A DECADE OF EXPERIMENTAL RESEARCH ON
SPATIAL MODELS OF ELECTIONS AND COMMITTEES

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Abstract

The Euclidean representation of political issues and alternative outcomes, and the associated representation of preferences as quasi-concave utility functions is by now a staple of formal models of committees and elections. This theoretical development, moreover, is accompanied by a considerable body of experimental research. We can view that research in two ways: as a test of the basic propositions about equilibria in specific institutional settings, and as an attempt to gain insights into those aspects of political processes that are poorly understood or imperfectly modeled, such as the robustness of theoretical results with respect to procedural details and bargaining environments. This essay reviews that research so that we can gain some sense of its overall import.
A Decade of Experimental Research on Spatial Models of Elections and Committees

A considerable body of political theory represents alternative outcomes or policies by a subset of n-dimensional Euclidean space, and assumes that we can represent individual preferences over these outcomes by quasi-concave utility functions with internal satiation points. By imposing this particular structure on alternatives and preferences, we can deduce a variety of substantively informative results, such as the Median Voter Theorem in elections and the generic emptiness of cores in cooperative political games (cf. Plott 1967, Schofield 1983).

Correspondingly, the special case of Euclidean preferences, introduced by Davis and Hinich (1966, 1968), form the basis for the most extensively developed models of two general classes of political institutions -- elections and committees. Although Euclidean preferences do not necessarily yield the most general models, they allow us to formulate estimatable statistical models of preference parameters (see Enelow and Hinich 1985 for a general survey). Furthermore, with them we secure a benchmark against which to compare other theoretical results, especially those that concern the relative importance and impact of alternative institutions in the determination of outcomes (cf. Kramer 1972; Shespol and Weingast 1981; Riker 1984).

This latter issue about the effects of alternative institutional structures is the take-off point for the literature this essay reviews -- experimental research on spatial models. Briefly, we can view that research, commencing with Fiorina and Plott's (1978) experiments on committee decision making, in two ways. On the one hand, we can interpret it as testing the basic propositions that such models offer about equilibrium outcomes in two specific institutions -- two-candidate, majority rule elections and n-person, majority-rule committees. Since an important part of this theoretical research consists of ascertaining the impact of procedures such as

the requirement of unusual majorities, issue-by-issue voting, and agendas, experimental research seeks to test the related theoretical propositions. On the other hand, we can view experimentation as providing insight into those aspects of political processes that are poorly understood or imperfectly modeled, such as the robustness of theoretical results with respect to particular procedural details and nuances in bargaining environments. This essay reviews that literature so that we can gain a clearer sense of the role of specific experiments with respect to these two views.

1. The Basic Theoretical Structure and Results

We begin by introducing some notation that allows us to formulate the central theoretical results of spatial models about Condorcet winners. These results are the focus of nearly all experimental research that we review. Briefly, if we let \( X \subseteq \mathbb{R}^n \) denote the set of feasible alternative outcomes (usually a closed and convex set, such as the set that corresponds to a budget constraint), and \( N = \{1,2,\ldots,n\} \) the set of voters or committee members, then the associated utility function for person \( i \in N \), \( u_i : X \to \mathbb{R} \), is written as a function of \( x = (x_1, x_2, \ldots, x_m) \), with the understanding that \( u_i(x) \) is characterized in particular by the ideal point of person \( i \), \( x^*_i = (x^*_i, 1, \ldots, x^*_i, m) \in X \).

Although theoretical results that seek generality impose the weakest assumptions possible on \( u_i \), experimental research is facilitated if intuitively simple forms are used. Hence, the majority of experimental research proceeds by using the Euclidean metric. Briefly, if \( A \) is a symmetric, positive-definite matrix, then, \( u_i(x) = -(x^*_i - x)A(x^*_i - x)' \). In this instance, it's indifference contours in \( X \) are concentric ellipses, where \( A \) determines the specific form and orientation of those ellipses. If \( A \) is a diagonal matrix (\( a_{ij} = 0 \) for all \( i \neq j \)), then a transformation of the axes renders A equal to the identity matrix 1, \( u_i(x) \) becomes...
the simple distance metric, and indifference contours are circles. In anticipation of some experimental results that we review, we note that one alternative to this formulation is the city block metric, defined by \( u(x) = \sum |x_i - y_i| \). In this case, indifference contours over \( X \) are concentric squares, with diagonals parallel to the coordinates of \( X \).

From the perspective of experimental research, the most relevant theoretical results concern the conditions for the existence of Condorcet winners -- outcomes that defeat all other alternatives in \( X \) in a majority vote. We cannot review that research in detail here. However, we should cite Plott's (1967) result for general quasi-concave and continuous preferences. If the number of voters is odd and if at most one person's ideal is at the presumed Condorcet point, then Plott gives necessary and sufficient conditions for the existence of that point in terms of the pairwise symmetry of voter utility gradients. This result shows that the existence of a Condorcet point is rare in multidimensional issue spaces. Indeed, if utility functions are everywhere differentiable, then generically Condorcet winners do not generically exist (Schafield 1983).

Despite these results, we can construct preference configurations that yield a Condorcet winner, and this theoretical research is an especially useful starting point for experimental research. In one dimension, Plott's symmetry condition reduces to Black's (1962) theorem, which is that if utility functions are single-peaked (strictly quasi-concave) and if the number of voters is odd, then the Condorcet winner exists and corresponds to the electorate's median ideal point. If the number of voters is even or if utility functions have "flat peaks," then a Condorcet winner may not exist, but there will exist a set of alternatives that cannot be defeated in a majority rule vote, called the majority rule core. In multiple dimensions when all voters have preferences based on Euclidean distance, the symmetry conditions for a Condorcet point are equivalent to the results of Davis, Degroot and Hinich (1972), who show that a necessary and sufficient condition for the existence of a Condorcet point is that there exist a median in all directions. Given this requirement's fragility, this also shows that Condorcet winners are rare. With respect to city-block preferences, the existence of a Condorcet winner is assured, but only if the dimensionality of \( X \) is one or two (Rae and Taylor 1971).

