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A Coevolutionary Model of Free Riding Behavior: Replicator Dynamics as an Explanation of the Experimental Results*

by

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Abstract

The predictions of current public goods theory are often inconsistent with the results of public goods experiments. In an effort to understand these anomalies, we propose a simple model of decision making based on evolutionary dynamics. The underlying model appears to be intuitively plausible, and is consistent with a variety of behavioral models. Despite the model's apparent simplicity, it produces a rich set of predictions. We find a striking parallel between the dynamics generated by this model and the experimental findings. In particular, the model predicts the decay toward free riding, as well as the patterns observed over the various group size and marginal return conditions.

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1. Introduction

A large set of experimental findings has been accumulated on the private provision of public goods. The robust finding across these studies is that cooperative behavior persists, despite strong incentives to free ride (for a review, see Dawes and Thaler, 1988). Furthermore, several experiments have failed to explain this anomaly in terms of rational optimizing behavior. In this paper, we develop a simple model of decision making based on evolutionary dynamics. We find that this model generates predictions that largely mirror the experimental results, and hence may provide a basis for understanding the findings of public goods experiments.

2. The Experimental Background

The typical public goods experiment has a simple design. Subjects are formed into groups of size g, and are each given an exogenous income of I. Each subject must then allocate I between a public and private good. After all individuals make their allocation decisions, each receives a payoff equal to

$$p_i = I - x_i + mg\bar{x},$$

where x_i is subject i's contribution to the public good, m is the previously known marginal per capita return from giving to the public good and $\bar{x}$ is the average contribution to the public good. The parameter m is chosen so that $0 < m < 1$ and $mg > 1$. Given this design, $x_i = 0$ (free riding) is the dominant strategy Nash equilibrium, and $x_i = I$ is the symmetric Pareto efficient allocation.

Experiments have uncovered two significant regularities in behavior. First, subjects on average allocate about half of their income to the public good in the initial play of either a single-shot (Marwell and Ames, 1981) or repeated game (Isaac, Walker and Thomas (1984); Isaac, McCue and Plott (1985); Andreoni (1988); and Isaac and Walker (1988)). Second, in iterated games provision "decays" toward the free riding equilibrium, although exact free riding behavior is seldom observed (Isaac, Walker and Thomas (1984); Kim and

Walker (1984); Isaac, McCue and Plott (1985); Andreoni (1988); and Isaac and Walker (1988)).

Recent work of Isaac, Walker and Williams (1990) has confirmed the above regularities, and has also produced some new anomalies. They found that in groups of size 4, 10, 40 and 100, subjects tend to give about the same fraction of their endowments to the public good in the first round. They also observed decay in giving, with the rate of decay inversely related to group size—larger groups had slower rates of decay. Finally, their results suggest that as the marginal return from contributing to the public good rises (even though the equilibrium is unchanged), decay markedly slows for all group sizes, and the paths of play became largely indistinguishable.

Existing models of rational Nash equilibrium play cannot account for the above results. Current public goods theory predicts that free riding should be far more prevalent than actually observed, and that cooperation should be more difficult to sustain in large groups. Moreover, the theory cannot easily explain the experimental observations on various marginal payoff conditions. In an effort to understand the above anomalies, we propose a simple model based on evolutionary dynamics. We find a striking parallel between the dynamics generated by this model and the experimental findings. In particular, the model predicts the decay toward free riding, as well as the patterns observed over the various group size and marginal return conditions.

3. The Model

The model derived here is based on a simple application of biological *replicator dynamics* (Schuster and Sigmund, 1983; and Hofbauer and Sigmund, 1988). Similar dynamics have been adopted by game theorist for the analysis of evolutionary games (e.g., Axelrod and Hamilton, 1981; Samuleson, 1988; and Nachbar, 1990). Underlying the dynamics is the notion that the use of a given strategy depends on its performance relative to the average payoff of all strategies. Strategies that are performing relatively poorly are replaced by those that perform relatively well.

