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Effects of uniform-allocation constraints in networked common-pool resource extraction games

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E-mail: andrew.schauf@ntu.edu.sg**Keywords:** common-pool resources, socio-ecological systems, evolutionary games, self-regulation, bipartite networks, networked commons

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**Abstract**

Communities that share common-pool resources (CPRs) often coordinate their actions to sustain resource quality more effectively than if they were regulated by some centralized authority. Networked models of CPR extraction suggest that the flexibility of individual agents to selectively allocate extraction effort among multiple resources plays an important role in maximizing their payoffs. However, empirical evidence suggests that real-world CPR appropriators may often de-emphasize issues of allocation, for example by responding to the degradation of a single resource by reducing extraction from multiple resources, rather than by reallocating extraction effort away from the degraded resource. Here, we study the population-level consequences that emerge when individuals are constrained to apply an equal amount of extraction effort to all CPRs that are available to them within an affiliation network linking agents to resources. In systems where all resources have the same capacity, this uniform-allocation constraint leads to reduced collective wealth compared to unconstrained best-response extraction, but it can produce more egalitarian wealth distributions. The differences are more pronounced in networks that have higher degree heterogeneity among resources. In the case that the capacity of each CPR is proportional to its number of appropriators, the uniform-allocation constraint can lead to more efficient collective extraction since it serves to distribute the burden of over-extraction more evenly among the network's CPRs. Our results reinforce the importance of adaptive allocation in self-regulation for populations who share linearly degrading CPRs; although uniform-allocation extraction habits can help to sustain higher resource quality than does unconstrained extraction, in general this does not improve collective benefits for a population in the long term.

1. Introduction

Common-pool resources (CPRs) are *non-excludable*, or freely accessible by all members of a community, and *rivalrous*, meaning that increased appropriation by one user can decrease the benefits that are available to other users [1]. The latter property distinguishes CPRs from *public goods*, which have provided the conceptual basis for an extensive literature exploring the role of networked population structure in the evolution of cooperative behavior [2–5]. In typical public goods game (PGG) models played out on complex networks, agents identify either as ‘cooperators’, who make voluntary contributions towards public goods, or as ‘defectors’ who free-ride on the efforts of others. Having assumed one of these identities, each agent is then assumed to apply the same strategy (either contribute something, or do not) to all the goods to which it has access within a network. Some variations upon the basic model have explored the possibility that agents might adaptively adjust their allocations of contributions based on the unique conditions of each public good [6–10]. However, by more completely relaxing the assumption that each agent must present itself either as a cooperator or as a defector in *all* games simultaneously, the mechanism by which cooperation spreads via imitation through a network would be removed. In contrast to these PGG models, the allocation aspects of individual choice have taken greater prominence in studies of networked CPR extraction games. At Nash equilibrium, for example,

agents are found to allocate their extraction efforts unevenly among resources on degree-heterogeneous networks [11]. In Pareto efficient states, which describe the ideal extraction levels by which a population would achieve its maximum possible collective wealth, all agents exert the same overall magnitude of extraction effort, but each allocates its effort unevenly among sources [11, 12]. Furthermore, if agents are inclined to practice *reallocation*—that is, adapting their behavior by shifting their extraction efforts from lower- to higher-quality sources while holding their overall extraction levels constant in magnitude—then they can attain more efficient and egalitarian outcomes at the population level [12].

