# The Dynamics of Continuing Conflict[*]

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## Abstract

There is a relatively small but growing literature in economics that examines conflictive activities using a framework in which agents allocate their resource endowments between wealth production and appropriation. To date, studies in this literature have employed a similar one period game theoretic framework. We propose a methodology to extend this literature to a dynamic setting, and illustrate it by modeling continuous conflict over renewable natural resources between two rival groups - an interesting topic in its own right. Recent case studies suggest that natural resource scarcities in less-developed countries (LDCs) lead to conflict, and predict more conflict in the future. However instances of conflict over resources in LDCs, absent resource scarcity, also exist. Thus, it appears that the role of renewable resources in conflict may be greater than simply a conflict trigger. Our model illustrates a complex non-linear dynamic interaction between the populations of the groups and the resource stock, with periods of heavy and light conflict. The system's steady states are identified, and comparative statics are computed. The system's global dynamics are investigated in simulations. Applications of our methodology to other types of conflict are discussed at a general level.

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[*] We thank Michael Baye, Elinor Ostrom, and especially Charles Anderton for valuable comments. We also thank seminar participants at the Eighth Junior Master Class for the Study of Formal Theory in Political Science at the University of Illinois, and the Indiana University workshop on Business Economics and Public Policy for useful discussions. The authors are responsible for any errors in the paper. Email: rreuveny@indiana.edu and jwmax@indiana.edu

# 1 Introduction

There is a relatively small but growing literature in economics, based on the seminal work of Hirshleifer (1988), which focuses on the allocation of an input endowment between wealth production and appropriation.[1] Hirshleifer develops a one period game theoretic framework that augments the standard economic theory of production and exchange by treating wealth appropriation as a basic economic activity. Production is peaceful, where as appropriation is conflictive. Models that adopt this framework typically include two rival groups. Each group's ultimate share of the contested wealth depends on its allocation of resources to appropriation. The contested wealth also depends on this allocation: the more resources allocated to conflict, the less resources available for production of the contested wealth. Each group maximizes its wealth by allocating its resources among production and appropriation with this basic tension in mind.

Hirshleifer's framework has been extended in various ways to include differentiation between defensive and offensive activities, trade, and the use of generalized or various specific functional forms. However, each of these extensions employs a one period game theoretic framework. Several authors in this literature are aware that this is a limitation of the approach, and have called for a dynamic extension of the basic approach (see e.g., Skaperdas, 1992; Hirshleifer, 1995; Grossman and Kim, 1995). The goal of this paper is to take an initial

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[1] Works in this area include, among others, Hirshleifer (1989, 1991, 1995), Skaperdas (1992), Grossman and Kim (1995), Skaperdas and Syropoulos (1996), Neary (1997) and Anderton et al. (1999).

step in answering these calls.[2]

We develop a relatively simple method to extend Hirshleifer's static framework to a dynamic setting, which acknowledges an important motivation for conflict. Parties fight over wealth not only for instant gratification, but also for the ability to invest their spoils in order to increase their own pool of resources, which will then be available for future productive and conflictive activities. In a dynamic setting, it is necessary to link each group's spoils to its resource pool in subsequent periods. At the same time, the production of wealth may require additional inputs that cannot be easily redirected for use in appropriation activities. Often the usage rate of these inputs has an impact on their availability in future periods. Thus, the model needs to distinguish between these two types of inputs and track their interactions and availability over time.

Our approach can be applied to various settings. For example, consider two nations that fight over some wealth. Both the victor and the loser (e.g., Germany after World War I) are likely to invest remaining resources to develop their economies, which in turn generates resources available for future conflict . It is unrealistic to assume that all economic resources are devoted to conflict activities. As such, the evolution of these resources will have to be tracked over time in order to determine the size of the wealth at stake at any point in time. Similarly, consider competition between two firms. The firm may compete over a pool of

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[2] We believe our study is the first to fully model conflict decisions in dynamic context based on Hirshleifer's initial work. However, it is worth noting that the issue of conflict dynamics has been considered in prior literature. Usher (1989) develops a fascinating model in which a society moves between anarchy and despotism. However, he provides no specific solution for the transition between these two states. Brito and Intriligator (1985) develop a two period game theoretic model which studies the circumstances under which conflict over the rights to a flow of a single good leads to the outbreak of war. This model is basically static, however, as the two periods game is played only once.

potential profits by investing in R&D and marketing strategies. The victor in any period (perhaps defined as one product cycle) will be better positioned to capture potential profits in subsequent periods. It will have greater resources to devote to product development and it may enjoy a greater level of customer loyalty. However, resources which could have been devoted to R&D will have to be combined with other inputs (e.g., basic labor, raw materials) in order to make the firm's product. In a dynamic setting, the evolution of these inputs must be tracked over time in order to reflect the totality of the potential profits at stake for each firm.

Keeping in mind that our methodology is generalizable, we apply it to conflict over harvested renewable resources. Several economic models of conflict use this case, among others, to motivate their analyses (e.g., Hirshleifer, 1995; Neary, 1997). At a basic level, we model conflict between two groups over harvested resources. The terms conflict and appropriation are used interchangeably throughout the paper. Conflict is assumed to cover a spectrum of activities such as threats, robbery, safeguarding, and attacks. To make the analysis tractable, we ignore the potentially destructive effect of conflict.[3] In each period, each group divides its labor endowment between harvest and conflictive activities, which ultimately result in a share of the combined harvest of the two groups. Periods are linked in two ways. First, the harvest of each group depends not only on its labor allocation but also on the resource stock, which changes over time depending on harvest activity and its own natural growth rate. Second, each group's population growth rate is positively related to

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[3] Such assumptions are unlikely to alter the qualitative results of our model, while adding considerably to its complexity. We return to this topic in the concluding section of the paper.

its ultimate share of the total harvest, as determined by its relative level of conflict activity. These links give rise to a complex dynamic interaction between conflict, harvest, population and natural resources.