Suppose there are N types of strategies interacting in a population of fixed size. Each type of strategy, i, always gives x_i dollars to the public good. Let d_i^t be the proportion of type i strategies in the population at time t. Then, the expected payoff to an agent with strategy type i is

$$p_i = I - x_i + mg \sum_{j=1}^{N} x_j d_j^t = I - x_i + mg\bar{x}^t,$$

where $\bar{x}^t$ is the average contribution at time t.

Replicator dynamics provide a natural algorithm for allowing the weights, d_i^t, to evolve over time. The dynamics are given by

$$d_i^{t+1} = d_i^t \frac{p_i}{\sum_{j=1}^{N} p_j d_j} = d_i^t \frac{I - x_i + mg\bar{x}^t}{I - \bar{x}^t + mg\bar{x}^t}. \tag{1}$$

This dynamic implies that the proportion of a given type of strategy increases (decreases) as it performs better (worse) than average. The above equation also incorporates a normalization factor so that the population remains constant. Types that are not initially in the population can never emerge, and those that have positive support will never disappear (although asymptotically they can go to zero).

A variety of arguments support the plausibility of a model based on replicator-like dynamics. The first is that agents are playing an evolutionary game (see Maynard Smith and Price, 1973). Over time, better strategies proliferate by some adaptive selection process. Alternatively, the model pursued here is one in which agents are concerned with their performance relative to the average. Experiments have indicated that subjects seem well aware of the free riding incentives from the start of the game, and that the observed cooperation cannot be attributed to rational strategic play (Andreoni, 1988). Other experiments suggest that the behavior of subjects is more consistent with the hypothesis that people enjoy being part of cooperative outcomes, but that they are unwilling to have their cooperation exploited by other more selfish players (Andreoni, 1990; and Kahneman and Knetsch, 1990). If this is true, then those who give above average will always feel exploited, and hence will switch to more selfish strategies. Moreover, the model's formulation mirrors prominent psychological theories of decision making based on "reference points" (See Ableson and Levi (1985) or Kahneman, Slovic and Tversky (1982) for reviews). According to these theories, strategies are adjusted more rapidly the more the outcome falls short of some reference point, such as the average.

4. Theoretical Predictions

Despite its simplicity, the model makes a number of predictions about the behavior of agents in the system. We will present six propositions. All of these propositions have rigorous proofs, although, for brevity, we will sometimes only outline the proofs (the full proofs can be obtained from the authors upon request). The propositions concern only those strategies that have positive support during the initial time period, and they assume that at least two such strategies exist.

Proposition 1. Any strategy that contributes less (more) than the average contribution will grow (decline) in proportion over time. (Proof: By Equation (1), $d_i^{t+1}/d_i^t >\ (=,<)\ 1$ as $x_i <\ (=,>)\ \bar{x}^t$.)

Proposition 2. The average contribution will fall over time. (Proof: At any point in time, the weights on those $x_i < \bar{x}^t$ increase, while the weights on those $x_i > \bar{x}^t$ decrease. Thus, the average must decline.)

Proposition 3. The system will converge asymptotically to the strategy that does the most free riding. (Proof (intuition): Proposition 1 indicates that the growth always occurs in strategies that give less than the current average. Proposition 2 indicates that the average will decline over time. Thus, as the average moves down, the weights will eventually be concentrated on only the strategy that gives the least to the public good.)

Propositions 1, 2, and 3 show that replicator dynamics will produce a decay towards free riding. Over time, the more cooperative strategies will be replaced by those that give less to the public good. Proposition 3 implies that given sufficient time the entire population will (asymptotically) use the one strategy initially present in the population which does the most free riding.

Proposition 4. Starting from the same initial distribution, the dynamics will be identical for $mg = k$ where k is a constant. (Proof: Equation (1) is identical for all systems in this case.)

Proposition 5. As mg increases, the rate of adjustment slows, and as $mg \rightarrow \infty$ then $d_i^{t+1}/d_i^t \rightarrow 1$. (Proof: It can be shown that the absolute value of $1 - d_i^{t+1}/d_i^t$ decreases as mg increases. For the latter statement, take the limit of equation (1)).

