Normative and descriptive approaches to multiattribute decision making*

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Abstract: The main problem in multiattribute decision making is deciding how to trade off increased value on one attribute for lower value on another attribute. Generally, there are two approaches to multiattribute decision making: Normative approach assuming that the rational behavior of decision maker is based on a set of axioms but descriptive approach based on pairwise comparisons. Firstly, the paper deals with the normative approach. It illustrates the utilization of normative approach on the example from the area of management of a purchase in a firm, in which the choice from the finite set of alternatives is necessary. Then, the same decision problem is solved with aid of analytic hierarchy process developed by Saaty. Finally, the comparison of the two approaches is made.

Key words: multiattribute decision making; normative approach; descriptive approach; utility function; decision tree; analytic hierarchy process; comparison

1. Introduction

The main problem in multiattribute decision making is deciding how to trade off increased value on one attribute for lower value on another attribute. This decision making requires the decision maker's judgment.

The attribute is the means to measure accomplishment of the fundamental objective. The term attribute will be used to refer to the quantity measured on an attribute scale. For example, if an objective is to maximize profit, then the attribute scale might be defined in terms of EUR.

2. Approaches to multiattribute decision making

Generally there are two approaches:[1]

(1) The normative approach. To multiattribute decision making assumes that rational behavior of decision maker is based on a set of axioms: Ordering and transitivity, reduction of compound uncertain events, continuity, substitutability, monotonicity, invariance, finiteness.[2] If decision maker accepts the axioms:

- It is possible to find a utility function for evaluation the consequences (the decision maker's subjective preference structure can be captured by an *m*-dimensional preference function);
- The decision making is consistent with maximizing of expected utility.

(2) The descriptive approach. It is based on pairwise comparisons of the alternatives for all attributes. The results are represented in a matrix. Different scales can be used. The weights for the attributes express their mutual importance. The weighted preferences have to be aggregated for each alternative. There are two different methods

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[1] More about two approaches see Beroggi (1999).

[2] More in detail see Clemen and Reilly, 2001, pp.572-578.

to it. The first one is to add up all the values in a row for each alternative and attribute—preferential aggregation within attribute. Another method is to put the matrices on top of each other and to add up the entries for each cell—preferential aggregation across attributes. The analytic hierarchy process (AHP) assumes that decision maker complies with reciprocal symmetry but not necessarily with intensity transitivity. To reduce the number of pairwise comparisons, an aggregation of attributes into hierarchical clusters may be done. Then, the relative importance of the attributes can be computed. Preferential aggregation leads to a preferential intensity matrix. Preferential aggregation is often assumed to be additive. On the basis of the preferential values, preferential orders for the feasible alternatives can be determined.

The descriptive approach can lead to inconsistencies in the assessments. The different methods of the resolution of inconsistencies are known (Beroggi, 1999, pp.69-73).

Firstly, the normative approach and an additive utility function will be described.

3. Multiattribute decision making using utility function

Firstly, suppose there is only one attribute *x*. The risk attitude of decision maker can be captured by a subjective preference function, named "utility function"—$u(x)$. The utility function represents a way to translate for example EUROs or other measurement units of the attribute into "utility units". If the utility function is concave, the decision maker is risk-averse; If it is convex, the decision maker is risk-seeking; If it is a straight line, the decision maker is risk-neutral (Raiffa, 1968; Brown, 2005). If more attributes are supposed, decision maker's subjective preference structure can be captured by an *m*-dimensional preference function, $u(x_1, x_2, ..., x_m)$, where x_1, x_2,…, x_m are attributes. This function will be called *m*-dimensional utility function, and an additive utility function will be used.

Example: A new car is needed in the company. This car will be used mainly to drive in the city. The decision maker has a basic idea of the brand and type of a car. There are 3 different types that have remained in his "closer" selection. Let us mark them A_1, A_2, A_3. Assume that there are only two parameters of the cars which are important to decision maker—the purchase price and the average fuel consumption for 100km in the city.

The decision maker has the values of the two parameters (see Table 1).

Table 1 The values of the parameters of the cars

	A_1	A_2	A_3
Purchase price (EUR)	9 200	8 300	8 000
Fuel consumption (l/100km)	8.2	8.5	9.0

One of the simple ways of reaching a "reasonable" solution will be demonstrated.