As shown in Section 3, the model has four steady states that exhibit either no population in one or both groups, or no resource stock. We focus on a fifth steady state in which both rival groups and the resource stock co-exist. Comparative static analysis on this steady state reveals that seemingly positive changes to the resource stock, such as raising its carrying capacity and growth rate, raise the level of conflict in the model. An increase in a given population's fertility also raises conflict, whereas an increase in harvesting efficiency raises conflict only when the resource stock is high. Finally, we find that groups that are more efficient at conflict allocate fewer resources to this activity, and enjoy greater wealth.

We study the model's global dynamics via numerical simulations in Section 4. The conflict parameterization draws on Hirshleifer (1989), and the resource and population parameterization draws on Brander and Taylor's (1998) work on Easter Island. The simulations reveal periods of heavy and light conflict. Interestingly, it is often the case that conflict reaches a peak when the resource stock is low. This is so despite the fact that we make no assumption that the two groups are competing *because* of scarce resources. This suggests that studies which examine only a short interval prior to the outbreak of violence might be incorrect in attributing those outbreaks to resource scarcity. We also find that prior population growth contributes to present conflict, and thus long-range planning is likely necessary to diminish conflict over natural resources. Increasing resource carrying capacity raises the system's volatility, while increasing its growth rate reduces system volatility. Raising harvesting efficiency raises system volatility, and could also reduce the resource to a level that cannot sustain the population, leading to system collapse.

In Section 5 we examine the model's implications for the works of Hirshleifer and Brander and Taylor. First, we examine the consequences of the model for a phenomenon Hirshleifer terms the Paradox of Power. Simply stated, this paradox implies that rival groups with disproportionate endowments (e.g., population) will have equal power in the sense that they will win an equal portion of the contested prize. Weaker groups devote proportionally more of their endowments to winning the prize than do stronger groups. We find that in our dynamic setting, the Paradox of Power is necessarily a short term phenomenon. Basically, this result arises from the fact that in splitting the prize, the less populous group wins more per capita, and therefore grows faster. Soon the populations of the two groups (and thus their effort endowments) equalize and the necessary precondition for the Paradox of Power (disproportionate effort endowment) evaporates.

Second, we address the question of whether conflict of the type we study could have taken place on Easter Island. The model of Brander and Taylor (1998) generates population trajectories which mimic evidence gleaned from historical studies on Easter Island. While these authors do not explicitly consider the possibility of conflict over the possession natural resource, there is abundant historical evidence which indicates that such conflict existed on the island (see, e.g., Keegan, 1993; Engleit, 1990).[4] Using a parametrization similar to that

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[4] Keegan (1993) also describes African Nguni conflicts over natural resources in the early 19th century.

used by Brander and Taylor, we generate trajectories that also mimic the historical evidence. Hence, conflict of the type we describe could have taken place on Easter Island, and could have contributed to the famous collapse of its civilization.

## 2 Model

Since Malthus' (1798) work on population growth, social scientists have been interested in the links between the level of renewable resources and conflict. In recent years political scientists have taken the lead in empirically documenting cases in which resource scarcity was a causal factor in conflict in less developed countries (LDCs). Scholars have suggested four channels through which this tends to happen: decline in economic performance, clashes due to population migration, weakening of political institutions, and exacerbation of existing socio-economic-political cleavages.[5] These specific event studies have neglected the interplay between resource scarcity and conflict that can take years to unfold. As such, several questions have been left unanswered. For example, once groups are in conflict, does a rise in the contested resource stock lead to increase/decrease of conflict? Does a decline in the population of one or both groups lead to a reduction in conflict? How does conflict affect the dynamic interplay between resource and population stocks? These studies also ignored the possibility that conflict could exist when no acute resource scarcities exist, which is studied in economic literature on conflict (e.g., territorial conflict, ethnic rivalry). In addition to illustrating our method, we also seek to shed light on these questions.

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[5] For a review of this literature see, for instance, Maxwell and Reuveny (2000). For compilations of case studies demonstrating these effects see, for instance, Myers (1993) and Homer-Dixon (1998). Tir and Diehl (1998) employ statistical methods. The issue is not without debate. For studies that doubt the importance of these channels, see Deudney (1990) and Simon (1996).

The term resource scarcity requires some clarification. From an economic perspective, all goods are scarce and prices reflect their relative scarcity. In the literature on renewable resources and conflict, goods are considered scarce when their quantities fall below levels that induce conflict activities. As noted in the political science literature, conflict over scarce resources is more likely when the resources are necessary for livelihood (e.g., food, water), or when the institutional arrangements required for prices to be able to mediate this scarcity (i.e., property rights and markets) are not well developed.

In a related study, Maxwell and Reuveny show that while resource scarcity can lead to conflict, the resulting conflict impacts the population and resource stocks, and as such may affect the likelihood of future conflict, once the present spell has subsided. One drawback of their study is the fact that conflict is assumed to be triggered at some exogenous level of resource scarcity, and end if resources rise above that level. We endogenize the conflict decision making. The origins of conflict are assumed to be exogenous. In principle, they could be any of the above resource scarcity related four channels, as well as others.

Our paper also draws from the literature on renewable resources and population dynamics. The studies of Prskawetz et al. (1994). Milik and Prskawetz (1996) and Brander and Taylor (1998) are most relevant to our work. Using a similar predatory-prey setting, these studies model the relationship between population and renewable resources, assuming that fertility grows with resource harvest. The model's global dynamics are then studied in numerical simulations. Brander and Taylor, in particular, use this setting to examine the collapse of Easter Island, offering an explanation in the spirit of scholars that link the collapse to man-made natural resource depletion. Their simulation generates resource and population trajectories that generally are in accord with historical data.

Population, resource and conflict dynamics in LDCs are complex. As such, our model is necessarily a simplification of reality. We make six assumptions. First, there are two rival groups, each dependent on the same contested renewable resource. Each group is modeled as a cohesive actor, ignoring in-group politics. Second, conflict effort ranges from zero to the total effort endowment of a group. Each period, the groups allocate effort between resource harvesting and appropriation, in order to maximize wealth. Third, the share of the total harvest won by each group is proportional to the ratio of each group's conflict effort to the sum of the conflict efforts of the two groups. Fourth, fertility is positively associated with per capita income (i.e., the per capita share of the total harvest won in the contest).[6] Fifth, actors care only about their current incomes. Finally, there are no exogenous interventions in the form of either humanitarian or military aid.

