that does lead to textbook Keynesianism.

<sup>25</sup> One may note here that an increase in the proportion of liquidity constrained voters has an ambiguous effect on the sensitivity of fiscal structure to transitory shocks.

hand term in equation (25)<sup>26</sup>. An analogous argument holds for  $\Delta t^e$  and for the change in the net deficit. This means that 'after Keynes', both the negative correlation between the  $(\Delta g_t - \Delta \hat{g}_t)$  and  $\Delta y_{t+s}^e$  (where a positive change in  $y$  is a good event) and the positive correlation of  $(\Delta t_t - \Delta \hat{t}_t)$  and  $\Delta y_{t+s}^e$  will be stronger. Thus the implied negative correlation for the deficit will also be larger.

A straightforward test for the presence of Keynesianism can then be implemented as follows. Consider the differential policy process  $D_g$  defined as

$$D_g = [(\Delta g - \Delta \hat{g}) \text{ 'after Keynes' }] - [(\Delta g - \Delta \hat{g}) \text{ in the counterfactual}] \quad (31)$$

where the counterfactual is a prediction of what ex ante transitory policy for the period after 1945 would have been if the data generating process governing pre-Keynesian policy had continued to apply. (This is the *change* in the difference between the counterfactual and the ex ante transitory policy shown in Figure 1.) Similarly, policy differential processes for  $D_t$  for taxation and  $D_{\Delta b}$  for the net deficit can be defined.

If an increase in  $y^e$  is 'good', our model then implies that  $D_g$  will be negatively correlated with  $\Delta y^e$  if Keynesian policy has been attempted 'after Keynes'. Such a counter-cyclical pattern for  $D_g$  would be revealed by the coefficient on  $\Delta y^e$  in a regression explaining  $D_g$ . Exactly the opposite, or a pro-cyclical pattern of  $D_t$  with respect to  $\Delta y^e$  would be observed for taxes. Because  $D_g$  moves counter-cyclically and  $D_t$  pro-cyclically,  $D_{\Delta b}$  will then behave countercyclically like spending. Note that since  $\Delta b - \text{rb}_t = \Delta g - \Delta t$ ,  $D_{\Delta b} = D_g - D_t$  and the three policy differential equations constitute a seemingly unrelated system. We also note for later use that in the event that  $y^e$  is measured using a deviation of unemployment rates from trend, so that a positive deviation is 'bad', then the above signs in the policy differential regressions will be reversed.

By construction, the policy differentials allow for the possibility that normal political responses to liquidity constrained voters leads governments to engage in Keynesian-like policy for motives that have nothing to do with Keynesian ideas. They do not explicitly allow for events like the rise of the welfare state after 1945 that may be unrelated to Keynesian ideas and may have affected the sensitivity of fiscal policies to transitory economic shocks. We allow for this possibility in the empirical work, in a manner discussed below.

#### 4.3 The structural/cyclical or automatic/discretionary decomposition in the present model

Before turning to the details of estimation, we draw out the differences between our permanent/transitory decomposition [in equations (28)-(30)] and the more traditional structural/cyclical or automatic/discretionary approach to the study of stabilization policy.

Schematically, we can represent the traditional approach to the study of, for example, the deficit by the following equation:

$$\Delta(g_t - \Delta t_t) = \Delta(\text{structural deficit}) + \Delta(\text{cyclical deficit}), \quad (32)$$

where the structural part is the change in  $\Delta(g_t - \Delta t_t)$  that would have occurred in the absence of transitory shocks to  $g$  or to  $t$ ; i.e., if  $y = 0$  in each case. In some variants of the approach, the structural deficit is identified with the changes in the values of  $g$  and  $t$  that are consistent with income being at some standard level, such as full employment. Most often the cyclical component is calculated purely as a residual, using the identity that actual year-to-year changes in policy equal the structural element plus the cyclical

<sup>26</sup> The change in the size of the coefficient depends essentially on the change in the absolute value of  $\phi X / \gamma b$ .

element.

For most stabilization studies, the structural part of the change in the deficit is regarded as the crucial aspect of 'discretionary' policy. For example, if the structural deficit increases when activity declines the government is judged to have attempted stabilization over and above the 'automatic' response (due to a rise in welfare or unemployment insurance payments or a decline in income sensitive taxes as activity declines.) It is implicit in this view that the automatic component is independent of attempts at Keynesian stabilization.<sup>27</sup> The value of this approach is its control for factors that might be regarded as unrelated to stabilization<sup>27</sup>. From our perspective, its weaknesses lie in its lack of clarity with respect to the distinction between trend and cycle, and in the implicit attempt to bypass the modeling of political behavior, as shown below.

To develop a basis for a comparison of approaches, rewrite the permanent or planned long run component of spending in our equation (28) as  $\hat{g}_t = \hat{g}(x_t^e, s_t)$ , where we explicitly distinguish between permanent income denoted  $x$ , and all other factors  $s$  affecting the permanent component of fiscal policies. Similarly, let the *ex ante* transitory component of spending be  $g_t^t = g^t(x_t^e, y_t^e, s_t)$  where  $y_t^e$  is the expected deviation of income from its longer run level. Then, equation (28) becomes

$$\Delta g_t = \Delta \hat{g}_t + \Delta g_t^t + \varepsilon_t^g. \quad (28')$$

Note that both  $\Delta \hat{g}_t$  and  $\Delta g_t^t$  can be nonzero for two types of reasons: (i) either because  $x^e$  or  $y^e$  has changed, or (ii) because other elements  $s$  determining  $\hat{g}$  and  $g^t$  have changed. Then to compare with the traditional approach, write the change in the permanent component of spending as  $\Delta \hat{g}_t = \Delta \hat{g}_t(\text{due to } \Delta x) + \Delta \hat{g}_t(\text{due to } \Delta s)$ . Taxes can be written analogously.

With this basis, the structural element in  $\Delta g_t$  in the traditional approach can be represented as  $\Delta g_t^*$ . This is the hypothetical change in  $g_t$  if  $y_t = 0$ . Typically this is estimated by forecasting the current value of  $g$  for a hypothetical (natural) level of unemployment using a regression of  $g_t$  on own lags and lagged values of the unemployment rate. The cyclical or automatic part is then calculated as a residual.

Using (28') and the definition of  $g^*$ ,  $\Delta g_t = \Delta g_t^* + \Delta \hat{g}_t(\text{due to } \Delta s) + \Delta g_t^t + \varepsilon_t^g$ . Analogously for taxes:  $\Delta t_t = \Delta t_t^* + \Delta \hat{t}_t(\text{due to } \Delta s) + \Delta t_t^t + \varepsilon_t^t$ . This implies that our analogue to (32) is:

$$\Delta[g_t - t_t] = [\Delta g_t^* - \Delta t_t^*] + [\Delta \hat{g}_t(\text{due to } \Delta s) - \Delta \hat{t}_t(\text{due to } \Delta s)] + [\Delta g_t^t - \Delta t_t^t] + [\Delta \varepsilon_t^g - \Delta \varepsilon_t^t], \quad (32')$$

where the first term in squared brackets on the right side is the change in the structural deficit, and the second term is the change in the cyclical deficit (as defined in the traditional literature).

Our differences with the traditional approach are now apparent. First, (32') illustrates that the automatic/discretionary approach attributes some elements of the permanent component to the cyclical deficit; these are the terms  $\Delta \hat{g}_t(\text{due to } \Delta s)$  and  $\Delta \hat{t}_t(\text{due to } \Delta s)$ . Unless these longer run elements have been accounted or controlled for properly, one cannot have confidence that the discretionary component or structural elements have been identified. Second, because government behavior is not explicitly addressed, the traditional approach includes all mistakes made by government in the cyclical component.

<sup>27</sup> It may be noted that being 'unrelated' does not by itself imply the absence of discretionary behavior that is correlated with economic activity; it is just unrelated to Keynesian stabilization. This point is usually not made in the automatic/discretionary literature.

A third difference in the traditional approach is not so easily captured. Our equations (28)–(30) allow for intertemporal optimization by political agents and the transition process between positions of long run equilibrium. This is absent from the traditional structure that assumes that  $g^*$  and  $t^*$  would have been picked this period if transitory shocks were absent. In other words, the traditional approach is not designed to allow for a dynamic adjustment process as a result of political optimization, but predicts simply on the basis of comparative statics. As a result the structural element derived will be biased to some unknown extent.

Perhaps the most serious difference is that we do not view the so-called structural part of policy as the appropriate focus for assessing Keynesianism. In our view, what is needed is a behavioral model of the *ex ante* transitory elements represented by  $\Delta g_t^t$  and  $\Delta t_t^t$  in (32'). Those who follow the traditional approach argue that it is first or structural element in (32) that captures the relevant part of the transitory component of planned spending. From our perspective, however, this is a (misspecified) part of the permanent component of *ex ante* spending. And if it is regarded as part of the transitory component, where then is the permanent part of *ex ante* policy?

## 5. Searching for Keynes in Four Steps

### 5.1 A brief look at the data

Before turning to the empirical implementation of the model of planned fiscal policies and the associated test for Keynesianism, it is interesting to look over the history of fiscal policy that we are exploring.

[Figure 2 here].

The annual fiscal data shown in Figure 2 are Gillespie's (1991) revised public accounts data, updated from 1990 and converted to a calendar year basis. In the figure, it appears that fiscal history is more volatile after WWII than before. However, the coefficients of variation given below the figure for the pre-WWI, interwar and post-WWII periods indicate that this is so only for the deficit. Both public expenditure and taxation are about as volatile after 1945 as they were in the pre-WWI period. Whether the relationship of instrument use to transitory economic shocks is the same after 1945 is, of course, another matter.

### 5.2 Estimation

Empirical implementation of our model and the associated test for Keynesianism may be conveniently summarized in four steps:

*Step 1: Estimate the long run or permanent components of  $\Delta g$ ,  $\Delta t$  and  $\Delta(\Delta b - rb_{-1})$*

The fiscal system is estimated using ordinary least squares by incorporating exactly the same set of explanatory variables in each equation, so that the estimates obey the adding up restrictions implied by the government budget restraint. The common variable set is appropriate since, at each point in time, all fiscal choices are the result of optimizing behavior by the same government.

In step 2 below, we will estimate the transitory components of policy together with the long run. However, to identify the *ex ante* transitory components separately, we need to disentangle the long run and so need the coefficients estimated from the long run structural equations. The permanent component is also of interest in its own right, and care is required deciding on the feasible and appropriate set of explanatory variables (the  $x$ ,  $y$  and  $s$  above) for the full - permanent plus transitory components - model.

The variables in the long run system should satisfy two key conditions. First, the explanatory variables should reflect only the permanent characteristics of government size. The long run equations should exclude transitory fiscal responses to the influences of the business cycle (if any) and the smoothing process by which the public economy evolves towards its long run equilibrium. Second, as a practical matter, each of our variables must be available for the entire one hundred and thirty one years of our sample period. The long period is needed for the estimation of the counterfactual. It follows that we are looking for a set of explanatory variables that are cointegrated with each of our three fiscal variables. While this requires the residuals of our estimated long run equations to be stationary, the residuals might be expected to show some evidence of serial correlation as part of the transitory response to the cyclical nature of the business cycle.

Perhaps because of the common pattern of growth throughout our long time period, the variables used to describe fiscal structure and all other variables in our long run analysis are found to be integrated of order one.<sup>28</sup> Hence both the time series characteristics of our data and the important fact that our theory is set out in terms of changes suggest that it is appropriate to estimate the long run equilibrium relationships in first difference form. This fits nicely with our model of the transitory component, which is also specified in first difference form. A consequence of such differencing is that we should expect to find  $R^2$  values that are somewhat lower than if the equations were estimated in levels.

Our estimates of the long run change in the real value of non-interest federal government spending per capita, real tax revenue per capita and real net deficit per capita are presented in Table 1. Accompanying estimates based on using two separate time periods for the regressions, before and after WWII, are given in Appendix Table 1. The explanatory variables in the tables reflect a compromise between the major hypotheses used to explain the growth and financing of government and data availability.