3.1 The additive utility function

Suppose that there are individual utility functions, $u_1(x_1)$, $u_2(x_2), \ldots, u_m(x_m)$ for *m* different attributes from x_1 to x_m. Assume that each individual utility function assigns values from 0 to 1 to the worst and best levels on that particular attribute.

The utility function is said to be additive if it can be expressed as the sum of the individual utility functions:

$$u(x_1, x_2, ..., x_m) = k_1 \ u_1(x_1) + k_2 \ u_2(x_2) + \ldots + k_m \ u_m(x_m) \qquad (1)$$

where k_1, k_2, …, k_m are the weights, and all weights are positive and they add up to 1.

3.2 Assessing individual utility functions

The so called proportional scoring will be used. The most desired value of the attribute will be assigned the utility 1, the least desired, the utility 0 (from the values of the attribute of all relevant alternatives). The utilities of the other alternatives will be calculated according to the relation:

Utility (the value of the attribute of alternative) = (the value of the attribute of alternative–the least desired attribute's value)/(the most desired attribute's value–the least desired attribute's value)

Let us calculate the utility for the price—$u_c(8300)$ and for the fuel consumption—$u_s(8.5)$, of the alternative A_2.

$$u_c(8300) = \frac{8300-9200}{8000-9200} = 0.750$$

$$u_s(8.5) = \frac{8.5-9.0}{8.2-9.0} = 0.625$$

Table 2 are the utilities for the prize and the fuel consumption of all alternatives, apparently $u_c(9200) = 0$, $u_c(8000) = 1$, $u_s(9.0) = 0$, $u_s(8.2) = 1$.

Table 2 The utilities for the price and the fuel consumption

	A_1	A_2	A_3
$u_c(A_i)$	0	0.750	1
$u_s(A_i)$	1	0.625	0

3.3 Assessing weights

All that left now is, to assign weight to the attribute according to the judgment of decision maker and the alternatives (final utilities of the alternatives) can be evaluated by:

$$u(A_i) = k_c \cdot u_c(A_i) + k_s \cdot u_s(A_i) \quad (2)$$

Generally, there are three methods of assessing the weights: pricing out, swing weighting and lottery weights (Clemen & Reilly, 2001, pp.614-620; Clemen, 1991). Suppose that according to one of their, the author obtained $k_C = 0.625$, $k_S = 0.375$.

Now the values of the additive utility function $u(A_i)$ of the alternatives can be calculated

$$u(A_1) = 0.625 \cdot 0 + 0.375 \cdot 1 = 0.375$$

$$u(A_2) = 0.62\,5 \cdot 0.75 + 0.375 \cdot 0.625 = 0.703$$

$$u(A_3) = 0.625 \cdot 1 + 0.375 \cdot 0 = 0.625$$

So, the best alternative is A_2.

3.4 Certainty versus uncertainty

Really, the decision problem in the example concerns decision making under certainty. When making decisions under certainty, we can use ordinal utility functions (value functions). Ordinal utility functions are only required to rank-order sure outcomes in a way which is consistent with the decision maker's preferences for those outcomes. There is no concern with lotteries or uncertain outcomes (Clemen, 2001, p.620).

If decision maker has to make a decision under uncertainty, he/she should use cardinal utility function. A cardinal utility function appropriately incorporates risk attitude of decision maker, so that lotteries are rank-ordered in a way that is consistent with decision maker's risk attitudes.

All of the weight-assessment methods can be used regardless of the presence or absence of uncertainty. If a specific multiattribute preference model was established, the weight-assessment methods amount to various ways of establishing indifference among specific lotteries or consequences. The weights can be derived on the basis of these indifference judgements.

The proportional-scores technique is a special case, and it may be used under conditions of uncertainty by a decision maker who is risk-neutral for the specified attributes.

The distinction between ordinal and cardinal utility models can be important in theoretical models of economic behavior, practical decision analysis applications rarely distinguish between the two (Clemen, 2001, p.621).

Example—continuing 1: Let's extend our example to one more attribute. Add the attribute: the costs of maintaining of the car during its life span. Suppose, the costs of maintaining are uncertain events and decision maker formulated the prior probability distribution for the outcomes of these uncertain events. The consequences of the outcomes in EUROs are also known. The structure of the decision problem with all needed data is in the decision tree in Fig. 1.

