# THUCYDIDES ON NASH VS. STACKELBERG:

# THE IMPORTANCE OF THE SEQUENCE OF MOVES IN GAMES

By George Tsebelis*

UCLA

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The paper provides a general characterization of the conditions in two-by-two games under which players will prefer to move first or second, or will be indifferent to the sequence of moves. The general result is that in games without a pure strategy equilibrium either there will be a struggle for the second move, or the players will agree on the sequence of play; a struggle for the first move is characteristic of games with two pure strategy equilibria; in games with one pure strategy equilibrium, players most of the time will be indifferent about the sequence of moves. Examples from Thucydides' History of the Peloponnesian War and other political and social situations illustrate how players manage to change the sequence of moves.

# THUCYDIDES ON NASH VS. STACKELBERG: THE IMPORTANCE OF THE SEQUENCE OF MOVES IN GAMES

In a period of "new institutionalism" in political science, one hardly has to argue about the importance of institutions. It is widely accepted that institutions shape political outcomes. Although now, after the behavioral revolution, the study of political institutions has re-entered the field of political science, in very few cases are we able to provide a theoretical understanding of the general properties of certain rules. The most frequently studied institutions are of course those of majority rule,[2] but this rule is most of the time embedded in a much more rich institutional framework,[3] about which we know very few propositions of general applicability—for example, that a close rule restricts the set of possible outcomes compared to an open rule, that exclusive jurisdictions of committees restrict outcomes even further, or that strategic voting leads to an outcome belonging to the top cycle.[6]

This paper provides a characterization of one important rule: the sequence in which two players play a game. The general solution to the problem of sequence in two-person games involves the payoffs of the players only indirectly, so it is not very helpful in answering whether a specific game can generate fights for whoever moves first or second. For this reason, I will use a simple two-by-two game, which I believe is a reasonable representation of common political situations, and I will provide a complete theoretical characterization of the conditions under which the two players will prefer to move first (struggle for leadership), prefer to move second (struggle for followership), or are indifferent to the sequence of moves. In other words, I will provide an exhaustive list of combinations of payoffs which will induce players to agree, disagree, or be indifferent about the sequence of playing the game, and if they disagree, the combinations of payoffs which will make each one of them prefer to play first or second.

1 For the transition from behavioralism to new institutionalism see March and Olsen (1984). The term of "new institutionalism" remains undefined in this paper because the authors, in order to make their point, pile up together approaches which are widely different.

2 See McKelvey (1976) and Schofield (1978).

3 See Shepsle (1979), Shepsle and Weingast (1984), FereJohn and Kreibel (1987), Hammond and Miller (1987)

4 See Shepsle and Weingast (1984).

5 See Shepsle (1979)

6 See McKelvey and Niemi (1978). For a general treatment of this literature which essentially stems from Arrow's (1951) theorem, see Riker (1982) and Schwartz (1986).

7 The general answer to the problem of sequence is: "Two person games can be partitioned in three classes: 1. If the beta-core is empty and the s-core is non-empty, there is a struggle for followership; 2. If the beta-core is non-empty and the s-core is empty, there is a struggle for leadership; 3. If the intersection of the beta-core and the s-core is non-empty, there is favorable area to cooperation by threats" (see Moulin (1981: 273). J. Hirshleifer (1985) on the other hand, through simulations came to the conclusion that positive correlation between the payoffs of the two players will give the advantage to the first mover, while negative correlation will give the advantage to the second mover.

Most but not all examples in this paper will be borrowed from the father of game theory, Thucydides. There are three characteristics of Thucydides which make me believe that he deserves that title. The first is that for Thucydides history is intelligible in terms of human actions, without any intervention by the supernatural. In modern terminology, outcomes are endogenous to his model.

The second is that he was interested in explaining the general through the particular, as he indicates in the first book of the History of the Peloponnesian War: "It will be enough for me, however, if these words of mine are judged useful by those who want to understand clearly the events which happened in the past, and which (human nature being what it is) will, at some time or other and in much the same ways, be repeated in the future. My work is not a piece of writing designed to meet the taste of an immediate public, but was done to last for ever" (Book 1: 22). In modern terminology, he was interested in historical questions as a means of finding theoretical answers.

Third, in each instance he tries to explain the actions of his actors as optimal choices given the circumstances, and not as mistakes. If the actors were misinformed, the reasons for their false beliefs are provided, so that their action becomes rational given their set of beliefs. As King Archidamus of Sparta puts it (Book 1: 84): "... there is no great deal of difference between the way we think and the way others think, and that it is impossible to calculate accurately events that are determined by chance. The practical measures that we take are always based on the assumption that our enemies are not unintelligent. And it is right and proper for us to put our hopes in the reliability of our own precautions rather than in the possibility of our opponent making mistakes." In modern terminology, Thucydides considers all interacting parties as rational players, and therefore, engages in equilibrium analysis.

All the examples borrowed from Thucydides are interpreted as games with complete information where sequence of moves is at stake. Obviously, this is not the only possible interpretation; in particular, one could use games with incomplete information as the basis of the analysis, but at the cost of simplicity and tractability of outcomes, and multiplicity of equilibria.[11] As an introduction to the importance of sequence, two examples demonstrate cases where the problem emerges.

1. The Revolt of Mytilene (Book III: 30). In the year 427 BC, Mytilene, who was an ally of Sparta, was under siege by the Athenian fleet. The Peloponnesians organized a fleet of forty ships to reinforce Mytilene. However, when the fleet arrived close to the island of Lesbos where Mytilene is located, it found that Mytilene had fallen into the hands of the Athenians. The Peloponnesians knew that the Athenians were not aware their fleet was so close to Lesbos. Therefore, Teutiaplus gave the

8 Relevant examples from other authors appear in the footnotes.
9 Maybe the title precursor would be more appropriate, since the date of birth and the parenthood of game theory can be attributed to the seminal work of Von Neumann and Morgenstern (1944).
10 For these two reasons, I believe, Thucydides has been called a "scientific historian" (for a discussion of the issue see Alker (1988) who, however, has a different point of view).
11 Typically, games with incomplete information, that is, games where the payoffs of the players are not common knowledge have an infinity of equilibria (see Kreps and Wilson (1982), Rubinstein (1985).

following advice regarding the appropriate course of action: "I propose that we should sail to Mytilene just as we are and before they know we are here. In all probability, since they have only just taken the city, we shall find that their precautions have been greatly relaxed; and this will certainly be so by sea, since they have no idea of having to face any possible attack, and where, in fact, our main strength happens to lie. It is likely, too, that their land forces, after their victory, will be dispersed about the houses in the city and not properly organized. So that if we were to attack suddenly and by night, I think that, with the help of those inside the town who are still on our side, we ought to be able to gain control of the place. Let us not be afraid of the danger, but let us remember that this is an example of the unknown factor in warfare, and that the good general is the one who guards against such unknown factors in his own case, but exploits them for attack in the case of the enemy." Teutiaplus' advice was not taken by the admiral, Alcidas, who sailed back to Sparta. It is easy to understand, however, why the plan had a high probability of success. It would have used an extraordinary combination of conditions which made it likely for the Athenians to be off guard and taken by surprise: attack at night, by sea (traditionally a weakness of the Peloponnesians), and by a fleet that as far as the Athenians were concerned was non-existent.

2. The Support to Eion (Book IV: 106-7). The event occurred in the year 423 BC, and this is the only expedition in which Thucydides himself participated, as a general of the Athenian army. [12] His description of the support of Eion goes as follows: "In this way the city was surrendered 13, and late on the same day Thucydides with his ships sailed into Eion. As for Amphipolis, Brasidas had just taken it, and he was between a night of taking Eion too. If the ships had not arrived so quickly to relieve it, it would have been in his hands by dawn. After this Thucydides organized the defence of Eion to keep it safe from any immediate attack by Brasidas and to secure it for the future." This account is terse, but one point is very clear: it is because the Athenians moved first that they obtained an important advantage over Brasidas—retention of the city of Eion.

One way of looking at these examples is as indications that the player who moved first (or wanted to move first), had (or would have had) a decisive advantage over the opponent. If a theory based on these two examples predicts that moving first provides a decisive advantage, then the theory cannot account for another story from Thucydides, where the Athenians won <u>although (or because</u>) they moved after the Peloponnesians at Cynossema in the year 411 BC (Book VIII: 105, emphasis mine): "Finally, however, the Peloponnesians in the confidence of their victory began to scatter in pursuit of individual ships and, over a considerable part of the line, to fall into disorder. <u>Seeing this</u>. Thrasybulus, and his men, instead of continuing to extend their line, turned about immediately and went into action against the enemy ships which were bearing down on them. After routing these, they fell upon that part of the Peloponnesian fleet which had been victorious, driving down upon them in their disorganized state and putting most of them to flight without any resistance being offered."

