Intergenerational Equity, Social Discount Rates and Global Warming

by

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Abstract

This article is about the logic underlying social discount rates. We argue that these rates are not ethical raw material, but are derived from the more fundamental notion of justice among generations. A number of approaches to the concept of intergenerational justice are discussed, and it is argued that the most compelling formulation available to us is the one long been in use among economists, namely, the Ramsey-Koopmans theory. This theory advocates that investment projects having long-run effects should be subjected to the same conceptual treatment as those that affect only the near future. We show that social discount rates depend on the numeraire, and that methods of estimating them depend on the institutional setting within which social cost benefit analysis is assumed to be undertaken. We also show that it is incorrect to advocate project-specific discount rates as a way of conserving environmental resources.

Social discount rates have universally been taken to be positive, on grounds that the rate of return on investment is positive. But if consumption and production activities give rise to environmental pollution as a by-product, the social rate of return on investment could be zero even when the private rate is positive; at the very least, the social rate would be lower than the private rate. The current practice among most global energy modellers of relying exclusively on (risk-free) market rates of return for estimating optimal carbon taxes is conceptually faulty. In the context of a formal model of environmental pollution we show how, even along an optimal programme, social rates of discount can be zero. We also demonstrate that in certain institutional settings, social discount rates can be negative.

CONTENTS

1. What are social discount rates and how might we estimate them?

Imagine a public authority engaged in an intertemporal optimization exercise - say, on the choice of the aggregate rate of investment in an economy. We will assume that the authority is morally at ease with the social welfare function it is charged with maximizing, and is confident that it has taken both technological and institutional constraints effectively into account. The institutional constraints consist of, among other things, the responses of private bodies and other public authorities to its decisions. So, depending on the context, the extent to which the authority in question can influence the economy may be great or small.

It is a well-known fact that, under certain circumstances, the optimal programme can be decentralized.[1] By this we mean that there is a set of accounting prices which, if used in the evaluation of small investment projects, could sustain the optimal programme (Arrow and Kurz, 1970; Little and Mirrlees, 1974).[2] Thus, investment projects are perturbations to the macro-economy (Section 5); the criterion for project selection is social profitability; and the social profitabilities of projects are estimated on the basis of accounting prices, not market prices.

Accounting prices are state-contingent. And it is on this point that the theory we are relying on is weak: the theory of choice under uncertainty, particularly the kind of uncertainty that makes possibilities in the distant future only dimly visible, is relatively undeveloped. Certain precautionary considerations, such as the value of keeping options open, do follow from the classical expected-utility theory (Arrow and Fisher, 1974; Henry, 1974); these features are, of course, a plus for the theory. On the other hand, the expected-utility theory runs into difficulties when used in the evaluation of projects with long-term effects. For example, it may simply not be possible today to evaluate environmental risks in the distant future. Bewley (1989) has developed an interpretation of Knightian uncertainty that offers a reason why we ought to be reluctant to undertake activities involving unevaluatable risks; that is, why the status

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[1] We are, of course, assuming that an optimum exists.

[2] The circumstances are those in which the Kuhn-Tucker theorem holds. Accounting prices are sometimes called "shadow prices" and also, by the technocratically minded, "Lagrange multipliers". In what follows, public policy choice will be interpreted as choice over investment projects.

quo should be favoured. But these are early days for such theories as Bewley's. At the moment, we do not have a theory, normative or otherwise, that would cover long-term environmental uncertainties in a satisfactory way. So, in what follows, we will ignore uncertainty. If this appears otiose, readers can imagine that we are working within the expected-utility framework.

Let a numeraire be chosen for the accounting price system. At any given date the social rate of discount is the percentage rate at which the accounting price of the numeraire declines. Formally, let R_t be the quantity of the numeraire you have to pay today for a unit of the numeraire to be delivered at time t along the optimal programme. Assuming continuous time and a differentiable accounting price, $-[dR_t/dt]/R_t$ is the social rate of discount at t.

The basis of the definition is this: a repeated use of social discount rates across time enables us to compute social discount factors, which can in turn be used to convert the spot prices of various goods and services in the future into their present-value prices. This means, of course, that, when we speak of a project's social profitability, as we have already done, we mean the present discounted-value of the project's flow of net social profits.

One immediate corollary is that social discount rates, typically, depend on the numeraire (Section 3). The reason is that, unless the optimal programme is a steady state, relative spot prices change over time. However, project selection is unaffected by the choice of numeraire. Projects that are socially profitable (resp. unprofitable) with one numeraire would remain socially profitable (resp. unprofitable) if the numeraire were something else.

A second immediate corollary is that, typically, social discount rates are not constant over time; they vary over time. In other words, except under special circumstances, the accounting price of a numeraire does not decline at a constant exponential rate (Section 3). The near-universal practice of using a constant discount rate in project evaluation (e.g. 5 percent per year applied to the net income of a project over its entire life) has grown out of a need for practical convenience, it is not a theoretical prescription.

A third immediate corollary is that social discount rates are not project-specific:

the same set of discount rates should be used in the evaluation of all projects. For example, if society were to value biodiversity greatly, it would, rightly, expect this concern to be reflected in the social profitability of such projects as those which protect biodiversity. It is on occasion suggested that preferential discount rates ought to be used in the evaluation of such projects, so as to make them look good. But the suggestion is wrong. If society were to value biodiversity greatly, this concern would be reflected in the accounting prices of biodiversity in the future; they would be high, and the magnitude of biodiversity loss in the future, even when discounted to the present, would be large.