To interpret experimental results correctly, however, it is important to understand that establishing an alternative as a Condorcet winner is not equivalent to predicting that alternative as the eventual outcome. The definition of the Condorcet winner refers only to a property of the social preference relation under pairwise voting and simple majority rule, and the social scientist's interest in it stems from the implicit judgement that if such a winner exists, then collective decision-making institutions ought to select it, whereas if an institution does not select it, then we ought to have good reasons for regarding the outcomes that are selected as legitimate social choices. Thus, our research typically begins with the question: Does institution yield the Condorcet winner as the eventual outcome if such a winner exists, and what kind of outcomes does this institution yield if there is no Condorcet winner?

Most theoretical and experimental research about voting, committees, and legislatures seek answers to this question. Hence, an important part of that research is the specification of the situation being modeled in such a way that we link individual decisions and collective outcomes to the concept of the Condorcet winner. This means that before making any prediction we must look at the game-form implied by the specific institutions that society uses to choose outcomes -- the strategies available to decision makers and the preferences over these strategies.
implied by preferences over outcomes in X.

This approach yields the principal, descriptive reason for interest in the concept of a Condorcet winner -- namely, the apparent variety of game-forms that render such winners predicted outcomes. Two important game-forms are two-candidate elections and n-person cooperative committees. With respect to elections, if X denotes the strategy spaces of each of two candidates who are competing for electoral victory under simple majority rule, if N is the set of voters, if all voters have perfect information about the candidates' strategies, if candidates have perfect information about voter preferences, and if all persons vote for the candidate whose strategy they most prefer, then a Condorcet winner, if it exists, is the unique Nash equilibrium strategy for both candidates in the corresponding two-candidate, zero sum game. With respect to cooperative committees, if N denotes the members of a committee that must choose an alternative in X, if the committee uses simple majority rule, and if the strategic character of procedures are described by a characteristic-function representation in which majority coalitions can secure anything in X and all other coalitions can secure nothing, then the Condorcet winner corresponds to the game's core -- to an outcome that no coalition has the means or the desire to upset.

Similar analysis also establishes the Condorcet winner as the predicted outcome for other institutional arrangements. For example, if a committee or legislature proceeds by an agenda in which alternatives or amendments to the status quo can be considered only "issue-by-issue," and if preferences correspond to unweighted Euclidean distance (A = I), then, regardless of whether voters act strategically or nonstrategically (in the sense of always choosing sincerely), the unique stable outcome is the one that corresponds to the median preference on every issue — the issue-by-issue median (if we look only at committees in which all members always vote sincerely, we can drop the requirement that A equal I to establish that a stable outcome exists, cf. Kramer 1972, but see Enelow and Hinich, 1983, for an analysis with uncertainty). Since Condorcet winners, if they exist, must correspond to a median in all directions, issue-by-issue voting yields Condorcet winners as eventual outcomes.

Not all game forms, however, imply the selection of Condorcet winners. If all voters are sincere, if one voter is exogenously selected as an agenda setter, and if this agenda setter can form an agenda of any type over a part of the feasible set (in this instance, a finite subset of R^m), then even if the Condorcet winner appears on the agenda, nearly any alternative can be made to prevail. On the other hand, if all voters are sophisticated in the sense that they look ahead to final consequences and vote accordingly, and if a Condorcet winner is on the agenda, then that winner will be selected (see Ordeshook and Schwartz 1987 for a review of these results and Levine and Plott, 1977, for some experimental verification). With respect to elections, if three or more candidates compete under plurality rule, then in even if the election concerns a single issue, the Condorcet winner generally is not an equilibrium strategy for any candidate — equilibrium strategies, if they exist, are lotteries (mixed strategies) over a subset of the issue space (Cox 1986).

These results leave us with five questions -- three theoretical and two experimental. The first experimental question is straightforward: If a Condorcet winner exists and if a game-form predicts that winner as the outcome, will it be selected? The theoretical questions engendered by these results are, first, if a Condorcet winner does not exist, what outcomes should we predict; second, what role does the concept of a Condorcet winner play in alternative institutional arrangements (under alternative game-forms); and what solution hypotheses are appropriate for games with empty cores or for game-forms that fail to yield a
plausible noncooperative equilibrium? The subsidiary experimental question is: What support can we find for theoretical extensions, and how might experimental research suggest viable theoretical approaches?

These questions organize the experimental research that we review. First, with respect to committees, that research includes:

1. testing the core (Condorcet winner) as a predictor in cooperative committee processes;
2. testing the robustness of the core with respect to variations in procedures;

Second, with respect to two-candidate elections and the relevance of Condorcet winners to final outcomes;

3. ascertaining whether candidates converge to Condorcet winners in two-candidate elections.
4. ascertaining whether candidates converge to Condorcet winners when the information of candidates and of voters is incomplete.