[Table 1 here]

The most prominent hypothesis associated with permanent government size is Wagner's Law, the hypothesis that as society develops it becomes increasingly complex, requiring government to play an increasing role in economic activity. As such the 'law' is usually associated with the proposition that the income elasticity of government spending is greater than one. Following Mueller (2003,509), we incorporate Wagner's Law by including both the forecasted change in real income per capita  $D(RYPCf)$  and in urbanization as determinants of each fiscal variable. (The equation used to forecast real income is presented in the Appendix and, like our model of long run fiscal structure in Table 1, it is estimated over the entire sample. Our estimates of the model when  $RYPCf$  is estimated over two periods, before and after world war two (WWII), are also discussed as we proceed.

Because urbanization is unavailable for our full time period we utilized a mirror image of it - the change in the percentage of the population in agriculture  $D(AGRIC)$ . We expect a positive coefficient on  $D(RYPCf)$  and a negative one on  $D(AGRIC)$  in the spending and deficit equations, and the opposite signs in the taxation equation. In both the government spending and tax equations the coefficients have their expected sign, but only in the tax equation is  $D(RYPCf)$  significantly different from zero, while  $D(AGRIC)$  is significant in the deficit equation and just misses significance at 10% in the expenditure equation.<sup>29</sup> When the model is estimated over two separate time periods,  $D(AGRIC)$  is significant for

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<sup>28</sup> As judged by Augmented Dickey-Fuller statistics. Interestingly, the fiscal structure can also be described as stationary in levels with a shift in the constant and the slope. Following Perron (1997, 358, model 2) in using the minimization of the t-statistic for testing of the null that the coefficient on the lagged dependent variable is equal to one, the apparent shift occurred sometime during WWII. For this reason we also estimated our model in two pieces, before and after WWII, as discussed further below..

<sup>29</sup>  $D(AGRIC)$  is significant for spending as well as for the deficit in the model estimated over two separate periods.

spending as well as for the deficit. See Appendix Table 1. In contrast, Mueller (2003, chap. 21) reports limited success in the literature as a whole with such a variable.

The change in population  $D(POP)$  is most often introduced into the long run fiscal equations as a test of the 'publicness' of government services. Given a fixed setup cost for government programs, 'publicness' in government services suggests that a larger population can be provided for at less than a proportionate expansion in expenditure, suggesting that the spending coefficient should be positive with an elasticity less than one. On the tax side, however, publicness means that a larger population lowers the marginal tax price of both current and future taxation and induces larger expenditure. In Table 1 we see that  $D(POP)$  has a positive and significant coefficient with an implied elasticity at the mean of about 3. This is similar to the effect on the deficit, with the effect on taxation being insignificant.

The change in the immigration rate  $D(IMRATIO)$  is another structural characteristic that may shape long run government size and financing in a country like Canada where immigration has been substantial, especially before the first world war (WWI). In our equations there is some tendency for government spending per capita to fall and current taxation to rise with higher rates of immigration. While these coefficients are negative for spending and the deficit and positive for taxation, they are insignificantly different from zero in Table 1.

Many studies of the long run size of government find that the percentage of the population over sixty-five has had a significant positive influence on government size. In addition, the age structure may influence the choice between debt and current taxation (see, for example, Cukierman and Meltzer 1989). In our analysis we can find a consistent time series only for its inverse, the percentage of the population younger than 18 years  $D(\%YOUNG)$ . Interpreting Figure 1 in this way, our results mirror earlier findings in that the recent aging of the population has resulted in more government expenditure per capita with essentially no change in per capita tax collections. Hence the aging of the economy is associated with the buildup of government debt, as also indicated in the results. The two period estimates in the Appendix reinforce the impression that these effects are likely the result of events following 1945.

A variable that is available over the long periods and has come into greater prominence in studies of government size is the degree to which an economy is exposed to foreign shocks (again see Mueller 2003). Defining the sum of exports and imports over GDP as openness  $D(OPEN)$ , greater openness is believed to expose the domestic economy to more shocks and result in a larger size of government as a form of insurance. From this basis Rodrik (1998) and others find a significant and positive relationship between the measure of fiscal size and openness. Our analysis finds no consistent sign nor significance to openness in any of our equations for the one period results. Somewhat intriguingly, in Appendix Table 1 a sign reversal is found across the two periods, with greater openness significantly increasing government size before WWII, but having a negative (though insignificant) effect in the later period.

Finally the equations include three dummy variables to control for the permanent influence of three important external shocks in our time period: world war I (WWI), world war II (WWII) and the period following the oil shock ( $POSTOIL$ , = 1 from 1974 on). We also include dummies  $WWI$ -aftermath (= 1 for 1919-21) and  $WWII$ -aftermath (=1 for 1946-49) to allow for a reduction in the *level* of the fiscal system following the wars. (Recall that the dependent variables are in first difference form). All of these variables have their expected signs. Wars increase the growth rate of expenditures and borrowing, and the immediate aftermath of war sees reductions in growth rates substantial enough that levels actually decline, at least after WWI.<sup>30</sup>

**See Appendix Table 1. In contrast, Mueller (2003, chap. 21) reports limited success in the literature as a whole with such a variable.**

<sup>30</sup> The estimates indicate that for WWI, the post-war downward adjustment in the level was insufficient to offset the

Overall, the long run equations in Table 1 (and in Appendix Table 1) explain a reasonable amount of the annual variation of the change in real per capita government spending, taxation and the net deficit and do so in a manner that is generally consistent with previous empirical work. As expected, the residuals do show some evidence of serial correlation while giving evidence of stationarity.

*Step 2: Estimate the transitory components of fiscal structure by estimating the full model (28)-(30) as a system over two periods, one 'before Keynes', and one 'after Keynes'.*

Here we seek estimates of the ex ante transitory components of fiscal structure in the competitive political equilibrium. The model must be estimated over two periods, one 'before' and one 'after' Keynes', so that the model of the first period can be used to construct a counterfactual. We use the period before 1939 as the period 'before Keynes' and the period from 1947 on as the period 'after Keynes'.

One procedure would be to use the residuals from the long run equations of Table 1 as our measure of transitory policy and estimate directly the equations for  $\Delta g_{t+s} - \Delta \hat{g}_{t+s}$ ,  $\Delta t_{t+s} - \Delta \hat{t}_{t+s}$ , and  $\Delta(\Delta b_{t+s} - r_{t+s} b_{t+s-1}) - \Delta(\Delta \hat{b}_{t+s} - r_{t+s} \hat{b}_{t+s-1})$  from (28)-(30) on this basis.<sup>31</sup> A better approach, in our view, is to estimate both long run and short run components together, allowing the two parts to interact. This should produce better estimates of the total effect on policy arising from each variable and hence better equation statistics. We then extract the transitory component from the more comprehensive model to construct the policy differential equations. A comparison of the role of the factors underlying the long run in Table 1 and the full model is a useful check on results. The sign restrictions on the coefficients on interest rates, on the changes in transitory and permanent shocks in (28)-(30) derived above provide a further way of assessing the estimation results.

Table 2 presents the OLS estimates of the full model for 1874-1938 and 1950-2000, adjusted for the nature of time dated dummy variables. It is important to note that a comparison of Table 1 and Table 2 indicates that we have included only the contemporaneous change in permanent income  $D(RYPCf)$  as a variable driving *both* components of the fiscal system. Other variables identified as part of the transitory component are interest rates, denoted  $r^e$  in the table, changes in transitory shocks  $\Delta y^e$  and changes in permanent income shocks  $\Delta x^e$ . The other variables in the long run model in Table 1 are regarded as affecting the permanent components of the system only. In effect this defines what part of the total is the short run and what part is the long run.

The coefficients on  $D(RYPCf)$  shown in Table 2 are those that apply only to the transitory part only. It is equal to the coefficient on this variable from the full model minus the coefficient in the long run model of Table 1, with the t-statistics shown appropriately recomputed.

Unconstrained distributed lags are used to model expectations with respect to interest rates, changes in transitory income, and changes in permanent income. We generally used the longest lag possible in each case, an approach that was tempered by degrees of freedom considerations and the need to include exactly the same set of variables in each OLS estimating equation (so that the budget restraint is enforced). Because of the use of lags to help capture the transitory components, the time periods are not exactly the same as those used in the long run model.

[Table 2 here]

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wartime increase, while for WWII, the post-war adjustment was more or less equal to the wartime increase. This suggests the presence of a Peacock-Wiseman (1961) displacement effect for WWI alone. For further discussion of this effect after WWI, see Dudley and Witt (2002).

<sup>31</sup> We used such a procedure in an earlier version of this paper.

Note that the F test reported in the last row of the table indicates that the equations for each fiscal variable do differ across time, at least at the 10% level. As expected,  $R^2$ 's and Durbin-Watson statistics are generally better in the full model in Table 2 (and in Appendix Table 2) than for the long run alone as in Table 1 (or Appendix Table 1).<sup>32</sup>

In discussing particular transitory variable results, consider first the coefficient estimates on the variables that also appear in the permanent component (shown in the lower half of the table). The coefficient signs are generally found to be the same as those in the long run models (in either Table 1 or Appendix Table 1). Note that as mentioned in relation to Appendix Table 1, the coefficient on  $D(\text{OPEN})$  is generally found to be positive in the first period and negative in the second period. Finally, the coefficient signs for these variables across Table 2 are more likely to be significant than in the long run model.

Second, the restrictions imposed by the model on the sum of the lags are supported generally, considering coefficients significant at 10% or better. The expected sign pattern of the sums of coefficients, based on equations (28)–(30), is indicated in Table 2 beside the explanatory variables representing expected interest rates and changes in expected transitory and expected permanent income. The expected pattern of the signs of significant coefficients is confirmed for spending in both periods (for the interest rate in the second period, but with all coefficients insignificant.) For taxation, the results are not as good: the sum of significant coefficients on interest rates has the wrong sign in the pre-Keynes period, and are insignificant in the period after WWII. Only the coefficient of transitory income is significant and positive as predicted and only for the first period. The sum of significant coefficients on permanent income for the deficit has the correct sign in both periods. Here the sum on interest rates is positive in the first period as predicted, and all coefficients on interest rates are insignificant in the second period.

It is interesting to note that the coefficients on  $\Delta y^e$  for the deficit indicate clearly that deficits became more sensitive to transitory shocks in the second period: the sum of significant coefficients on changes in transitory income switches from positive to negative (the predicted sign), and is about 15 times larger in absolute value in the second period. But while the deficit is more sensitive to transitory shocks 'after Keynes' than before WWII, it is not correct to conclude on this basis alone that Keynesianism is present in the data.

*Step 3: Use the transitory part of the 'before Keynes' (1874-1938) estimates of (28)–(30) to forecast into the period 'after Keynes' (1951-2000), so as to generate a counterfactual explanation of fiscal structure for the period 'after Keynes'. Compute the policy differentials,  $D_y$ ,  $D_b$ , and  $D_{\Delta b}$ , which compare the counterfactual with the best "in period" model of transitory fiscal plans.*

We next forecast the transitory component found for the earlier 1874-1938 time period into the period 'after Keynes'. These counterfactuals use the Table 2 (or Appendix Table 2) estimated coefficients for  $r^e$ ,  $\Delta y^e$  and  $\Delta x^e$  and constant terms for each instrument, adjusted to remove the effect of the current value of  $D(\text{RYPCf})$  on long run values.<sup>33</sup> We then use the second set of estimates in Table 2 for the period after WWII to estimate the transitory component for each instrument 'after Keynes'. The policy differentials can then be computed by subtraction. These results are the first differences of the level form of the differentials identified in Figure 1, and are graphed in Figure 3 for visual inspection. Here the similarity between the estimated policy differentials for the deficit and for spending is striking, as is the absence of

<sup>32</sup> The taxation equation for 1950-2000 performs with mixed results in this respect.