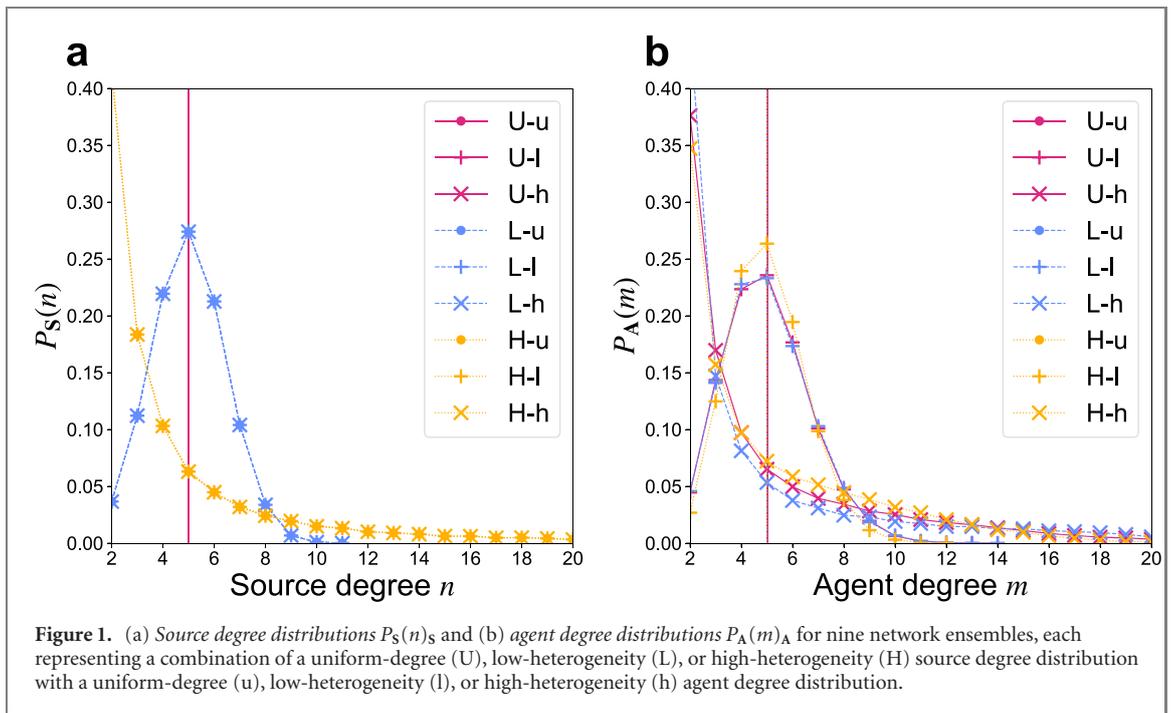
However, various ‘laboratory-in-field’ experiments [13–15] have provided empirical evidence that real-world CPR appropriators, when confronted with multiple resources and changing conditions, do *not* necessarily practice the kind of targeted reallocation described above. Specifically, in response to the degradation of one resource, many individuals tend to reduce extraction not just from the newly degraded resource, but also from other resources. Rather than channelling ‘individual payoff-maximizing behavior’ through reallocation, these individuals reduced the overall magnitude of their extraction efforts [13]. In this way, these individuals seemed to treat a structured environment consisting of multiple distinct resources as if it were a single entity, applying caution to this environment as a whole, even when targeted reallocations of effort might have yielded greater payoffs in the short term. If indeed CPR appropriators behave as did the individuals observed in those studies and interpret the degradation of a single resource as a warning signal to reduce overall extraction, then this could serve to prevent a more widespread cascade of resource depletion from spreading, and so could have implications for the population’s ability to conserve resource quality and effectively self-regulate. In a CPR context, then, behavior that serves to balance an individual’s short-term self-interest with longer-term sustainability of the collective system may not necessarily take the form of obedience to some pre-defined cooperative rule, as if often assumed in PGG or even CPR models [16–18]. Instead, it could involve an adaptive tendency to respond to *local* signs of resource degradation by moderating one’s overall extraction more generally. Aside from these potential advantages, it seems that a coarser-grained perspective—by which an individual effectively treats its environment of resources as a single entity rather than as a multiplicity of distinct parts—could also potentially emerge as a result of various cognitive or logistical limitations placed upon CPR users in some situations [19, 20]. Whatever the origin of this behavior might be, the current study is motivated by the intriguing contrast between the highly non-uniform allocations of effort exhibited by payoff-maximizing agents within computational models and the observed behavior of many real-world CPR appropriators who seemingly tend to treat multiple resources as a unified whole.