Proposition 6. As mg increases, the decline in average contributions slows. (Proof (intuition): As in the proof of Proposition 5, a larger mg slows the changes in d_i^{t+1} relative to d_i^t, and thus starting from the same initial conditions the convergence is slowed.)

Propositions 4, 5, and 6 indicate that a critical determinant of the system's behavior is the quantity mg. Intuitively, these propositions hold because as mg increases, the agent's own contribution makes less difference to the agent's total payoff (see equation (1)). Hence the agent's relative performance is not closely tied to the agent's strategic choice. Although the optimizing model would predict, for instance, that free riding will increase as g increases, this model does not: since all strategies perform about the same, there is a smaller incentive for any agent to change his or her strategy.

Figures 1 and 2 show the model's predicted average provision of public goods as a proportion of the Pareto efficient amount. It is assumed that there are 20 different strategy types, evenly distributed between 0% and 100% giving. Figure 1 uses a uniform initial distribution of strategies, while in Figure 2 the initial distribution is concentrated around 50% provision.[1] The resulting dynamic paths are shown for a variety of group sizes, with marginal return set at $m = .3$.

5. Discussion

The model developed here is a very simple replicator dynamic. However, more complex versions of evolutionary dynamics are likely to lead to the same qualitative results. For example, if agents who perform worse than average only switch to the average strategy (as opposed to a distribution across all of the above average strategies), then similar dynamics result. If agents follow a complicated adaptive scheme based on imitation and innovation driven by a genetic algorithm model (e.g., Miller, 1988), comparable dynamics also occur. Given the simple structure of the public goods game, it is likely that the general results obtained here hold across a wide class of such models. Although the simple replicator model does not have any explicit parameters, more complex versions of these models do have parameters for, say, altering the speed of adjustment of the strategies. Such models could be fit empirically to the experimental data.

Despite its simplicity, the qualitative predictions of the replicator-based model appear to capture most of the results from the public goods experiments. In particular, the model's predictions about free riding decay, and the effects of group size and marginal returns, correlate well with the experimental data. The experimental evidence reported in Isaac, Walker and Williams (1990) seems to indicate that the distribution of strategies, as measured by first period choices, is relatively uniform, with some concentration around giving proportions that are multiples of ten. As the game is iterated, the proportion of subjects that free ride increases, and average giving decays toward free riding. Moreover, Isaac and Walker (1988) and Isaac, Walker, and Williams (1990) find that the larger the group size and the larger the marginal returns, the slower the decay. This too is consistent with the replicator-based model of free riding.

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[1] In particular, the distribution has 95.25% of the weight on the 50% strategy, while the remainder is evenly distributed across the other types.

There is one regularity found by Isaac and Walker (1988) and Isaac, Walker and Williams (1990), however, that is not captured by this simple model of decision making. Comparing separate groups in which mg is identical, they find that decay is slower for groups with higher m and lower g, than in groups with lower m and higher g. While the difference is not vast, it is persistent. This is clearly at odds with proposition 4 above, which indicates that the dynamics depend only on mg, and indicates the potential benefit of more detailed research into the evolutionary models of giving. For instance, the above model is based on the assumption that large and small groups sample the population at the same rate, that is, the dynamic is based on the population average, regardless of group size, g. However, subjects in small groups have less exposure to different "types" of strategies, hence sample the population at a slower rate. As a result, their convergence to free riding may be slower, even though mg is the same. Generalizing and developing this aspect of the model may yield predictions consistent with this last experimental result.

6. Conclusion

We have developed a simple model of behavior based on evolutionary dynamics. The underlying model appears to be intuitively plausible, and is consistent with a variety of behavioral models. Despite the model's apparent simplicity, it produces a rich set of predictions. Moreover, these predictions closely coincide with the experimental results, which vis-à-vis optimization models appear anomalous. This indicates that the standard techniques for analyzing public goods games may, in practice, be failing to capture some important qualities of these games. This in turn suggests that our understanding of the private provision of public goods may be improved by more careful research into evolutionary game theory, and by theory and experiments that examine the motives, decision processes, and dynamics of public goods games.