These assumptions are generally consistent with a focus on LDCs. Many LDCs exhibit a positive relationship between income and fertility, depend on the environment for their livelihoods, and have weak property rights institutions. The possibility of conflict is consistent with the breakdown of institutions, and further supports our assumption that individuals care only about their current incomes. Furthermore, agents that care about their future incomes, or the incomes of future generations, would need to follow optimal rules of allocating effort between productive and appropriative activities across time. The ability to do so

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[6] This positive relationship is reasonable in light of our focus on LDCs. It is also assumed by Sato and Davis (1971), Prskawetz et al. (1994), Milik and Prskawetz (1996) and Brander and Taylor (1998).

assumes an understanding of the dynamic behavior of rivals, resources, and population. We believe that this requirement is unrealistic in the context of conflict in LDCs.

The basic conflict interaction in the model follows Hirshleifer's work. The model features two rival groups with population sizes of $R_1(t)$and $R_2(t)$, respectively, in period $t$. Actors from each group harvest from a single renewable resource stock. The two groups then engage in conflict over the total harvested resource each period. Labor is allocated each period between the productive (harvesting) activity ($E$) and the appropriative activity ($F$), in order to maximize the group's wealth. Each population's labor endowment is fully utilized (i.e., population size equals the labor force) and labor is the only input in production or appropriation. Thus, we can write $R_1(t) = F_1(t) + E_1(t)$ and $R_2(t) = F_2(t) + E_2(t)$.

The outcome of the struggle depends on the relative allocation of labor resources to appropriative activities. We define $P_1(t)$ and $P_2(t)$ as the contest success functions as follows:

$$P_1(t) = \frac{\alpha_1 F_1(t)}{\alpha_1 F_1(t) + \alpha_2 F_2(t)} \tag{1}$$

and

$$P_2(t) = \frac{\alpha_2 F_2(t)}{\alpha_1 F_1(t) + \alpha_2 F_2(t)}, \tag{2}$$

where $\alpha_1$ and $\alpha_2$ denote the relative efficiency of conflict effort of the two groups, respectively.[7]

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[7] One could also use other contest success functions, such as a logistic function. We intend to study the effects of different contest success functions, which are suggested by Hirshleifer and others in our future research.

With the labor that is allocated to harvesting, the groups harvest (or produce) the amounts $H_1(t)$ and $H_2(t)$ of resources, respectively, in each period. The total harvested resource, $H(t) = H_1(t) + H_2(t)$, is contested by both groups. The income of each group is given by the portion of the total contested harvest it wins.[8] That is:

$$Y_1(t) = P_1(t)\,H(t) \tag{3}$$

$$Y_2(t) = P_2(t)\,H(t) \tag{4}$$

The harvesting technology is modeled as in Brander and Taylor (1998).[9]

$$H_1(t) = \beta S(t)\,E_1(t) \tag{5}$$

$$H_2(t) = \beta S(t)\,E_2(t) \tag{6}$$

Expressions (5) and (6) introduce the notion that the harvest of a renewable resource depends on the contested resource stock ($S$), the amount of labor allocated to harvesting ($E_1$ or $E_2$), and a parameter denoting the efficiency of harvesting ($\beta$).[10] To simplify the notation, hereafter we drop the time dependency of variables (i.e., $S(t)$ becomes simply $S$).

The total contested harvest ($H$) is written using (5) and (6):

$$H = S\beta\,(E_1 + E_2) \tag{7}$$

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[8] Note that this assumption is conceptually equivalent to assuming that each group tries to consume its own harvest, but that the harvest is also subject to possible appropriation from the rival group.

[9] This harvesting technology was first proposed by Schaefer (1957), and is now common in the natural resource literature (e.g., Clark, 1990: Chapter 1).

[10] To make the model analytically tractable, we assumed the harvesting technologies of two rival groups do not differ. In addition to aiding tractability, there seems to be no a priori reason to assume that two neighboring groups should possess significantly different harvesting technologies.

Substituting (7), (1) and (2) into (3) and (4), respectively, we obtain the following expressions for each group's income:

$$Y_1 = \left( \frac{\alpha_1 F_1}{\alpha_1 F_1 + \alpha_2 F_2} \right) S\beta \left( E_1 + E_2 \right) \tag{8}$$

$$Y_2 = \left( \frac{\alpha_2 F_2}{\alpha_1 F_1 + \alpha_2 F_2} \right) S\beta \left( E_1 + E_2 \right) \tag{9}$$

Each group maximizes its income by choosing how much labor to allocate to appropriation and to harvesting, subject to the constraint $F_i + E_i = R_i$, where $i = \{1, 2\}$. When optimizing, each group assumes the other group does not change its own choices, that is, the two groups are assumed to engage in a Cournot-Nash type conflict.[11] Performing the optimization for group 1 yields its reaction function defined by

$$\frac{F_1}{F_2} = \frac{\alpha_2 \left( E_1 + E_2 \right)}{\alpha_1 F_1 + \alpha_2 F_2}. \tag{10}$$

Similarly, the reaction function of group 2 is defined by:

$$\frac{F_2}{F_1} = \frac{\alpha_1 \left( E_1 + E_2 \right)}{\alpha_1 F_1 + \alpha_2 F_2}. \tag{11}$$

Solving (10) and (11) for $F_1$ and $F_2$, and substituting them in (8) and (9), we obtain the Cournot-Nash income solutions in each period:

$$Y_1 = \frac{\sqrt{\alpha_1}}{\left( \sqrt{\alpha_1} + \sqrt{\alpha_2} \right)} S\beta \frac{(R_1 + R_2)}{2} \tag{12}$$

$$Y_2 = \frac{\sqrt{\alpha_2}}{\left( \sqrt{\alpha_1} + \sqrt{\alpha_2} \right)} S\beta \frac{(R_1 + R_2)}{2}. \tag{13}$$

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[11] Hirshleifer (1988, 1989, 1991, 1995) examines several types of conflict including Cournot and Stackelberg, and a few other contest success functions.