Third, with respect to situations in which preferences are configured so that no Condorcet winner describes the social preference order under simple majority rule;

5. identifying possible bounds on the strategies of election candidates, such as those hypothesized by concepts like the uncovered set;
6. developing and testing alternative cooperative solution hypotheses, such as the Competitive Solution.
7. ascertaining the circumstance under which alternative procedural restrictions, such as issue-by-issue voting or the requirement of unusual majorities, induce equilibria;

2. Condorcet Winners in Simple Committees

The seminal experimental essay dealing with spatial preferences is Fiorina and Plott’s (1978) study of cooperative committees. Using the configuration of ideal points \((x_1,\ldots,x_6)\) shown in Figure 1a and a payoff structure that induces circular indifference contours so that the ideal point of voter 5 is a Condorcet point, the broad outline of their procedure is as follows: Beginning with a status quo point (the extreme upper-right corner of the grid), any subject can be recognized by the chair (experimenter) to offer a feasible alternative as a proposed change in the status quo. If this motion is seconded, the alternative is put to a majority vote against the status quo, with the winner becoming the new status quo. This process is repeated until some subject makes a motion to adjourn that is seconded and approved by a majority. Fiorina and Plott consider four experimental contexts, distinguished by two variables: High versus low payoffs, and experiments in which subjects could discuss alternatives after motions are seconded versus experiments in which all discussion is precluded. (In all of the experiments that we review, to insure that it is the spatial dimensions and not mere monetary rewards that are the “issues” under consideration, subjects are prohibited from revealing anything about the cardinal nature of their payoffs or from devising schemes for dividing their winnings at the termination of an experiment.) Figure 1a summarizes the final outcomes of the “high payoff” experiments; Figure 1b reports the “low payoff” outcomes (no general pattern of significant differences between the “with communication” and “without communication” is apparent in their data).

One reaction to these data is that, because only a minority of outcomes correspond to the Condorcet point, we ought to reject the simple hypothesis that “Cooperative majority rule committees choose Condorcet winners if such a winner exists.” On the other hand, we can see that most outcomes -- at least for the high
payoff experiments — are "close" to the Condorcet point in terms of Euclidean
distance, and that this point appears to serve as an important focus for most
committees. Indeed, Figure 2b summarizes the outcomes of a procedurally equivalent
series of experiments using committees with from twenty three to forty five subjects
(Plott 1977). Figure 2a shows the distribution of ideal points for the largest
committee. Six of nine outcomes now correspond identically to the core. Hence, the
deviations that Figures 1a and 1b report may be merely the artifact of a small
experimental committee (which are magnified by decreasing the subjects' monetary
incentives).

There are, however, some aspects to the details of Fiorina and Plott's
procedures that warrant emphasis. First, the method whereby subjects are recognized
to make motions appears to place the experimenter in too central a role. Do
subjects perceive the recognition rule as random or as biased towards particular
subjects, such as those who have not previously made a motion? Second, because
subjects do not know each other's preferences (even ordinally), it is more
appropriate to describe the corresponding game-form as a game of incomplete
information. Our concern here is the fact that with incomplete information, other
institutions occasion seemingly perverse predictions about Condorcet winners
(Ordeshook and Palfrey 1988) and there is no guarantee, a priori, that Fiorina and
Plott's procedure avoids such possibilities. Third, the game-form implied by a
sequential parliamentary procedure should be modeled, since that form need not
correspond to one in which we predict a Condorcet winner. Indeed, coupled with the
subjects' incomplete information, we can speculate that the predictions rendered by
a carefully drawn (and probably intractable) model of the situation will include but
will not be limited to the Condorcet point. In a series of finite-alternative
experiments, however, Salant and Goodstein (1987) confront many of these issues.
directly with a revised parliamentary procedure. Their conclusion that “committees ... are much more likely to select the Condorcet point when “high intensity” preferences are used (p. 17),” nevertheless, reaffirms Fiorina and Plott’s conclusions, although they accept the necessity for introducing the notion of a threshold in order to explain deviations from predicted outcomes.

The objections to Fiorina and Plott’s procedure are summarized by the concern that the implied noncooperative extensive-form game of incomplete information is not known to yield a Condorcet winner as an equilibrium outcome. Rather than revise this procedure incrementally so that we can deduce predicted outcomes directly from its game-form, however, Berl et al (1976) report on a series of experiments using the preference configuration in Figure 3 in which a procedure is implemented that attempts to approximate the rules identifying a cooperative extensive form game as closely as possible (although this essay has a earlier publication date than that of Fiorina and Plott, the inspiration for this research came from earlier drafts of Fiorina and Plott’s essay). In this instance, subjects are informed about each other’s ordinal preferences, and in lieu of a specific parliamentary procedure, subjects can discuss alternatives freely, without interference from the experimenter. And any majority at any time is free to end the experiment after reaching an agreement. The data that Figure 3 portrays strengthens Fiorina and Plott’s conclusions about the attractiveness of the core.

One objection to both the Fiorina-Plott and Berl et al experiments, nevertheless, is that with circular indifference contours, the Condorcet winner is necessarily “central” — the distribution of ideal points must be radially symmetric so that the Condorcet winner corresponds to the ideal point that lies on the intersection of the straight lines connecting the ideals of diametrically opposite voters (assuming that at most one voter’s ideal is at the presumed Condorcet point).
Thus, such a point may be chosen because it is deemed equitable by subjects or because it is a "natural" focal point. Fiorina and Plott, however, also report on experiments using elliptical indifference contours, and the results (see Figure 4) corroborate their earlier conclusions.* For a more dramatic asymmetry in the location of the core with respect to ideal points, Berl et al provide a series of three-person experiments in which city-block utility functions are induced, and in which, with an appropriate rotation of the axes, the core gives the appearance of benefitting one or two specific subjects and thus does not appear to be equitable. Figures 5a and 5b summarize their results. Finally, Figure 6 summarizes the outcomes of some experiments reported by Plott (1978, 1979) using the Fiorina-Plott procedures applied to rhomboid indifference contours with the orientations shown in the figure. In general, all of these variations confirm the attractiveness of the core as a solution hypothesis.