<sup>33</sup> We adjust the constant term in analogous fashion, though this is not essential. The difference in the constant for the transitory and permanent components will be absorbed into the constant of the later policy differential equations.



any obvious trend in these data.<sup>34</sup>

[Figure 3 here]

*Step 4: Regress  $D_g$ ,  $D_n$ , and  $D\Delta b$  on transitory shocks to which the government may have responded in formulating stabilization policies, making due allowance for the possibly unrelated (to Keynesianism) rise of the welfare state after 1945.*

To test for the presence of a Keynesian element in fiscal policies, we regress the policy differentials on the change in both a transitory shock and a variable that allows for the rise of the welfare state. Unconstrained lags in these variables are incorporated to allow for decision and bureaucratic delays.

Given the importance attached to unemployment in public debate, we use the deviation of unemployment from its long run trend to measure the shock to which stabilization policy (if it exists) responds. More specifically, our measure of the change in that shock,  $\Delta y_u^e$ , is the *change* in the difference between the current unemployment rate and the lagged value of a four year moving average of unemployment rates.<sup>35</sup> In other words,  $\Delta y_u^e$  = the change in the unemployment rate less the change in its lagged 4 year moving average.

Recall that the policy differentials are defined in terms of year over year changes in policy relative to the counterfactual, so here we are explaining changes in policy with changes in the policy makers' environment. Note also that a positive change in  $\Delta y_u^e$  is 'bad', in contrast to the meaning of the shock  $\Delta y^e$  used in Tables 1 and 2. Hence a positive sign on the sum of lags of  $\Delta y_u$  in the policy differential equation for spending, a negative sign for taxation, and/or a positive sign in the deficit equation means that *planned* transitory policy 'after Keynes' exhibits greater textbook Keynesian response to expected shocks than 'before Keynes', and thereby provides statistical evidence of explicit attempts at Keynesian stabilization.<sup>36</sup>

To allow for the maturation of the welfare state over the post-war period and its effects on fluctuations in fiscal instruments following transitory economic shocks, we employ an additional variable in the regressions.  $D(\text{autoT})$  = (the lagged value of the four year moving average of the ratio of personal income tax revenue to total tax revenue) **times** (the contemporaneous change in real income per capita). The idea here is that the sensitivity of the fiscal system to economic shocks gradually increased after the war along with the role of the personal income tax in the system. What matters in this respect is the product of the growing importance of the income tax and the total change in income. This variable is highly correlated with an analogous variable defined as the ratio of personal transfers to total government spending times the unemployment rate, so that it can be used to reflect the sensitivity of fiscal policy to economic fluctuations on both sides of the budget.<sup>37</sup> Note that we do not use the shock to unemployment  $\Delta y_u^e$  in allowing for the role of the welfare state in the present context, as the income tax and transfer payments respond to any change in economic conditions, and not just to expected deviations from some counterfactual level.

<sup>34</sup> In fact, the policy differentials are clearly stationary using the Dickey-Fuller test at 10% (with constant, no trend).

<sup>35</sup> The moving average includes the current value and three lags, equally weighted.

<sup>36</sup> The unemployment rate cannot be used at prior stages since it is available only from 1919, while estimation begins with data for 1870.

<sup>37</sup> Over 1950-2000,  $D(\text{autoT})$  and its analogue for transfers to persons -  $D(\text{autoG})$  = (the lagged value of the four year moving average of the ratio of transfers to persons to non-interest spending) **times** (the contemporaneous change in the unemployment rate) - had a correlation of -0.71. So  $D(\text{autoT})$  and its lag can serve as a reasonable proxy for the effects of the welfare state on both sides of the budget. Moreover, the correlation of  $D(\text{autoG})$  with  $\Delta y_{ut}^e$  is 0.88, while the correlation of  $D(\text{autoT})$  with  $\Delta y_{ut}^e$  is -0.61. Hence to reduce the problem of collinearity in identifying the Keynesian element, we used just  $D(\text{autoT})$  together with  $\Delta y_u^e$  in the policy differential regressions.

In these regressions,  $D(\text{autoT})$  is *not* a measure of the automatic or cyclical part of fiscal policy (using the traditional terminology.) No doubt there were many discretionary decisions that lay behind the evolution of the welfare state since 1945 and some of these may well have been correlated with current economic conditions. *What matters is whether or not these decisions were unrelated to attempts at Keynesian stabilization.* However even this is not simple to judge; one can argue that Keynesian thinking lead to, or made easier to adopt, the economically sensitive welfare institutions after 1945. For these reasons, then, we estimate the policy differential equations with and without  $D(\text{autoT})$ . Comparison of the shorter equations with the full policy regressions illustrates the effect on the results of allowing for the (assumed) unrelated rise of the welfare state.

Finally, we also estimate the equations using  $D(\text{autoT})$  above with the estimated ex ante transitory components of fiscal structure 'after Keynes' from Table 2 as the dependent variable. (Hence this third set of equations does not use the 'before Keynes' counterfactual.) Comparison with the other equations indicates how the use of the counterfactual affects our conclusions regarding the presence of Keynesian stabilization.

Finally, to allow for lags in decision making, we use two lags on  $\Delta y_u$  and one on  $D(\text{autoT})$ . Further lags lead to degrees of freedom problems. To preserve the adding up of the policy differentials due to the government budget restraint, all variables are entered into each equation. With this background, the results of the OLS estimation of these three sets of policy differential equations are given in Table 3.

[Table 3 here]

The quantitative importance of our results is considered in a later section. Here we base our conclusions on a consideration of the sign on the sums of the significant coefficients. To aid in assessing the results, we note here that for the spending regression, Keynesianism is judged to be present if the sum of the significant coefficients on  $\Delta y_u^e$  and its lags is positive. A positive sum indicates that spending rises more (relative to the counterfactual) the larger is the increase in the rate of unemployment above its moving average. Sums of lags of significant coefficients on  $D(\text{autoT})$  that are negative indicate that as a result of the welfare state, the induced rise in spending is larger the deeper is the fall in real per capita income. In these respects, signs for the taxation regression are opposite to those for spending, and signs for the net deficit are the same as spending.

In panel 1 of Table 3, where the complete policy differential regressions control for the unrelated role of the welfare state, we see that while there is likely little Keynesianism in spending, there appears to be some attempt at Keynesian stabilization via taxation and the deficit. In making this judgement, we take into account that the coefficient on the current value of  $\Delta y_u^e$  in the spending equation is close to being significant at 10%.

Panel two of the table presents the policy differential regressions when the welfare state is viewed as part of the Keynesian revolution and  $D(\text{autoT})$  is therefore absent from the regressions. Here we see that stabilization via deficit financed tax cuts is more strongly evident in the data (judging again by the sum of significant coefficients). However, when the role of the welfare state is not separated out using  $D(\text{autoT})$ , ex ante spending after Keynes is more pro-cyclical (or less counter-cyclical) than in the counterfactual since the sum of significant coefficients is negative.

Panel 3 of Table 3 presents results where the counterfactual is ignored altogether. In other words these regressions test for the correlation of the ex ante transitory components 'after Keynes' with economic shocks. Judging by significant coefficients, these results look traditionally Keynesian; this is, spending in response to shocks is countercyclical, and more strongly so than in panel 1. The effect of the welfare state

(represented by  $D(\text{autoT})$ ) is to increase spending when income falls as expected (i.e., the coefficient on  $D(\text{autoT})-1$  is negative and significant). Taxation is also countercyclical, though less clearly so than in panel 1, while the effect of the welfare state is to reduce taxes when income falls. Finally, the net deficit in panel 3 is also clearly countercyclical, with the coefficient on  $D(\text{autoT})$  negative as expected. Note that these results indicate that  $D(\text{autoT})$  appears to be doing what it is supposed to in capturing the effect of the welfare state on the cyclical sensitivity of the transitory components, even though this may not always appear to be so on the basis of the results in panel 1.

Looking across the panels of Table 3, the presence of the counterfactual has made the most difference when assessing Keynesianism in spending. The presence of the counterfactual in the policy differentials washes out what in effect appears to be evidence of Keynesianism if only the post WWII period is looked at in isolation.

### 5.3 Sensitivity

Table 3a provides estimates analogous to those in Table 3, but where all underlying relevant relationships have been estimated in two parts, essentially before and after WWII. In particular, the equation used to construct a forecast of long run income is estimated for 1870-1938 and 1947-2000, and the long run model of fiscal structure is estimated for 1872-1938 and 1948-2000. Table 2 is then re-estimated (see Appendix Table 2) and the succeeding steps recomputed.

The major difference between Tables 3 and 3a is in the results for non-interest spending, in the first row of the panels. Here we see in panel 1, in contrast to the previous results, that non-interest spending also exhibits a significant Keynesian countercyclical element (the sum of significant coefficients is positive). Here the role of the welfare state is also apparent, with the expected sign (the sum of significant coefficients on  $D(\text{AutoT})$  is negative) in contrast to the results in Table 3.

The results for taxation and the deficit are similar to those in Table 3. In relative terms, however, the Keynesian countercyclical element in taxation appears stronger while the net deficit seems somewhat weaker. Moreover, for the deficit as well as taxation,  $D(\text{autoT})$  always has its expected sign, in contrast to panel 1 of Table 3 where the wrong sign appears on  $D(\text{autoT})$  in the deficit regression.

All of the signs of coefficients in panel 2 of Table 3a are as expected. In particular, the sum of significant coefficients for spending is positive instead of negative as in Table 3, indicating that the larger is the increase in unemployment rate above its moving average, the larger is the increase in spending above the counterfactual increase 'after Keynes'. The countercyclical behavior of taxation and the net deficit is also stronger in panel 2 of Table 3a than in panel 2 of Table 3. In Table 3a, the sum of significant coefficients on  $\Delta y_u^e$  and its lags are of the expected sign and generally larger in magnitude than in the corresponding regression in Table 3.

On balance, when considering Tables 3 and 3a as a whole, there does appear to be convincing evidence of Keynesian stabilization present. Panel 1 in both Tables shows countercyclical fiscal action over and above the action that would have been expected in the absence of Keynes and even after controlling for the effects of the welfare state on the sensitivity of fiscal policy to economic shocks. This Keynesian element appears most clearly as deficit financed tax changes.

Assuming that the welfare state is part of the Keynesian enterprise leads generally to larger estimated countercyclical responses, as one would expect. It also appears that when the signs of the coefficients within each panel are compared across tables, the results in Table 3a, based on the estimation of all relationships over two separate periods, give better results. This assessment is reinforced by the last set of results concerning quantitative magnitudes to which we now turn.

#### 5.4 Significance versus importance

The statistical significance of an effect is one thing and its economic significance is another. It is possible that Keynesian stabilization was implemented but in magnitudes that were, for all practical purposes, irrelevant. In our results, however, the magnitudes involved are substantive. To see this, consider the effect on fiscal policy of an increase in the shock to unemployment by one standard deviation of  $y_u^e$  ( $= 1.5$  percentage points) over the 1952-2000 estimation period used in Tables 3 and 3a. The sample's period statistics are given in Tables 4 and 4a along with the calculated magnitude of the effect produced on policy. A one standard deviation increase in the shock to unemployment reduces taxation by between 21 and 27 dollars per capita relative to the counterfactual. This is the change in the Keynesian element of taxation. The deficit increases by even more, between 60 and 86 dollars per capita, while spending rises by less. A good guess for spending is between -4 dollars and 17 dollars, although this requires us to take into account a coefficient that is close to but not significant at 10%.

Overall, it appears that deficit financed tax cuts have been the primary element in Keynesian stabilization. To get a feel for the scale of the magnitude involved, we can compare the policy changes in Table 4 with a similar standard deviation change in transitory income. On this basis an increase in the deficit of 86 dollars corresponds to about 20% of a standard deviation shock to real per capita income.