A fourth immediate corollary is that, because social discount rates connect accounting prices at different dates, they are a derived notion. Reflecting as they do both the possible and the desirable, social rates of discount are not ethical raw material; they are endogenously determined within the optimization exercise.[3]

Thus far an optimizing institution. An extreme special case of it is one where the choice of investment projects is conceived of as a sequence of reforms. The thought-experiment here is that small investment projects are chosen one at a time, in a sequential manner. Accounting prices in this scheme of things are estimated from the prevailing structure of production and consumption, at each stage of the sequence; they are not obtained from an optimal programme. However, social rates of discount in this context connect accounting prices at different dates in the same manner as in the optimizing institution: at any date the social discount rate is defined as the percentage rate at which the accounting price of the numeraire declines.

Consider now a sequence of choices: at each stage small investment projects are evaluated at the marginal social valuations of goods and services at that stage, and only socially profitable projects are accepted. Such a process of choice is called a "gradient process". Under certain conditions (viz. when the economy satisfies strong convexity properties), a sequence of project selections obeying a gradient process eventually leads

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[3] Arrow et al. (1996) provides a summary of the various guidelines we possess on how social discount rates can be estimated from observable data.

the economy to the optimal programme.[4]

There is a third institutional framework within which project choice can be studied. Here the focus is on some small sector of an economy. Assume that project selection in the sector is conducted optimally. Being a small sector, decisions made within it leave the rest of the economy unaffected. This is the world of partial equilibrium. Social rates of discount in such exercizes are exogenously given: they are not determined from the optimization exercise, they are inferred from a forecast of the behaviour of the rest of the economy. Early models of global warming were cast in such a mould (e.g. Nordhaus, 1977). These models were instructive. But "partial equilibrium" models are an inappropriate arena in which to build models of global warming, because global warming is expected to have economy-wide effects on production possibilities; indeed, world-wide effects. This means that the welfare economics of global warming needs to be developed in the context of optimizing economies. This is done in Nordhaus (1994) and Nordhaus and Yang (1996).[5]

We began by observing that if public authorities are to optimize, they have to be able to evaluate alternatives by means of a social welfare function. A prior question is whether a public authority is at all needed for reaching good investment decisions. To ask the question another way, does mere temporal distance between us and future people provide a reason why we cannot trust our private preferences to be ethical? In short, does temporal distance imply empathetic distance?

2. Us vs. the future: parental concern as a recursive relation

A vast literature on optimal economic development (e.g. Chakravarty, 1969) would suggest that it does provide a reason. The literature gives the impression that future generations are at risk from the present's lack of a natural concern for them; so that if deliberative collective action were not taken, say, through the agency of a public authority, the present would blow its inherited wealth. The thought here is that future

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[4] See Uzawa (1958) and Arrow and Hurwicz (1958) for a formal demonstration; and see Dasgupta, Marglin and Sen (1972) and Ahmad and Stern (1990) for the development of practical methods for project evaluation and policy reform based on the finding.

[5] Other well-known and instructive models of the economics of global warming are Cline (1992), Manne and Richels (1992), and Peck and Teisberg (1992).

generations cannot bargain with us, nor can they remind us of our obligations to them. Their absence today is a reason why they are at risk from our rapacity.

We are talking here of a possible lack of both moral and instinctive motivation to care for future generations, we are not referring to the constraints that may prevent us individually from investing the amounts we would ideally like to invest (e.g. because capital markets are imperfect, or because many of our actions give rise to current and future externalities, or whatever; see Section 5). And when we recognise this distinction (i.e. the distinction between what we desire on reflection and what we are able to achieve), we begin to doubt if we are as rapacious as the literature on economic development often suggests we are. There are people who feel that their ancestors saved too much for the future; that they should have sacrificed less for their descendents. To be sure, it is not unacknowledged by such people that, in the process, their ancestors also damaged many ecosystems. But this could have been due to ineffectual property rights over such resources, it need not have reflected a lack of concern on the part of the ancestors for their descendents. For example, the standard of living in Western Europe has increased more than eight-fold since the start of the Industrial Revolution. This happened through an accumulation of physical, human, intellectual, and organizational capital (and, almost certainly, a concommitant decumulation of natural capital). To look at the contemporary world, gross investment rates in East Asian countries have been well in excess of 30 percent for some time now. It is doubtful if this has all been due to the visible hand of government, or from life-cycle, precautionary motives. But if this doubt is warranted, where does the the intergenerational concern come from?

Some economists (e.g. Dasgupta, 1974; Barro and Becker, 1989) have explored the thought that, leaving aside their external effects (e.g. contributing to global warming by one's consumption and production activities), savings decisions do not involve social ethics. They have suggested that considerate parents take into account the well-being of their children when deciding how much to save. If they are in addition thoughtful parents, they would know that the welfare of their children will depend upon the well-being of their children, that the welfare of their children will in turn depend upon the well-being of their children, and so on, down the generations. In short, or so it has been argued, there is a natural recursion of well-being interests along a family line.[6] Thus, thoughtful parents can be expected to take account of the interests of their distant descendents indirectly, even when they are directly interested only in their own children.