In the following, we investigate the consequences of uniform-allocation constraints within networked CPR extraction games. While previous work has explored a case where agents place *greater* emphasis upon allocation [12], this new case represents the opposite extreme wherein agents’ flexibility of allocation is removed altogether. Real-world CPR appropriators (at least to the extent that their adaptive behavior can be meaningfully characterized in terms of the relative emphasis they place upon *allocation* versus overall *magnitude* of effort) will likely fall somewhere on a continuum between these two limiting cases. However, by studying the consequences of these two extreme bounds to *individual* behavior, we hope to delineate the boundaries of a corresponding continuum of *collective* outcomes, between which more realistic systems would presumably lie. Following previous work that focused on adaptive reallocation [12], we apply a heterogeneous mean-field perspective [21, 22] to predict the population-level changes that result from a uniform-allocation constraint on networks with different degree distributions. We then validate these predictions using the ensemble mean results of direct computations on individual network realizations. Our results show that the uniform-allocation constraint drastically reduces the efficiency of collective extraction, particularly in networks with high heterogeneity among CPR degrees (that is, the numbers of appropriators of each resource). In the special case that a network’s multiple CPRs have capacities that are proportional to their degrees, however, the constraint can instead lead to increased efficiency. There, the improvement occurs precisely because the *uniform-allocation* constraint achieves a similar end in that case as does gain-seeking *reallocation*: it redistributes the burden of over-extraction more evenly among the system’s multiple CPRs. Our findings thus strongly reinforce the importance of agents’ flexibility of allocation in networked CPR extraction games. Within the linearly degrading CPR model considered here, a uniform-allocation constraint on individual behavior can indeed help to sustain higher resource quality, but it does so without improving collective wealth in the long run.

2. Methods

2.1. Summary

We consider games played on bipartite networks that link populations of 50 **agents** to 50 common-pool **sources**. The degree of an agent a is denoted by $m(a)$ and the degree of a source s is denoted by $n(s)$; all networks share mean agent degree $\langle m \rangle = 5$ and mean source degree $\langle n \rangle = 5$. Nine network ensembles are



generated, each representing a combination of one of three levels of heterogeneity among source degree (Uniform-degree, Low-heterogeneity, or High-heterogeneity) and one of three similar levels of heterogeneity among agent degree (uniform-degree, low-heterogeneity, or high-heterogeneity), resulting in the **source degree distributions** $P_S(n)$ and **agent degree distributions** $P_A(m)$ shown in figure 1. Within these networks, each agent a decides how much **extraction effort** $q(a, s)$ to apply to each of its affiliated sources s . For each unit of extraction effort exerted upon a source s , the source returns a benefit of a magnitude determined by its **quality**, $b(s)$. Source quality decreases linearly as the **collective extraction** $\vec{q}(s)$ applied to the source increases:

$$b(s) = \alpha - \beta \vec{q}(s), \quad (1)$$

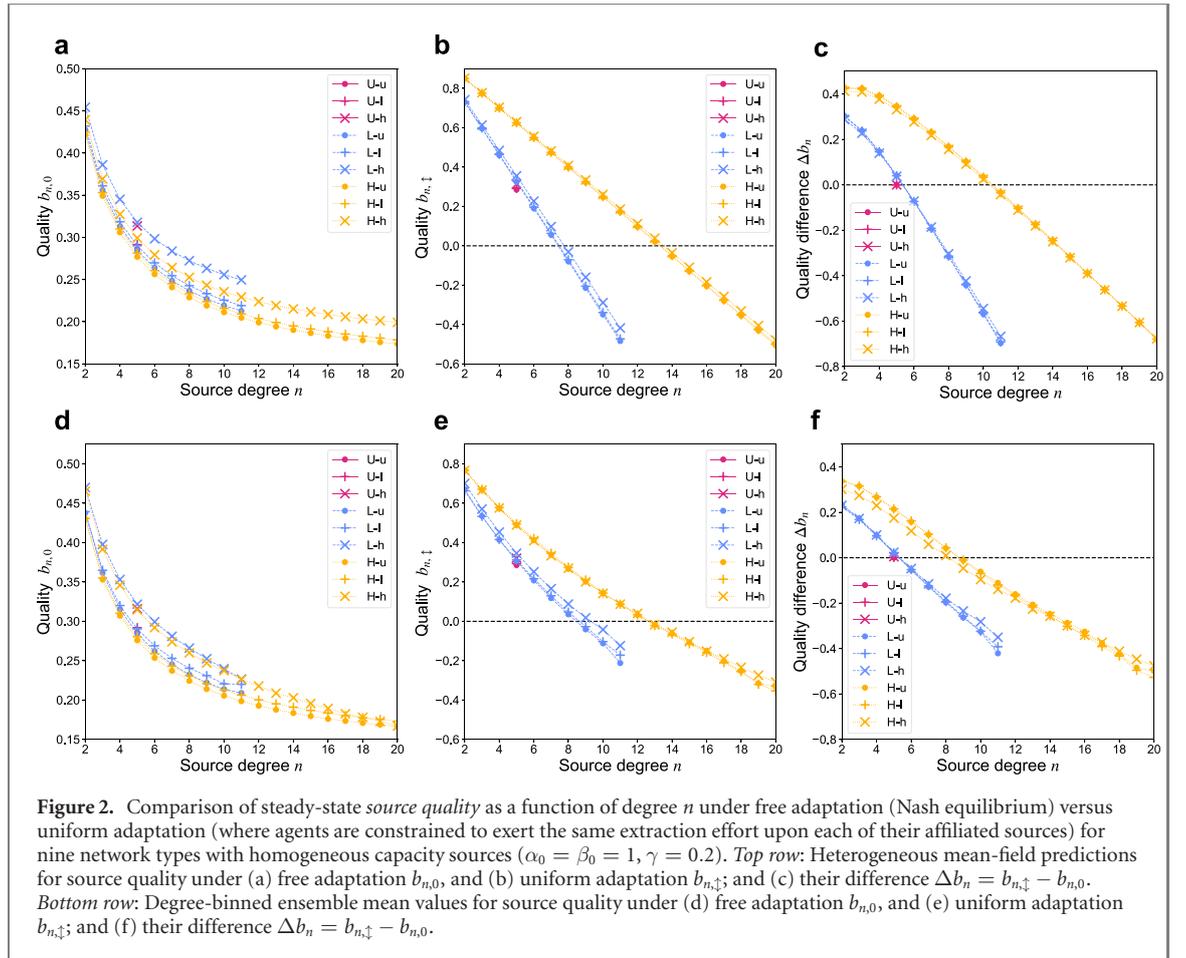
where α and β are positive parameters. The benefit extracted by an agent from a source is proportional to the agent's effort and the source's quality. From the total benefits accumulated from its multiple affiliated sources, each agent deducts a cost that increases quadratically with respect to its total **individual extraction** $\vec{q}(a)$, such that it receives a net **payoff** of

$$f(a) = \left[\sum_{s \in S_a} q(a, s) \cdot b(s) \right] - \frac{\gamma}{2} \vec{q}(a)^2, \quad (2)$$

where γ is a non-negative cost parameter. Each agent a adapts its extraction levels $q(a, s)$ from each of its affiliated sources s based on current resource conditions at rates proportional to the increase in payoff they expect to gain thereby, as in a replicator rule. We study the steady-state patterns of extraction that emerge when agents practice *uniform adaptation*, that is, when they are constrained to allocate their extraction efforts uniformly among their affiliated sources. Focusing on how agents and sources of different degrees are affected by these constraints, we apply a heterogeneous mean-field approach to estimate source quality and agent payoffs, and compare these predictions to the corresponding mean values computed on ensembles of individual networks. Nash equilibria, which are the steady states approached if agents adapt freely without a uniform-allocation constraint, serve as a basis for comparison. We consider two scenarios, each representing a different type of relationship between the degradation characteristics of sources and their network degrees: first, the case of **homogeneous capacity** sources that all share the same value of $\beta \equiv \beta_0$ regardless of degree, and then the case of sources with **degree-proportional capacity**, for which $\beta = \beta_0 \langle n \rangle / n(s)$. Further details are elaborated in the following.