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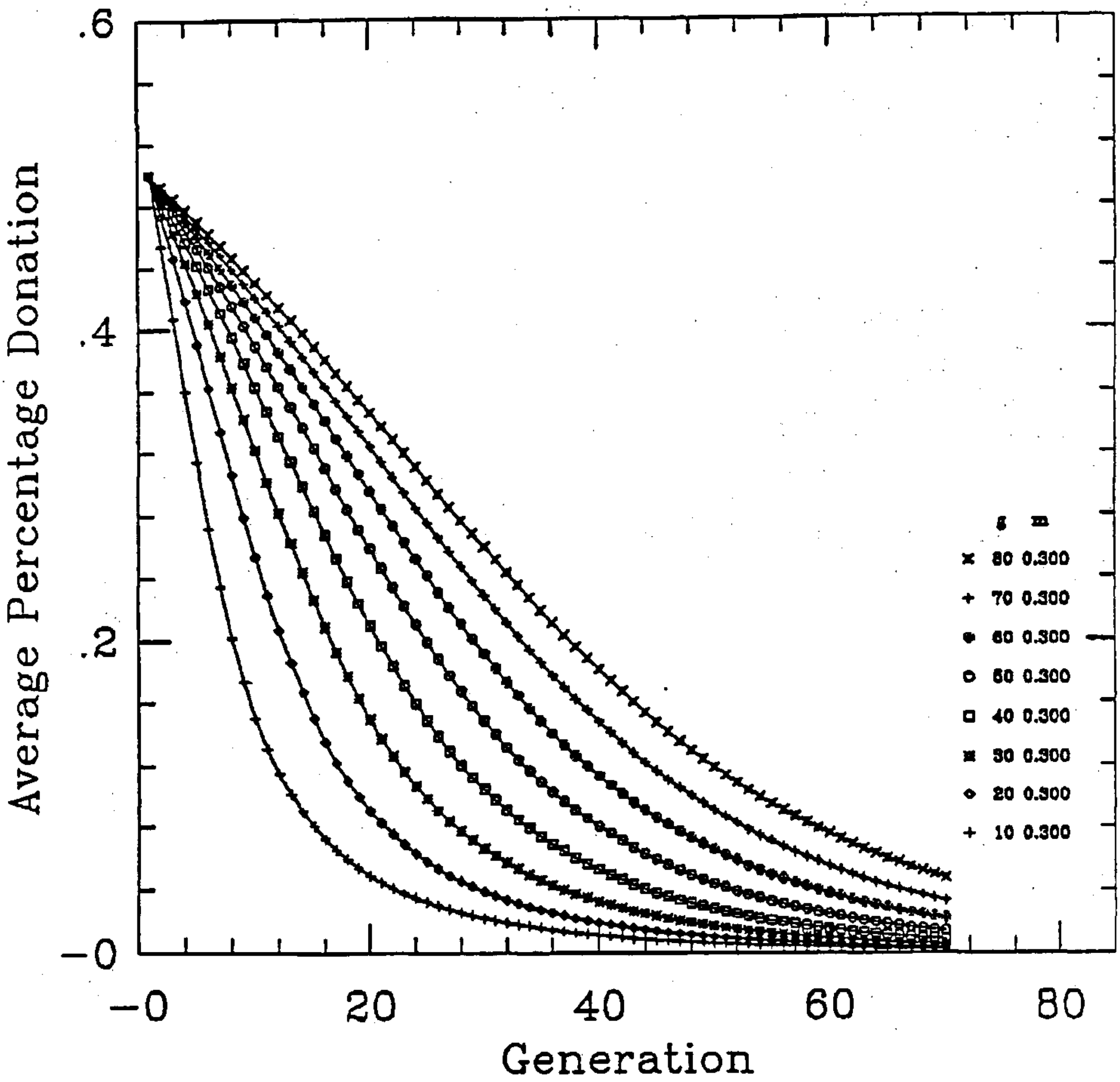