This particular example describes a surprise attack: the surprise comes from the player who moves <u>second</u>. Some terminological clarification

12 He spent in exile most of the time of the war.

13 He refers to the capture of Amphipolis by Brasidas and the Peloponnesian army.

is in order. This clarification is necessary because even military or academic experts call surprise attacks "first srikes."[14]

There two ways to define sequence (or priority) of moves. The first is temporal: whoever moves before the opponent moves first. The second is informational: if one of the players moves while having accurate information about the opponent's move, then this player moves second. Often the two definitions produce the same sequence of moves. For example, when a head of state responds to the statement of another head of state, both in temporal and in informational terms the sequence of moves is clear: the respondent moves second. However, it is frequently the case that the two definitions produce different sequences of moves. Consider a sealed auction: one participant gives his sealed bid one day, while a second one hour before the deadline. In temporal terms there is a sequence, while in informational terms, the two players move simultaneously. Suppose now that the participant who moved first (in temporal terms) was aware (through espionage) of the amount the opponent was going to offer; in this case the informational sequence is exactly opposite to the temporal sequence. It must be intuitively clear at this point that the way the two players behave depends on informational and not on temporal priority. Later in the paper it will become clear why game theory adopts the terminology which calls a surprise attack a second move although this terminology conflicts with immediate intuition.

Consequently, the crucial question which discriminates between the first and second move in a game is whether or not a player has accurate information about the opponent's move before she selects her strategy. If a player has accurate information about the opponent's strategy, then she is in fact moving second in the game. The implication is that the surprise attack advocated by Teutiaplus was in fact a second move, despite intuitive expectations.

Thucydides explains the logic of surprise attack in a way that makes precisely this point. In the year 422 BC, Brasidas explained to his soldiers why they should make a surprise attack against the Athenians in the battle of Amphipolis (Book V: 9, emphasis mine): "It is, according to my calculations, because they dispise us and because they have no idea that anyone will come out to fight them that the enemy have come up to the position in which they are and are now looking carelessly about them in no sort of order. But success goes to the man who sees more clearly when the enemy is making mistakes like this and who, making the most of his own forces, does not attack on obvious and recognized lines, but in the way that best suits the actual situation. And it is by these unorthodox methods that one wins the greatest glory; they completely deceive the enemy, and are the greatest possible service to one's own side. So, at this juncture, while they are still confident and unready, while they are thinking, so far as I can see, more of slipping away than of standing their ground, in this moment where their spirits are relaxed, and before they have time to pull themselves together. I propose to charge out at the double with my own troops against the enemies center, taking them, if possible, by surprise..."

Later in Thucydides' account (the year 414 BC) the renegade Alcibiades pushes the argument favoring a second move even further when he advises the Spartans to fortify Decelea in Attica (Book VI: 91): "The surest way of

14 Sometimes a surprise attack is called in military terminology "first strike." For a similar use in academic writings, see Schelling (1960: 207).

harming an enemy is to find out certainly what form of attack he is most frightened of and then employ it against him. He is likely to know himself more accurately than anyone else where his danger lies, and that is why he is frightened."

Another commonly held belief is that the player who is better informed about his opponent's intentions will have a decisive advantage over the opponent. This belief leads to opposite prescriptions than those of the "first strike" theory: it implies that players who move second have the decisive advantage. However, as the Eion example indicates, sometimes moving first makes a player better off.

One can find multiple examples where the sequence of moves was important, such as when Kennedy declared unilaterally that the U.S. would not tolerate Soviet missiles in Cuba,[15] or when Reagan refused to reveal U.S. strategy when asked what he would do against Qadhaffi. Some public policies (like laws on drugs) are announced before they become effective, or even before they are voted into law, while there is absolute secrecy about other policies (like the equivalence of the dollar to the yen).

Comparing the behavior of the players in these examples indicates that the Peloponnesians could have moved second, after knowing what their opponents were doing, and taken them by surprise; on the other hand, the Americans during the Cuban missile crisis moved first, informed the Soviets about their precise intentions, and let the Soviets move second by letting them know the strategy of their opponent; while Kennedy preferred to move first in the Cuban case, Reagan preferred not to move first in the Libya case; national authorities sometimes prefer to inform the public about their intentions, and sometimes not.

Why is it that in one case a player considered the knowledge of his opponent's moves an advantage, in another case he refused to reveal his strategy, and in yet another case he gave the opponent precise information? Why is it that it was important for the Peloponnesians to move after knowing what the Athenians were doing in Mytilene, while the Athenians tried hard to move first in Eion? In other words, why did the Peloponnesians want to move second, while the Athenians wanted to move first? Why is it that in the first case the player who moves second extracts all the benefits, while in the second case all the benefits go to the first mover? What are the characteristics of games where the first mover has the advantage, and what are the characteristics of games where all the advantages go to the player who moves second? Under what conditions will players fight for the first move, and under what conditions will they fight for the second? Is it always the case that players will fight about the sequence of moves, or is it possible that in some cases the sequence of play does not matter? What are these cases? The second section of this paper will provide answers to all these questions for two-by-two games. The third section will carry the argument one step further. It will use specific theoretical and historical examples to examine the question: if there is a struggle for leadership or for followership, how do the players assure that they will move first or second? The question of what kind of institutions and under what conditions will regulate the sequence of moves will not be answered in this paper.

15 As it will become clear later, this account of the crisis assumes that it can be accurately described as a chicken game. This assumption is common in the relevant literature, but by no means unanimous.

## SECTION II: NASH AND STACKELBERG EQUILIBRIA

INSERT TABLE 1

Consider a two by two game: two players must choose between two strategies each. The Row player chooses between Top and Bottom, while the Column player chooses between Left and Right. Table 1 presents a generic two-by-two game. The payoffs of the Row player are indexed by 1, while the payoffs of the Column player are indexed by 2. The outcome of this game is a Nash equilibrium, that is, a pair of strategies such that each player is better off by sticking to her strategy than by unilaterally deviating from it. If, for example, in Table 1 $a_1>c_1$ and $a_2>b_2$, then the outcome $(a_1,a_2)$ is an equilibrium outcome, and strategies Top and Left are an equilibrium pair of strategies. In the Game in Table 1 the two players are assumed to move simultaneously.

Suppose now that player 1 (the Row player) moves first, and player 2 (the Column player) moves second, knowing the choice of strategy of player 1.[16] Again, we can calculate the Nash equilibrium of this new game, and this equilibrium need not be the same as that of the two players moving simultaneously. The Nash equilibrium in the sequential game where player 1 moves first and player 2 moves second is called a Stackelberg equilibrium, where player 1 is the leader and player 2 the follower.

Finally, in the case where player 2 moves first and player 1 moves second, the Nash equilibrium is also called a Stackelberg equilibrium, with player 2 the leader and player 1 the follower. From now on, we will describe the sequences of moves by referring to the outcome of the game in the case where player 1 moves first as the Stackelberg equilibrium where player 1 is the leader; the outcome of the game in the case where the two players move simultaneously is the Nash equilibrium; and the outcome of the game in the case where player 2 moves first is the Stackelberg equilibrium where player 2 is the leader.

INSERT FIGURE 1

Figure 1 presents the three games in extensive form. The first representation corresponds to the case of simultaneous moves (Nash equilibria).[17] The second representation corresponds to the game where player 1 moves first (Stackelberg where player 1 is the leader), while the last representation indicates sequential moves where player 2 moves first (Stackelberg where player 2 is the leader).[18] Figure 1 indicates that the sequence of moves is determined by whether an actor knows the move of his opponent: the player who moves first is the player who moves without knowing his opponent's strategy; if both players move without knowing what the opponent is doing, then the players move simultaneously. We can now understand that in the initial examples the Peloponnesians in Mytilene, the Athenians at Cynossema, and Reagan against Qadaffi all moved second, that

16 The normal form representation of this game is a two by four matrix, which is omitted as non necessary for what follows.

17 The line connecting the two nodes representing the choice of player 2 is an information set, indicating that player 2 has to move without knowing player 1's move.

18 Games with a sequence of moves can be described in normal form by a two by four matrix, since the player that moves first continues to have two pure strategies, but the player who moves second has now 4 pure strategies (see Moulin (1981)). I will not introduce this description, because it is not necessary for what follows.

is, chose the Stackelberg equilibrium where they were followers, while the Athenians in Eion and the Americans in the in the Cuban crisis moved first, that is, chose the Stackelberg equilibrium where they were leaders.

If the Nash and the two Stackelberg equilibria are the same, the players will be indifferent to the sequence of moves. If, however, the two Stackelberg equilibria are different from the Nash equilibria and from each other, then the sequence of moves matters. We will call the case where both players prefer to have the first move, as in the Eion example, the struggle for leadership, while the case where they both want to move second, as in the cases of Mytilene, Cynossema, or Libya, the struggle for followership.