2.2. Agent-resource affiliation networks

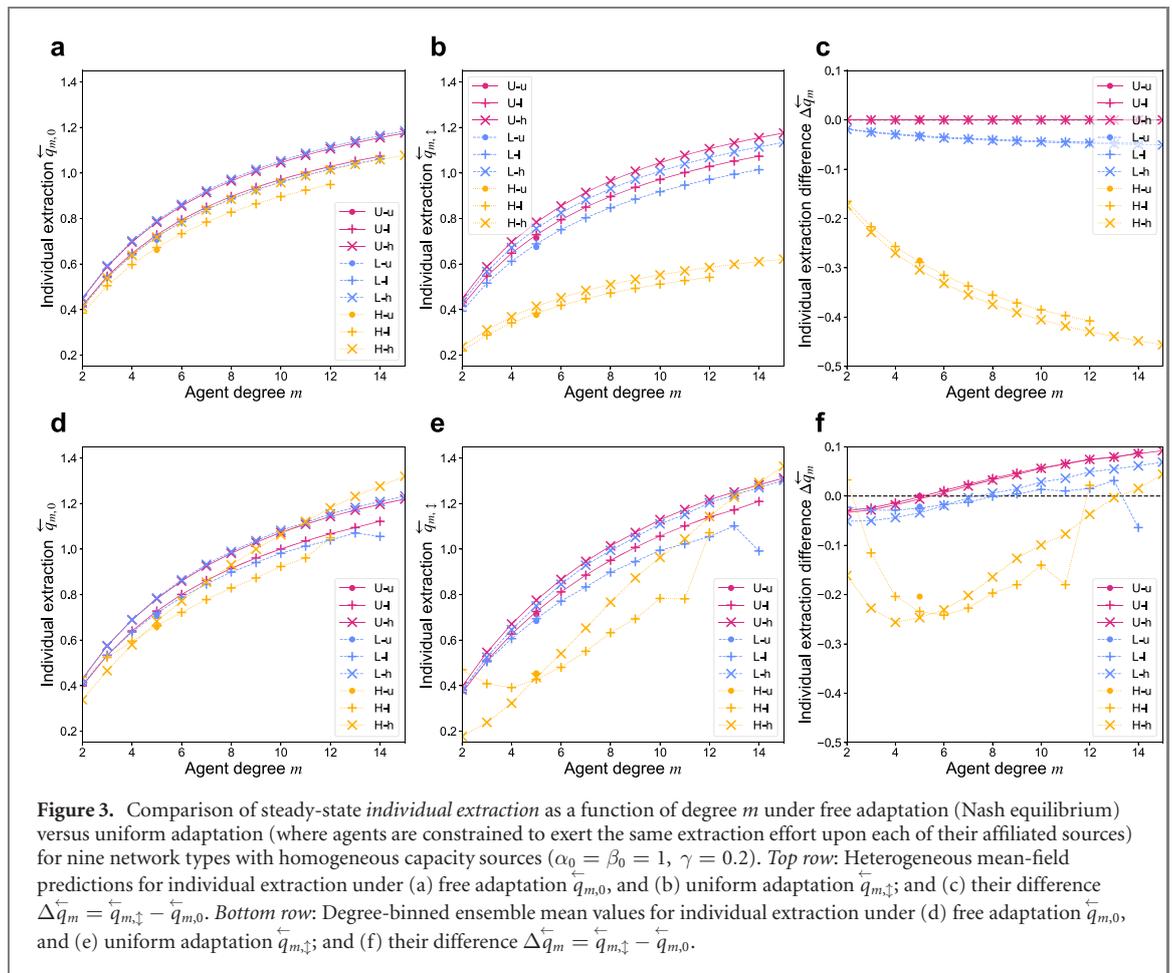
We consider games involving populations \mathbf{A} of agents whose access to a set \mathbf{S} of sources are defined by bipartite networks; a link between an agent and a source indicates that the agent can extract from the source. Links are assumed to be determined by some exogenous factors that remain static in time. The set of agents linked to a particular source s is denoted as \mathbf{A}_s , while the set of sources linked to a particular agent a is denoted as \mathbf{S}_a . The degree of an agent a is given by $m(a)$, while for a source s , degree is indicated by $n(s)$. We