Not surprisingly, the measure of the responsiveness of fiscal structure that includes the maturation of the welfare state is even more substantial. For example, if the estimates in panel 2 of Table 3a are used (where the role of the welfare state is not explicitly controlled for), the deficit response to the same one standard deviation shock rises to \$125 per capita over and above what would have happened in the counterfactual. This is about 29% of one standard deviation in transitory income. The value, -\$60, found for the change in spending based on using the estimates in panel 2 of Table 3, is implausible. Thus when one considers consistency across panels 1 and 2 in Table 4, the greater consistency of the results based on Table 3a reinforces our previous call for greater weight to be given to these than those based on Table 3. This in turn suggests that there is also some Keynesianism in spending, though less than that in taxation and the deficit.

#### 6. Conclusions and Directions for Further Work

While the General Theory has undoubtedly exerted a profound influence on how economists, politicians, voters and policy makers think about intervention, whether policy has actually changed in response to Keynesian ideas is another matter. In this paper, we consider what elements are necessary for any 'search for Keynes'. The task requires the construction of a counterfactual that illustrates what planned policy would have been 'after Keynes', had Keynesian stabilization not been attempted, and that separates *ex ante* policy choices into permanent and transitory components.

Several steps are required. These include the modelling of the long run evolution of fiscal structure, a dynamic voting model that can be used to place restrictions on estimating equations for the *ex ante* or planned transitory parts of the fiscal system, the construction of a counterfactual world that allows us to compare estimates of planned policy 'after Keynes' to what would have happened (after Keynes) in the absence of Keynesian stabilization and, finally, allowance for the effect of the welfare state in the policy differential equations. On most of these steps little work has been done, either in Canada or elsewhere. Together they take one on a fascinating tour through the methodology and substance of fiscal history.

The separation of the public finances into longer run and transitory components constitutes a second general theme of the paper. This is another area in which there has been too little work. We have taken the

view that it is better to model the longer run directly rather than to resort to one of the usual smoothing procedures. To do so we derive an explicit model of the transitory part of fiscal actions by intertemporally extending the familiar probabilistic voting framework.

Our results indicate the presence of significant and substantial Keynesian elements in the transitory components of fiscal structure (relative to the counterfactual), even when it is assumed that the welfare state matured for reasons that were completely unrelated to Keynesianism. Whether the attempts at Keynesian stabilization we have identified have actually influenced economic activity is another matter that we have not attempted to deal with here.

These conclusions are generally different from those of previous studies, which on balance suggest that if Keynesian stabilization was explicitly attempted in Canada, it was either badly timed, small in magnitude, or both. In considering our evidence, one should note that earlier studies do not formally model policy actions as equilibrium outcomes of a political process and so cannot distinguish between *ex ante* and *ex post* policy actions. Moreover, they do not deal with the problem of distinguishing between permanent and transitory elements in the time series evolution of policy actions and therefore do not control for (possibly volatile) intertemporal adjustments of the longer run path of public policy.

In addition to further work on the permanent components of public policy, issues that remain to be addressed in future research and which constitute separate studies in their own right include: the disaggregation of policy instruments by type of expenditure and tax base; the investigation of Keynesianism in monetary policy; a reconsideration of the role of electoral cycles and political partisanship in the context of our explicitly dynamic model, and the investigation of asymmetries in the response to positive and negative shocks, in order to see if the extent of fiscal conservatism was less pronounced in the 20th century than in the 19th as Buchanan and Wagner (1977) claim.<sup>38</sup> In carrying out any of these projects, it will be necessary to continue to pay attention to the modelling of both permanent and transitory components of policy actions.

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<sup>38</sup> Our results concerning the short run effects of transitory shocks, in Table 2 (and Appendix Table 2) are consistent with this view.

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**Figure 1**

**Planned Fiscal Policy 'Before' and 'After' Keynes**

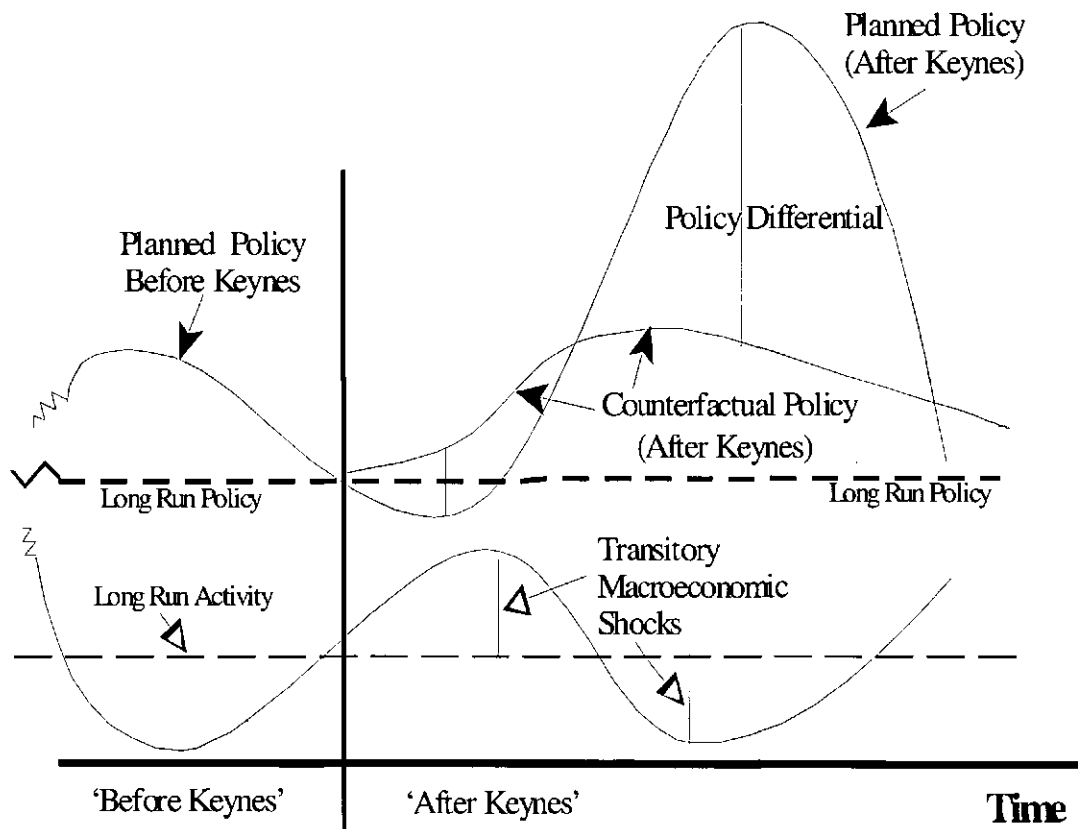
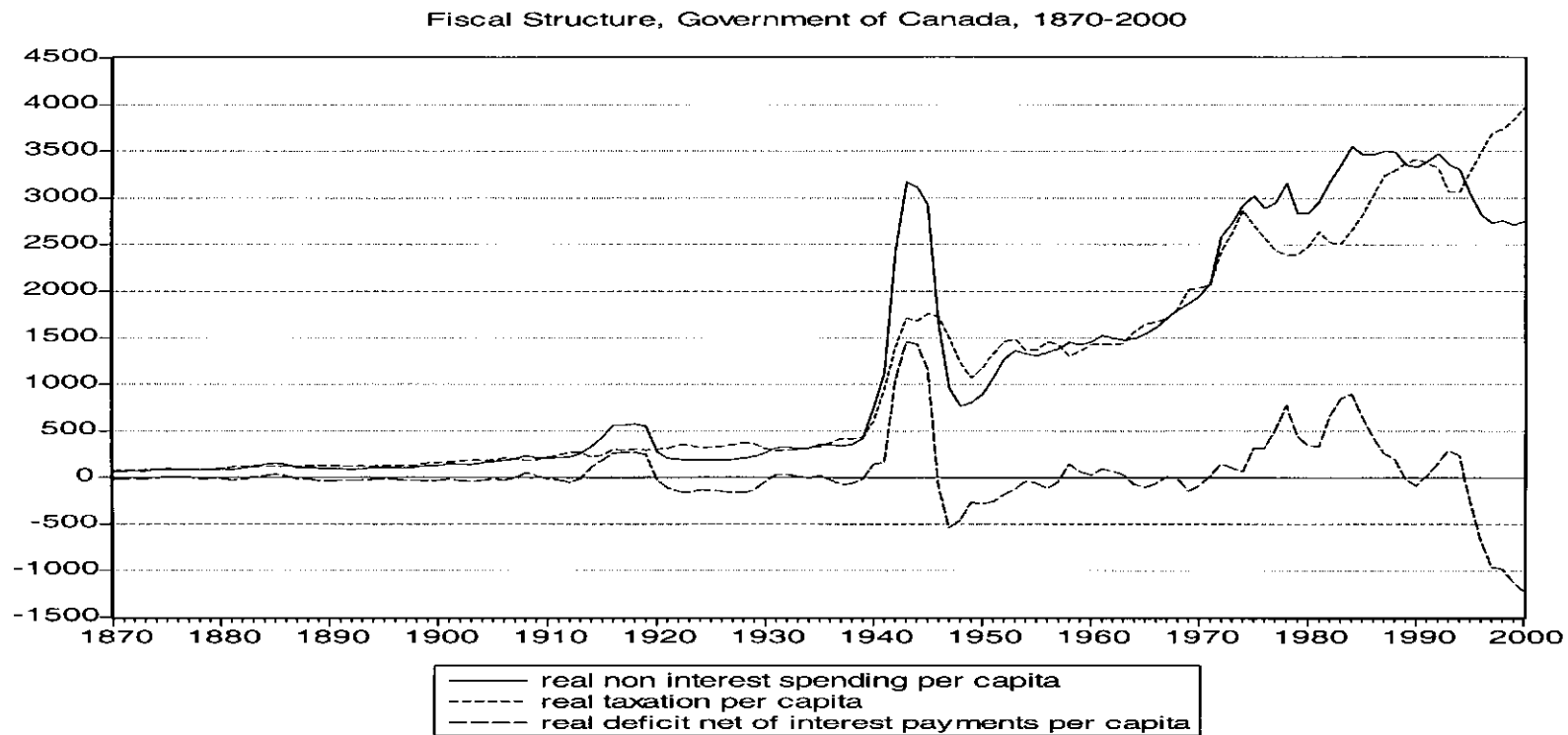


Figure 2



Fiscal History, Government of Canada, 1870 - 2000\*

Variable	Period		1871-1913		1920-1938		1947-2000	
			mean	coef of var	mean	coef of var	mean	coef of var
growth of real income per capita ( $\dot{x}/x$ )			0.022	2.35	0.006	13.97	0.023	1.04
growth of real non-interest public spending per capita ( $\dot{g}/g$ )			0.034	2.71	-0.023	-8.27	0.01	10.57
real non-interest spending per capita ( $g$ )			130.61	0.38	263.66	.26	2320.90	0.39
real tax revenue per capita ( $t$ )			142.27	0.35	339.06	0.11	2315.20	0.37
real deficit net of interest payments per capita ( $\Delta b - rb_1$ )			-11.66	-1.75	-75.40	-0.97	5.74	77.20

\* Notes to Table: coef of var = coefficient of variation. Figures may not add due to rounding. Deficits are defined to be net of interest payments to the private sector and net of return on investments in the private sector.