All the games I will describe will be represented by Table 1. In each case, I will provide the names of the players, the two strategies available to each one of them, and the order of the payoffs for each player. The reader can then refer to Table 1 and solve the game, compare the Nash and Stackelberg equilibria, and understand which player decided to alter the sequence of moves and why. The remainder of this section will provide an exhaustive characterization of the conditions under which struggle for leadership or for followership in two-by-two games occurs.[20]

**THEOREM 1.** Two-by-two games without ties in the payoffs can be partitioned into three classes: games with one pure strategy equilibrium; games with two pure strategy equilibria (which also have one mixed strategy equilibrium); and games without pure strategy equilibria (which have one mixed strategy equilibrium).21

Proof: See Moulin (1981: 162-64).

**THEOREM 2.** In two-by-two games with two pure strategy equilibria, if one equilibrium Pareto dominates the other, the players are indifferent to the sequence of play; if no equilibrium is Pareto superior, then there is a struggle for leadership.

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19 An equivalent way of defining struggle for leadership or for followership in terms of the games of Figure 1 is to assume that each one of the players is given the choice between the three games: if both players choose to play the same game, then, they agree on sequence; if player 1 prefers game 1B and player 2 prefers game 1C there is a struggle for leadership; if they have the revers order of preferences there is a struggle for followership.

20 The general examination of the conditions under which there would be a struggle for the leadership or for the followership in a two person game, would proceed the same way, by comparing the payoffs of each player in the case of Nash or Stackelberg equilibrium. If we index the Stackelberg equilibrium payoffs by the leader first and then the name of the player, and Nash equilibrium payoffs by the player, then the condition for struggle for the leadership is $S_{1,1}>N_1$, and $S_{2,2}>N_2$ (unless one of the two Stackelberg equilibria Pareto dominates the other). It can be shown that whenever a pure strategy equilibrium exists, $S_{1,1}>=N_1$, and $S_{2,2}>=N_2$. However, this proposition does not discriminate between the case where there is a struggle for leadership or the players are indifferent about the sequence of moves, or agree that one of them to move first.

21 In the case of two by two games with simultaneous moves, each player has two pure strategies (to select each one of the two options) and an infinity of mixed strategies (a probability distribution over her pure strategies).

Proof: There are two possible orders of payoffs which produce two pure strategy equilibria. (1). If $a_1>c_1$, $d_1>b_1$, $a_2>b_2$, $d_2>c_2$, then the equilibria are $(a_1,a_2)$, and $(d_1,d_2)$. (2). If $a_1<c_1$, $d_1<b_1$, $a_2<b_2$, $d_2<c_2$, then the equilibria are $(b_1,b_2)$, and $(c_1,c_2)$.[22] Without loss of generality, consider the first pair of these equilibria. Both players know that the outcome of the game will be either the first or the second equilibrium. If one of these equilibria gives higher payoffs than the other to both players, then both players have the incentive to select the corresponding strategy regardless of whether they move simultaneously or sequentially, and there is no conflict about sequence. If, however, one equilibrium is better for one player and worse for the other, then in a sequential game the first mover can essentially select the equilibrium by choosing the corresponding strategy, knowing that when her opponent's turn comes, she will select her best response to this strategy.

Examples: The support to Eion described in the first section of this chapter is a case of a game with two pure strategy equilibria. Eion can be occupied by either of the two opponents, and if it is occupied and fortified, the other prefers not to engage in a battle. The examples can be multiplied: any occupation of a strategic position can be represented as a game with two equilibria. Negotiations also can be seen as games with multiple equilibria. When the government in Poland insists that the workers should stop their strike first and negotiate later, while the workers want the reverse, each actor asks the other to recognize his right to select the equilibrium (that is, the leadership in the game).

On the other hand, the collective action problem is usually represented by the advocates of collective action not as a prisoners' dilemma game (where defection is the dominant strategy for each potential participant), but as an assurance game, where there are two equilibria--mutual cooperation and mutual defection. In this representation, one equilibrium Pareto dominates the other, and cooperation is the rational solution, no matter who moves first. I will return to the problem of collective action from a different perspective in the next part of this paper.

In a more general way, examples with two equilibria in which moving first provides a decisive advantage are frequently games of chicken or battle of the sexes, while examples with two equilibria where sequence is immaterial are cases of the assurance game.

THEOREM 3. In two-by-two games without pure strategy equilibria, there is never a struggle for leadership.

Proof: There are two possible orders of payoffs which produce no pure strategy equilibria: (1). $a_1>c_1$, $d_1>b_1$, $a_2<b_2$, $d_2<c_2$, and (2). $a_1<c_1$, $d_1<b_1$, $a_2>b_2$, $d_2>c_2$.[23] Without loss of generality, consider the first combination of payoffs. The possible Stackelberg equilibria with player 1 as leader are $(c_1,c_2)$ and $(b_1,b_2)$. The possible Stackelberg equilibria with player 2 as leader are $(a_1,a_2)$ and $(d_1,d_2)$. No matter which the actual combination in a specific game, since there is no pure strategy equilibrium at least one of the players will have the incentive to move second. For example, if the choice is between $(c_1,c_2)$ and $(a_1,a_2)$, player 1 prefers to move second.

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22 The reader can verify this statement by testing the alternatives for the number of equilibria.

23 The reader can verify this statement by testing the alternatives for the number of equilibria.

Example: An interesting generic example of a game without a pure strategy equilibrium is the game of compliance and monitoring. According to Thucydides, the profound cause of the Peloponnesian war was that the Athenian empire represented a threat to Sparta. The Athenian empire was composed of cities annexed by Athens as well as by cities which joined the alliance out of their own free will. For the former, especially when the war started and the Athenians were fighting on land and sea, the possibility of choosing either to declare independence or to join the Peloponnesians presented itself. Schematically, one can represent the game of Athenian hegemony by Table 1. The Athenians have two strategies, to monitor their allies or not; the allies also have two options, to revolt or not. It is preferable for the Athenians not to monitor if their allies are not revolting, and to monitor if they are revolting. It is preferable for the allies to revolt if the Athenians are not monitoring, and not to revolt if the Athenians are monitoring. This is a game without pure strategy equilibria, and therefore, the first mover does not have the advantage.[24] In this game the Athenians find themselves in a problematic situation, since their forces are not enough to monitor all their allies. Therefore, areas which are far away from the Athenian fleet can enjoy the advantage of the second move and revolt. In order to solve the problem of sequence of moves the Athenians try to rely on their reputation for toughness. One of the most famous debates in Thucidydes is the Mytilenian debate which took place in the year 427 BC (Book III: 36-49). After Mytilene has revolted and surrendered, the Athenians debate whether they should kill all the men and sell the women as slaves, or kill only the leaders of the revolt. The accuracy of the reported debate has been questioned by historians because the discussion does not involve any moral argument; it is a pure discourse on self-interest and reputation building. One side wants to be tough in order to provide an example to all possible challengers; the other side says that this strategy will lead future challengers to fight to the end since they know what the consequences of surrendering will be.[25]

In the case of no pure strategy Nash equilibrium at least one player will want to move second. But under what conditions will the other player accept the first move, and under what conditions will he engage in a struggle for followership? The following theorem answers this question.

THEOREM 4. In two-by-two games without pure strategy equilibria a player accepts the lead when the corresponding Stackelberg equilibrium is his second best outcome; otherwise, there is a struggle for followership.

Proof: Consider the first order of payoffs which generates a game without pure strategy Nash equilibrium in the proof of Theorem 5. Assume that the two Stackelberg equilibria are the following: $(c_1,c_2)$ and $(d_1,d_2)$. It follows that $c_1>b_1$, and $d_2>a_2$; consequently, player 2 prefers to follow. The only way that player 1 will accept the lead is if $c_1>d_1$. The only possible combination of payoffs which generates this outcome is:

$a_1>c_1>d_1>b_1$ for player 1, and $b_2>a_2$, and $c_2>d_2>a_2$ for player 2.

24 For a detailed analysis of cases of games without a pure strategy equilibrium which present the peculiar characteristic that changes in the payoffs of a player do not affect his behavior, see Tsebelis (1989).

25 If the reader was not persuaded so far by my arguments about Thucydides being the father of game theory, a reading of the Mytilenian debate will dissipate any doubts.

In this case, player 1 accepts leadership, and the the outcome is $(c_1,c_2)$, that is, his second best choice. The argument can be repeated for the other combinations of Stackelberg equilibria, and Table 2 summarizes the results.

INSERT TABLE 2.