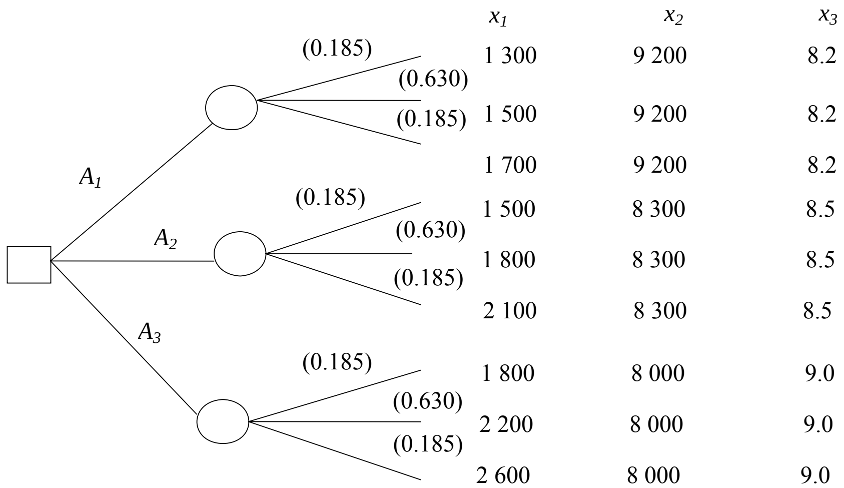


Fig. 1 Decision tree

In Fig. 1: x_1 is cost of maintaining; x_2 is purchase price; x_3 is fuel consumption.

Fig. 2 is the decision tree of the decision problem with the utilities calculated by the proportional scoring method (Terek, 2007). We will consider: The price the weight k_c = 0.6, the fuel consumption the weight k_s = 0.3 and the costs of maintenance the weight k_n = 0.1.

The 3-dimensional additive utility function is calculated according to:

$$u(x_1, x_2, x_3) = 0.1u_n(x_1) + 0.6u_c(x_2) + 0.3u_s(x_3) \qquad (3)$$

Finally, the expected utilities EU are calculated:

$$\mathrm{EU}(A_1) = 0.185 \cdot 0.400 + 0.630 \cdot 0.385 + 0.185 \cdot 0.369 \approx 0.385$$
$$\mathrm{EU}(A_2) = 0.185 \cdot 0.722 + 0.630 \cdot 0.699 + 0.185 \cdot 0.676 \approx 0.688$$
$$\mathrm{EU}(A_3) = 0.185 \cdot 0.662 + 0.630 \cdot 0.631 + 0.185 \cdot 0.600 \approx 0.631$$

So the highest utility value has the alternative A_2.

3.5 Interactions of attributes

Until now, suppose that we are able to model preferences accurately with additive utility function. Really, we need it for the additive independence.

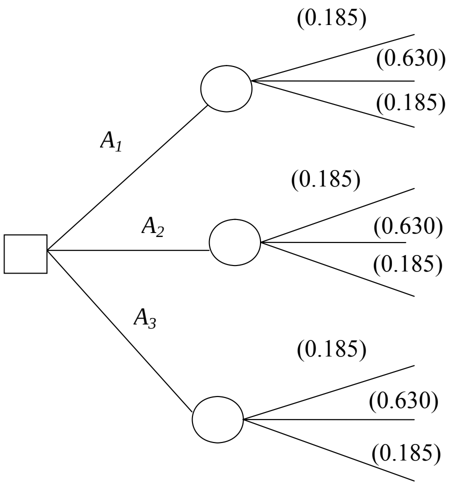
Generally, the situations in which attributes interact are more subtle. For example, two attributes may be substitutes for each other. On the other hand, attributes could be complementary (for example, the success of various phases of a project).

Such interactions cannot be modeled with aid of the additive utility function. The additive model is simply an additive combination of preferences for individual attributes. To capture the interactions as well as risk attitudes, we must think more generally.