To see how the argument works, consider generation t (≥ 0) of a family line, or dynasty. The present is denoted by $t = 0$ and time is discrete. For simplicity of exposition, assume that population is constant; and to make certain there are no hidden externalities, let us normalise the population size to one in each period. A person lives for precisely one period. Assume now that each person's well-being depends on her own consumption level and on her offspring's well-being. Denote the well-being of generation t by W_t. Writing by C_t her consumption of goods and services, we may express W_t as a function of C_t and W_{t+1}:

$$W_t = W(C_t, W_{t+1}) \qquad (1)$$

This is a recursive relation. The problem is that, as it stands, it is far too general to offer us any insights. So we seek to simplify. The simplest thing by far is to assume that W_t is of the form

$$W_t = U(C_t) + \phi W_{t+1}, \text{ with } 1 \geq \phi > 0, \qquad (2)$$

where $U(C_t)$ is an increasing function of C_t and ϕ (a constant) is a time discount factor.[7]

Repeated use of expression (2) then implies that

$$W_0 = \Sigma_0^{\infty}\phi^t U(C_t), \qquad (3a)$$

and, more generally, that for generation t (≥ 0),

$$W_t = \Sigma_t^{\infty}\phi^{(\tau-t)}U(C_\tau). \qquad (3b)$$

Even though (3b) looks like the classical-utilitarian formula for generation t, it is not so: the function U(.) does not necessarily possess the interpretation that classical utilitarians placed on the notion of utility.

Parental concerns in formula (3a) extend into the indefinite future. However, it has on occasion been suggested, if only implicitly (Rawls, 1972), that this reflects more concern than is displayed typically by parents for the welfare of their descendents. With

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[6] We will use the terms "well-being", "welfare", and "utility" interchangeably here.

[7] There is no discounting if $\phi = 1$.

this in mind, alternative motivation assumptions have been explored in the economics literature (Dasgupta, 1974). A simple form of a truncated W_t is:

$$W_t = U(C_t) + \phi U(C_{t+1}), \qquad \text{for } t \geq 0. \tag{4}$$

A natural generalization of (4) involves parents caring only about their own consumption and the consumption rates of the T generations that are to follow them (Arrow, 1973):

$$W_t = \Sigma_t^{t+T} \phi^{(\tau-t)} U(C_\tau).^{8} \tag{5}$$

Schelling (1995: 396) has suggested a different formulation. He has speculated that: "... time may serve as a kind of measure of 'distance'.... Beyond certain distances there may be no further depreciation for time, culture, geography, race, or kinship."

Phelps and Pollak (1968) had explored a similar idea. They assumed:

$$W_t = U(C_t) + \kappa \Sigma_{t+1}^{\infty} \phi^{(\tau-t)} U(C_\tau),$$

$$\text{where } 1 \geq \phi > 0 \text{ and } 1 > \kappa > 0. \tag{6}$$

(6) reduces to Schelling's suggested form if $\phi = 1$.

Observe that, in (6), the marginal rate of substitution between U at t+1 and U at t+2, when evaluated at t, differs from their rate of substitution when evaluated at t+1. This is another way of saying that expression (6) embodies incoherent concerns among generations. So the inevitable, next question is: how would the incoherence be expected to be accomodated?

It is difficult to imagine that generations can reach a binding agreement over such an object as aggregate saving. So one is led to study non-cooperative equilibria of intergenerational savings games. In the context of a simple, one-commodity production model (Ramsey, 1928), Phelps and Pollak (1968) did just that. They showed that if intergenerational concern is reflected by expression (6), stationary non-cooperative equilibrium savings rules are Pareto-inefficient. Dasgupta (1974) used the simpler form, (4), to show that all non-cooperative equilibrium savings rules are Pareto-inefficient.

We would appear to have reached an impasse. We have to go elsewhere if are to arrive at a language for justice among generations.

We began this account with parental concerns. But if parents are thoughtful, they

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[8] (5) reduces to (3) if $T = \infty$.

will ask what concern they ought to display toward their descendents. This leads to welfare economics. So we turn to it.

3. Intergenerational social welfare functions and the way they yield social discount rates[9]

Rawls (1972) offered us reasons for taking seriously extreme equity in the distribution of what he called "primary goods". The concept of primary goods has spawned a large philosophical literature since his book was published. However, in their attempts to apply Rawls' insights to the problem of optimal saving, economists cut through philosophical knots by interpreting W_t in (3)-(5) as a primary good. As is well known, Rawls' theory of justice, when used among contemporaries, yields the (lexicographic) maxi-min principle of distribution of primary goods. What is less well appreciated is that Rawls shied away from the principle in his discussion of justice among generations. Nevertheless, it is instructive to study intergenerational maxi-min savings policies, if only to check whether Rawls was correct in discarding the maxi-min principle.

Arrow (1973) and Solow (1974a) explored savings behaviour when each generation invokes the intergenerational maxi-min principle. They and Dasgupta (1974) proved that if parental concerns extend only to a finite number of descendents (as in expression (5)), the principle implies either a stagnant economy (this is so if ϕ is small), or a programme of savings and dissavings that would be revoked by the generation following any that were to pursue it (this is so if ϕ is not small). In the latter case, the maxi-min programme is dynamically inconsistent. The Rawlsian route would appear to be unpromising.

But justice among generations had been the object of inquiry among economists long before Rawls wrote on it. The framework in use was, and continues to be, that adopted by Ramsey (1928) and developed by Koopmans (1960, 1972). Ramsey took a straightforward utilitarian approach to the problem, while Koopmans greatly enlarged the scope of utilitarianism by adopting an axiomatic approach to the matter of intergenerational justice. We turn to Koopmans' formulation.

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[9] This section is taken from Dasgupta (1994) and Dasgupta and Mäler (1995).

Time is taken to be continuous. Let population at date t ($\geq$ 0) be denoted by L_t. We take it as exogenously given.[10] For expositional ease, we will imagine that if C_t is aggregate consumption at t, the well-being of the representative person at t is a continuous, increasing function of per capita consumption, C_t/L_t. We denote this by $U(C_t/L_t)$. We next assume that aggregate well-being at t is a function solely of $U(C_t/L_t)$ and L_t. We write this as $W_t = W(U(C_t/L_t), L_t)$.[11] The planning horizon is from the present to infinity.