FIGURE 1

Uniform Initial Distribution

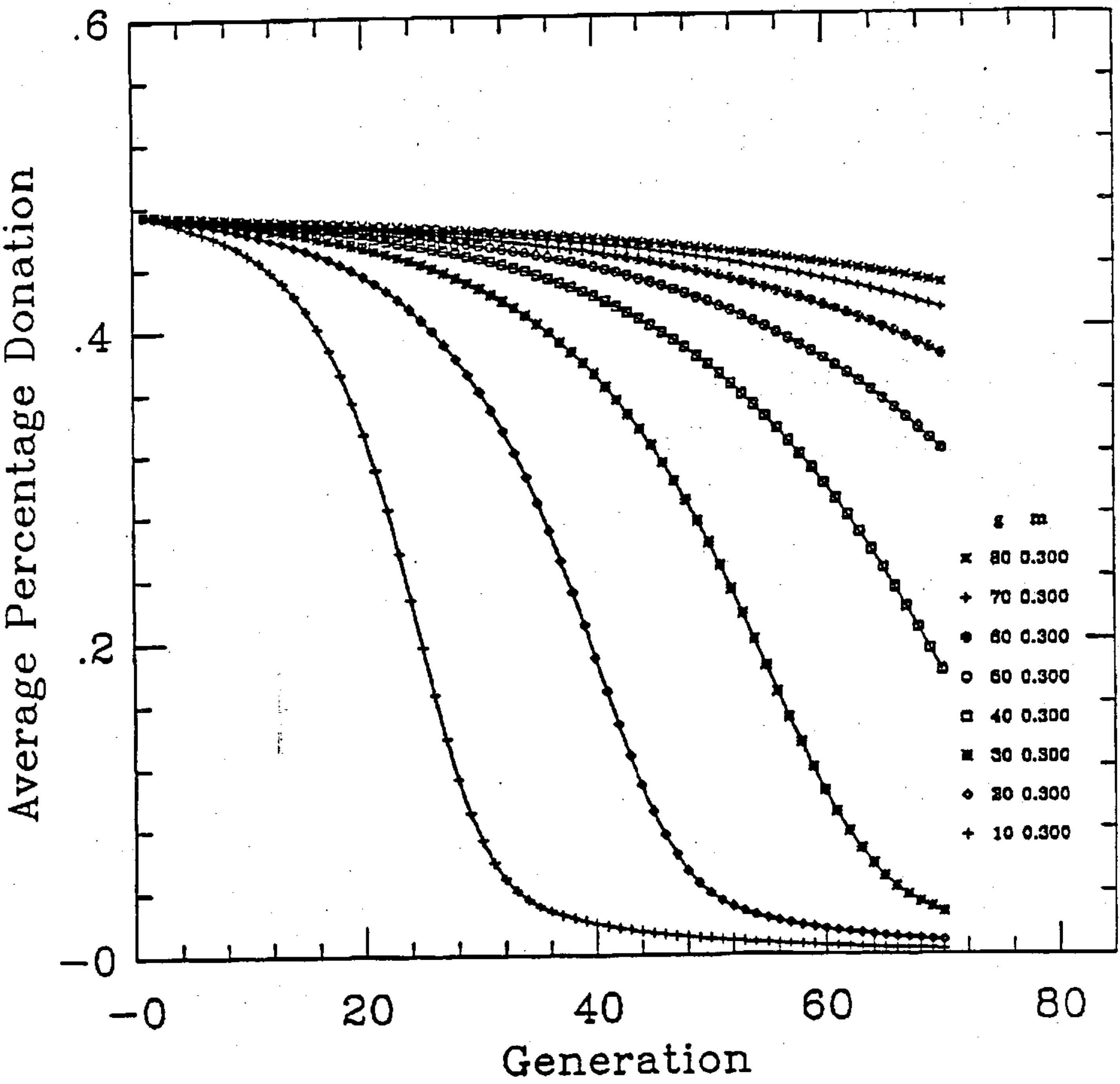

FIGURE 2

Biased Initial Distribution