It is possible to think about many attributes at once. The multiattribute utility theory concepts using only two attributes will be presented. The ideas can be extended to more attributes. Generally, there are two approaches to assessment a multiattribute utility function. The first approach consists in the direct assessment of the multiattribute utility function, and the other approach consists in the thinking about a multiattribute utility function that is composed of the individual utility function:

$$u(x,\ y)\ =\ c_1 + c_2 u_X(x) + c_3 u_Y(y) + c_4 u_X(x) u_Y(y) \qquad (4)$$

The importance of any such formulation is that it eases the assessment. This way of modeling multiattribute utility function sometimes is called separability. The separability requires some conditions. These conditions concern how the preferences interact among the attributes.



$u_n(x_i)$	$u_c(x_2)$	$u_s(x_3)$	$u(x_1, x_2, x_3)$
1	0	1	0.400
0.846	0	1	0.385
0.692	0	1	0.369
0.846	0.750	0.625	0.722
0.615	0.750	0.625	0.699
0.385	0.750	0.625	0.676
0.615	1	0	0.662
0.308	1	0	0.631
0	1	0	0.600

Fig. 2 Decision tree with utilities

3.6 Independence conditions

3.6.1 Preferential independence

The mutual preferential independence is needed for separability. An attribute *Y* is said to preferentially independent of *X* if preferences for specific outcomes of *Y* do not depend on the level of attribute *X* (Clemen & Reilly, 2001, p.647).

Mutual preferential independence holds for many people and many situations, or that at least it is a reasonable approximation. It is like decomposability property for an objective’s hierarchy.

The mutual preferential independence is the condition that is needed for the ordinal additive utility function. For the decision making under uncertainty, mutual preferential independence is not strong enough. It is necessary, but not sufficient condition for obtaining separability of a cardinal multiattribute utility function (Clemen & Reilly, 2001, p.647).

3.6.2 Utility independence

An attribute *Y* is considered utility independent of attribute *X* if preferences for uncertain choices involving different levels of *Y* are independent of the value of *X*. Imagine assessing a certainty equivalent for a lottery involving only outcomes in *Y*. If our certainty equivalent amount for the *Y* lottery is the same no matter what the level of *X*, then *Y* is utility independent of *X*; If *X* also is utility independent of *Y*, then the two attributes are mutually utility independent (Clemen & Reilly, 2001, p.648).

Utility independence is analogous to preferential independence, except that the assessments are made under uncertainty.

Almost all reported multiattribute applications assume utility independence and thus are able to use a decomposable utility function (Clemen & Reilly, 2001, p.648).

3.6.3 Determining if independence exists

We can imagine the series of pairwise comparisons that involve one of the attributes. With the other attribute fixed at its lowest level, the decision maker should decide which outcome in each pair does he/she prefer, and then, change the level of the fixed attribute. If the comparisons will be the same regardless of the fixed level of the other attribute, then preferential independence holds.[3]

If the preferential independence is studied, the dialogue concerns the comparisons between sure outcomes of *Y* for fixed values of *X*. The study on the utility independence is the same, except that the pairwise comparisons would be comparisons between lotteries involving attribute *Y*.

To establish mutual preferential or utility independence, the roles of *X* and *Y* would have to be reversed to determine whether pairwise comparisons of outcomes or lotteries in *X* depended on fixed values for *Y.* If each attribute turns out to be independent of the other, then mutual utility or preferential independence holds (Clemen & Reilly, 2001, p.650).

3.6.4 Additive independence

If we want to model preferences with additive utility function, the additive independence is needed. Suppose *X* and *Y* are mutually utility independent and decision maker is indifferent between lotteries *A* and *B*:[4]

- A: (x_-, y_-) with probability 0.5;
  (x_+, y_+) with probability 0.5;
- B: (x_-, y_+) with probability 0.5;
  (x_+, y_-) with probability 0.5;

If this is the case, then EU(A) = EU(B) and the additive utility function can be used.[5]

In this case, changes in lotteries in one attribute do not affect preferences in lotteries in other attribute. For utility independence, changes in sure levels of one attribute do not affect preferences for lotteries in the other attribute.

3.6.5 Three or more attributes

When a decision problem involves three or more attributes, modeling preferences is more difficult. Under certain conditions, the multiplicative utility function can be used.

3.6.6 When independence fails

Suppose we want assess a two-attribute utility function over attributes *X* and *Y*. The author has found that

[3] The example of a dialog between analyst and decision maker can be seen in Keeney and Raiffa (1976).
[4] The worst level of the attribute is assigned by "–", and the best level, by "+".
[5] The proof see in Clemen and Reilly (2001, p.652).

neither *X* nor *Y* is independent of each other. A reasonable utility function can be obtained by two ways.