Let the set of feasible consumption programmes be denoted by Γ_C. We will suppose that Γ_C is uniformly bounded. Admittedly, this smacks squarely against current models of economic growth (Barro and Sala-i-Martin, 1995; Romer, 1996), which devise macroeconomic policies that would sustain unbounded consumption programmes. But the idea of unbounded consumption is science fiction. It ignores the environmental resource-base upon whose services all production and consumption ultimately depends. This base is very much finite in extent. For example, Vitousek et al. (1986) have estimated that 40 percent of the net energy created by terrestrial photosynthesis (i.e. net primary production of the biosphere) is currently being appropriated for human use. This is, of course, a very rough estimate. Moreover, net terrestrial primary production is not given exogenously, nor is it fixed; it depends in part on human activity. Nevertheless, the figure does put the scale of the human presence on the planet in perspective.[12]

Corresponding to each feasible C_t is a W_t. So let Γ_W be the set of feasible W_ts. For the moment, we will take it that Γ_W is also uniformly bounded.[13] Imagine next that the public authority has been provided with an ordering over Γ_W. Koopmans (1960) showed that if the ordering satisfies a set of plausible assumptions, it can be represented by the

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[10] Ethical foundations of population policies have been explored in Dasgupta (1969, 1994).

[11] So as to concentrate on intergenerational matters, we are therefore ignoring intragenerational distributional issues here.

[12] Indeed, no ecologist we know thinks that a population of some 10 billion (which is a reasonable projection for world population in the year 2,050) can support itself indefinitely at a standard of living of today's representative West-European.

[13] Unlike uniform boundedness of Γ_C, this is not a requirement dictated by ecology.

"utilitarian" form

$$_0\int^\infty L_t\, U(C_t/L_t)\, e^{-\delta t})dt, \text{ where } \delta > 0.^{14} \qquad (7)$$

Expression (7) may look like classical utilitarianism, but it is not. There is nothing in the Koopmans axioms that forces a classical-utilitarian interpretation upon U. It is a numerical representation of an ordering, nothing more. Economists continue to refer to U as utility, but that is more from habit than anything else.

Expression (7) involves discounting aggregate utility at a constant rate ($\delta > 0$) and integrating over their present-values. If aggregate utility at $t = 0$ were the numeraire, δ would be the social rate of discount. It is often called the rate of pure time preference. Koopmans' work is illuminating because it shows whether we should discount future utilities is something that ought to be derived from more fundamental ethical principles. It represents a radical break with a philosophical tradition, stretching from Ramsey (1928) to Parfit (1984), that has warned us against discounting future utilities without providing serious arguments.

When conducting experiments with alternative assumptions embodied in expression (7), it makes sense to go beyond the Koopmans axioms and consider also the case, $\delta = 0$. This way we are able to test models to see what they imply about social cost-benefit analysis. On the other hand, purposeless generality should be avoided. So we will assume that $U(C/L)$ is strictly concave, to give shape to the idea that intergenerational equity is valued as an ethical goal (see below).

It will make for expositional simplicity now if we were to suppose that population is of constant size. So we will do so. In which case we may as well normalise and set $L_t = 1$. Thus, aggregate well-being at t is $U(C_t)$. Now recall the definition of social discount rates. If, instead of U, consumption were chosen as the numeraire, the social rate of discount would be

$$\rho_t = \rho(C_t) = \delta + \alpha(C_t)[dC_t/dt]/C_t, \qquad (8)$$

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[14] This is not strictly true, since Koopmans assumed constant population. But his analsyis extends to the present case. Moreover, Koopmans worked with discrete time. In Koopmans (1972) the ethical axioms are imposed directly on Γ_C, a considerable generalization.

where $\alpha(C_t) > 0$ is the elasticity of marginal utility.[15] $\rho(C_t)$ is sometimes called the consumption rate of interest (Little and Mirrlees, 1974). In applied cost-benefit analysis, aggregate consumption is often taken to be numeraire (e.g. Dasgupta, Marglin and Sen, 1972).

In a discrete time formulation, $\rho(C_t)$ would be the marginal rate of substitution between consumption at dates t and t+1, minus one. Along an optimal programme, this must equal the marginal rate of transformation between consumption at dates t and t+1, minus one. The latter is sometimes called the social rate of return on investment. Write it as r_t. Then, along an optimal programme in continuous time, we would have

$$r_t = \delta + \alpha(C_t)[dC_t/dt]/C_t. \tag{9}$$

This is the famous Ramsey Rule.

All this is familiar (see e.g. Arrow and Kurz, 1970; Dasgupta and Heal, 1979; Lind, 1982). But it is as well to remind ourselves that if consumption were expected to grow for a period, the consumption rate of interest would be positive even if δ were zero, an illustration of the fact that social discount rates depend on the numeraire.

Iso-elastic utility functions offer a simple, flexible form of U. Consider the form

$$U(C) = - C^{-(\alpha-1)}, \qquad \text{where } \alpha > 1.^{16} \tag{10}$$

In this case the social welfare function in (7) depends only on two parameters: α and δ. They reflect different concerns. α is an index of the extent to which intergenerational equity in the distribution of consumption is valued (see below). δ is, as we have seen, more directly interpretable: the larger is δ, the lower is the weight awarded to future generations' utilities relative to that of the present generation.