Although we might quibble about the procedural dissimilarities in these experiments, there is a more serious issue concerning the special role of Euclidean preferences and spatial alternative sets. The experimental data that we have reviewed thus far supports the hypothesis that if preferences and alternatives are spatial, if a Condorcet winner exists, and if procedures imply a game-form (or "approximate game form") that links this winner to the core, then that winner is the final outcome, either identically or approximately. McKelvey and Ordeshook (1981), however, offer a series of finite-alternative core experiments in which with one preference configuration the core is selected less than half the time whereas with another configuration the core is selected every time (see also Eavey and Miller, * This figure and Figure 7b are drawn according to the reported data since in each case one observation in the original figure is inconsistent with that data.
A variety of straightforward hypotheses [e.g., considerations of equity, indifference to payoff magnitudes, the experience of subjects, and the completeness of information] fail to explain this discrepancy. These experiments, then, raise the question as to whether spatial configurations with Condorcet winners can be found that will yield equivalent discrepancies. Thus, we must ask: Is the support that the core receives in the Fiorina-Plott and Berl et al experiments an artifact of the fortuitous selection of ideal point configurations, or is there something special about spatial representations? The first part of this question can only be 'answered' by additional experimentation, although the evidence to date strongly suggests that the core's support in spatial experiments is not an artifact. The second part suggests that there is a missing element to our explanations of experimental outcomes -- that a satisfactory explanation must also tell us why people react to a spatial structure differently than they react to other topological representations of alternatives and preferences.

Despite these caveats, experimental research supports the prediction that, at least in alternative sets with a spatial structure, the core (Condorcet winner) of a committee game emerges as the final outcome. Indeed, because they employ different procedures, the comparison of the Fiorina-Plott with the Berl et al experiments suggests (but does not establish) that this prediction is robust to slight variations in procedural details. As further evidence for this suggestion, McKelvey and Ordeshook (1984) offer a series of experiments that graft an important variation of the Fiorina-Plott parliamentary procedure onto their own design. Specifically, these experiments use an identical parliamentary procedure in which subjects must formally make motions to change the status quo, and so forth, with the exception that a motion can differ from the status quo on only one issue. That is, voting must proceed 'issue-by-issue' with the opportunity to reconsider issues in any
sequence. And as before, two types of experiments are considered: Those that allow
discussion and those that preclude discussion among subjects. Thus, agreements to
trade votes across issues implicitly (as is possible in the Fiorina-Plott
experiments via the opportunity to make motions that differ from the status quo on
both issues simultaneously) or explicitly are prohibited in the first case but not
in the second. Theoretically, however, we know that if preferences are
characterized by simple Euclidean distance, then regardless of whether voters are
sincere or strategic, and regardless of whether discussion is or is not permitted,
the core and the Condorcet winner remain equivalent (Kramer 1972). Figure 7a and 7b
plot the results of these experiments with discussion permitted and with it not
permitted. Generally, this data can be seen to support the theoretical prediction
of the core and the insensitivity of this prediction to possibilities for
communication.

3. Condorcet Winners in Simple Elections

Turning now to elections, it is surprising how little experimental research
there is to report here even though our prediction — that candidates will converge
to a Condorcet point if it exists — is identical to the prediction that we make in
the committee setting. McKelvey and Ordeshook (1982) provide a series of two-
candidate, two-dimensional elections with a Condorcet winner — the same preference
configuration as the one shown in Figures 7a and 7b — and, thus, with a unique Nash
equilibrium for the corresponding two-candidate zero sum game. In these
experiments, however, only the candidates are subjects — voters are artificial
actors that simply vote for the candidate closest to their ideal. Each experiment
consists of a sequence of trials in which the same two subjects choose strategies
after observing each other's previous choice and the electoral outcome. Figures 8a
and 8b summarize the candidates' choices in the first 5 and last 5 trials, and they
In the preceding experiments, the subjects (candidates) are perfectly informed about the preferences of each member of the electorate, and the simulated electorate is assumed to be informed about the candidates' issue positions -- the voters choose on the basis of the candidates' actual strategies. In reality, of course, electorates typically are poorly informed about the policies that candidates will implement if elected, and candidates may have only noisy data about voter preferences. As a partial response to such considerations, Plott (1977) offers a series of ten experiments in which candidates are uninformed about the preferences of voters (the number of voters varies in these experiments between nineteen and forty one). Voters know the positions of the candidates and candidates are allowed to query the electorate about preferred directions of change (e.g. "How many would like me to move to the point ?"). Also, a poll is conducted periodically in which the electorate is asked "If the election were held now, how many would vote for?" Although Plott does not show that voting sincerely in the polls is a dominant strategy, if we assume that this is so, then each candidates' equilibrium strategy corresponds to the Condorcet point. Figure 2a shows the distribution of ideal points for the largest experiment, and Figure 9 shows the final experimental outcomes.

These experiments, however, do not confront perhaps the most important feature of information in elections — the low levels of information of voters. Correspondingly, a series of theoretical results by McKelvey and Ordeshook, in conjunction with a series of experimental tests of those results, ascertain the robustness of spatial election theory to such incomplete information. We cannot review the details of their theoretical analyses here (for a more comprehensive review see McKelvey and Ordeshook 1985b, 1987). Briefly, those analyses develop two
general classes of models. The first model applies ideas taken from the rational expectations theory of markets to hypothesize that voters use a variety of readily available cues, such as public opinion polls and interest group endorsements, to estimate which candidate in a unidimensional or multidimensional spatial environment is closest to their ideal point. Specifically, the electorate is assumed to consist of both informed and uninformed voters, where informed voters know the candidates' issue positions (in the unidimensional case some portion of informed voters can correspond to a unitary actor that simply endorses the candidate closest to its ideal point), and uninformed voters must rely on cues—endorsements and public opinion polls of the entire electorate—to make their decisions.