We next characterize the dynamic path of population. We assume that population grows according to the equation $\frac{dR_i}{dt} = \delta_i R_i$, $i = \{1, 2\}$, where the growth rate of population ($\delta_i$) rises with per capita income. Specifically, $\delta_i = \varepsilon + \varphi_i$, where $\varepsilon$ denotes the natural net birth rate (i.e., the difference between natural birth and mortality rates), and $\varphi_i = \phi \frac{Y_i}{R_i}$ captures the positive dependence of fertility on the per capita income ($\phi > 0$). Heerink (1994) finds support for this assumption in some LDCs.[12] An alternative interpretation of the dependency of fertility on resource consumption may be that natural resources are essential for procreation. For instance, when food or water decline, fertility will decline. We adopt the convention that $\varepsilon$ is negative, hence, population will decline to zero for sufficiently low rates of fertility, which in turn implies a lower harvest.

Incorporating the fertility assumption into the differential equations of population, we obtain the following two population differential equations.

$$\frac{dR_1}{dt} = R_1(\varepsilon + \phi \frac{Y_1}{R_1}) \tag{14}$$

$$\frac{dR_2}{dt} = R_2(\varepsilon + \phi \frac{Y_2}{R_2}) \tag{15}$$

Note that in previous papers based on Hirshleifer's work, the total available effort was exogenous and static. Here, the total available effort is dynamic, determined by equations (14) and (15).

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[12] A possible criticism of this assumption is that it does not describe higher income countries where fertility seems to decline with consumption, as explained by the theory of demographic transition. Since we focus on LDCs, this criticism applies less in our case. Note, while accepted by many economists, the theory of demographic transition is not without its critics (e.g., Abernethy, 1993). Note also that one could assume that $\varepsilon$ and $\phi$ differ across groups. This would complicate the model without adding much insight since, again, there is no a priori reason to assume that the rival groups differ in these respects.

The growth rate of the resource is given by the difference between its underlying biological growth and total harvesting. As is standard, we assume that the natural growth of the resource is given by the familiar logistic growth, the term inside the square brackets in (16) below.[13] Combining the logistic resource growth assumption with the harvesting functions results in the following resource differential equation:

$$\frac{dS}{dt} = \left[rS\left(1 - \frac{S}{K}\right)\right] - \beta S E_1 - \beta S E_2 \tag{16}$$

where, $r$ is the intrinsic rate of growth of the resource, and $K$ is the resource carrying capacity. The term in square brackets represents the period $t$ natural growth rate of the resource, which is growing with both $r$ and $K$.[14] Noting that $E_1 = R_1 - F_1$ and $E_2 = R_2 - F_2$, Equation (16) can be re-written as follows:

$$\frac{dS}{dt} = \left[rS\left(1 - \frac{S}{K}\right)\right] - \beta S\left(\frac{R_1 + R_2}{2}\right) \tag{17}$$

Substituting (12) and (13) into (14) and (15), the system of equations (14), (15) and (17) describes the evolution of the system in terms of $S$, $R_1$, and $R_2$. To the best of our knowledge, due to its highly non-linear nature, this system does not have an analytical solution. Consequently, we first examine the system's steady states, and then investigate its global dynamic behavior using numerical simulations.

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[13] For details on the logistic growth equations, see Clark (1990: 10).

[14] The logistic function implies that the natural growth rate of the resource stock will be greatest when the stock is low. As the stock rises to its carrying capacity growth will slow, eventually to zero (at $S(t) = K$).

## 3 Steady State and Comparative Statics

The steady state solutions are found by setting the time derivatives of $R_1, R_2$ and $S$ in (14), (15), and (17) to zero.

$$R_1\left(\varepsilon+\phi\frac{\sqrt{\alpha_1}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)}S\beta\frac{(R_1+R_2)}{2R_1}\right)=0 \tag{18}$$

$$R_2\left(\varepsilon+\phi\frac{\sqrt{\alpha_2}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)}S\beta\frac{(R_1+R_2)}{2R_2}\right)=0 \tag{19}$$

$$rS\left(1-\frac{S}{K}\right)-\beta S\left(\frac{R_1+R_2}{2}\right)=0 \tag{20}$$

This system of equations (18), (19) and (20) has five solutions. The first steady state, in which $R_1 = 0, R_2 = 0$,and $S = 0$, depict a situation in which both populations have declined to zero following exhaustion of the natural resource. In the second steady state, $R_1 = 0, R_2 = 0$,and $S = K$ which describes a situation in which both populations have declined to zero before the resource has been depleted, and the resource recovers to its carrying capacity. The next two steady states are "semi-corner" solutions. In one steady state $R_1 = 0, R_2 = R_2^*$,and $S = S^*$, and in the other $R_1 = R_1^*, R_2 = 0$,and $S = S^*$, where superscripts denote some constant positive level.[15] The fifth steady state is denoted as an "internal" solution, since it depicts a situation in which $R_1 > 0, R_2 > 0$, and $S > 0$. In this case, the solutions are given by:

$$R_1=\frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi}+1\right)\frac{\sqrt{\alpha_1}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)} \tag{21}$$

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[15] Consider, for example, the semi-corner steady state when $R_2 = 0$ and $R_1 = R_1^*$. In this case, the model implies that in order to maximize its payoff, which is given in Equation (8), group 1 needs to allocate all of its labor effort to harvesting.

$$R_2 = \frac{2r}{\beta}(\frac{2\varepsilon}{K\beta\phi}+1)\frac{\sqrt{\alpha_2}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)} \tag{22}$$

$$S = \frac{-2\varepsilon}{\beta\phi} \tag{23}$$

The effort allocations for appropriation and harvesting are then given by:

$$F_1 = \frac{R_1}{2}\sqrt{\frac{\alpha_2}{\alpha_1}} \tag{24}$$

$$F_2 = \frac{R_2}{2}\sqrt{\frac{\alpha_1}{\alpha_2}} \tag{25}$$

$$E_1 = \frac{R_1}{2}\frac{\left(2\sqrt{\alpha_1}-\sqrt{\alpha_2}\right)}{\sqrt{\alpha_1}} \tag{26}$$

$$E_2 = \frac{R_2}{2}\frac{\left(2\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)}{\sqrt{\alpha_2}} \tag{27}$$

In the internal steady state it must be the case that $\frac{2\varepsilon}{\beta K\phi}+1>0$, which implies that the steady state resource stock is below its carrying capacity (observe from (23) that $S=\frac{-2\varepsilon}{\beta\phi}$). If this condition does not hold, (23) implies $S>K$, and (21) and (22) imply $R_1<0$ and $R_2<0$. Then, the system collapses to the corner steady state $S=K, R_1=R_2=0$. If the rate of population growth of one of the groups becomes negative due to a low income (and therefore low fertility), the system will collapse to a steady state with only one group. If the resource is exhausted before one or both of the populations are exhausted, the system will collapse to the steady state with zero population and resource stock.