Let $\delta = 0$. As an example, assume that $\alpha = 2.5$ (a not implausible figure if U(C) were to be based on revealed preference). If, over a period, the rate of growth of consumption at t along the optimal programme is, say, 0.02 percent per year, then $\rho_t = 0.05$ per year during that period. That the social rate of discount depends upon the numeraire is even now not appreciated in a good deal of the environmental literature

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[15] See, for example, Arrow and Kurz (1970). $U'(C_t)$ is the accounting price of consumption at date t in terms of utility at t. So $\exp(-\delta t)U'(C_t)$ is its present-value price in terms of utility at t = 0. The percentage rate of decline of this expression is $\rho(C_t)$.

[16] This function is unbounded below and so violates the Koopmans axioms.

that is critical of social cost-benefit analysis (e.g. Daly and Cobb, 1991). Modern philosophers writing on the matter also often do not appreciate it. They argue that δ should be zero and then criticize the practice of discounting future flows of consumption in social cost-benefit analysis (Parfit, 1984; Cowen and Parfit, 1992).

It will be noticed that the larger is α, the more egalitarian is the optimal consumption path. As $\alpha \rightarrow \infty$, the social welfare function in (7) looks more and more like the intergenerational maxi-min principle. This in turn means that, even in productive economies, optimal growth in consumption is slow if α is large (equation (9)). In the limit, as $\alpha \rightarrow \infty$, optimal growth is zero, a supremely egalitarian outcome. From equation (9), we can see why the consumption rate of interest is bounded (and how it manages to equal the social rate of return on investment) even in these extreme parametric terrains. (On this, see Dasgupta and Heal, 1979, Chapters 9-10.)

In equation (10), $U(C)$ is unbounded below. If $\delta = 0$, this ensures that very low consumption rates are penalized by (7). On the other hand, if δ were positive, low consumption rates by generations sufficiently far in the future would not be penalized by (7). This means that, unless the economy is sufficiently productive, optimal consumption will tend to zero in the very long run. As an illustration of how critical δ can be, Dasgupta and Heal (1974) and Solow (1974a) showed in a stylized economy with exhaustible resources that optimal consumption declines to zero in the long run if $\delta > 0$, but increases to infinity if $\delta = 0$. This was the substance of Solow's remark (Solow, 1974b) that, in the economics of exhaustible resources, the choice of δ can be a matter of considerable moment.

One way of preserving something like a balance among the generations would be to maximize expression (7), subject to the constraint that the well-being of no generation falls below some acceptable level. The weakness of this approach is that side constraints do not admit tradeoffs between competing goals: the accounting price of a side constraint is nil when the constraint is non-binding, but is positive whenever it binds. For this reason, welfare economists typically avoid side constraints as a device for avoiding morally indefensible outcomes.

An alternative would be to weaken the Koopmans-axioms in some directions even while strengthening them in others, so as to build in directly the idea that the dictates of intergenerational fairness prohibit zero or 'near-zero' consumption level for any generation. One way to do this would be to regard aggregate well-being as a weighted sum of expression (7) and the long-run average well-being over time.[17] The latter term could even be approximated by minimum requirements for long-run stocks of environmental resources, such as air- and water-quality, biodiversity, soil quality, and so forth. Such a hybrid formulation attempts to account for both basic human needs and resource constraints.

4. The atmosphere as a global commons, or, can social discount rates ever be zero or negative?

Production and consumption activities involve the emission of greenhouse gases as a by-product; and the atmosphere is a sink for greenhouse gases. So the atmosphere is a global commons. There is enormous uncertainty about the nature and spatial distribution of the economic impacts of global warming, but there is little divergence of opinion among experts that, in the long run, there will be significant economic impacts if greenhouse emissions continue to increase at current rates (IPCC, 1996).[18] It might then be thought that, taken together, these features imply market failure with a vengeance, and that large (Pigovian) taxes on carbon emissions would be required to remove the distortion. But simulations undertaken on a model of an optimizing world economy in Nordhaus (1994) suggest that the social costs of doing much today and in the near future to reduce the rate of global warming would far exceed the benefits: optimal carbon taxes in Nordhaus' simulations are quite low.

Why is this? There are two interrelated reasons. First, Nordhaus' most favoured specification has it that total factor productivity increases over time (although at a declining rate - the rate halving every 70 years). This assumption, among others, implies that the effect of global warming on world output is small: a doubling of CO_2 concentration (corresponding to something like a 3°C increase in average global temperature) would result in a mere 1.33 percent loss in world output of goods and

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[17] Radner (1967), Chichilnisky (1994), Beltratti, Chichilnisky, and Heal (1995) provide alternative rationales for this when U is bounded.

[18] Schelling (1992) and Ehrlich and Ehrlich (1996) provide excellent summaries of what we know and what we do not know.

services (Nordhaus, 1994: 51). Secondly, social rates of discount in Nordhaus' optimizing world economy are throughout positive and non-negligible, and they make the small losses in world per capita consumption in the distant future due to global warming look really small today.[19]

The first of Nordhaus' assumptions mentioned above is followed with little questioning by modern growth theorists, but we doubt if it is prudent to postulate everlasting increases in total factor productivity, let alone in per capita output. To do so would be to place an enormous burden of proof on an experience which is not much more than a few hundred years old. Extrapolation into the past is a sobering exercise: over the long haul of time (say, a few thousand years), the rate of growth of per capita income has not been much more than zero.

Then there are scientific reasons why we should be wary of relying on unspecified technological progress that is postulated to overcome ecological constraints. But the limits to growth are not so much that we will run out of resources as that the vast array of ecological services we rely on are finite in scope. For these reasons, it is not unreasonable to assume no technological change when evaluating the long term consequences of economic policies.