Example: One of the immediate reasons for the Peloponnesian War, according to Thucydides, was the dispute over Corcyra in 433 BC (Book I: 31-55). Corcyra was an independent island which was in conflict with Corinth, a member of the Peloponnesian alliance. Corcyra asked Athens for help. The Athenians debated the issue publicly, because an attack against a member of the Peloponnesian alliance was a violation of existing treaties. Finally, they decided to send thirty ships to help Corcyra, without attacking the Corinthians. When the Athenian ships arrived, they found the Corcyraeans in retreat. However, now the balance of forces had shifted in favor of the Corcyraeans, and the two fleets were facing each other without sincerely intending to start an engagement. The Corinthians wanted to return home, but were afraid the Athenians considered the treaty broken and so would intercept them on their way back. They sent messengers to the Athenians (messengers who according to Thucydides did not carry a herald's wand) to say (Book 1: 53):

"Athenians you are putting yourselves in the wrong. You are starting a war and you are not abiding by the treaty. We are here in order to deal with our own enemies, and now you are standing in our path and have taken arms against us. Now if your intention is to prevent us from sailing against Corcyra or anywhere else that we wish, if, in other words, you intend to break the treaty, then make us who are here your first prisoners, and treat us as enemies."

The Athenians replied that they would not prevent them from sailing in any other direction, but that if they sailed against Corcyra they would fight back. After this answer, the Corinthians began to prepare for their voyage home.

The game theoretic account of these events is that the Athenians had to choose between two strategies, attacking and not attacking the Corinthians, while the Corinthians had a choice between proceeding and retreating. Their preferences were the following: the Athenians prefer to attack if the Corinthians proceed ($a_1>c_1$), but not to attack if they retreat ($d_1>b_1$). The Corinthians prefer to proceed if the Athenians do not attack ($c_2>d_2$), but if they are to be attacked by the Athenians they prefer to have an attack without provocation, that is, while retreating, so that it will be obvious to all Greeks that the Athenians had broken the treaty ($b_2>a_2$).

This is a game without a pure strategy equilibrium, so none of the two fleets wants to move first. That is why the two fleets stood in front of each other without an engagement or any other indication of their intentions. The situation can be a struggle for followership, or a case where one of the opponents will agree to move first. In order to find out, we have to examine further the payoffs of the two opponents. The very presence of the Athenian fleet indicates that the Athenians prefer to attack a retreating Corinthian fleet rather than not attack a fleet that sails against Corcyra ($b_1>c_1$). Moreover, we know from Thucydides' account that the Corinthians who intend to retreat prefer not to be attacked rather than to be attacked by the Athenian fleet ($d_2>b_2$). The order of payoffs of the two opponents is given by the fourth line of Table 2. This line indicates that there is no struggle for followership, and that the situation can be resolved if player 2 moves first.

The remainder of the story describes how the two opponents arrived at the mutually beneficial outcome. The Corinthians through their messengers offered the first move to the Athenians. The fact that the messengers did not carry a herald's wand was carefully planned, so that the Athenians would not have to respect them because of their mission and would reveal their true preferences. The Athenians rejected leadership in the game, and so the Corinthians made the first move, leading to the mutually beneficial outcome (retreat, no attack).

COROLLARY 1. In zero-sum games without a pure strategy equilibrium, there is a struggle for followership.

Proof: A zero-sum game without a pure strategy equilibrium cannot be represented by the order of payoffs of Table 2, so there is a struggle for followership.

Two cases where modelling is frequently done as a zero-sum game are war and elections. In both these situations one of the opponents wins and the other loses. According to our corollary, in both these situations whoever moves first is bound to lose. The reader may be surprised by this statement. After all, it is well known that taking one's opponent by surprise is a good method for increasing one's chances of winning in war situations. The contradiction is only apparent. Here, the definition of a second move is one which is done with accurate information of what the opponent is doing. And this is precisely the essence of taking your opponent by surprise: you move knowing that he does not expect you to move, and so he continues his usual activities.

An example from Thucydides (Book II: 71-78) indicates how two opponents made a series of moves which took their opponent by surprise. Each time that one was innovating, that is, using a strategy which would lead to victory given what the opponent was doing, the other took measures to cancel and reverse the advantage. In the third year of the Peloponnesian War (the year 429 BC), the Spartans laid siege to the city of Plataea, which was an ally of Athens. The Spartans started building a mound up against the city wall. When the Plataeans saw the mound growing, they started building a wooden wall on top of their own wall; moreover, they demolished a part of their own wall against which the mound was being built, and started carrying away the loose earth. The Spartans started filling up the gap by packing clay tightly inside reed wattles. However, the Plataeans dug a tunnel underneath the mound and carried the material of the mound away again; moreover, they built an inner wall in case they were defeated, so that the enemy would be obliged to start the mound maneuver all over again. The Peloponnesians brought siege engines, which the Plataeans destroyed. The Peloponnesians tried to burn the city down, and if the wind had been favorable, Plataea would not have survived. After this last failure the Peloponnesians built a wall around the city, and started the siege of Plataea.[26]

26 A more recent and theoretical example of the advantage provided to the second mover can be provided by voting models which assume sequence in moves; these models come invariably to the conclusion that the incumbent who runs on her record finds herself in disadvantage (See McKelvey (1979), Kramer (1975), and Glaser Grofman and Owen (1989)).

THEOREM 5. If the game in Table 1 has one pure strategy equilibrium, then at least one of the players has a dominant strategy.[27]

Proof: Without loss of generality, consider that $(a_1,a_2)$ is an equilibrium.[28] By the definition of equilibrium, it follows that $a_1>c_1$ and $a_2>b_2$. There are three possible cases: (1) if $b_1>d_1$, then top is the dominant strategy for player 1; (2) if $c_2>d_2$, then left is the dominant strategy for player 2; (3) otherwise, there are two equilibria. The reason is for (3) is that in this case, both $b_1<d_1$ and $c_2<d_2$ hold, and these are the conditions for $(d_1,d_2)$ to be a Nash equilibrium. QED.

THEOREM 6. In a two-by-two game with one pure strategy equilibrium, the Nash equilibrium is identical with the Stackelberg equilibrium, with the player who has a dominant strategy following.

Proof: Without loss of generality[29] assume that player 1 has a dominant strategy, and the equilibrium is $(a_1,a_2)$. It follows that $a_1>c_1$, $b_1>d_1$, and $a_2>b_2$. If player 2 moves first, he knows that player 1 will always answer by his dominant strategy, so essentially he has to choose between $a_2$ and $b_2$. Since by assumption $a_2>b_2$ the Stackelberg equilibrium is $(a_1,a_2)$. QED.

COROLLARY 2. In a two-by two game with one pure strategy equilibrium if both players have dominant strategies, then sequence does not matter.

Proof: From Theorem 6 follows that each one of the two Stackelberg equilibria is identical to the Nash equilibrium.

Example: In the beginning of the Peloponnesian war the Athenian coalition had a clear superiority in naval forces, while the Peloponnesian alliance was invincible on land. So, each one of the players had a dominant strategy: confront the opponent on his own field. The outcome was that for several years each one of the opponents chose his strategy independently of the other. The Athenians with their fleet raided the coastline of Peloponnesos, while the Spartans with their troops camped outside Athens and devastated the land.

THEOREM 7. In a two-by-two game with a unique Nash equilibrium, if there is a distinct Stackelberg equilibrium, it does not include any of the Nash equilibrium strategies.

Proof: Without loss of generality assume that player 1 has a dominant strategy, and the equilibrium is $(a_1,a_2)$. It follows that $a_1>c_1$, $b_1>d_1$, and $a_2>b_2$. The existence of a distinct Stackelberg equilibrium implies that player 1 moves first (see Theorem 6). If player 1 chooses strategy "top", player 2 would answer "left" (because $a_2>b_2$); consequently $(b_1,b_2)$ cannot be a Stackelberg equilibrium. On the other hand, player 1 would never choose "bottom" if he anticipated the answer "left" (since $a_1>c_1$); therefore $(c_1,c_2)$ cannot be a Stackelberg equilibrium. The only remaining candidate is $(d_1,d_2)$ which does not include any of the Nash equilibrium strategies. QED.

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27 Dominant is the game theoretic word for an unconditionally best strategy, that is a strategy which provides higher payoffs no matter what the opponent does. Theorem 1 gives the important distinctive characteristic of 2*2 games, which as we will see simplifies drastically the solution of the problem of sequence.

28 If instead of $(a_1,a_2)$, there is another equilibrium, one can rearrange the names of the strategies, so that the top left quadrangle is the equilibrium.

29 One can always obtain such a result by permutation of players, and/or strategies.

Theorems 5, 6, and 7 indicate that in two-by-two games with one pure strategy equilibrium one has to focus only on the possible differences between the payoffs of the Nash equilibrium on the one hand and the Stackelberg equilibrium with the player with dominant strategy leading on the other, in order to understand the importance of sequence. There are three possible cases: 1. The two equilibria are identical; in this case sequence does not matter. 2. SE1,1>NE1 and SE1,2>NE2; in this case both players agree that player 1 should move first. 3. SE1,1>NE1 and NE2>SE1,2; in this case there is a struggle for leadership. The remaining Theorems discriminate between these three cases.