Equations (24) and (25) imply that the system exhibits conflict in steady state. This result differs from Maxwell and Reuveny (forthcoming), where conflict was temporary. Ignoring strategic interactions, they assume that conflict arises when the level of resource per capita falls below a threshold. In their model, conflict raises the mortality rate. Consequently, it raises the level of resource per capita and the system returns to peace. The conflict we study here is less drastic, *i.e.*, the destructive effects of conflict are ignored. However, as we show in simulations of the model below, the steady state conflict in (24) and (25) is much lower than the maximum attained level of conflict. Hence, we get a similar flavor to Maxwell and Reuveny in that maximum conflict levels do not arise in the steady state.

When the resource carrying capacity ($K$) or the intrinsic growth rate ($r$) rise, *ceteris paribus*, equations (21), (22) (23) imply that each group's steady state population rises and the resource stock does not change. At the same time Equations (24) and (25) imply that the total amount of resources devoted to conflict rises. The reason for this result is clear. When $r$ or $K$ rise, the natural growth rate of the resource $rS\left(1-S/K\right)$ rises, which increases the yield from each group's harvesting activity. This in turn raises the level of the contested prize (the total harvest) and each group's share of the prize. As a result each group's population rises as does the total resources devoted to conflict. Thus, if group rivalries are not driven solely by resource scarcities, interventions of the type mentioned here may actually raise the level of observed conflict activity.

As the fertility parameter $\phi$ rises, Equation (23) implies that the steady state resource stock falls, and Equations (24) and (25) imply that the steady state allocations of effort to conflict rise. Hence, the model implies that the effort allocated to conflict grows with group's size. The effect of an increase in the natural net mortality rate ($\varepsilon$) is naturally opposite.

The effect of $\beta$ on conflict is given next:

$$\frac{\partial F_i}{\partial \beta} = \frac{\sqrt{\alpha_j}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right) K\beta^3\phi}(-4\varepsilon - K\beta\phi) \quad i, j = \{1, 2\}; i \neq j \tag{28}$$

Recalling (23), the sign of (28) is positive if in steady state $S > K/2$, and negative if in steady state $S < K/2$. That is, if the steady state $S$ is relatively high (low), raising harvesting efficiency raises (lowers) conflict. From (23), the steady state $S$ rises with net mortality rate, and falls with harvesting efficiency and fertility. Hence, when the mortality rate is high, and harvesting efficiency and fertility are low, a rise in harvesting efficiency is more likely to raise conflict. In particular, as conflict intensifies, one could expect the net mortality rate to rise. While this effect is not included here, the model implies that technological progress in harvesting efficiency will intensify conflict.

The effects of the conflict parameters on the steady state are in the spirit of Hirshleifer, but with a few differences. Equation (24) implies that when group 2 gets better at conflict ($\alpha_2$ rises), group 1 allocates more people to conflict:

$$\frac{\partial F_1}{\partial \alpha_2} = \frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi} + 1\right)\frac{0.5\sqrt{\frac{\alpha_1}{\alpha_2}}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)^2} > 0 \tag{29}$$

Intuitively, this is so because as group 2 gets better at conflict ($\alpha_2$ rises), its marginal return to harvesting has risen relative to conflict (as it retains more of its harvest). It then allocates more effort to harvesting. This results in $\frac{\partial E_2}{\partial \alpha_2} > 0$ and $\frac{\partial F_2}{\partial \alpha_2} < 0$. The improved conflict efficiency of group 2 lowers the marginal return to group 1 from harvesting (as it retains less of its harvest). This results in $\frac{\partial E_1}{\partial \alpha_2} < 0$ and $\frac{\partial F_1}{\partial \alpha_2} > 0$. However, unlike Hirshleifer's model, the strength of these effects grows with $K$, $r$ and $\phi$, and falls with $\varepsilon$.

Finally, we derive the comparative statics effect of the conflict parameters on the groups' populations and incomes, which are not studied by Hirshleifer as his effort endowments are exogenous. Equations (11), (12), (21) and (22) imply that an increase in $\alpha_1$ raises group 1's population and income, and an increase in $\alpha_2$ reduces group 1's population and income. Hence, a group that becomes better at conflict is able to sustain a higher income and population. But when its rival gets better at conflict, the group's population and income will be smaller.

## 4 Dynamics

To the best of our knowledge, our system of non-linear differential equations for $R_1, R_2$ and $S$ does not have an analytical solution. Two methods may be used in such cases to learn about the system's dynamics, local stability analysis and numerical simulations. A local stability analysis involves linearizing the system around each steady state and finding its eigenvalues. This method is not tractable analytically in our case since the system's characteristic equation is cubic.[16] Consequently, we find a global solution to the system via numerical simulations. It should be noted that these two methods are related. As time passes, if the system tends toward a steady state in a particular simulation, then it is likely that this steady state is stable for the parameters used in that simulation.

In order to solve the system numerically, we must settle on a particular parameterization or group of parameterizations. There are, of course, many sets of parameters from which

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[16] See, for example, Spiegel (1968: 32) for the general solution of a cubic equation. Note that since the system is of an order higher than two, the phase diagram approach is not appropriate here.

one could choose. It is also clear that any such set is arbitrary to some extent and the solution may only apply to that set. We are aided in our choice of parameters by the fact that our model extends and integrates two important works which themselves investigate parameterized models, namely, Brander and Taylor (1998) and Hirshleifer (1989, 1991). We base our parametrization on these works. We refer the reader to these two studies for a fuller discussion. Here we briefly describe these parameters.