The second of Nordhaus' assumptions we have alluded to is also standard. We do not know of any exercise in project evaluation that has used a zero, not to mention negative, rate of interest for discounting future flows of a project's income. But are consumption rates of interest inevitably positive? As we will see below, standard models of economic growth have features built into them that make consumption interest rates positive. But standard models are an inadequate vehicle for obtaining insights into social discount rates when production and consumption activities involve externalities that filter into the distant future through the accumulation of some "public bad". Since global warming is a prime example of such externalities, we will use it as a backdrop for our discussion.

Consider first a world economy where project choice is seen as policy reform

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[19] The rate is 5.9 percent per year in 1995 and declines to about 4.4 percent per year in 2075, as growth in per capita world output slows in the second half of the next century (Nordhaus, 1994: 91).

(Section 1).[20] Assume that the economy is otherwise laissez faire. If global warming is expected to lead to declines in (weighted) global consumption over some extended period in the distant future, then from expression (8) we would conclude that, over this same extended period, consumption rates of interest could well be negative. Suppose, for example, that $\delta = .01$ per year and that global consumption would be expected to decline at 2 percent a year for a period beginning 30 years from now if emissions of greenhouse gases were to continue at their laissez faire rates. If $\alpha(C_t) = 2.5$ in expression (8), the consumption rate of interest would be -0.04 per year from year 30 until the end of the period in question. Long lasting projects that contribute to further global warming would not look so good because, from our current viewpoint, future losses due to global warming would be amplified; they would not be reduced to negligible figures by the relentless application of positive discount rates.

But what about an optimizing economy? Here, the Ramsey Rule (equation (9)) comes into play and matters become more complex. In standard models of economic growth, the social rate of return on investment equals the private rate (there are no externalities), and the private rate equals the marginal productivity of capital. As the latter is positive, the left-hand-side of equation (9) is positive. Since the consumption rate of interest (the right-hand-side) equals this, it must be positive as well.

Matters would be different if production activities were to contribute to the accumulation of some "public bad". The social rate of return on investment would differ from its private rate; and so, the fact that the private rate is positive would not mean that the social rate is positive. Imagine, for example, that laissez faire policies in the past have resulted in the accumulation of a large stock of the public bad (e.g. CO_2 concentration). Assume that the government now wants to pursue an optimal economic policy. If emissions are a by-product of production, one way to lower emissions would be to reduce output.[21] And one way to reduce output would be to leave idle some of the

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[20] This paragraph is taken from Dasgupta and Mäler (1995: 2,401).

[21] Others would be to treat emissions before they disperse (e.g. the use of stack-gas scrubbers for removing the SO_2 produced when coal is burnt); use cleaner, but more expensive technologies (e.g. the use of clean energy); and clean up the pollution after it has been emitted (e.g. artificially aerate eutrophied bodies of water; sequester additional CO_2 from the atmosphere by planting trees; and so forth). Dasgupta

existing stock of physical capital. However, if it is optimal to do this for a while, then, during that while there would be no sense in accumulating capital. This is another way of saying that gross investment in physical capital should be zero; which, in turn, is another way of saying that all output ought to be consumed. At the margin, the social value of consumption would then not equal the social value of investment. So, the accounting price of consumption would differ from the accounting price of investment. It is obvious also that, if capacity were left idle, the social rate of return on investment would be zero. This means that if output or investment is numeraire, the social rate of discount would be zero. We will confirm this in the context of a formal model.

5. A not-too-far-fetched model of global warming to illustrate that social discount rates can be zero even in an optimizing world

(i) The basic model

The model economy we will consider is a simple generalization of Ramsey (1928) and a special case of Nordhaus (1994).[22]

Population is constant, there is no technological change, and there is no uncertainty.[23] Let K denote the quantity of physical capital in existence and $\bar{K}$ the quantity utilized in production. So,

$$\bar{K} \leq K. \tag{11}$$

Gross output, Y, is

$$Y = \sigma \bar{K} e^{-\pi E}, \tag{12}$$

where $\sigma, \pi > 0$, and E is the stock of a public bad (e.g. CO_2 concentration).[24]

(1982, Ch. 8) analyses a model of pollution containing all these options.

[22] But with a difference: Nordhaus does not allow for the possibility that the optimal economy can have idle capacity.

[23] Variables will be represented by upper-case letters and constants by lower-case ones. Therefore, we may as well suppress the time-subscript from the former.

[24] A major weakness of this model of global warming, and indeed all models with which we are familiar, is that the dynamic processes underlying them display no irreversibilities. Thus, for example, catastrophies are not a possibility in climate models. In other words, there are no naturally occuring bifurcations.

A simple formulation would have it that it is changes in the level of the public bad that are bad for production. A natural way of formulating this is to write $Y = \sigma \bar{K} \exp(-\pi\ [dE/dt])$, where [dE/dt] denotes the absolute value of dE/dt. But our aim here is to make a number of analytical points, not to simulate the

Capital used in production depreciates at a constant rate, ϵ (> 0). To make the points we would like to raise in a forceful manner, we take it that idle capital does not depreciate. Therefore, if C denotes aggregate consumption, accumulation of physical capital is governed by:

$$dK/dt = \sigma\bar{K}e^{-\pi E} - C - \epsilon\bar{K}. \qquad (13)$$

We assume capital cannot be consumed. So gross investment must be non-negative. This means,

$$\sigma\bar{K}e^{-\pi E} \geq C. \qquad (14)$$

In order to present the simplest possible model of the problem in hand, we will suppose that emissions of the public bad are a proportion, ß, of current output, and that the only way emissions can be lowered is by reducing output. In short, ß is the emission-output ratio.[25] The public bad is assumed to depreciate at a percentage rate, γ (> 0).[26] So we have

$$dE/dt = \sigma ß\bar{K}e^{-\pi E} - \gamma E. \qquad (15)$$

Thus far the physical model. We will now study production and consumption activities in two institutions: the *laissez faire* economy and the optimal economy.