DEFINITION. A player has a super-dominant strategy if a strategy produces for him outcomes better than any other combination of strategies in a game. Using this definition we can partition the games with one pure strategy equilibrium into games with super-dominant strategies, and games without super-dominant strategies.

THEOREM 8. If one player in a two-by-two game has one super-dominant strategy, then the sequence of moves does not matter.

Proof: The player with the super-dominant strategy will choose this strategy even if he moves first, and the other player will answer by his best response. Consequently they will arrive at the Nash equilibrium. QED.

In the case of super-dominance, the Nash (and Stackelberg) equilibrium is by definition either the best or the second-best preferred outcome by the player with the super-dominant strategy. Therefore, the only remaining case for examination is when the Nash equilibrium outcome is the third preference of the player with a dominant (but not super-dominant) strategy.

THEOREM 9. In a two-by-two game where one only player has a dominant (but not super-dominant strategy), if the Nash and the Stackelberg equilibrium are different from each other, then if the Stackelberg equilibrium Pareto dominates the Nash equilibrium the two players will agree to let the player with dominant strategy move first; otherwise there will be a struggle for leadership.

Proof: Without loss of generality assume that player 1 has a dominant strategy, and the equilibrium is $(a_1,a_2)$. From the existence of a distinct Stackelberg equilibrium, it follows that this equilibrium is $(d_1,d_2)$ (Theorem 7). The following relationships between payoffs follow:

$a_1>c_1$, $b_1>d_1$, (dominance)

$a_2>b_2$ (Nash equilibrium)

$d_1>a_1$, $d_2>c_2$ (distinct Stackelberg equilibrium).

Or, equivalently $b_1>d_1>a_1>c_1$, $a_2>b_2$, and $d_2>c_2$. There are two possibilities: if $d_2>a_2$ both players prefer the Stackelberg over the Nash equilibrium, they will agree for player 1 to move first; if $a_2>d_2$ both players prefer to move first, so there will be a struggle for leadership.

Table 3 summarizes the cases where there is a unique pure strategy Nash equilibrium, and where the players will agree on a particular sequence, or struggle for leadership.

Example: In the year 431 BC, when the Peloponnesians under Archidamus were about to invade the Athenian countryside, "...Pericles... one of the ten Athenian generals, realizing that the invasion was coming, suspected that Archidamus, who happened to be a friend of his, might possibly pass by his estates and leave them undamaged. This might be either from a personal wish to do him a favour, or as the result of instructions given by the Spartans in order to stir up prejudice against him... He therefore came forward first and made a statement to the Athenians in the assembly, saying that,... in case the enemy should not lay waste his estates and houses, like those of. other people, he proposed to give them up and make them public property..."(Book II: 13). It is difficult to reconstruct the complete game from this excerpt. However, it is reasonable to assume that Pericles would prefer to keep his land than give it up to the Athenians, so he had a dominant strategy. Archidamus' preferences are not completely revealed in the text. However, we have enough information to infer that the Nash equilibrium of the game is: Archidamus spares the property of his friend and Pericles keeps it. Pericles for political reasons prefers the Stackelberg equilibrium where he gives up the land, and Archidamus destroys it. If Archidamus preferred the Nash equilibrium (keep, not-destroy) then he would try to move first; if however he preferred the Stackelberg equilibrium, he would have left Pericles make the first move. The most plausible inference is that there was a struggle for leadership and Pericles through his public statement made the first move. Thucydides does not follow up on this story; however, he does not mention later any exception by the Peloponnesians to the strategy of laying waste to the land.

**INSERT TABLE 3**

To summarize: In games with two pure strategy equilibria where none of them Pareto dominates the other, it pays to move first, and therefore, there will be a struggle for leadership (otherwise sequence is immaterial); in games without pure strategy equilibria it pays to move second, and therefore, most of the time there will be a struggle for followership (see Table 2 for the cases of agreement on sequence); in cases with one pure strategy Nash equilibrium, most of the time sequence does not matter (see Table 3 for the exceptions, that is, cases of agreement on sequence, or cases of struggle for leadership).

This summary provides all the necessary information to answer the questions <u>when</u> and <u>in what direction</u> players will try to influence the sequence of moves. There are two questions which logically follow; the first is <u>how</u> can players affect this sequence?

For the second question, one has to observe that, given that players will try to affect the sequence of moves, institutions which regulate this sequence will demonstrate certain characteristics: for example, institutions which guarantee simultaneity of moves will be generally impartial, except in cases where both players would like a particular sequence of moves, in which case they will be inefficient; institutions which give the first move to one actor will systematically favor this actor in cases where moving first represents an advantage and handicap her in the cases where she prefers to move second (for example, in zero-sum-games). So, the second question is what kind of institutions regulate the sequence of moves, and with what consequences?

Thucydides helps to answer only the first of the two questions. His actors operate in an anarchic environment where winning is not everything, it is the only thing. Therefore, in the remainder of this paper, I will focus exclusively on the question of how actors influence the sequence of moves in a game. I will leave the problem of institutions regulating sequence for another study.

30 For example rules of impartiality guaranteeing fair competitions like sealed auctions, or submission of routines in skating or diving ahead of time (rules that eliminate the advantage of the second mover); rules that regulate circulation (elimination of three lane two direction roads which invite drivers to play the game of chicken); rules that give the first move

## SECTION III: STRATEGIC CHANGE OF SEQUENCE

In this section I will provide all the mechanisms I could find in Thucydides by which one of the opponents influences the sequence of moves for strategic purposes. Therefore, the character of the section will be more inductive and less systematic than the previous sections. Some of the methods or examples may overlap, and no claim of exhaustiveness is made.

In a very abstract way, Thucydides examines two kinds of conflictual situations. If the conflict can be avoided, then the game is a game with two equilibria, and the first mover has (most of the time) a decisive advantage; these situations lead to a struggle for leadership. If the conflict cannot be avoided, or if it is an ongoing conflict (like a battle), then the game is zero-sum and the second mover has a decisive advantage; these situations lead to a struggle for followership.

In all these cases, struggle for leadership or struggle for followership can be resolved in two different ways. The first and obvious is temporal priority. If one of the players manages to move before or faster than the other, then she assures the first move (like the Athenians in Eion).

The second and by far the most frequent way of influencing sequence is by manipulation of the information of the adversary. For example, a unilateral and credible statement of intention assures the first move to a player, while <u>accurate</u> information about the opponent's intentions assures the second move. In the previous sentence the word accurate is underlined, because if the information is inaccurate, then the actor who believes she moves second and will surprise the opponent is, in fact, taken herself by surprise (that is, moves first).

Very few of the examples provided by Thucydides revolve around temporal priority. Most of the cases demonstrate strategic use of information. Intentions are revealed either truthfully (to assure first move) or falsely (to assure second move). In all cases, measures are taken so that the opponent will believe the information.

In all these cases, the game ceases to be a simple two-by-two game with complete information. Each player has additional strategies and most of the time does not know the payoffs of the opponent. The accurate representation and solution of such games presents the problem, besides the complications introduced by incomplete information, that the whole strategy space of either player cannot be exhaustively described; each opponent innovates and introduces new options into her strategy space. Therefore, instead of providing a complete game-theoretic account of the situation, I will use the analysis of games with complete information introduced in section II to account for the reasons why each opponent tries to change the sequence of moves strategically. The rest of this section is organized in two parts: (1). Ways to move first and (2). Ways to move second.

<u>1. Ways to move first:</u>

like power of agenda setting to one actor, like the nomination to the supreme court in the US, or the nomination of the prime minister in European countries.

31 Most of the methods can be found in Schelling's seminal book (1960). What I add here, is a more formal presentation centered around the problem of sequence of moves in Thucydides.

-Credible Commitment. Before the game, one of the opponents can make a statement or send a signal that she will follow one particular strategy. If this statement or signal is believed, the opponent has to select his best response to this particular strategy. Imagine the two drivers heading towards each other in the original version of chicken. If one of them takes out the steering wheel and throws it out the window, the other is going to yield. It is easy to give examples of credible threats, but it is much more difficult to find the general conditions of how a commitment can be made credible, that is, what is the particular characteristic of a statement that makes it believable by the opponent. I will examine some cases.

-Public Statement. It is difficult for an actor, especially a public figure, to renege on a statement made in public. Pericles' public statement about the ownership of his land is an example of credible commitment through a public statement (Book II: 13). Pericles solves the (probable) struggle for leadership problem by making his public statement first.