**Table 1**

**The Long Run or Permanent Fiscal System: Government of Canada, 1871- 2000**  
(OLS estimation. Newey-West corrected t-statistics in brackets)

<b>Independent variables</b>	<b>Change in Real Public Non- interest Spending per capita</b>	<b>Change in Real Taxation per capita</b>	<b>Change in Real Net Deficit per capita</b>
<b>D(RYPCf)</b>	0.06 (0.86)	0.12* (3.46)	-0.06 (-0.75)
<b>D(AGRIC)</b>	-8370.3 (-1.65)	-1667.7 (-0.741)	-6702.6** (-1.84)
<b>D(IMRATIO)</b>	-2333.5 (-1.37)	439.75 (0.51)	-2773.2 (-1.35)
<b>D(% YOUNG)</b>	-128.02* (-3.16)	-26.47 (-0.66)	-101.55** (-1.80)
<b>D(POP)</b>	0.31* (3.55)	0.02 (0.24)	0.29* (2.48)
<b>D(OPEN)</b>	-66.57 (-0.12)	-175.75 (-0.55)	109.18 (0.20)
<b>WWI</b>	111.64* (3.89)	26.05 (1.33)	85.59* (2.79)
<b>WWI-aftermath (1919-21)</b>	-76.79 (-1.64)	24.94 (1.50)	-101.73* (-2.50)
<b>WWII</b>	277.29* (3.81)	126.18 (4.61)	151.11* (2.43)
<b>WWII-aftermath (1946-49)</b>	-584.02* (-3.38)	-181.86 (-4.17)	-402.16* (-1.98)
<b>POSTOIL (1974 on)</b>	-76.24 (-1.82)	-6.31 (-0.47)	-69.94 (-1.28)
<b>Constant</b>	-97.51* (-2.90)	-8.17 (-0.45)	-89.34* (-2.66)
<b>Obs.</b>	130	130	130
<b>R<sup>2</sup></b>	.495	.452	.275
<b>DW</b>	1.54	1.37	1.41
<b>ADF (at 10%)</b>	-3.69*	-4.19*	-3.55*

**Notes to Table 1:** \* (\*\*) = Statistically significant at 5% (10%).

Newey-West corrected t-statistics in brackets.

D = first difference of indicated variable. R<sup>2</sup> is unadjusted. DW = Durbin-Watson statistic.

ADF (at 10%)=Augmented Dickey-Fuller statistic for equation residuals (at 10%),  
with a constant and no trend (using the default procedure in *Shazam* version 9).

Variable definitions: see Appendix. All estimation done using *Shazam* version 9.

**Appendix Table 1**  
**The Long Run or 'Permanent' Fiscal System In Two Separate Periods:**  
**Government of Canada, 1872-1938 and 1948-2000**  
(OLS estimation. Newey-West corrected t-statistics in brackets)

	1872 - 1938	1948-2000	1872 - 1938	1948-2000	1872 - 1938	1948-2000
Independent variables	Change in Real Public Non-interest Spending per capita	Change in Real Public Non-interest Spending per capita	Change in Real Taxation per capita	Change in Real Taxation per capita	Change in Real Net Deficit per capita	Change in Real Net Deficit per capita
D(RYPCF)	-0.11* (-2.54)	-0.01 (-0.18)	0.04* (4.52)	0.17* (3.23)	-0.15* (-3.38)	-0.18* (-2.43)
D(AGRIC)	-2612.4* (-2.26)	-8740.5** (-1.72)	473.1 (1.90)	3241.7 (0.76)	-3085.5* (-2.48)	-11982* (-2.03)
D(IMRATIO)	328.5 (0.72)	-5371.2 (-1.58)	634.9* (2.01)	10117. (1.84)	-306.4 (-0.49)	-15488* (-2.55)
D(% YOUNG)	18.6 (0.87)	-152.8* (-3.72)	-25.0 (-1.16)	1.91 (0.04)	43.65 (1.56)	-154.7* (-2.67)
D(POP)	0.06 (1.61)	0.04 (0.29)	-0.01 (-0.37)	-0.18 (-1.34)	0.06 (1.55)	0.22 (1.04)
D(OPEN)	496.7* (2.35)	-658.4 (-0.71)	143.9* (2.01)	-474.7 (-0.47)	352.7 (1.55)	-183.8 (-0.15)
WWI	40.42* (2.39)		15.2 (1.42)		25.23 (1.26)	
WWI-aftermath (1919-21)	-154.8* (-6.63)		24.92* (2.55)		-179.7* (-8.29)	
WWII-aftermath (1946-49)		-124.6* (-3.03)		-249.3* (-4.73)		124.7* (2.68)
POSTOIL (1974 onward)		-74.07 (-1.85)		-50.93 (-1.12)		-23.14 (-0.51)
Constant	-0.22 (-0.03)	8.34 (0.13)	-1.34 (-0.21)	104.2** (1.71)	1.12 (0.12)	-95.89 (-1.08)
Obs.	67	53	67	53	67	53
R <sup>2</sup>	.68	.29	.50	.38	.73	.29
DW	2.50	1.86	1.87	1.25	2.30	1.48
ADF (at 10%)	-4.06*	-4.20*	-3.11*	-3.01*	-3.18*	-3.59*
P-Value for shift across time	0.04		0.03		0.20	

**Notes to Appendix Table 1:** See Notes to Table 1. \* (\*\*) = Statistically significant at 5% (10%).

'P-value for shift across time' = P-value for F test of null that equations as a whole are not different across time.

**Table 2**  
**The Full Model of Transitory and Permanent Components of Fiscal Structure: Government of Canada, 1874-1938 and 1950-2000**  
 OLS estimation. Newey-West corrected t- statistics in brackets)

Variables (expected signs) <sup>a</sup>	Lags	Public Non- interest Spending 1874 -1938 $\Delta g$	Public Non- interest Spending 1950 - 2000 $\Delta g$	Variables (expected signs) <sup>a</sup>	Taxation 1874-1938 $\Delta t$	Taxation 1950-2000 $\Delta t$	Variables (expected signs) <sup>a</sup>	Net Deficit 1874-1938 $\Delta b-rb_{-1}$	Net deficit 1950-2000 $\Delta b-rb_{-1}$
$r^e$ (+)	0	4.50 (5.21)*	9.04 (0.87)	$r^e$ (-)	0.99 (2.43)**	4.72 (0.78)	$r^e$ (+)	3.56 (5.58)*	4.32 (0.42)
	1	-0.55 (-0.93)	-8.04 (-1.25)		-0.44 (-1.23)	4.89 (0.55)		-0.10 (-0.17)	-12.92 (-1.31)
	2	-0.13 (-0.28)	1.80 (0.32)		-0.43 (-1.55)	8.11 (1.38)		0.30 (0.64)	-6.31 (-0.74)
$\Delta y^e$ (-)	0	-0.01 (-1.01)	-0.09 (-1.78)**	$\Delta y^e$ (+)	0.03 (3.14)*	-0.004 (-0.08)	$\Delta y^e$ (-)	-0.04 (-2.29)**	-0.08 (-1.71)**
	1	0.04 (3.89)*	-0.18 (-2.24)**		-0.02 (-1.75)**	0.04 (0.63)		0.06 (3.52)*	-0.23 (-3.08)*
	2	-0.05 (-2.36)**	-0.09 (-1.34)		-0.02 (1.21)	0.07 (0.85)		-0.03 (-1.42)	-0.17 (-1.26)
$\Delta x^e$ (-) <sup>c</sup>	0	-0.37 (-7.28)* <sup>b</sup>	0.05 (-0.79) <sup>b</sup>	$\Delta x^e$ (+)	-0.04 (-1.57) <sup>b</sup>	0.02 (0.45) <sup>b</sup>	$\Delta x^e$ (-)	-0.33 (-4.65)* <sup>b</sup>	0.05 (0.79) <sup>b</sup>
	1	0.10 (5.15)*	-0.08 (-1.90)**		0.001 (0.09)	0.0 (1.05)		0.10 (3.85)*	-0.12 (-2.29)**
	2	0.01 (0.57)	-0.14 (-2.68)*		0.03 (1.66)	0.07 (1.35)		-0.02 (-0.57)	-0.20 (-2.79)*
D(AGRIC)		-6049.5 (-5.88)*	-1155.8 (-0.25)		392.81 (0.89)	-3855.9 (-1.08)		-6442.3 (-5.07)*	2700.1 (0.66)
D(IMRATIO)		3197.8 (4.50)*	-1919.4 (-0.29)		795.42 (1.78)**	-1876.6 (-0.41)		2402.5 (2.38)*	-42.77 (-0.01)
D(%YOUNG)		-23.86 (-1.60)	-174.44 (4.22)*		-39.56 (-2.24)**	-37.70 (-1.00)		-15.69 (-0.77)	-136.7 (-3.45)*
D(POP)		0.13 (3.94)*	-0.19 (-1.73)**		-0.02 (-1.03)	-0.322 (-3.57)*		0.15 (3.62)*	0.13 (1.05)
D(OPEN)		642.9 (4.23)*	-2778.1 (-3.65)*		177.1(2.28)**	-1074.2 (-1.17)		465.8 (2.17)**	-1693.9 (-1.32)
WWI or POSTOIL		104.5 (7.52)*	-129.95 (-3.01)*		32.05 (2.58)*	-91.0 (-1.78)**		72.44 (3.74)*	-38.92 (-1.00)
WWI-aftermath		-146.4 (-7.22)*			39.90 (3.74)*			-186.3 (-7.51)*	
Constant		-42.89 (-3.83)*	171.40 (3.07)*		-8.42 (-1.49)	68.8 (1.43)		-34.47 (-3.02)*	102.6 (1.69)
Obs. R <sup>2</sup>		65 0.88	51 0.49	Obs. R <sup>2</sup>	65 0.67	51 0.24	Obs. R <sup>2</sup>	65 0.85	51 0.58
D.W.		2.04	2.18	D.W.	2.17	1.56	D.W.	2.01	2.08
ADF (10%)		-4.18*	-3.44*	ADF (10%)	-3.75*	-4.41*	ADF (10%)	-3.65*	-4.62*
P-value for shift across time		0.0006			0.09			0.03	

**Notes to Table 2:** See notes to Table 1. \*(\*\*) significantly different from zero at 5% (at 10%). P-value for shift across time = P-value for F test of null that equations as a whole are not different across time.

(a) Expected signs on coefficients refer to sum of coefficients on corresponding lags. The sign predictions given in Table 2 are derived from equations (28)-(30) of the text.

(b) Note that the coefficient and standard errors on the first of the  $\Delta x^e$  terms is adjusted to remove the long run effect of  $\Delta x_t$  estimated using the long run model in Table 1, as discussed in the main text.

(c)  $x^e$  = the predicted or permanent value of income from a 'long-run equation' for real income (gnp) per capita. Explanatory variables include the same level values of variables used in the long run model of fiscal structure:- AGRIC, IMRATIO, POP, %YOUNG, OPEN, WWI, WWII, POSTOIL - as well as U.S industrial production.  $\Delta y^e$  = [the actual change in real gnp per capita -  $\Delta x^e$ ].  $r^e$  = [the actual long term government bond rate - the actual rate of inflation]. For the equation estimate, see the Appendix, part 1.