McKelvey and Ordeshook (1985b) report on two unidimensional experiments. In these experiments, each of which consist of a series of eight elections, candidates are completely uninformed about voter preferences. Each election begins with the selection of issue positions by the two candidates, followed by two polls of the electorate in which voters must identify the candidate they think they most prefer, followed by a vote to determine that election's winner. Half of the electorate (plus one) is told the candidates' strategies in each election, whereas each of the remaining voters is told (1) which candidate is to the left and which is to the right on the issue (via the interest group endorsement); (2) the outcome of each poll; and (3) the location of his or her ideal point relative to the entire electorate (e.g., whether it falls in the 30th percentile, counting from the left, the 35th percentile, etc.). Theoretically, this information is sufficient for the realization of a rational expectations equilibrium in which candidates converge to the electorate's median preference and all voters vote as if they had complete information about the candidates.

In the actual experimental data uninformed voters vote "correctly," given their
information, 84.9 percent of the time; and Figures 10a and 10b, which graph the candidates' choices in each election, document their rapid convergence to the electorate's median preference. It appears, then, that the Median Voter Theorem, given an appropriate structure to incomplete information, is robust to such information. However, McKelvey and Ordeshook also report on a series of two-dimensional experiments, run an average of nine election periods, that are far less conclusive. From the perspective of uninformed voters, the details of these experiments differ from their unidimensional counterparts as follows: there is no interest group rendering an endorsement, the electorate is divided into three groups—each consisting of informed and uninformed subparts—a poll within each group is reported to the electorate, and each voter is told the position of his or her ideal point relative to the median preference within each group. Despite these changes, which are necessitated by the theoretical model, uninformed voters vote correctly 85.5 percent of the time. Only three of the six candidates, however, can be said to have converged to positions close to the overall median (see McKelvey and Ordeshook 1985a for a detailed analysis of the data).

We can hypothesize, of course, that multidimensional contests (as compared to unidimensional ones) present candidates with a sufficiently difficult decision environment so that even several election outcomes do not allow them to judge the directions in which they ought to move their issue positions (since 60-90 subjects are required to implement the experimental design, each experiment requires 3-4 hours to administer, which precludes additional trials). Thus, we can also conjecture that if the experiments are run longer, the candidates would converge—that additional data on victories and defeats would decrease the degrees of freedom that candidates have for choosing strategies. This conjecture, however, remains merely an unsubstantiated speculation.
Although subjects participate in a sequence of experiments so that they can learn to process the information made available to them, McKelvey and Ordeshook's first theoretical model in fact focuses on a single election and on the value to voters of cues that arise in a campaign. Much of the empirical literature on voting, however, suggests that voters vote retrospectively. Briefly, the traditional retrospective voting hypothesis is that, in lieu of any knowledge about policies advocated by candidates and implemented by incumbents, voters know only their stream of income or welfare (cf., Key 1966; Fiorina 1981). The empirical question, then, is whether retrospective voting in a spatial context generates signals that lead incumbents to implement Condorcet winners as policy. A second model and series of experiments is developed, then, which only permit voters to choose retrospectively (Collier et al 1987; Grey et al 1986).

In this second set of experiments, each subject that plays the role of a voter is assigned a spatial utility function (simple Euclidean distance in either one or two dimensions) that is not revealed to anyone. Instead, the incumbent (initially selected at random) chooses a spatial position that is translated, via the assigned utility functions, into a payoff and it is this payoff only that is revealed to each voter. Voters must then choose between reelecting the current incumbent and electing the challenger. Notice that with this procedure, a challenger does not adopt a policy until he or she becomes an incumbent. Thus, election campaigns are nonexistent, voters must act on the basis of the past stream of payoffs associated with the two potential incumbents, and the sole source of information available to candidates is the history of spatial positions adopted by incumbents and the votes those spatial positions earned incumbents.

Evaluating voting behavior in this context is difficult. The absence of structure in each experiment means that there are a great many alternative decision rules that voters could reasonably employ (e.g., vote to reelect the incumbent if the incumbent's policy yields a payoff greater than the discounted average of the past, or greater than the discounted average of the challenger's past, or if the payoff is no worse than the payoff from the previous incumbent, etc.). But with respect to the more important question of what policies incumbents choose, these experiments suggest that, despite the limited information available to subjects, Condorcet winning spatial positions remain powerfully attractive to the candidates. Looking at both the unidimensional and the two-dimensional experiments, Table 1 reports the results of the simple regression \( y = a + bx + e \), where \( x \) is the location of the electorate's median preference and \( y \) is the actual position adopted by the incumbent (in the case of two dimensions, each dimension is treated as a separate source of data for the regression). Notice that if incumbents converge identically to the median, then we would have \( a = 0 \) and \( b = 1 \). The data that this table reports shows that such perfect convergence does not occur, but they confirm that, on average, the candidates do approach the median as the experiment proceeds.

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<th>Table 1: Estimates of Regression</th>
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At least for the limited information structure imposed by these experiments, then, the Condorcet winner remains attractive to candidates. This does not mean that the sole sufficient condition for the median's attraction is that it merely exist. The experiments reviewed in this section support the proposition that, experimentally, candidates will converge to equilibrium strategies -- that the definition of rational choice offered by noncooperative game theory has empirical foundation. But we cannot conclude that Condorcet winners prevail if the situation's game form is changed so that equilibrium strategies correspond to something other than that winner. Consider, for example, the fact noted earlier that even if an election is unidimensional or if the distribution of ideal points is radially symmetric, equilibrium strategies in general cannot correspond to the Condorcet winner if there are more than two candidates. The experimental evidence that Figure 11 summarizes supports this view. This figure shows the positions of victorious candidates in a series of three-candidate elections (Plott 1977), and although the data is near the Condorcet winner, a comparison of Figures 11 and 9 reveals that outcomes are more dispersed in three-candidate elections than in their two-candidate counterparts. Thus, the mere addition of a candidate changes equilibrium strategies, that this change is reflected in the experimental data.