Brander and Taylor choose parameters so as to roughly mimic historical estimated information about Easter Island. The carrying capacity, $K$, is set to 12,000. The resource growth rate, $r$, is set to 0.04 per decade, the natural mortality rate of the population, $\varepsilon$, is set to -0.1 per decade, implying that without harvesting the population will eventually disappear. The fertility parameter, $\phi$, is set to 4, and the harvesting efficiency parameter, $\beta$, is set to 0.00001. The initial populations are set to 40 each, and the initial resource stock is set at 12,000. The conflict efficiency parameters are taken from Hirshleifer, and in the base case both are set to 1.[17]

We start with a base case and then change the model's parameters, one at a time. In each case, the parameters not mentioned are set at their base levels. In general, we are more interested in the qualitative nature of the results, rather then their exact numerical realization. However, we also compare our results to documented information regarding

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[17] These numbers imply that population would decline (rise) if the resource were below (above) 50 percent of carrying capacity. In addition, as noted by Brander and Taylor, the estimated initial population for the island ranges from around 20 to 100 and more. The initial population on our "island" is double that of Brander and Taylor. We also report a case with the same initial population as in Brander and Taylor (i.e., 40), but the results are virtually identical.

Easter Island.

Figure 1 presents the base case for group 1's population ($R_1$), people allocated to conflict ($F_1$), the resource stock ($S$), and income ($Y_1$).[18] As shown, the system cycles toward a steady state. Income is a leading indicator of population and conflict, while $S$ leads $R_1$ and $F_1$. This is so because income affects fertility, and a rise in $S$ raises harvest, income and fertility. $F_1$ is high when $S$ is low, which deserves further comment. Many studies link resource scarcities to conflict. Our simulations point out that both the past and current resource use contribute to conflict. The past resource feeds population growth, which heightens resource use. Hence, the model highlights the importance of the timing of policies to alleviate conflict. For example, measures of birth control need to applied when the size of population is relatively small, and harvesting control need to be applied when the resource stock is relatively large. It is also worth noting from Figure 1 that it is frequently the case that the level of observed conflict is often at its peak when the level of resources reach their trough. Thus the model generates outcomes in accord with the notion that resource scarcity induces conflict even though we do *not* assume that the conflict activities modeled here are driven by resource scarcity. Thus one must be cautious in drawing causal links between resource scarcity and conflict solely from the observation that conflict is high when per capita resources are low.

[Insert Figure 1 here: Base Case]

Figure 2 investigates the impact of conflict efficiency. We change $\alpha_1$ to 1.25 and $\alpha_2$ to 0.75,

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[18] In this case the two groups are similar in every respect. The values for group 2 are therefore identical, and are not shown. In addition, we plot income at ten times its actual level to be able to view it on the same graph with the other variables.

both from a value of 1, making group 1 better at conflict than group 2. Recall that in Figure 1, both groups allocated the same effort to conflict. In Figure 2, the less conflict-effective group allocates more effort to conflict, while the more conflict-effective group allocates less effort to conflict, at each point in time. What is less clear from Figure 2, but is discernible from the underlying data, is that the relative allocation of effort between conflict and harvesting is not constant. The share of people allocated for conflict for group 1 begins at .436, and declines to .387 over three periods. The adjustment is in the opposite direction for the less effective group. Its allocation share begins at .563, and declines to .645 over three periods. The causal factor in the adjustment is the fact that the marginal return to the two activities is affected not only by $\alpha_1$ and $\alpha_2$, but also by $S$, $R_1$ and $R_2$.

[Insert Figure 2 here: Impact of Fighting Efficiency]

Figures 3 and 4 show the impact of raising $r$ and $K$, respectively, on $R_1$, $S$, $F_1$, and $Y_1$. In Figure 3, $r$ is increased to 6% from 4%. In Figure 4, $K$ is increased to 20,000 from 12,000. Increasing $r$ or $K$ has no effect on the steady state resource, but it raises the populations. Comparing Figures 1 and 3, a higher $r$ makes the dynamics more damped, but raises the level of conflict at each point in time. Comparing Figures 1 and 4, a higher $K$ makes the system less damped, but again raises the level of conflict at each point in time. Since at time zero the populations are low and $S < K$, the resource grows above its initial level. This raises harvesting and income. As such, population and conflict rise beyond their base case levels. The greater population, in turn, leads to a dramatic depletion of the resource. These movements are not disposed of after the initial cycle, and subsequent cycles are once again more severe than in the base case.

[Insert Figure 3 here: Impact of Resource Growth Rate]

[Insert Figure 4 here: Impact of Carrying Capacity]

Figures 5 and 6 demonstrate the effect of a change in harvesting efficiency. In Figure 5, $\beta$ is increased from 0.00001 to 0.000015. In steady state, the system tends toward lower resource stock, population and conflict, compared with the base case. In Figure 6, $\beta$ is reduced from 0.00001 to 0.0000075. In steady state, the system tends toward higher levels, compared to the base case. With a high $\beta$, the system is much less damped than with a low $\beta$.

[Insert Figure 5 here: Impact of Increasing Harvesting Efficiency]

[Insert Figure 6 here: Impact of Decreasing Harvesting Efficiency]

Figure 7 demonstrates the effect of increasing harvesting efficiency beyond its level in Figure 5, to 0.0001. In this case the system ends up in a steady state with $S = K$, and $R_1 = R_2 = 0$. With a high $\beta$, population and conflict rise rapidly at the beginning of the simulation. The resource stock declines quickly and population growth turns negative, leading to population collapse. Conflict subsides, going to zero faster than the population. Since both populations go to zero before the resource is fully diminished, the resource is able to return to its carrying capacity. Unfortunately, at this point the system no longer has any human population.

[Insert Figure 7 here: Population Collapse]

# 5 The Paradox of Power and Easter Island

This section evaluates the implications of our model to the works of Hirshleifer and Brander and Taylor.