Appendix Table 2

**The Full Model of Transitory and Permanent Components of Fiscal Structure: Government of Canada, 1874-1938 and 1950-2000**  
**Using Two Period Estimation for Expected Long Run Income and for Long Run Components of Fiscal Structure**  
(OLS estimation. Newey-West corrected t-statistics in brackets)

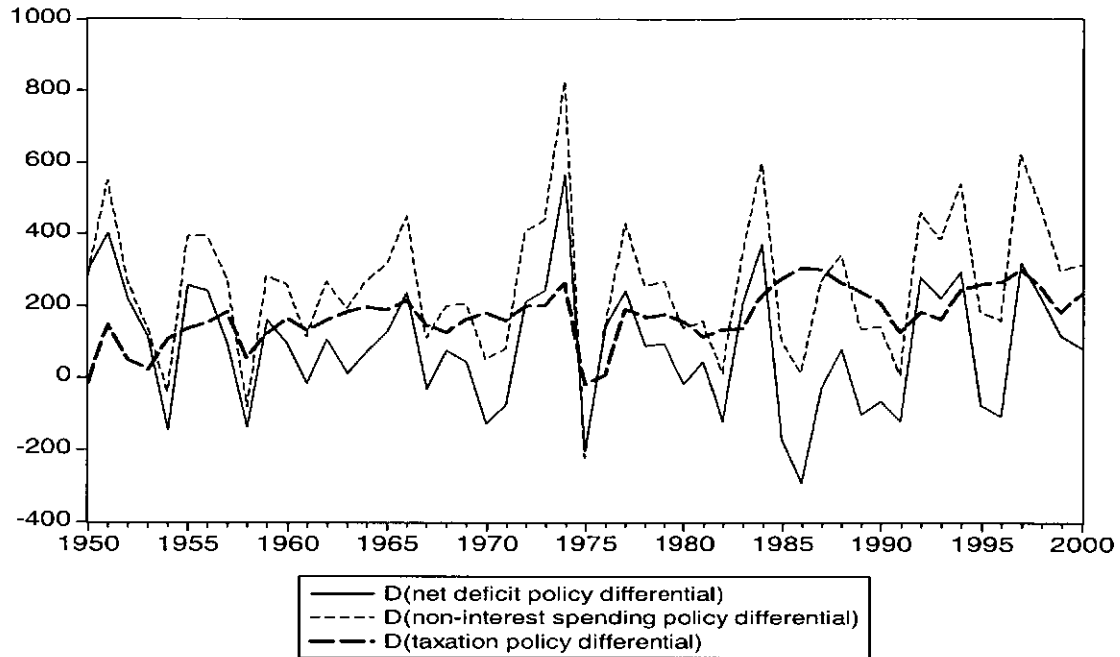
Variables (expected signs) <sup>a</sup>	Lags	Public Non- interest Spending 1874-1938 $\Delta g$	Public Non- interest Spending 1950-2000 $\Delta g$	Variables (expected signs) <sup>a</sup>	Taxation 1874-1938 $\Delta t$	Taxation 1950-2000 $\Delta t$	Variables (expected signs) <sup>a</sup>	Net Deficit 1874-1938 $\Delta b - rb_{-1}$	Net deficit 1950-2000 $\Delta b - rb_{-1}$
$r^e$ (+)	0	3.92 (4.67)*	-0.54 (-0.05)	$r^e$ (-)	1.04 (2.61)*	-0.89 (-0.10)	$r^e$ (+)	2.88 (5.13)*	0.34 (0.04)
	1	-0.25 (-0.45)	-12.02 (-2.10)*		-0.77 (-1.85)**	6.13 (0.80)		0.51 (0.75)	-18.15 (-1.97)**
	2	-1.05 (-1.52)	10.08 (1.65)**		-0.64 (-1.59)	10.09 (1.55)		-0.41 (-0.65)	-0.01 (-0.002)
$\Delta y^e$ (-)	0	-0.02 (-0.94)	-0.10 (-1.85)*	$\Delta y^e$ (+)	0.02 (1.96)**	.004 (0.08)	$\Delta y^e$ (-)	-0.04 (-2.00)*	-0.10 (-2.27)*
	1	0.005 (0.17)	-0.10 (-1.38)		-0.01 (-1.00)	0.08 (1.95)**		0.02 (0.65)	-0.18 (-2.50)*
	2	-0.02 (-0.74)	-0.20 (-1.99)**		-0.008 (-0.60)	0.02 (0.20)		-0.17 (-0.47)	-0.21 (-1.56)
$\Delta x^e$ (-) <sup>c</sup>	0	-0.006 (-0.19) <sup>b</sup>	0.052 (0.67) <sup>b</sup>	$\Delta x^e$ (+)	0.01 (1.97)** <sup>b</sup>	-0.06 (-1.20) <sup>b</sup>	$\Delta x^e$ (-)	-0.02 (-0.58) <sup>b</sup>	0.11 (1.67)** <sup>b</sup>
	1	0.04 (3.08)*	-0.04 (-0.74)		-0.01 (-1.18)	0.06 (1.36)		0.05 (3.51)*	-0.10 (-2.10)*
	2	0.01 (0.63)	-0.11 (-2.45)		0.004 (0.36)	0.07 (1.49)		0.008 (0.37)	-0.18 (-2.98)*
<b>D(AGRIC)</b>		-3039.3 (-3.83)*	-394.36 (-0.75)		408.10 (1.81)**	-4071.8 (-0.99)		-3447.4 (-3.97)*	3677.5 (0.78)
<b>D(IMRATIO)</b>		1485.9 (2.00)*	2838.7 (0.40)		845.62 (2.43)*	3533.5 (0.74)		640.32 (0.82)	-694.76 (-0.09)
<b>D(%YOUNG)</b>		7.51 (0.42)	-145.53 (-3.69)*		-33.33 (-1.88)**	-25.57 (-1.04)		40.84 (1.45)	-117.95 (-2.77)*
<b>D(POP)</b>		0.05 (1.41)	-0.12 (-0.98)		-0.003 (-0.21)	-0.28 (-3.28)*		0.06 (1.32)	0.16 (1.06)
<b>D(OPEN)</b>		709.54 (3.73)*	-2008.3 (-2.44)*		213.68 (3.27)*	-569.21 (-0.81)		495.86 (2.24)*	-1439.0 (-1.39)
<b>WWI or POSTOIL</b>		76.34 (8.05)*	-1.4.36 (-2.86)*		20.76 (2.23)	-77.22 (-2.42)		55.58 (4.51)*	-27.15 (-0.92)
<b>WWI-aftermath</b>		-136.88 (-7.26)*			19.75 (1.96)**			-156.62 (-6.86)*	
<b>Constant</b>		-17.87 (-2.44)*	155.80 (2.23)*		-2.93 (-0.60)	55.62 (1.07)		-14.95 (-1.58)	100.19 (1.43)
<b>Obs. R<sup>2</sup> D.W. ADF (10%)</b>		65 0.81 2.26 -3.56*	51 0.42 2.06 -3.54*	<b>Obs. R<sup>2</sup> D.W. ADF (10%)</b>	65 0.68 1.99 -4.25*	51 0.48 1.65 -4.22*	<b>Obs. R<sup>2</sup> D.W. ADF (10%)</b>	65 0.81 1.97 -2.97*	51 0.55 2.04 -5.28*
<b>P-value for shift across time</b>		<b>0.02</b>			<b>0.05</b>			<b>0.05</b>	

Note: See Notes to Table1 and Table 2.

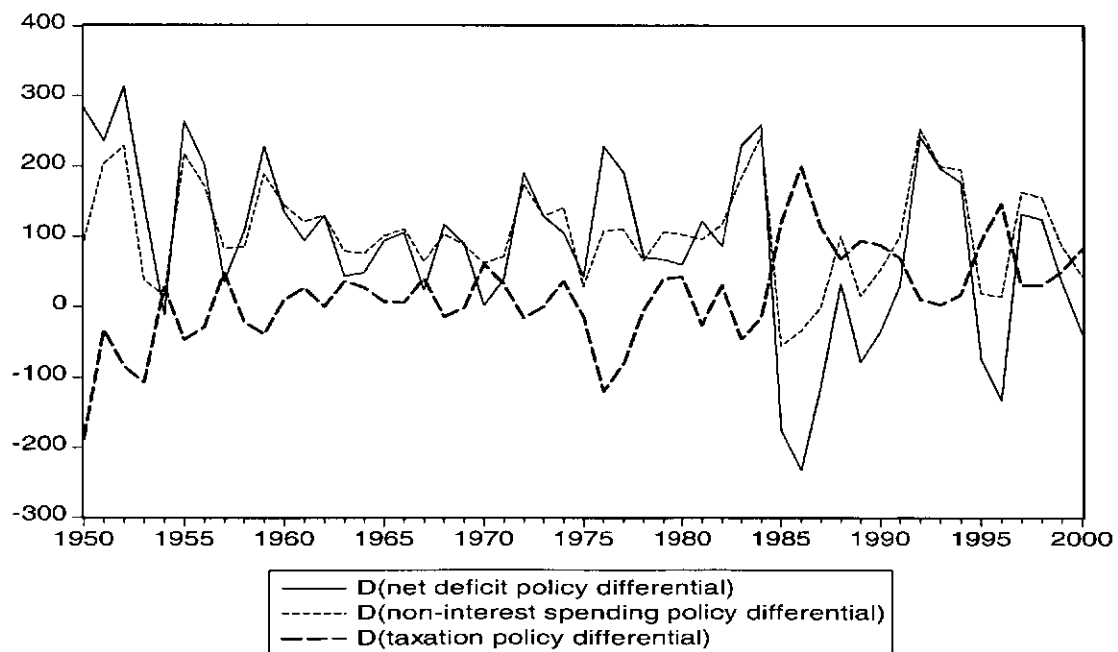
**Figure 3**

**Policy Differentials (In First Difference Form), Government of Canada, 1950–2000**

**A. Basing estimation of long run income and long run fiscal structure on entire sample from 1870**



**B. Basing estimation of long run income and long run fiscal structure on two periods, before and after WWII**





**Table 3**  
**Policy Differential Equations Dg, Dt, DΔb, 1953-2000.**

(OLS estimation. Newey-West corrected t-statistics in brackets. Hansen stability tests as indicated. [Sum of lagged coefficients in square brackets])

**(1) Assuming that Keynesianism and the rise of the welfare state are unrelated**

	$\Delta y_u^e$	$\Delta y_u^e - 1$	$\Delta y_u^e - 2$	D(autoT)	D(autoT) -1	Statistics
Dg: Real non-interest spending per capita	-43.69 (-1.64)	3.25 (0.175)ns	41.0* (3.26) [0.57]	0.39* (2.16)	-0.14 (-0.69) [0.249]	R <sup>2</sup> = 0.36 DW = 1.99 Obs = 48
Dt: Real taxes per capita	-12.85 (-1.36)	-13.85** (-1.69)	-1.55** (-0.22) [-25.2]	0.04 (0.60)	0.114* (2.32) [0.15]*	R <sup>2</sup> = 0.36 DW = 0.99 Joint and Constant = ns
DΔb: Real net deficit per capita	-30.84 (-1.35)	17.11 (1.02) ns	39.45* (3.37) [25.7]	0.35* (2.42)	-0.25 (-1.44) [0.1]	R <sup>2</sup> = 0.36 DW = 1.77

**(2) Assuming that the welfare state is part of Keynesianism**

	$\Delta y_u^e$	$\Delta y_u^e - 1$	$\Delta y_u^e - 2$	Statistics
Dg: Real non-interest spending per capita	-77.26* (-4.13)	12.95 (0.63)ns	37.28* (2.57) [-27.03]*	R <sup>2</sup> = 0.30 DW = 1.89
Dt: Real taxes per capita	-19.14* (-2.98)	-23.51* (-3.68)	-0.45 (-0.6) [-43.1]*	R <sup>2</sup> = 0.31 DW = 0.94 Constant and Joint = ns
DΔb: Real net deficit per capita	-58.12* (-3.18)	36.46** (1.93) ns	37.74* (3.05) [16.08]	R <sup>2</sup> = 0.310 DW = 1.72

**(3) Without using the counterfactual (using only the estimated transitory component 'after Keynes')**

	$\Delta y_u^e$	$\Delta y_u^e - 1$	$\Delta y_u^e - 2$	D(autoT)	D(autoT) -1	Statistics
Transitory real non-interest spending per capita	-2.60 (-0.33)	-7.57 (-0.76)	28.64* (4.06) [18.47]	0.019 (0.34)	-0.23* (-2.64) [-0.211]*	R <sup>2</sup> = 0.44 DW = 1.40
Transitory real taxes per capita	-2.92 (-0.42)	-10.13 (-1.48)	-7.37 (-1.38) [-20.4]	0.05 (0.95)	0.08** (1.77) ns [0.13]**	R <sup>2</sup> = 0.33 DW = 0.45 Constant and Joint = ns
Transitory real net deficit per capita	0.32 (0.03)	2.56 (0.26)	36.01* (3.95) [38.9]	-0.03 (-0.44)	-0.31* (-3.86) [-0.34]*	R <sup>2</sup> = 0.58 DW = 1.27

**Notes to Table 3:** See Notes to Table 1. **\*(\*\*)** significantly different from zero at 5% (10%)

" **ns** " = Coefficient **not stable** according to Hansen (1992) at 10% or less. All other coefficients are **stable**. Constant terms are not reported.  
Sum of similar coefficients in square brackets; indicated significance is for test of null that sum of coefficients is zero.