The first possibility is to perform a direct assessment. Pick the best and worst (*x*, *y*) pairs, assign then utility values of 1 and 0, and then use the reference gambles to assess the utility values for other points. The second approach is to transform the attributes and proceed to analyze the problem with the new set. The new set of attributes must capture the critical aspects of the problem, and they must be measurable (Clemen & Reilly, 2001, p.660).

Now the descriptive approach to multiattribute decision making will be studied.

4. Analytic hierarchy process

There are a lot of descriptive preference aggregation methods. The methods of the classic ELECTRE, PROMETHEE and a lot of others are well-known and applied.[6] From this large set of methods, the author will describe more in details with the analytic hierarchy process (AHP), developed by Thomas L. Saaty, described in Anderson, Sweeney, Williams, Martin (2008, pp.744-756). The method is designed to solve complex multiattribute decision problems. AHP requires the decision maker to provide assessments about the relative importance of each attribute and then specify a preference for each decision alternative using each attribute. The output of AHP is a ranking of alternatives reflecting the decision maker preferences.

Example—continuing 2. The problem given in Example—continuing 1 will be solved with the aid of the Saaty method.

Firstly, the preferences for the attributes have to be determined. We need to construct a matrix of the pairwise comparison ratings. Suppose we will use the comparison scale in Table 3 (Anderson, Sweeney, Williams & Martin, 2008, p.747).

Intermediate judgments in Table 3, such as "moderately to strongly more important" are also possible and will receive a numerical rating 4.

Table 3 Comparison scale for the importance of attributes

Verbal judgment	Numerical rating
Extremely more important	9
	8
Very strongly more important	7
	6
Strongly more important	5
	4
Moderately more important	3
	2
Equally important	1

Then, the decision maker has to realize the pairwise comparisons. In each comparison he/she must select more important attribute and then express a judgment of how important the selected attribute is.

Suppose the judgments of decision maker are showed in Table 4.

The numbers in Table 4 express the preferences of attributes. For example, the number 3 in the third row and first column means that for the decision maker, attribute x_3 is moderately more important to attribute x_1 (numerical

[6] A concise overview of descriptive preference aggregation methods can be found in Vincke (1992).

rating of preference is equal to 3). Logically, attribute x_1 is for him less important than x_3 (numerical rating is equal to 1/3).

Table 4 Judgments of decision maker concerning weights

	x_1	x_2	x_3
x_1	1	1/6	1/3
x_2	6	1	2
x_3	3	1/2	1
Sum	10	10/6	10/3

The pairwise comparison matrix for attributes is showed in Table 4. On the basis of comparison matrix, the calculation of the weight (priority) of each attribute is possible. This procedure of AHP is referred to as synthesization. The following simple procedure was used.

(1) Divide each element in the pairwise comparison matrix by its column total.

(2) Compute the average of the elements in each row. These averages provide the weights for the attributes (see Table 5).

Table 5 Weights of attributes

	x_1	x_2	x_3	Weight
x_1	0.1	0.1	0.1	0.1
x_2	0.6	0.6	0.6	0.6
x_3	0.3	0.3	0.3	0.3

In the next step of AHP, the consistency of the judgments has to be verified. For example, if attribute x_2 compared to attribute x_3 has a numerical rating 2, and attribute x_3 compared to attribute x_1 has numerical rating 3, then perfect consistency of attribute x_2 compared to attribute x_1 would have a numerical rating of 6 (2×3). Some degrees of inconsistency can be expected. In Anderson, Sweeney, Williams and Martin (2008, pp.750-752), a simple method of verifying consistency is described. If the degree of inconsistency is unacceptable, the decision maker should review and revise the pairwise comparisons before continuing with the AHP analysis.

In this example, the ideal consistency of weights was met. Then, the use comparison procedure to determine the priorities for the cars, each of the attributes is needed. And the corresponding scale has to be created. In the example, the author will use the scale in Table 6 (Anderson, Sweeney, Williams & Martin, 2008, p.752).

The pairwise comparisons of the alternatives according to the price will be realised. Suppose the judgments of decision maker are showed in Table 7.

In Table 8, the synthesization is done.