-Destruction of alternative options. Another means of making a credible commitment is to destroy the alternatives. Consider the following account by Thucydides (Book IV: 11): In 425 BC the Spartans had surrounded the Athenian army by land and sea at Pylos. The geography of Pylos was such that the Spartans could not attack with all their naval forces and they had to bring up a few ships at a time. According to Thucydides' description, Brasidas, who was in command of a Spartan trireme, "when he saw that, because of the difficult nature of the ground, the captains and steersmen, even at points where it did seem possible to land, were hanging back for fear of damaging their ships, he shouted out to them, asking them what was the point in sparing ships' timbers and meanwhile tolerating the existence of an enemy fortress in their own country[32], telling them to break up their ships so long as they forced a landing, and appealing to the allies, in return for all the benefits they had received from Sparta, to sacrifice their ships now for her sake, to run them around, to make a landing some way or other, and to overwhelm the place and its defenders."

Brasidas' perception of the situation is that there are two equilibria in this game, either the Peloponnesians land and the Athenians retreat, or the Peloponnesians retreat and the Athenians keep the territory. So he wants his men by crashing their ships against the coast to give the clear signal that they will be satisfied only with the equilibrium where the Peloponnesians land and the Athenians retreat.[33]

A different way of selecting between equilibria is by elimination of some of the opponent's options, so that the one left leads to the preferred equilibrium. In 411 B.C., when some Athenian citizens were favoring the replacement of the democratic regime by an oligarchy, they were afraid that they would be penalized if they spoke against the regime (Book VIII: 67). So they brought on the floor "one proposal and one only, which was that any Athenian should be allowed to make whatever suggestions he liked with impunity; heavy penalties were laid down for anyone who should bring a case

32 Pylos is in the Peloponnese.

33 According to the remainder of Thucydides account (Book IV: 12), Brasidas' perception of the situation was not correct, because defending the particular spot of land was a dominant strategy for the Athenians, and so instead of retreating when they saw the signal of determination from some Spartan ships, they selected their dominant strategy, defended the land and won.

against such a speaker for violating the laws or who should damage him in any way. Now was the time for plain speaking..."

-Rejection of intermediate options. A case more general than the destruction of available options is the rejection of intermediate options which are available before the game starts. Consider the following game: Player 1 has to choose between an outside option (that is, an outcome that is certain), or to play a simultaneous move two-by-two game with player 2. The simultaneous move game has two equilibria--($a_1$,$a_2$) and ($d_1$,$d_2$)--neither of which dominates the other. According to the second section of this paper, in such a game the players will want to move first, that is, they will engage in a struggle for leadership. Consider also that the value of the outside option for each one of the players is ($e_1$,$e_2$), where $a_1>e_1>d_1$. Figure 2 presents the game in extensive form, and summarizes the assumptions concerning the payoffs. How are the two players going to play this game? In particular, will the existence of the outside option help the players select between the two equilibria of the subgame with the simultaneous moves?

INSERT FIGURE 2

Consider that you are player 2 and you are informed that it is your turn to play. You know that your opponent has refused to accept the outside option, that is, has rejected a payoff of $e_1$. This choice indicates he is not willing to receive $d_1$ , which by definition is less. Therefore, you know with certainty that your opponent will select the strategy which leads to the equilibrium ($a_1$,$a_2$), and you will make the corresponding choice. So, the refusal of player 1 to accept the outside option acts as a signal of his intentions, and leads to the selection of one equilibrium.[34] When strikers lie down in front of the trains which transport strike breakers they send the message that they are determined to win, because they are rejecting the option of keeping their lives and losing their jobs. One can give a similar interpretation to the previous example by Thucydides, where Brasidas asks his countrymen to give up the option of keeping the ships and not landing in order to demonstrate their determination to win.

-Nested Games. Sometimes one or both actors are involved in games in other arenas and with other actors at the same time they are playing against each other. I have called such situations where the move of one actor in one arena has consequences in other arenas nested games. In such situations it is possible for one actor to claim that he has no control over the choice of his strategy; alternatively, the actor can ask a third actor to select the equilibrium and take all the necessary steps so that the selection will be to his advantage.

An example where an actor has no control over his choice of strategy is provided by the debate of the Athenian generals in the Sicilian expedition (Book VII: 48). In the year 413 B.C., after a series of defeats in Sicily, the Athenian generals discuss whether they should return home. The general Nicias, who opposed the Sicilian expedition from the beginning, argued that the army should remain in Sicily because the Athenians would not approve of the withdrawal: "For his own part, therefore, knowing the

34 In game theoretic terms, the game presented in Figure 2 has two subgame perfect equilibria. However, only ($a_1$,$a_2$) is the iterated dominant strategy (or sophisticated) equilibrium, that is, the equilibrium which survives successive elimination of dominated strategies. For the definition of subgame perfect equilibrium and sophisticated equilibrium see Moulin (1981).

Athenian character as he did, rather than be put to death on a disgraceful charge and by an unjust verdict of the Athenians) he preferred to take his chance and, if it must be, to meet his own death himself at the hands of the enemy". Nicias did not proceed to make his reasoning public in order to demonstrate to the enemy (the Syracusans) his determination to select the equilibrium. On the contrary, he did not want the enemy to know that the Athenians even considered the possibility of withdrawal, "for they would then find it much harder to do so secretly, when they did decide upon this step." However, one can multiply the cases where actors involved in chicken games try to make the point that they do not control the process of strategy selection and they are forced to select one particular strategy.

An example where the two actors compete for the favor of a third actor who will select the equilibrium is provided by the arguments in front of the demos of Athens as to who will lead different expeditions. In 425 B.C., the Athenians were discussing the expedition of additional forces against Pylos, and Cleon criticized Nicias for not capturing the Spartan forces on an island close to Pylos. He went so far as to argue that he would have done so if he had been in command (Book IV: 28). Nicias answered that as far as he was concerned, Cleon "could take out whatever force he liked and see what he could do himself. Cleon's first impression was that this offer was made only as a debating point, and so he was ready enough to accept it; but when he realized that the command was being handed over to him quite genuinely, he began to back out of it, saying that it was Nicias, not he, who was general. He was now indeed thoroughly scared, since he never imagined that Nicias would have gone so far as to give up his post to him. Nicias however, repeated his offer and called the Athenians to witness that he was standing from the command in Pylos. The Athenians behaved the way that crowds usually do. The more that Cleon tried to get out of sailing to Pylos, the more they encouraged Nicias to hand over his command, and they shouted at Cleon, telling him that he ought to sail. The result was that Cleon, finding that there was no longer any possibility of going back on what he had said, undertook to go on the voyage."

The account indicates that Cleon believed leading the expedition was a dominant strategy for Nicias, so it did not matter what he said, and he could try to appear tough in front of the demos. However, Nicias preferred Cleon to lead the expedition. In the real game, there were two equilibria (either Nicias or Cleon leading), and Nicias saw to it that the demos would select the equilibrium that he preferred.

## 2. Ways to move second:

-Gathering of intelligence. Since by definition moving second means moving after having accurate information about what the opponent is doing, gathering intelligence is an effective way to move second in zero-sum situations. During the Sicilian expedition, Nicias is reported to have collected information about the enemy from his own private sources (Book VII: 48).

35 Consider the Daniloff affair: the reaction of the American public opinion was in fact strengthening the position of the Reagan administration which could persuasively argue that any result short of unconditional release would not be acceptable by the American people, and would create domestic problems. Similarly, I have argued, that Belgian elites sometimes initiate conflict claiming that they cannot control their followers, in order to force their opponents to aquiesce (see Tsebelis 1989).

-Misinformation of the opponent: From the previous account of the importance of sequence in zero-sum games it becomes clear that whoever moves second wins. On the other hand, a misinformed player moves first (trying to react to what the situation is according to his mistaken beliefs) and gives the opportunity to the opponent to move second. In 411 B.C., the Peloponnesian fleet was sailing to Syme, and as a result of rain and bad visibility the ships lost contact with each other. In the dawn, the Athenians saw only the left wing of the Peloponnesian fleet, and attacked the enemy. The Athenians were quite successful, until the rest of the Peloponnesian fleet appeared and surrounded them on all sides (Book VIII: 42).

One further step is to make strategic use of misinformation, that is, provide false information which (if believed) would make the opponent take a certain course of action, and at the point when this action is undertaken, attack and take the opponent by surprise. One critical step in the whole maneuver is to present the misinformation in such a way that it will be believed. For example, after their victory at sea (413 B.C.) the Syracusans wanted to prevent the Athenians from moving during the night and getting over the most difficult part of the route unopposed (Book VII: 74). So, they sent some soldiers who pretended that they were friendly to the Athenians (in fact, they were Nicias' own informers), asking the Athenian soldiers to tell Nicias not to lead the army during the night since the Syracusans were guarding the roads. The Athenian generals "on the strength of what they were told, put off the retreat for the night, in the belief that the information was genuine."