3. Elections without Pure Strategy Equilibria

Much of the early research on spatial election models focused on sufficient conditions for the existence of readily identifiable pure equilibrium strategies for candidates. Indeed, owing to the obsession of this research with uncovering such conditions, it is safe to infer that many scholars believed at the time that little could be said about political outcomes if such strategies did not exist. However, beginning with McKelvey and Ordeshook's (1976) development of the admissible set and
Cramer's (1977) application of the minmax set, and proceeding to the formulation and application of the uncovered set (Miller 1980), we now have good theoretical reasons for supposing that candidates in two-candidate elections without a Condorcet winning platform will not diverge "much" from the median voter preference on each spatial dimension (for specific bounds on the divergence see McKelvey 1987). The uncovered set is especially important in this regards since all strategies outside of it are necessarily dominated by strategies within it, in which case the support set of mixed strategy equilibria must fall within the uncovered set.

As an initial test of these ideas, McKelvey and Ordeshook (1982) offer a series of five- and nine-voter, two-candidate experiments. Figures 12a and 12b summarize the choices of candidates in the first and last five rounds of the five-voter experiments; Figures 12c and 12d summarize the nine-voter experiments (recall that the voters here are artificial and that only candidates are actual subjects). Three important conclusions are evident from these data. First, the candidates' strategies are not scattered randomly across the entire issue space; rather, they are constrained for the most part to Pareto-optimal positions. Second, the dispersion of positions decreases as the experiments proceed. Although the candidates do not appear to be converging to a specific point (compare these figures with Figure 8b), there is an evident tendency for candidates to select positions near the "middle" of the space — positions that fall generally in the support set of a mixed strategy solution to the game. Finally, the dispersion of positions is less with nine voters than with five voters. Hence, since, in general, the size of the uncovered set decreases as the density of ideal points increases, these data are consistent with the hypothesis that candidates are attracted to positions within this set.
4. Unrestricted Majority Rule and Committees without cores

Although there are sound reasons for supposing that the description of mixed minmax strategies drawn from a static description of an election is of limited empirical relevance owing to the sequential nature of campaigns, the uncovered set itself is important because it places limits on undominated spatial positions. This fact, in turn, provides a theoretical basis for supposing that candidates will not "wander too far." But matters are far less clear for cooperative committees without cores. Although we can show that various cooperative solution concepts such as the V-set necessarily predict outcomes that are uncovered, our confidence in these concepts does not equal our confidence in the core or in the hypothesis that people will not choose dominated alternatives. Consequently, experimental research into cooperative spatial committees without cores serves more of an exploratory function -- testing tentative ideas and suggesting alternative approaches. Unfortunately, owing to the specific concerns of individual researchers, this research comprises something less than a well integrated experimental package. Indeed, procedures often are incomparable even though the experimental evidence suggests that variations in these procedures may be of profound significance.

One model of cooperative spatial committees without cores is McKelvey, Ordeshook, and Winer's (1978) competitive solution. Briefly, the invention of competitive solution is a response, first, to the fact that concepts such as the main-simple V-set and the various bargaining sets are typically empty for spatial games without cores, and second, to the unreasonableness of the supposition that the absence of a core implies "chaos" in simple majority rule institutions. To test this solution hypothesis, McKelvey, Ordeshook, and Winer offer a series of spatial experiments in which: subjects are told the ideal points of other players, and everyone knows that all indifference contours are circles; free and open discussion
is permitted — no specific parliamentary voting mechanism is employed — with the
sole prohibition that subjects cannot reveal their actual monetary payoffs; and an
experiment ends when some majority indicates that it has reached an agreement, and
the members of that majority sign a document to that effect. Figure 13 reports the
outcomes of six experimental sessions. Similarly, Figures 14a through 14d report on
a series of sixty experiments reported by Ordeshook and Winer (1980) that use
weighted voting (the weights are the numbers beside each committee member’s ideal
point). In general, these data support the competitive solution and thereby
disconfirm the supposition that the absence of the core implies chaos.

Laing and Olmstead (1978) report on a series of spatial experiments that use
different configurations of ideal point but identical procedures with one exception
subjects play a sequence of otherwise independent games. Their results, then,
are difficult to interpret because of the possibility of multiple-trial agreements
among subjects. Indeed, Laing and Olmstead exclude two experimental sessions out of
19 from their analysis owing to explicit agreements to choose successively the ideal
points of different players. With this qualification in mind, Figure 15 summarizes
the outcomes from one of their preference configurations. This particular
configuration is especially interesting because it does not appear to yield a
competitive solution, and thus serves as a counter-example to the conjecture that
this solution exists in general for all simple games. To the extent that the
outcomes in Figure 15 are close to an “approximate” solution, then, suggests that
although the competitive solution captures essential features of bargaining with
spatial preferences, this concept either needs refinement or that a new hypothesis
must be formulated that is consistent with the competitive solution’s predictions.

Laing and Olmstead raise a theoretical problem for the competitive solution in
particular, but with a series of finite alternative (non-spatial) experiments
FIGURE 14a: Weighted Voting Test of Competitive Solution
(Ordeshook-Winer 1980)

FIGURE 14b: Weighted Voting Test of Competitive Solution
(Ordeshook-Winer 1980)
FIGURE 14c: Weighted Voting Test of Competitive Solution
(Ordeshook-Winer 1980)

FIGURE 14d: Weighted Voting Test of Competitive Solution
(Ordeshook-Winer 1980)
designed to discriminate among cooperative game-theoretic solution hypotheses, McKelvey and Ordeshook (1983) show that this solution need not predict outcomes. In fact, the data they report suggest that the cardinality of utility plays an important role in the patterns of outcomes. Miller and Oppenheimer (1982) attempt a systematic study of the effects of the cardinality of preferences, and they also show that cardinality affects the performance of the competitive solution. It is evident, then, that much critical theorizing remains before any definitive conclusions can be reached concerning the specific outcomes that prevail in a cooperative environment.