**Dg (Dt, DΔb )** = estimated change in the transitory component of real non-interest spending per capita (real taxes per capita, real net deficit per capita) **minus** the forecast of the same variable 'after Keynes' based on our model of the transitory component 'before Keynes'

**Δy<sub>u</sub><sup>e</sup>** = first difference of (the current unemployment rate **less** the lagged value of the four year moving average of the unemployment rate).

**D(autoT)** = first difference of (the lagged value of the four year moving average of the ratio of personal income tax revenue to total tax revenue) **times** (the contemporaneous change in real income per capita).

**Table 3a**  
**Policy Differential Equations Dg, Dt, DΔb, 1953-2000, Based on Two Period Estimation at All Stages**  
(OLS estimation. Newey-West corrected t-statistics in brackets. Hansen stability tests as indicated. [Sum of lagged coefficients in square brackets])

**(1) Assuming Keynesianism and the rise of the welfare state are unrelated**

	$\Delta y_u^e$	$\Delta y_u^e -1$	$\Delta y_u^e -2$	D(autoT)	D(autoT) -1	Statistics
Dg: Real non-interest spending per capita	-11.03 (-1.79)	8.42 (1.14)	22.3* (5.48) [19.65]	0.06 (1.29)	-0.20 (-3.12) [-3.29]*	R <sup>2</sup> = 0.56 DW = 1.83 Obs = 48
Dt: Real taxes per capita	0.47 (0.05)	-17.96** (-2.25)	-8.77 (-1.42) [-26.3]	-0.05 (0.70)ns	0.10 (1.66)ns [0.05]	R <sup>2</sup> = 0.36 DW = 0.73 Joint and Constant = ns
DΔb: Real net deficit per capita	-11.49 (-0.87)	26.38** (2.17)	31.04* (3.75) [45.9]	0.11 (1.12)	-0.30* (-3.31) [-0.19]**	R <sup>2</sup> = 0.56 DW = 1.30 Constant and variance = ns

**(2) Assuming the welfare state is part of Keynesianism**

	$\Delta y_u^e$	$\Delta y_u^e -1$	$\Delta y_u^e -2$	Statistics
Dg: Real non-interest spending per capita	-12.26 (-1.69)	24.75* (3.88)ns	23.95* (6.22) [36.43]*	R <sup>2</sup> = 0.48 DW = 1.49
Dt: Real taxes per capita	2.58 (0.36)	-25.81* (-4.11)	-9.36 (-1.56) [-32.6]**	R <sup>2</sup> = 0.34 DW = 0.77 Constant and Joint = ns
DΔb: Real net deficit per capita	-14.84 (-1.13)	50.56* (4.77)	33.31* (4.44) [69.0]*	R <sup>2</sup> = 0.50 DW = 1.17 Constant = ns

**(3) Without using the counterfactual (using only the estimated transitory component 'after Keynes')**

	$\Delta y_u^e$	$\Delta y_u^e -1$	$\Delta y_u^e -2$	D(autoT)	D(autoT) -1	Statistics
Transitory real non-interest spending per capita	-7.63 (-1.29)	11.40 (1.48)	25.10* (5.22) [28.86]**	0.05 (1.21)	-0.10 (-1.68) [-0.05]	R <sup>2</sup> = 0.50 DW = 1.80
Transitory real taxes per capita	1.24 (0.14)	-17.10** (-2.22)	-9.17 (-1.58) [-25.0]	-0.01 (0.11)ns	0.07 (1.26) ns [0.07]	R <sup>2</sup> = 0.34 DW = 0.76 Constant and Joint = ns
Transitory real net deficit per capita	-8.87 (0.71)	28.50** (2.33)	34.27* (3.91) [53.9]**	0.06 (0.67)	-0.18** (-2.01) [-0.12]	R <sup>2</sup> = 0.52 DW = 1.33 Constant and variance = ns

**Notes:** See Notes to Table 1 and Table 3. \*(\*\*) significantly different from zero at 5% (10%).  
Estimation for D(RYPCf) and for long run fiscal structure conducted in two periods, before and after WWII.

**Table 4**

**Estimated Changes in Keynesian Stabilization Policy Given a One Standard Deviation Increase in the Shock to Unemployment**

(Based on the use of coefficients significant at least at 10% only)

Assuming one standard deviation change in shock to unemployment	Using Table 3	Using Table 3a
<b>Using Panel 1 (full policy differential regressions, welfare state assumed unrelated to Keynesianism)</b>	<b>real dollars per capita</b>	<b>real dollars per capita</b>
Change in Real Spending per capita	62 (-4) <sup>a</sup>	17
Change in Real Taxation per capita	-21	-27
Change in Real Net Deficit per capita	60	86
<b>Using Panel 2 (the welfare state assumed part of Keynesianism)</b>		
Change in Real Spending per capita	-60	55
Change in Real Taxation per capita	-64	-39
Change in Real Net Deficit per capita	24	126

**Note:** (a) using a coefficient that is almost significant at 10%

**Table 4a: Sample Statistics, 1951-2000.**

Variable	Mean	Minimum	Maximum	Standard deviation
predicted real gnp per capita $x^e$ (RYPCf in Table 1)	14963	8342	24948	4826
estimated transitory income $y^e = \text{actual real gnp per capita} - x^e$	-34.4	-1178	723	<b>438</b>
unemployment shock $y_u^e = \text{actual rate of unemployment} - \text{lagged moving average of unemployment rates}$	0.23	-2.3	3.5	<b>1.5</b>
population	22963 million	14009	30731	4904
real spending per capita	2438	1071	3552	835
real taxation per capita	2401	1302	3970	822
real net deficit per capita	37	-1218	898	444

**Note:** All figures in real dollars per capita except for population

## Appendix

### 1. List of Variable Names and Data Sources

**AGRIC** = proportion of the labor force in agriculture. 1871-1926: Urquhart, (1993), 24-55; 1926-1995 Cansim series D31251 divided by D31252; 1996-2001: Cansim II series V2710106 divided by V2710104.

**$\Delta b$**  = real primary deficit per capita. Calculated as  $t - g$ .

**$\Delta \hat{b}$**  = permanent or long run real deficit per capita calculated from Table 1.

**Dg (Dt, DAb)** = estimated change in the transitory component of real non-interest spending per capita (real taxes per capita, real net deficit per capita) **minus** the forecast of the same variable 'after Keynes' based on our model of the transitory component 'before Keynes'

**D(autoT)** = first difference of (the lagged value of the four year moving average of the ratio of personal income tax revenue to total tax revenue) **times** (the contemporaneous change in real income per capita). Personal Income tax collections: 1880-1989, Gillespie, pp.285; 1990-2001: CANSIM II V156116.

**Exports and Imports** = exports and imports. 1870-1926, Urquhart, (1993) Table 1.4; 1927-1960, Leacy, et al, 1983, Series G383, 384; 1961-2001: CANSIM series D14833 & D14836.

**GNP** = gross national product in current dollars. 1870-1926: Urquhart (1993), pp. 24-25 (in millions); 1927-1938: Leacy et al (1983), Series E12, p.130; 1939-1960 *Canadian Economic Observer, Historical Statistical Supplement 1986*, Statistics Canada Catalogue 11-210 Table 1.4. CANSIM D11073 = GNP at market prices. 1961-2001 Cansim I D16466 = Cansim II V499724 (aggregated from quarterly data).

**GOV** = total government expenditure net of interest payments. 1870-1989: Gillespie (1991), pp.284-286; 1990-1996: Public Accounts of Canada 1996-97; 1997-2000: Federal Government Public Accounts, Table 3 Budgetary Revenues Department of Finance web site, September 2001. To this we add the return on government investment (**ROI**) and subtracted out interest paid to the private sector. Data on **ROI**: 1870 to 1915: Public Accounts 1917 p.64; 1915-1967: Dominion Government Revenue and Expenditure: Details of Adjustments 1915-1967 Table W-1; 1916-17 to 1966-67: Securing Economic Renewal - The Fiscal Plan, Feb 10, 1988, Table XI; 1987-88 to 1996-97: Public Accounts 1996, Table 2.2. Interest on the Debt (**ID**) was subtracted out (with adjustment for interest paid to the Bank of Canada (**BCI**) ultimately returned to the government). Data on **ID**: 1870-1926: Historical Statistics of Canada, Series H19-34: Federal Government budgetary expenditures, classified by function, 1867-1975; 1926-1995: Cansim D11166. 1996-2000: Cansim D18445. Finally, data for **BCI**: copied by hand from the Annual Reports of The Bank of Canada, Statement of Income and Expense, Annually, 1935-2000. Net Income paid to the Receiver General (for the Consolidated Revenue Acct). Note: all government data had to be converted from fiscal to calendar years. For the Canadian Government the fiscal year ended on March 31 from 1908 onward and ended on June 30 for 1907 and before. Conversion to calendar year made accordingly.

**g** = real government spending per capita. Calculated as  $GOV/(P*POP)$

**$\hat{g}$**  = long run or permanent size of government expenditure per capita. Calculated as the forecast value of long run government expenditure (presented in Table 1).

**IMMIG** = immigration numbers. 1868 – 1953: Firestone (1958), Table 83, Population, Families, Births, Deaths; Updated by Cansim D27 (1955 to 1996). Cansim Sum of X100615 (Females) plus X100614 (Men) for 1954; 1997-2001, Cansim D27 (sum of quarters).

**IMRATIO** = number of immigrants as a fraction of total population. Calculated as **IMMIG/POP**.

**IPIUS** = Index of Industrial Production for the United States. 1870-1929: Table A15. NBER, Nutter; 1930-1970, Table A16. (BEA) Bureau of Economic Analysis; 1971-1995: Cansim D360048 (1987=100); 1996-2000, U.S. Department of Commerce, Business Cycle Indicators, Index of Industrial Production 1992=100.

**OPEN** = openness. Calculated as **(Exports + Imports)/GNP**.

**%YOUNG** = percentage of the population below 17. 1870-1920 Leacey et al (1983). Interpolated from Census figures Table A28- 45 sum of columns 29, 30, 31, and 32 all divided by 28 (adjusted to make 1921 the same); 1921-2001 Cansim C892547.

**P** = GNP deflator before 1927 and GDP deflator after (1986 = 100). 1870-1926: Urquhart, (1993), 24-25; 1927-1995 (1986=100): Cansim data label D14476; 1996-2001 Cansim D140668. All indexes converted to 1986 = 100 basis.

**POP** = Canadian population. 1870-1926: Urquhart, (1993), 24-25; 1927 - 1995: CANSIM data label D31248; 1995 - 2001: Cansim D1 (average of four quarters).

**POSTOIL** = dummy 1 for 1974 to 1980, 0 otherwise

**RYPC** = real income per capita. Calculated as  $\text{GNP}/(\text{POP} \cdot \text{P})$ .

$\mathbf{x}^e$  = **RYPC<sub>f</sub>** = forecast value of permanent real income per capita from the following long run estimate (1870-2000):

**RYPC** =  $10.31 + 0.0003 \cdot \Delta \text{IPIUS} - 3.93 \cdot \Delta \text{AGRIC} - 0.01 \cdot \Delta \% \text{YOUNG} + 4.05 \cdot \Delta \text{IMRATIO} - 0.00003 \cdot \text{POP} + 0.15 \cdot \Delta \text{OPEN} + 0.126 \cdot \text{WWI} + 0.112 \cdot \text{WWII} + 0.11 \cdot \text{OILPERM}$

**R<sup>2</sup> = 0.997; D.W. = 0.799; \* = significantly different from zero at 1%**  
**ADF test statistic on residuals = -3.3205 (10% critical value = -2.57)**

**r** = long term government bond rate. 1870-1913: Rich (1988), Average Yield on Dominion Government Bonds - Table 7-4, p.201; 1914-1919. Homer (1973) Province of Ontario Bonds Annual Average, %, p. 484; 1920 - 1958: Homer (1973), Long Term Dominion of Canada Dollar Bonds Annual Average, % p. 484; 1959-1995: Gov't of Can. Bond Yield Ave 5-10 year, Cansim series B14030; 1995-2001: updated by hand as average of 12 months Cansim B14030.