(ii) The laissez faire economy

There are a continuum of identical households distributed uniformaly over the unit interval. Households are infinitely-lived. The representative household's intertemporal utility function is

$$\int_0^\infty e^{-\delta t}U(C)dt, \text{ where } \delta > 0 \text{ and } U(C) \text{ is an increasing, strictly concave function.} \qquad (16)$$

Households own identical amounts of physical capital. As there is no environmental policy in the *laissez faire* economy, the market price of E is zero: it is a pure public bad. The private rate of return on investment is, therefore, $\sigma e^{-\pi E}$. Moreover, $\bar{K} = K$.

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world. So we do not try to build in such complications.

[25] In their article on optimal environmental taxation, Ulph and Ulph (1994), quite rightly, relate emissions (of CO_2) to the use of fossil fuels in production, not to production itself.

[26] Thus, the rate of sequestration is not determined within the model. Under current conditions, the mean residence time of CO_2 in the atmosphere is a century or more. This suggests $\gamma \leq 0.01$ per year.

A rational expectations equilibrium of the laissez faire economy is a time path of C, K, E, and Y, such that, it is the consequence of the following maximization problem:

Choose C so as to maximize (16), subject to conditions (13)-(15) being satisfied with rational expectations, and $\bar{K} = K$; with K_0 and E_0 as initial values of the two stock variables. (17)

Our treatment will be informal. Moreover, we will confine ourselves to stationary rational expectations equilibria. Let P be the spot price of consumption in terms of utility numeraire. It is routine matter to confirm that the stationary point(s) must satisfy the conditions:

$$\sigma K e^{-\pi E} = C + \epsilon K \tag{18}$$

$$\sigma \beta K e^{-\pi E} = \gamma E \tag{19}$$

$$U'(C) = P \tag{20}$$

$$\sigma e^{-\pi E} = \delta + \epsilon \tag{21}$$

$$\sigma K e^{-\pi E} = Y. \tag{22}$$

Equations (18)-(22) are five in number, and there are five unknowns: C, K, E, Y, and P. Assume $\sigma > \delta + \epsilon$ (i.e. the economy is productive). It is then immediate that the unique stationary value of E (call it E**) is

$$E^{**} = \pi^{-1} \log [\sigma/(\delta + \epsilon)]. \tag{23}$$

And so forth. Let C**, K**, Y**, and P** denote the stationary values of the remaining variables. We wish to compare them with their optimal stationary values. We turn to this.

(iii) The optimal economy

The public authority respects household preferences. Its maximizing problem is:

Choose C and $\bar{K}$ so as to maximize (16), subject to conditions (11)-(15), with K_0 and E_0 as initial values of the two stocks. (24)

Let Q and S denote the auxiliary variables associated with equations (13) and (15), respectively. If utility is numeraire, Q is the (spot) accounting price of investment and S is the (spot) accounting price of environmental pollution. We will confirm that $Q > 0$ and $S < 0$.

The present-value Hamiltonian, H_0, of problem (24) can now be expressed as:

$$H_0 = e^{-\delta t}U(C) + e^{-\delta t}Q(\sigma\bar{K}e^{-\pi E} - C - \epsilon\bar{K}) + e^{-\delta t}S(\sigma\beta\bar{K}e^{-\pi E} - \gamma E)$$
$$+ e^{-\delta t}M(K - \bar{K}) + e^{-\delta t}N(\sigma\bar{K}e^{-\pi E} - C) \tag{25}$$

where

$$M \geq 0, \text{ and } M(K - \bar{K}) = 0 \text{ with complementary slackness,} \tag{26a}$$

and

$$N \geq 0 \text{ and } N(\sigma\bar{K}e^{-\pi E} - C) = 0 \text{ with complementary slackness.} \tag{26b}$$

Routine calculations show that the optimal policy must satisfy the following conditions:

$$U'(C) = Q + N \tag{27}$$

$$\sigma e^{-\pi E}(Q + \beta S + N) = M + \epsilon Q \tag{28}$$

$$(dQ/dt)/Q = \delta - M/Q \tag{29}$$

and $$(dS/dt)/S = \delta + \sigma\pi\bar{K}e^{-\pi E}[(Q/S) + \beta + (N/S)] + \gamma. \tag{30}$$

From equations (26a) and (26b), we can conclude that if $M = 0$, then $N > 0$. Notice too that when $\bar{K} < K$, equation (29) reduces to

$$dQ/dt = \delta Q. \tag{31}$$

So, if investment or output is numeraire, the social rate of discount is zero whenever it is optimal to hold idle capacity. Notice as well that when $N > 0$ (i.e. it is optimal to consume all output), consumption and investment have different accounting prices. From equation (27) we see that, relative to utility, the spot price of consumption is $(Q + N)$, which is greater than Q.