## 5.1 The Paradox of Power in a Dynamic Setting

In Figures 1-7, the two groups initially had equal effort endowments (i.e., population size). Conflicting groups may, of course, differ in their initial population. We have conducted two simulations to investigate this case. In the first simulation, presented in Figure 8, the only change from the base case (Figure 1) is that the initial population of group 1 is set to 100. The initial population of group 2 is set as in the base case (40). In the second simulation, presented in Figure 9, we keep this initial population size differential, and assume further $\alpha_1 = 1.25$ and $\alpha_2 = 0.75$.

Hirshleifer uses the term Paradox of Power to denote his finding that a disparity in effort endowments between two groups does not imply a difference in their power. Here, power is captured by the income that each group realizes in the contest. In his model, the groups' incomes are the same, although their effort endowments are different. This result arises because the party with a lower effort endowment devotes relatively more of it to conflict, while the party with a higher endowment devotes relatively less of it to conflict.[19]

We now examine the implications of the Paradox of Power in our dynamic setting, where the gains from conflict are "invested" to raise the effort stock. In our model, a difference in

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[19] Hirshleifer finds that this result is generally robust to both different behavioral assumptions regarding the conflict rules of engagement (e.g., Cournot or Stackelberg) and different contest success functions.

the groups' effort endowments means a difference in initial populations (recall that fertility rises in the per capita income gained from conflict). In order to investigate the Paradox of Power, we plot the relative allocations of effort to conflict ($\frac{F_1}{R_1}$ and $\frac{F_2}{R_2}$) and the per capita incomes ($\frac{Y_1}{R_1}$ and $\frac{Y_2}{R_2}$).

Hirshleifer's Paradox of Power is not a steady state result in our model. When the groups' conflict efficiencies are identical (Figure 8), the groups' steady state populations are identical, as in Figure 1. The incomes per capita equalize after around 10 periods. Hence, the steady states incomes $(Y_1, Y_2)$ also equalize. When group 1 is more efficient in conflict (Figure 9), its steady state population is larger than that of group 2, as in Figure 2. While the per capita incomes equalize, the total income of group 1 is higher. Hence, if the groups are identical except for their effort endowment, their steady state incomes and incomes per capita equalize. If the groups differ in their conflict efficiency, their steady state incomes per capita equalize, but the total income of the more conflict-efficient group is higher.

[Insert Figure 8 here: Paradox of Power with Equal Conflict Efficiencies]

[Insert Figure 9 here: Paradox of Power with Unequal Conflict Efficiencies]

Figure 8 illustrates that initially group 2 (the effort-poor group) devotes relatively more effort to conflict than does group 1, and its per capita income is higher than that of group 1. As a result, the population of group 2 grows faster than that of group 1. As time passes, the extent to which group 2 allocates relatively more effort to conflict diminishes, while that of group 1 rises. Eventually, the relative effort allocations equalize across groups. At this point, the populations of the two groups are also equal, determined by the resource and the population parameters of the model. Hence, the group with a low effort endowment has caught up with the group with the high effort endowment.

Figure 9 illustrates the same phenomenon, except that group 2 is, in addition to being initially endowed with less effort than group 1, is also less effective at conflict. In this case, the relative effort allocations to conflict and the populations differ across groups. Group 1 allocates a lower population share to conflict than group 2, and its income per capita is lower. However, note that the income per capita of both groups tend to equalize, and the steady state relative allocations of effort to conflict are the same as in Figure 2 (i.e., 0.387 for group 1 and 0.645 for group 2), regardless of the initial effort endowment, again pointing out that the Paradox of Power result is not a steady state result.

The disappearance of the Paradox of Power in steady state is driven by two factors. The first is the Paradox of Power itself. Namely, the effort-poor group generates a greater return on its conflict allocation (i.e., a greater per capita income). The second factor is that the returns to investment in effort (i.e., population) growth are rising in terms of per capita income (i.e., fertility rises with per capita income). The latter factor is plausible for our setting. While exceptions to this assumption may exist, we can think about non population-related cases where it may also be reasonable.[20]

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[20] For example, *once proper economic and political institutions are in place*, LDCs with low per capita income tend to generate greater returns on investments than DCs. They subsequently grow faster, so that the per capita capital disparity between DCs and LDCs declines.

## 5.2 Easter Island

The simulations' resource and population parameters, and the model's assumptions on fertility and the agents' time horizon are based on Brander and Taylor's (1998) study of Easter Island. Their framework, however, did not include conflict (although their paper briefly alludes to the possibility of conflict). In this context, therefore, it is interesting to examine the effect of conflict on an Easter Island-like society, comparing our simulation results to available historical data on Easter Island.

One needs to be careful in comparing our model with Brander and Taylor's, since conflict is not the only difference between them. Brander and Taylor's agents derive utility from a harvested good and a manufactured good. Our agents derive utility only from the share of the harvested good won by each group. It is clear that the agents on Easter Island had more than one good in the economy. Yet that resource played a crucial role in the inhabitants' livelihoods as other goods were linked to it (e.g., wooden fishing boats, tree trunks used as rollers to transport the island's famous statues). Similarly, Easter Island may have not experienced conflict over resources from day one. However, several researchers note that there was conflict over resources on the island. With these differences in mind, it is still interesting to compare the resource and population trajectories of the two cases, just as it is interesting to compare the economic development of two nations.

Figure 10 presents the resource and the total island population in Brander and Taylor's and our models. Period 0 corresponds to year 400-700 AD.[21] The two models generate

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[21]

fluctuating trajectories which tend toward a steady state. However, there are also differences in the dynamics. In our case, population peaks at 14,000 around period 50, and then declines to around 2,000 around period 130 (year 1700). In Brander and Taylor's case, population peaks at around 10,000 people around period 80, and then declines to 3,800 around period 130.[22] In our case, the resource reaches a minimum of around 3,000 units around period 80, where as in Brander and Taylor's case it reaches a minimum of around 5,000 units around period 110. Compared with Brander and Taylor, then, our system is less damped and exhibits larger fluctuations.