The experimental evidence on cooperative games without cores, then, yields the following general conclusions. First, the absence of a core does not imply incoherence or chaos — there are patterns to the data that warrant explanation. Second, 'classical' solution hypotheses such as the V-set and the various bargaining sets are, as presently formulated, wholly inadequate. Third, the competitive solution captures an important feature of bargaining and coalition formation, but, for theoretical reasons, it cannot be regarded as a satisfactory general hypothesis. Finally, although the distribution of outcomes is somewhat sensitive to the cardinality of preferences when the core is not empty (cf. Salant and Goodstein 1987), cardinality appears to be more important when the core is empty.

5. Committees and the Consequences of Institutions

The experiments that the previous section reviews concern a particular institution — unrestricted majority rule -- but it represents merely one institution drawn from a potentially infinite variety that might describe actual political systems. And alternative institutions can have significant theoretical consequences: They can change the nature of the core, they can induce a non-empty core for cases in which no such equilibrium exists under unrestricted majority rule,
or they can render the core empty. An especially graphic illustration of the first possibility is offered by Kormendi and Plott (1982). Using a preference configuration and procedures that match the open-discussion experiments of Fiorina and Plott (1978) but which employ a non-monetary preference inducement mechanism (student grades were dependent on their performance as subjects), Figure 16a reports the results of seven experimental outcomes, all clustered tightly about the core.

In Figure 16b, however, voter 4 is the only subject allowed to make motions. Hence, voter 4 must be a member of any coalition to upset the status quo, in which case the core corresponds to all outcomes that lie on the dashed line connecting 4's ideal point to the original core. Similarly, in Figure 16c, voter 4's role is given to voter 5 so that the core now corresponds to the dashed line shown there. Figure 16c especially shows how this simple procedural variation, by changing the core, changes the pattern of outcomes.

Laing and Slotznick (1987a) consider a different mechanism for inducing a nonempty core — four-fifths majority rule: Referring to the five-person preference configuration in Figure 17, the interior hatched quadrilateral (called the "heartland" by Laing and Slotznick) corresponds to those points that are Pareto optimal for every winning coalition (every coalition with four or five members). Under simple majority rule, such points, if they exist, correspond to the core, whereas for the particular preference configuration in this figure, simple majority rule yields an empty core (indeed, this Figure depicts Laing and Olmstead's example of a spatial game with no competitive solution). With unusual majorities, however, we cannot identify a core without referring to a status quo point, since we must also identify the outcomes that blocking coalitions can ensure. Suppose, then, that the point s' is the status quo. The two 'lenses' attached to this point denote the alternatives that defeat s' under four-fifths rule, and the intersection of these
lenses with the heartland is the core of the corresponding game. If these lenses fail to intersect the hatched region, as is the case with a status quo at s^2, the core is empty. Hence, theoretically, we predict outcomes "near" or in the intersections of these lenses with the heartland when the status quo is s'; and although there is no specific prediction for this game when there is no core, we should anticipate a wider dispersion of outcomes when the status quo at s'.

Laing and Slotznick report on ten experiments using the preference configuration in Figure 17, as well as two other configurations, in conjunction with the open discussion procedures that McKelvey, Ordeshook, and Winer (1978) apply to their test of the competitive solution. Figure 17 reports their data, where x's denote outcomes with the status quo at s' and o's denote outcomes with the status quo at s'. In this instance, it is apparent that the evidence is merely suggestive and not conclusive. The x's do in fact tend to cluster more strongly than the o's, but only one x is in the core itself. The particular feature of Laing and Slotznick's procedure, however, that may account for these discrepancies is the one we cite earlier: namely, subjects play a series of experiments and are thereby allowed the opportunity to reach multi-trial agreements. Indeed, two experimental series are deleted from the study owing to the explicit appearance of such an agreement. Grether et al (1981), for example, report on a series of single-trial experiments using unanimity rule that strongly support the theoretical prediction that unanimity renders the status quo the unique core outcome. Laing and Sloztnick (1987b), on the other hand, describe a series of unanimity experiments in which, although the core preforms better that all other hypotheses considered, sharp deviations from the core are evident. Again, though, they employ a procedure in which subjects play a sequence of games in which multi-trial agreements are possible.
The role that a status quo plays depends, of course, on the procedural choice context. In Laing and Slotznick's experiments, that role is determined by the four-fifths voting requirement. An alternative possibility is that committees proceed by an endogenously determined agenda in which the status quo enters the agenda, as in the Fiorina-Plott experiments, as the first motion to be voted on (forward agendas), or as the alternative entered in the agenda's final stage (backward agendas). In a forward agenda, the status quo, theoretically at least, should play little role in the final outcome, whereas in the backward agenda, only outcomes that defeat the status quo should be selected. Wilson (1986) documents this fact in a series of experiments played by subjects, without discussion, over computer terminals. Figures 18a and 18b summarize his findings and show that with a backward agenda, outcomes always fall in the lenses that correspond to alternatives that defeat the status quo; but with forward agendas, no such pattern is evident.