What is net national product (NNP) along the optimal programme? NNP is the linearised Hamiltonian (Dasgupta and Mäler, 1991; Mäler, 1991). Denote the current-value of NNP by Z. From equation (25) we conclude that, if consumption is numeraire,

$$Z = C + [Q/(Q + N)]dK/dt + [S/(Q + N)]dE/dt. \tag{32}$$

Macroeconomic programmes consist of projects; and projects, taken together, make a macroeconomic programme; for recall that a project is a perturbation to a macroeconomic programme. Write $I = dK/dt$ and $J = dE/dt$. We will call any feasible, one-period perturbation, (dC, dI, dJ), to a macroeconomic programme an elementary project. All feasible investment projects can be viewed as a compound of elementary projects. In Section 1 we recalled that accounting prices can be used for sustaining an optimal programme in a decentralized environment. In the present context, this means that small investment projects could be decomposed into its constituent, elementary projects and evaluated on the basis of a repeated use of (30) as the criterion of choice. Alternatively (and equivalently), we could avoid decomposing the projects by evaluating them directly on the basis of their contribution to the present-discounted value of the flow of Z. Either way, the set of projects that would be chosen would comprise the optimal programme. The marginal project would contribute nothing to NNP.

At a stationary point of the optimal programme, $\bar{K} = K$ and $Y > C$. So from conditions (26) and equations (18)-(19) and (27)-(29), we may conclude that the stationary values must satisfy the conditions:

$$\sigma K e^{-\pi E} = C + \epsilon K \tag{33}$$

$$\sigma ß K e^{-\pi E} = \gamma E \tag{34}$$

$$U'(C) = Q \tag{35}$$

$$\sigma e^{-\pi E}(Q + ßS) = M + \epsilon Q \tag{36}$$

$$\delta = M/Q \tag{37}$$

$$\delta + \sigma\pi K e^{-\pi E}[(Q/S) + ß] + \gamma = 0 \tag{38}$$

$$\sigma K e^{-\pi E} = Y. \tag{39}$$

Equations (33)-(39) are seven in number and there are seven unknowns, Q, S, C, K, E, Y, and M. Routine manipulations with them yields

$$\sigma(\delta + \gamma)e^{-\pi E} = (\delta + \epsilon)\pi\gamma E + (\delta + \epsilon)(\delta + \gamma). \tag{40}$$

Since $\sigma > (\delta + \epsilon)$, equation (40) has a unique solution. Call it E^*. Write $X \equiv Q/S$. From equations (33)-(39) it is an easy matter to confirm that the stationary value of X (call it X^*), is

$$X^* = (\delta e^{\pi E^*} - \sigma)/\sigma ß < 0.$$

And so forth for the rest of the variables. Let C^*, K^*, Y^*, Q^*, S^* and M^* denote the unique stationary values of the remaining variables. The social rate of discount in the stationary state is δ, independent of numeraire.

C. A Comparison

On using equations (23) and (39), we conclude that

$$E^{**} > E^*,\ K^{**} > K^*, \text{ and } Y^{**} > Y^*. \tag{41}$$

Thus, in the long run, the laissez faire economy, compared to the optimising economy, suffers from greater pollution, has a larger stock of physical capital; and produces more output. In short, the laissez faire economy is too large: it should cut down its economic activities.

What of consumption though? When comparison is limited to stationary states, does the laissez faire economy consume too much or too little? The answer is not unambiguous. However, it is easy to show that, if ϵ is "small", then $C^{**} > C^{*}$; or, in the long run, consumption level in the laissez faire economy is too high.

Suppose, instead, that the economy has been run along laissez faire lines for a long while. The government now wants to manage it optimally. If the laissez faire economy had tracked a rational-expectations equilibrium path, the initial values of the two state variables in problem (24) would be K^{**} and E^{**}. But the idea of a rational-expectations equilibrium of a laissez faire economy appears unreasonable to many economists. If it were to be dispensed with as a descriptive construct, there would be no natural initial values of K and E in problem (24): we could entertain the possibility that both are arbitrarily large.

Since the optimal economy transits to lower values of the two stocks (K^{*} and E^{*}), we would expect an initial phase when capacity is not utilized in full and when all output is consumed. During this phase the social rate of discount (in investment numeraire) would be zero. It is only when the stocks of both physical capital and environmental pollution are decumulated sufficiently that full capacity would be optimal.

One can prove this for the case, $K_0 >> K^{*}$ and $E_0 > E^{*}$ in problem (24).[27] But we have not been able to prove that there is a transient phase like this when the initial values are K^{**} and E^{**}; that is, we have not been able to piece together a complete narrative of phases. But it is a possibility that simulations with the model would uncover. This is the next task.

6. Conclusions

This article has been about the logic underlying social discount rates. We have

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[27] The intuition is that, as the inherited stock of physical capital is "very" large, output at full capacity is very large and, so, adds to the stock of pollution. Holding idle capacity and simultaneously consuming all output is a way of reducing the stocks of both types of capital.

argued that these rates are not ethical raw material, but are derived from the more fundamental notion of justice among generations. A number of approaches to the concept of intergenerational justice were discussed, and it was argued that the most compelling formulation available to us is the one long been in use among economists, namely, the Ramsey-Koopmans theory. This theory advocates that investment projects having long-run effects should be subjected to the same conceptual treatment as those that affect only the near future. We have shown that social discount rates depend on the numeraire, and that methods of estimating them depend on the institutional setting within which social cost benefit analysis is assumed to be undertaken. We have also shown that it is incorrect to advocate project-specific discount rates as a way of conserving environmental resources.

Social discount rates have universally been taken to be positive, on grounds that the rate of return on investment is positive. But if consumption and production activities give rise to environmental pollution as a by-product, the social rate of return on investment could be zero even when the private rate is positive; at the very least, the social rate would be lower than the private rate. The current practice among most global energy modellers of relying exclusively on (risk-free) market rates of return for estimating optimal carbon taxes is conceptually faulty. In the context of a formal model of environmental pollution we have shown how, even along an optimal programme, social rates of discount can be zero. We also demonstrated that in certain institutional settings, social discount rates can be negative.

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