These results are intuitive. The people on our island are more dependent on the resource than on Brander and Taylor's island. Conflict intensifies their harvesting in the beginning of the simulation. The population then rises quickly, and the resource quickly declines. As a result, population also declines. The same dynamics are also present on Brander and Taylor's island, but it is more damped since these agents do not engage in conflict and also derive utility from a second good whose production in the model does not require the resource.

Finally, it is interesting to compare our results to available information on Easter Island. It is harder to compare the resource in the simulation to the real world. As both our model and Brander and Taylor's model are stylized, the resource represents an ecological complex (i.e., soil, forestry, vegetation, water). Nonetheless, as they do, we could discuss popula-

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The date the island was first settled varies across studies. Brander and Taylor use the date 400 AD, Gowdy (1998) and Bahn and Flenley (1993) argue the date is 700 AD, and Brown and Flavin (1999) use the date 500 AD.

[22] In both cases, the model stops being applicable in the late 1700s when the island is no longer a closed system.

tion. The available information on the island is estimated based on various archeological inquiries. We know that when the island was discovered in 1722, the Dutch Admiral Rogeveen estimated that there were 3,000 people on the island, whereas the British Captain Cook estimated in 1774 that there were 2,000 people. The estimated maximum population ranges from 7,000 to 20,000, whereas the timing of this maximum is thought to be in the range of 1200 to 1500 AD.[23]

Given these ranges, and the range of years during which people arrived on the island, our simulation results cannot be rejected as a possible description of the main social forces operating on the island. Of course, the model of Brander and Taylor is also plausible. Hence, we are left with two alternative explanations of the historical rise and decline of Easter Island. One explanation is based on the standard economic paradigm of production and exchange. The second explanation is based on Hirshleifer's competing paradigm of production and appropriation.

## 6 Conclusions

There exists a relatively small but growing literature in economics that examines agent decisions to allocate resources between productive and appropriative (or conflictive) activities in a static setting. There also exists a considerable literature aimed at documenting empirically observed causal links from natural resource scarcity to conflict. The current paper advances both literatures by examining the *interplay* between conflict decisions and the level

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[23] For different population estimates about Easter Island see e.g., Ponting (1991), Bahn and Flenley (1992), Van Tilberg (1994) and Brander and Taylor (1998).

of available natural resources in a dynamic setting.

We modeled two groups that compete for a good harvested from a renewable resource stock. The framework developed assumes that fertility rises with per capita consumption and agents maximize their current consumption, clearly assumptions which are more appropriate for LDCs than for DCs. As the harvested good affects both groups' fertility, fluctuations in the resource stock (which affect the harvest rate) affect population levels. Importantly, since the amount of harvest consumed is affected by the harvest and by appropriative decisions, these decisions affect, and are affected by, each groups' allocation decision.

We summarize our findings in two categories, steady states and dynamics. The model has five steady states. We focused our analysis on the sole interior steady state, which features positive population and resource stocks. In the internal steady state, there is conflict. The conflict-related comparative statics around the internal steady state were investigated. The level of conflict rises with the resource carrying capacity, intrinsic growth rate, and fertility. A rise in harvesting efficiency may cause more or less conflict, depending on the models' harvesting efficiency, natural mortality rate and the resource carrying capacity. An increase in one group's conflict efficiency causes that group to reduce the effort it devotes to conflict, while the other group increases its conflict effort. The more conflict-efficient group's income rises, while the income of its rival drops. The other four steady states exhibit no conflict. In two steady states, only one group survives, where as in the other two steady states, both groups die due to resource depletion. In one of these cases, the resource reaches a zero stock before population dies, and in the other the resource reaches its carrying capacity.

The model's dynamics are complex and potentially unstable. We investigated the model's global dynamics via numerical simulations, with parameterization based on the models that inspired it. In the simulations, the system tended toward either the internal steady state or the corner steady state with zero population, and resource at carrying capacity. The relative allocation of effort to conflict of each group is not constant along the transition path. A higher intrinsic resource growth rate dampens the system. An increase in the resource carrying capacity or harvesting efficiency, on the other hand, makes the system less damped. A too large or small harvesting efficiency may drive the system to a corner steady state with zero population, where the resource is either nonexistent or at its carrying capacity.

We also revisited the insights of two important works that have motivated our paper, namely Hirshleifer and Brander and Taylor. We first examined the model's implications for Hirshleifer's Paradox of Power. We found that the basic conclusion underlying his analysis (namely, resource disparity among competing groups) does not hold in the long run, since the Paradox of Power is self-correcting. Along the transition path, the smaller group allocates relatively more effort to conflict, which allows it to win more resource (in per capita terms), which then causes its population to grow more rapidly, until its population equalizes with the population of the initially larger group. We then compared our population trajectories with the known history of Easter Island, as was done in Brander and Taylor. We found that our framework generates simulation results which approximate known information about Easter Island. Hence, our results could be considered as a plausible explanation to what happened on the island. In other words, the people on Easter Island devoted a portion of their labor endowment to conflict, resulting in great population and resource fluctuations over time.

As our work represents an initial step in the study of conflict dynamics, research extensions are numerous. First, the agents in the model care about current income. While we find this assumption generally appropriate for LDCs under a conflict situation, it would be interesting to find whether the spirit of our results remains when agents care about future incomes. Second, we assumed that fertility rises with income per capita. This assumption, while appropriate for some LDCs, ignores the possibility of demographic transition. Incorporating a decline in fertility above some income is an interesting extension. Third, our agents consume one good and use one production factor. It is natural to add more goods and factors, which will remove pressure from the resource. However, it is not clear whether the nature of our results will disappear in this case. Fourth, the conflict in the model did not have destructive effects. One way to alter this assumption is to make the population mortality rate grow with conflict efforts, and the resource carrying capacity and growth rate decline with these efforts. We suspect these changes would dampen the dynamics, but will not change our basic results. Fifth, future work could employ modifications considered by Hirshleifer and others including a Stackelberg protocol, distinguishing between defensive and offensive activities, and using different conflict success functions. Finally, the steady state in our model differs from that in Hirshleifer's model. This finding suggests that the application of our technique to models that have applied Hirshleifer's work is in order.

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