From the previous account it becomes clear that strategic manipulation of information is crucial in order to determine the sequence of moves from which the outcome of the game is determined. Thucydides (Book VIII: 50-51) provides an account of the consecutive maneuvers of misinformation and treason between two individuals without any scruples: Phrynichus and Alcibiades. These two leaders find themselves in a deadly confrontation with each other. In their game they try to involve three different armies, the Persians, the Spartans and the Athenians, in order to exterminate each other.

-One actor is collective. A more original way of solving the problem of sequence occurs when one or both actors involved are collectivities which can react as unitary actors, or as individual actors. In this case, individual actors cannot undertake strategic initiatives, and as a result they leave the first move to the unitary actor. So, in a confrontation between a unitary and a collective actor, if the collective actor solves the problem of collective action ahead of time the game results in a Nash equilibrium, while if the collective actor is unable to solve the collective action problem, the game results in a Stackelberg equilibrium with the constituent units of the collective actor as followers.

This analysis is particularly relevant in two cases: First, in games with two Nash equilibria, the unitary actor may try to prevent a collective actor from solving the collective action problem, so that he will be able to select the most profitable Nash equilibrium. This is the game theoretic approach to the "divide and rule" maxim. Second, it is possible to imagine cases where a collective actor will strategically choose to be divided so that he will have the second move, or that the opponent will actually want him to solve his collective action problem in order to avoid a Stackelberg equilibrium solution.

Thucydides makes use of situations where actors are kept from solving their collective action problem purposefully twice. Since the methods of keeping the opponents divided are different, I will describe both cases: In the year 415 B.C., when the Athenians make the Sicilian expedition, the Syracusans try to unite all the Sicilians against the Athenian threat. Here is their argument (Book VI: 77): "We have in front of us the example of the Hellenes in the mother country who have been enslaved through not supporting each other; now we find the Athenians employing the same sophistries against us... Are we waiting until we are taken separately, city by city, though we are well aware that this is the only chance that they have of conquering us and we see that this is just the method they are adopting—sometimes trying to create dissention among us by their arguments, sometimes stirring up wars among us by holding out hopes of an alliance with them—doing, in fact, all the harm they can by using the most flattering language possible on every particular occasion?"

We can use the game of Figure 2 to explain exactly how the solution of the collective action problem is equivalent to the selection between the two Nash equilibria of a game. Consider that the Sicilians play a game with the Athenians where they have the strategies of resisting or not resisting, while the Athenians have the strategies of attacking or not attacking. There are two equilibria in this game, the first (preferred by the Athenians) is attack and no resistance, while the second is no attack because of anticipated resistance. Consider also that before playing this game, the Sicilians have the option of uniting or not. Then the selection or non selection of the option operates as a signal for the Athenians as to which equilibrium strategy the opponent will select. So, divided Sicilian nations will accept the attack with no resistance equilibrium, while a united Sicily signals the selection of the second equilibrium strategy (resistance, no attack).

In the second example, the opponents are kept from coordination not by persuasion and promises, but by fear and misinformation. In 411 B.C. some of the Athenians start thinking, under the influence of Alcibiades, that an oligarchy would be the best way to solve all their problems. The public is divided between the supporters of democracy and the supporters of oligarchy, who conspire against the regime. However, the supporters of the democracy are much weaker than their relative numbers for the following reason (Book VIII: 63): "Throughout the democratic party people approached each other suspiciously, everyone thinking that the next man had something to do with what was going on. And there were in fact among the revolutionaries some people whom no one could ever have imagined would have joined in an oligarchy. It was these who were mainly responsible for making the general mass of people so mistrustful of each other and who were of the greatest help in keeping the minority safe, since they made mutual suspicion an established thing in the popular assemblies." The same method of inducing compliance is used by practically all military regimes through their internal security police and informers.

Unfortunately, to my knowledge, there is no example in Thucydides where a collective actor keeps itself divided in order to exploit a second move. The case that comes immediately to mind would be armies being divided in order to win a battle (which is a zero-sum game, where moving second gives the decisive advantage). However, in this case the division of the army is information for the enemy, who is able to move knowing the condition of the opponent.

## CONCLUSIONS

I examined the problem of the importance of sequence in two-by-two games and made three points. The first point was to correct two wide-spread false beliefs. The first mistake is the confusion between first move and surprise attack; closer game theoretic scrutiny reveals that a surprise attack is a second move, since accurate information about the opponent's situation is available. The second mistaken belief is that it is always better to have information about the opponent's intentions. I demonstrated situations where it is better for a player to move first, that is, to publicize his intentions.

The second point is an exhaustive characterization of the conditions under which it is preferable or indifferent for players to move first or second. The conclusions were the following: in games with two Nash equilibria, if one equilibrium Pareto dominates the other, sequence is immaterial; otherwise, there is a struggle for leadership. In games without a pure strategy Nash equilibrium either there is a struggle for followership, or the two players can agree on the sequence (the cases are presented in Table 2). In games with one Nash equilibrium, most of the times sequence is immaterial (the exceptions are presented in Table 3 and include cases where there is struggle for leadership and cases where both players agree on who moves first).

The third point was to use examples from Thucydides in order to investigate how it is possible for actors to alter strategically the sequence of moves. Most of Thucydides' examples can be classified in two broad categories: either a battle is going on or is unavoidable, in which case the game is zero-sum, or one of the opponents gives up, in which case there are two equilibria. As I have shown, the first case typically leads to struggles for followership, while the second to struggles for leadership. Thucydides' examples indicate that actors who want to move first use credible commitments, public statements, destruction of alternative options (their own or the opponent's), rejection of intermediate options, or nested games. Actors who want to move second use the gathering of intelligence, misinformation of the opponent, or the fact that they are collective and not unitary actors.

36 However, there other cases, where actors choose strategically not to unite, in order to select the Stackelberg equilibrium instead of the Nash, and promote their interests. For example Przeworski and Wallerstein (1982, 1988) present a model of capitalism, where, because capital makes not collective but individual decisions (each firm sets its investment rate separately), the Stackelberg equilibrium that results with business as followers is better than all the Nash equilibria. Similarly, Bates and Contreras (1988) dispute Krasner's (1973) findings concerning coffee trade on the grounds that the appropriate equilibrium is not Nash but Stackelberg, since business is divided. Unfortunately this example does not have sufficient structure in order to decide whether lack of coordination of business is in fact a strategic move.

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TABLE 1

Generic game with simultaneous moves.

| | | Player 2 | |
|---|---|---|---|
| | | Left | Right |
| | Top | $a_1, a_2$ | $b_1, b_2$ |
| Player 1 | Bottom | $c_1, c_2$ | $d_1, d_2$ |

TABLE 2

Conditions for agreement on sequence in games without pure strategy Nash equilibrium. (In the absence of these conditions, struggle for followership)

| Player 1 | Player 2 | Leader | Outcome |
|---|---|---|---|
| $a_1>c_1>d_1>b_1$ | $b_2>a_2$, $c_2>d_2>a_2$ | 1 | $(c_1,c_2)$ |
| $d_1>b_1>a_1>c_1$ | $c_2>d_2$, $b_2>a_2>d_2$ | 1 | $(b_1,b_2)$ |
| $d_1>b_1$, $a_1>c_1>b_1$ | $b_2>a_2>c_2>d_2$ | 2 | $(a_1,a_2)$ |
| $a_1>c_1$, $d_1>b_1>c_1$ | $c_2>d_2>b_2>a_2$ | 2 | $(d_1,d_2)$ |
| $c_1>a_1>b_1>d_1$ | $a_2>b_2$, $d_2>c_2>b_2$ | 1 | $(a_1,a_2)$ |
| $b_1>d_1>c_1>a_1$ | $d_2>c_2$, $a_2>b_2>c_2$ | 1 | $(d_1,d_2)$ |
| $b_1>d_1$, $c_1>a_1>d_1$ | $a_2>b_2>d_2>c_2$ | 2 | $(b_1,b_2)$ |
| $c_1>a_1$, $b_1>d_1>a_1$ | $d_2>c_2>a_2>b_2$ | 2 | $(c_1,c2)$ |