In the case of the costs of maintaining, we can not work directly with the probability distributions, but only with some their characteristics. The author will use the expected values. So, firstly the expected values of the costs of maintaining for each alternative will be computed:

$$E(A_1) = 1500$$

$$E(A_2) = 1800$$

$$E(A_3) = 2200$$

Then, we can proceed as in the case of the price.

Suppose the judgments of decision maker are in Table 9.

Table 6　Comparison scale for the preference of alternatives

Verbal judgment	Numerical rating
Extremely preferred	9
	8
Very strongly preferred	7
	6
Strongly preferred	5
	4
Moderately preferred	3
	2
Equally preferred	1

Table 7　Judgments of decision maker concerning alternatives according to price

	A_1	A_2	A_3
A_1	1	1/4	1/8
A_2	4	1	1/2
A_3	8	2	1
Sum	13	13/4	13/8

Table 8　The priorities of alternatives according to price

	A_1	A_2	A_3	Priorities
A_1	1/13	1/13	1/13	1/13
A_2	4/13	4/13	4/13	4/13
A_3	8/13	8/13	8/13	8/13

Table 9　Judgments of decision maker concerning alternatives according to costs of maintaining

	A_1	A_2	A_3
A_1	1	2	8
A_2	1/2	1	4
A_3	1/8	1/4	1
Sum	13/8	13/4	13

In Table 10, the synthesization is done.

By the same procedure also the priorities of alternatives according to the fuel consumption were obtained. The results are in Table 11.

Now, an overall priority ranking can be computed. For each alternative, the weighted sum of priorities will be computed:

(1) For alternative A_1 : $0.1 \cdot 8/13 + 0.6 \cdot 1/13 + 0.3 \cdot 9/13 \approx 0.3154$

(2) For alternative A_2 : $0.1 \cdot 4/13 + 0.6 \cdot 4/13 + 0.3 \cdot 3/13 \approx 0.2846$

(3) For alternative A_3 : 0.1 · 1/13 + 0.6 · 8/13 + 0.3 · 1/13 ≈ 0.4000

So, the best alternative is A_3 . A different result as before was obtained.

Table 10 The priorities of alternatives according to costs of maintaining

	A_1	A_2	A_3	Priorities
A_1	8/13	8/13	8/13	8/13
A_2	4/13	4/13	4/13	4/13
A_3	1/13	1/13	1/13	1/13

Table 11 The priorities of alternatives according to fuel consumption

	A_1	A_2	A_3	Priorities
A_1	9/13	9/13	9/13	9/13
A_2	3/13	3/13	3/13	3/13
A_3	1/13	1/13	1/13	1/13

5. The comparison of the normative and descriptive approaches

The descriptive approach can lead to inconsistencies in the assessments. In descriptive approach, the alternatives' near replicas can lead to structural instability. Functional stability is affected by dependencies among attributes and alternatives. Numerical instability can occur due to changes in parameters (Beroggi, 1999, p.111). The advantage of the methods of descriptive approach is their relative simplicity in comparison with the procedures of normative approach. In normative approach, a probability distribution of the outcomes of the uncertain events can be explicitly included to the decision making procedure. In descriptive approach, only some characteristics of the probability distributions, calculated before the decision making procedure is applied can be explicitly used. In the application of Saaty method, in the example, the means of the distributions calculated before the method was applied.

In the presented example, the author used the linear individual utility functions. Generally, the individual utility functions in normative approach can be linear or non-linear. If a cardinal utility function is applied, the normative approach will be able to determine the attitude of decision maker toward the risk. The individual utility function is linear when the decision maker is risk neutral and non-linear when he/she is risk-averse or risk-seeking. When the ordinal utility function is applied, the individual utility functions can also be linear or non-linear. This information is explicitly included to decision making process. In the author's opinion it is a great advantage of the normative approach.

The normative approach is sometimes criticized because decision makers do not always behave rationally. Some behavioral paradoxes are known.[7] These are situations in which intelligent people make decisions that violate one or more of the axioms, and make decisions which are inconsistent with the expected utility. These paradoxes do not invalidate the idea that we should still make decisions according to the expected utility (Clemen & Reilly, 2001, p.588).

(to be continued on Page 43)

[7] See for example Clemen and Reilly, 2001, pp.578-582.