**r<sup>e</sup>** = real rate of interest. Calculated as long term government bond rate - the actual rate of inflation (using **P**).

**Taxes** = aggregate tax revenues plus return on investment. 1870-1995: Sum of the fourteen different categories of taxes collected by Gillespie(1991); 1996-2000 updated and annualized [(1/4), (3/4)] from column entitled total budgetary revenues in *Fiscal Reference Tables*, Federal Government Public Accounts, Table 3 Budgetary Revenues Department of Finance web site, September 2001. **ROI**, which was excluded by Gillespie from both revenue and expenditure, was added. For source see GOV above.

$t$  = real taxes per capita. Calculated as  $\text{Taxes}/(\text{P} \cdot \text{POP})$

$\hat{t}$  = long run or permanent taxes per capita. Calculated as forecast of long run tax equation from Table 1.

**URATE** = the unemployment rate. 1870 - 1918: unavailable; 1919-1995: Cansim D767863; 1996-2001: Cansim D980745 (annual average of monthly data).

**WW1** = dummy 1 for the years 1914-1918.

**WWI aftermath** = dummy 1 for the years 1919-1921.

**WW2** = dummy 1 for the years 1939-1945.

**WWII aftermath** = dummy 1 for the years 1946-1949.

$y^e$  = transitory income per capita. Calculated as  $\text{RYPC} - x^e$ .

$\Delta y_u^e$  = change in the transitory shock to unemployment. Measured as the change in the deviation of URATE from the lagged value of its four year moving average.

## 2. Linearization of First Order Conditions (23)

We proceed by taking Taylor series approximations of the left hand side of equations (23) for the policy variables  $g$  and  $t$ . Here we show the detail for the case of government spending only and expand about the long-run path of  $g$ , given the expected values of the permanent influences on government size, when the expected transitory deviations from the equilibrium path are equal to zero. That is, we linearize the first order equations around the long-run expected paths for  $g$ ,  $\hat{g}_{t+s} = g(x_{t+s}^e, y_{t+s}^e=0, \dots)$  and  $\hat{g}_{t+s-1} = g(x_{t+s-1}^e, y_{t+s-1}^e=0, \dots)$ , defined by solving the Lagrangian (15) for policies of period  $t+s-1$  and  $t+s$  under the specified conditions. For this purpose we consider the left side of (24) to be a function of expected quantities  $x^e$  and  $y^e$ .

Terminating the expansion after the set of second order terms, we find that the Taylor series expansion of the left hand side of (23), with becomes

$$\begin{aligned} & \approx [\ln[\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s})]] - \ln[\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s-1})] \\ & + [\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s})]^{-1} \langle [\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s})](g_{t+s}^e - \hat{g}_{t+s}) + [\gamma E_t \partial \mu_g^1 / \partial y_{t+s}^e](y_{t+s}^e) \rangle \\ & - [\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s-1})]^{-1} \langle [\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s-1})](g_{t+s-1}^e - \hat{g}_{t+s-1}) + [\gamma E_t \partial \mu_g^1 / \partial y_{t+s-1}^e](y_{t+s-1}^e) \rangle. \end{aligned} \quad (\text{A1})$$

where we have simplified by assuming that the indirect utility function is separable in the policy instruments so that the cross partial terms are all zero.

To evaluate this expression, we assume that the expected marginal utility generated by a change in the policy variable is linearly related to the expected level of the state variables  $y_{t+s}^e$  and  $x_{t+s}^e$  as in  $E_t \mu_g^1 = a + b y_{t+s}^e$  and  $E_t \mu_g^2 = c + d x_{t+s}^e$ , where the scalars  $a$  and  $c$  are positive and both  $b$  and  $d$  are negative. (The analogous assumptions about the effects of taxes and debt are also made.) Additional government spending then generates positive utility along the equilibrium path, but produces less additional value for

larger positive values of both transitory and permanent income. These are assumptions of convenience in the present discussion - the empirical work allows  $b$  and  $d$  to take either sign.

Because the difference in logarithms equals the rate of growth of the inside variables, the first term of the expansion in (A1) when evaluated at  $\hat{g}_{t+s}$  and  $\hat{g}_{t+s-1}$  becomes

$$\frac{\gamma b(a-a) + d(x_{t+s}^e - x_{t+s-1}^e)}{\gamma E_t \mu_g^1(0) + E_t \mu_g^2(x_{t+s-1}^e)} = \frac{d(x_{t+s}^e - x_{t+s-1}^e)}{\gamma a + (c + d x_{t+s-1}^e)}. \quad (A2)$$

Repeating the use of linear marginal utility to evaluate the first order derivatives in (A1)<sup>1</sup> from (24) and combining these with (A2), our linear approximation to (25) becomes

$$\begin{aligned} & \frac{d(x_{t+s}^e - x_{t+s-1}^e)}{\gamma a + c + d x_{t+s-1}^e} + \frac{\gamma b(\partial y / \partial g_{t+s}^e)}{\gamma a + c + d x_{t+s}^e} (g_{t+s}^e - \hat{g}_{t+s}) - \frac{\gamma b(\partial y / \partial g_{t+s-1}^e)}{\gamma a + c + d x_{t+s-1}^e} (g_{t+s-1}^e - \hat{g}_{t+s-1}) \\ & + \frac{\gamma b}{\gamma a + c + d x_{t+s}^e} y_{t+s}^e - \frac{\gamma b}{\gamma a + c + d x_{t+s-1}^e} y_{t+s-1}^e = \rho - r_{t+s} + D[\ln \psi_{t+s}]. \end{aligned} \quad (A3)$$

It may be noted here that while similar variables have coefficients with the same general form across time in (A3), all coefficients are time dated and become equal only if the state variables are equal in adjacent periods. The same general form holds for  $t_{t+s}$ .

We can rearrange (A3) to solve explicitly for  $g_{t+s}^e$  (and for  $t_{t+s}$ ). Using  $X_{t+s}^e$  to represent the time dated term representing the weighted sum of the first derivatives,  $X_{t+s}^e = \gamma a + c + d x_{t+s}^e > 0$ , we have

$$\begin{aligned} g_{t+s}^e - \hat{g}_{t+s} &= \frac{[\rho - r_{t+s} + D \ln(\psi_{t+s})]}{(X_{t+s}^e)^{-1} \gamma b (\partial y / \partial g_{t+s}^e)} - \frac{1}{(\partial y / \partial g_{t+s}^e)} \left[ y_{t+s}^e - \frac{y_{t+s-1}^e}{X_{t+s-1}^e (X_{t+s}^e)^{-1}} \right] \\ &- \frac{d(x_{t+s}^e - x_{t+s-1}^e)}{\gamma b (\partial y / \partial g_{t+s}^e) X_{t+s-1}^e (X_{t+s}^e)^{-1}} + \left[ \frac{(g_{t+s-1}^e - \hat{g}_{t+s-1}^e)}{X_{t+s-1}^e (X_{t+s}^e)^{-1}} \right] \left[ \frac{(\partial y / \partial g_{t+s-1}^e)}{(\partial y / \partial g_{t+s}^e)} \right]. \end{aligned} \quad (A4)$$

To interpret this equation, note that since  $X_{t+s}^e = \gamma a + c + d x_{t+s}^e$  and  $X_{t+s-1}^e = \gamma a + c + d x_{t+s-1}^e$ , then  $X_{t+s-1}^e (X_{t+s}^e)^{-1} \approx 1$ . In addition, we assumed earlier that  $\partial y / \partial g_{t+s-1}^e \approx \partial y / \partial g_{t+s}^e$ . Using these approximations and rearranging, (A4) reduces to

$$\Delta g_{t+s}^e = \Delta \hat{g}_{t+s} + \frac{[\rho - r_{t+s} + D \ln(\psi_{t+s})]}{(X_{t+s}^e)^{-1} \gamma b (\partial y / \partial g_{t+s}^e)} - \frac{(y_{t+s}^e - y_{t+s-1}^e)}{(\partial y / \partial g_{t+s}^e)} - \frac{d(x_{t+s}^e - x_{t+s-1}^e)}{\gamma b (\partial y / \partial g_{t+s}^e)}.$$

where  $\Delta g_{t+s}^e = (g_{t+s}^e - g_{t+s-1}^e)$ . Equation (A5) is equation (25) in the text.

<sup>1</sup> Here  $\partial E_t \mu_g^1 / \partial g_{t+s} = \gamma b (\partial y / \partial g)$  is assumed to be independent of the size of  $y^e$ ;  $\partial E_t \mu_g^2 / \partial g_{t+s} = d(\partial x_{t+s}^e / \partial g_{t+s}) = 0$ ;  $\partial E_t \mu_g^1 / \partial y_{t+s} = \gamma b$ ; and  $\partial E_t \mu_g^2 / \partial y_{t+s} = 0$ .



### 3. List of Definitions and Assumptions Used in the Derivation of Equations (25) and (26)

$$\lambda = N_1/N > 0$$

$$\alpha_h = \partial F_h / \partial (E_t(U^h(k))) = 1/(\varphi_{\max} - \varphi_{\min})$$

$$\theta_1 = N_1 \lambda \alpha_1$$

$$\theta_2 = N_1(1-\lambda) \alpha_2$$

$$\gamma = N_1 \alpha_1 / (N_1 \alpha_1 + N \alpha_2) < 1$$

$$\mu^1(x, y) = \mu^1(x) + \mu^1(y) = \mu^2(x) + \mu^1(y).$$

$$\partial(\theta_2 E_t \mu^2) / \partial b_t = 0$$

$$E_t \mu^1_g = a + b \cdot y^e_{t+s} > 0; \quad a > 0, b < 0$$

$$E_t \mu^2_g = c + d \cdot x^e_{t+s} > 0; \quad c > 0, d < 0$$

$$E \mu^1_t = e + f \cdot y^e_{t+s} < 0; \quad e < 0, f > 0$$

$$E \mu^2_t = h + k \cdot x^e_{t+s} < 0; \quad h < 0, k > 0$$

$$\partial(\theta_1 E_t \mu^1) / \partial b_t \neq 0$$

$$\partial(\theta_2 E_t \mu^2) / \partial b_t = 0$$

$$\partial y / \partial g_{t+s-1} = \partial y / \partial g_{t+s} > 0$$

$$\partial y / \partial t_{t+s-1} = \partial y / \partial t_{t+s} < 0$$

$$(\partial^2 y / \partial t_{t+s}^2) < 0$$

$$X^e_{t+s} = \gamma \cdot a + c + d \cdot x^e_{t+s} > 0$$

$$X^e_{t+s-1} = \gamma \cdot a + c + d \cdot x^e_{t+s-1} > 0$$

$$X^e_{t+s-1} (X^e_{t+s})^{-1} \approx 1$$

$$Z_{t+s} = (X^e_{t+s})^{-1} \gamma \cdot b \cdot (\partial y / \partial g_{t+s})^e < 0$$

$$W_{t+s} = (V^e_{t+s})^{-1} \gamma \cdot f \cdot (\partial y / \partial t_{t+s})^e > 0$$

$$V^e_{t+s} = \gamma \cdot e + h + k \cdot x^e_{t+s} < 0.$$