TABLE 3

Conditions for agreement on sequence or struggle for leadership in games with a unique pure strategy Nash equilibrium. (In the absence of these conditions sequence is immaterial)

| Player 1 | Player 2 | Leader | Outcome |
|---|---|---|---|
| $b_1>d_1>a_1>c_1$ | $d_2>a_2>b_2,\ d_2>c_2$ | 1 | $(d_1,d_2)$ |
| $a_1>c_1>b_1>d_1$ | $c_2>b_2>a_2,\ c_2>d_2$ | 1 | $(c_1,c_2)$ |
| $d_1>b_1>c_1>a_1$ | $b_2>c_2>d_2,\ b_2>a_2$ | 1 | $(b_1,b_2)$ |
| $c_1>a_1>d_1>b_1$ | $a_2>d_2>c_2,\ a_2>b_2$ | 1 | $(a_1,a_2)$ |
| $d_1>a_1>c_1,\ d_1>b_1$ | $c_2>d_2>a_2>d_2$ | 2 | $(d_1,d_2)$ |
| $c_1>b_1>d_1,\ c_1>a_1$ | $d_2>c_2>b_2>a_2$ | 2 | $(c_1,c_2)$ |
| $b_1>c_1>a_1,\ b_1>d_1$ | $a_2>b_2>c_2>d_2$ | 2 | $(b_1,b_2)$ |
| $a_1>d_1>b_1,\ a_1>c_1$ | $b_2>a_2>d_2>c_2$ | 2 | $(a_1,a_2)$ |
| $b_1>d_1>a_1>c_1$ | $d_2>b_2,\ a_2>d_2>c_2$ | struggle | ? |
| $a_1>c_1>b_1>d_1$ | $b_2>a_2,\ b_2>c_2>d_2$ | struggle | ? |
| $d_1>b_1>c_1>a_1$ | $c_2>d_2,\ c_2>b_2>a_2$ | struggle | ? |
| $c_1>a_1>d_1>b_1$ | $d_2>c_2,\ d_2>a_2>b_2$ | struggle | ? |
| $a_1>c_1,\ a_1>d_1>b_1$ | $c_2>d_2>a_2>d_2$ | struggle | ? |
| $b_1>d_1,\ b_1>c_1>a_1$ | $d_2>c_2>b_2>a_2$ | struggle | ? |
| $c_1>a_1,\ c_1>b_1>d_1$ | $a_2>b_2>c_2>d_2$ | struggle | ? |
| $d_1>b_1,\ d_1>a_1>c_1$ | $b_2>a_2>d_2>c_2$ | struggle | ? |

FIGURE 1

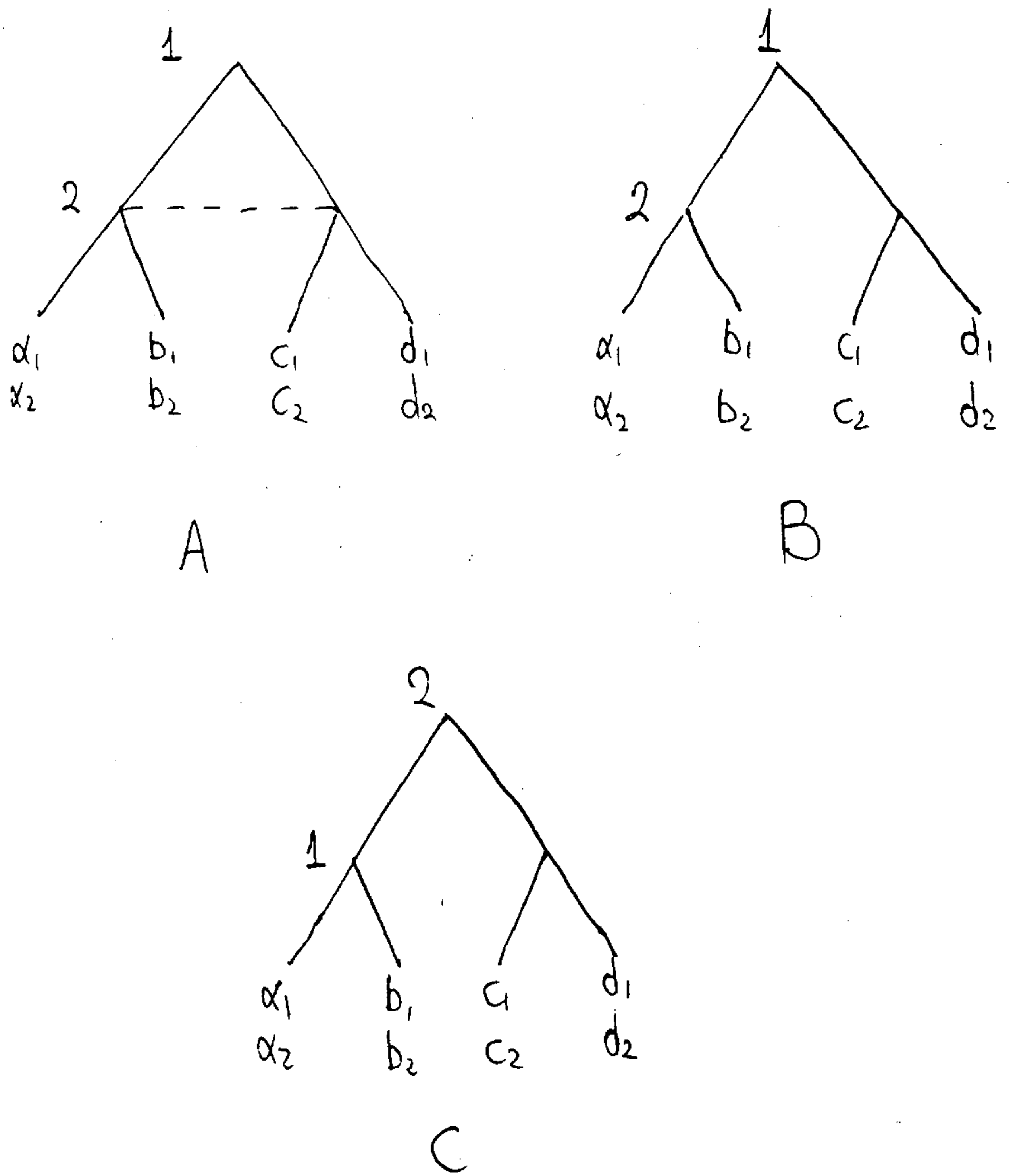
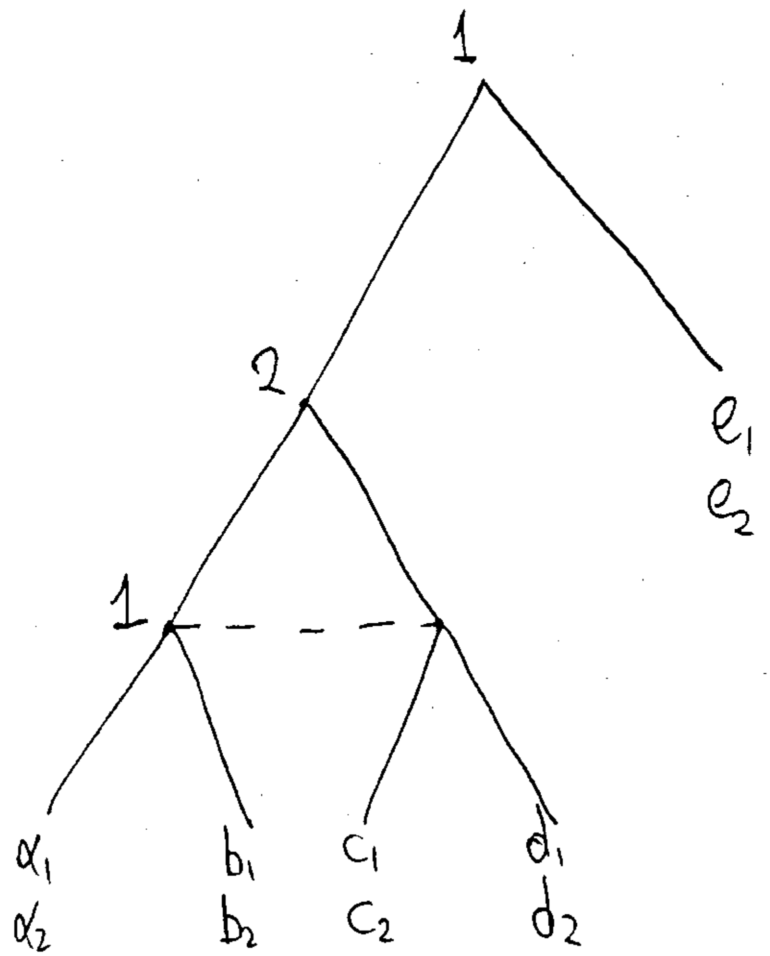

1
2
$a_1$ $b_1$ $c_1$ $d_1$
$a_2$ $b_2$ $c_2$ $d_2$
A
1
2
$a_1$ $b_1$ $c_1$ $d_1$
$a_2$ $b_2$ $c_2$ $d_2$
B
2
1
$a_1$ $b_1$ $c_1$ $d_1$
$a_2$ $b_2$ $c_2$ $d_2$
C

FIGURE 2




Assumptions: $(a_1, a_2)$, $(d_1, d_2)$ equilibria

$a_1 > e_1 > d_1$