By precluding the possibility of direct communication and discussion, Wilson ensures that the agenda formed by a decentralized and computerized process is adhered to by committee members. Unfortunately, we have little theoretical knowledge about optimal strategies for endogenous agenda setting (for some results, see McKelvey 1986) and thus we cannot interpret further the dispersion of outcomes that Figure 18b reports. We can, however, offer one speculation: If communication and coordination are allowed, other aspects of procedural structure should be less influential. Correspondingly, McKelvey and Ordeshook (1984) study a different procedural variant that induces a stable point for a preference configuration that does not yield a core under unrestricted majority rule.

Theoretically, if voting proceeds issue-by-issue without discussion, and if indifference contours are circles, then the unique stable point is the issue-by-issue median (Kramer, 1972). Figures 19a and 19b summarize the outcomes of the
experiments designed to test this hypothesis. The procedures adopted here correspond identically to those that Fiorina and Plott use when subjects are not allowed to hold any discussion, with the exception that motions can differ from the status quo on only one dimension. The comparison of these figures with the data that Figure 13 reports when unrestricted majority rule is in force reveals the influence on outcomes of an issue-by-issue constraint. Notice in particular, that unlike the experimental outcomes that prevail with unrestricted discussion (see Figure 13), nearly all outcomes here lie in the interior pentagon of the figures. Further, when preferences are rotated so as to move the location of the issue-by-issue median, the mean outcome tracks that point (compare Figures 19a and 19b).

The theoretical prediction that follows from Kramer's analysis supposes that committee members cannot communicate to form binding agreements. If such agreements are allowed, then the issue-by-issue procedural context as well as the specific orientation of ideal points in the space become theoretically irrelevant, and the committee is effectively acting under unrestricted majority rule. With this argument in mind, McKelvey and Ordeshook also report on an experimental series that requires issue-by-issue voting, but which permits subjects to discuss matters freely prior to each vote. Figures 20a and 20b summarize their results, and the comparison of these data to the data in Figures 13, and Figures 18a and 18b is informative. Specifically, notice that the dispersion of outcomes in Figures 20a and 20b falls somewhere between the dispersion in Figure 13 and the dispersions in Figure 19a and 19b, and the mean outcome moves away from the issue-by-issue stable point. The inference drawn from these data is that as the constraints on the imposition of a procedure such as issue-by-issue voting are relaxed -- as the costs of reverting to unrestricted majority rule are decreased -- majority coalitions that find it in their interest to bypass restrictive procedures will in fact bypass them. And if
these costs are zero, outcomes should follow the pattern in Figure 13.

6. Conclusions

It has been slightly more than ten years since Fiorina and Plott and Berl et al published their experimental results about cores and Condorcet winners. This research and the research proceeding from it tells us, first, that the concept of the core, at least in the context of spatial preferences and majority rule, is predictive for a wide range of institutions. Second, the Condorcet winner as an equilibrium strategy for candidates in two-candidate elections has impressive credentials as a predictive concept. There are disquieting notes to be sure, such as those finite alternative experiments in which cores are not chosen by subjects. But the experiments run to date leave no doubt that the development of political and economic theory must heed the definition of rational action encapsulated by these concepts.

Although experimental research gives us confidence in our theoretical research in situations in which a core exists, it can make a more impressive contribution. With the discovery that Condorcet winners are rare with spatial preferences and that cyclic social preferences can extend across the entire space of feasible alternatives (McKelvey 1976, 1979), some scholars were led to the belief that political processes are inherently unstable and unpredictable (Riker 1984). Others, believing that only the possibility of rendering unique predictions is removed if a game's core is empty, and the like, began the development of alternative equilibrium notions such as the uncovered set and the competitive solution. Experimental research supports this second view. Additionally, it has guided the development of new equilibrium concepts, and has led us to reject outright ideas such as the V-set and the bargaining set, and to seriously question the generality of alternative cooperative solution hypotheses such as the competitive solution.

Finally, experimental research contributes in a natural way to our understanding of the role of procedures and institutions in economic and political processes. Experimental research, by its very nature, is intimately involved with institutional design. Every set of instructions and every variable considered in the implementation of an experiment, is an aspect of institutional design, which renders the experimentalist especially sensitive to institutions and their implications. Not unsurprisingly, then, experimental research, taken as a whole, gives us a view of the sensitivity and insensitivity of outcomes to alternative formal and informal procedures. The data, for example, suggest that knowledge about the preferences of others plays little if any role in committees with unrestricted majority rule; procedures such as issue-by-issue voting can restrict outcomes and induce stability, but only to the extent that those procedures also limit the ability of committee members to communicate directly and negotiate agreements; the requirement that only unusual majorities can modify the status quo works in a theoretically proscribed way; theoretical predictions work better in larger committees; and the theory of two-candidate elections is robust to important forms of incomplete information.

Experimentation also makes us sensitive to procedural effects about which we know little theoretically. For example, Hoffman and Plott (1983) suggest that formal procedures can significantly effect the consequences of informal pre-meeting discussions. Quoting their conclusion at length, they surmise that 'If the rule is simple majority, with no Robert's Rules provisions, the coalitions formed during the pre-meeting discussions simply vote in the agreed alternative with very little discussion or debate. Under the Robert's Rules provisions which can for discussions and the orderly submission of amendments, the proposal advanced by the pre-formed coalition is immediately challenged by amendment. During the ensuing discussion
information is generated which reveals the potential advantages of other possibly successful motions ... It seems that the institutional arrangements foster a search and informational gathering activity (p. 34)."

In summary, then, the experimental research that this essay reviews gives us considerable confidence that the large body of theoretical research into spatial models of committees and elections is not without sound empirical content. Certainly, those models may have to be elaborated before we can apply them directly to the study of, for example, the U.S. Congress, Parliaments, and actual elections. But those same experiments, in addition to revealing fruitful avenues of opportunity for the theorist, also suggest how theoretical models can be adapted to environments outside of the laboratory.

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