consider nine ensembles of 10^3 networks each, all of which have 50 agents, 50 sources, and share mean degrees $\langle m \rangle = 5$ and $\langle n \rangle = 5$. All network realizations thus possess the same total number of links, but differ in terms of how these links are distributed among agents and sources. Each ensemble represents a particular combination of one of three types of source degree heterogeneity (**U**: uniform-degree, **L**: low-heterogeneity, or **H**: high-heterogeneity) with one of three similar types of agent degree heterogeneity (**u**: uniform-degree, **l**: low-heterogeneity, or **h**: high-heterogeneity) [12, 23]. These distributions were selected to represent qualitatively distinct types of diversity among the degrees of sources and agents. In networks where agent (source) nodes exhibits a uniform-degree distribution, all agent (source) nodes carry the same number of links. In low-heterogeneity distributions, node degrees vary within a narrow range around the mean degree, leading to peaked, normal-type distributions. High-heterogeneity networks have highly skewed, scale-free-type degree distributions, where the vast majority of nodes have below-average degrees while a few nodes possess degrees of a higher order of magnitude. We label as **L-h**, for example, the ensemble of networks generated to have low-heterogeneity source degree distributions and high-heterogeneity agent degree distributions. Degree histograms, averaged over each ensemble, provide a representative *source degree distribution* $P_S(n)$ and *agent degree distribution* $P_A(m)$ for each network type (figure 1).

2.3. CPR extraction game

Each agent selects a level of **extraction effort** $q(a, s)$ to apply towards each of its affiliated sources $s \in \mathbf{S}_a$. We denote a source's total **collective extraction** by $\vec{q}(s) = \sum_{a \in \mathbf{A}_s} q(a, s)$, and an agent's total **individual extraction** by $\overleftarrow{q}(a) = \sum_{s \in \mathbf{S}_a} q(a, s)$. For each unit of extraction effort exerted upon a source s , an agent receives a benefit $b(s)$. The magnitude of $b(s)$ describes the source's **quality**, which degrades linearly with respect to collective extraction (equation (1)). Defining quality in this way assumes that each CPR will consistently supply a benefit of magnitude $b(s)$ in return for each unit of extraction effort applied by an appropriator, and that these benefits are not contingent upon any other variables affecting resource quality or availability (by contrast with some previous models [24]). The characteristics of each source, as described by the parameters α and β that specify



this linear dependence, are assumed to remain constant in time. Realistic systems potentially exhibit variability among these source characteristics; following previous work [12], we limit our analyses to two distinct scenarios. First, we consider games where networks have *homogeneous capacity*, that is, where parameter values are independent of source degree: $\alpha_n \equiv \alpha_0$ and $\beta_n \equiv \beta_0$ for all sources, where α_0 and β_0 are positive constants. We then consider an opposite extreme case: *degree-proportional capacity* sources, where parameters are set such that sources degrade linearly with respect to the collective extraction *per link* that is exerted upon them: $\alpha_n \equiv \alpha_0$ and $\beta_n = \beta_0 \langle n \rangle / n$. Following previous work on networked CPR extraction games [11, 12], extraction is associated with a cost that is proportional to the square of individual extraction, $\bar{q}(a)^2$ (equation (2)). In addition to modelling increasing marginal costs (i.e., diminishing marginal utilities) associated with the act of extraction itself, this convex cost term could also be interpreted as reflective of the escalating, informal social costs often discussed in the context of CPR self-regulation ('graduated sanctions' [1, 25]).

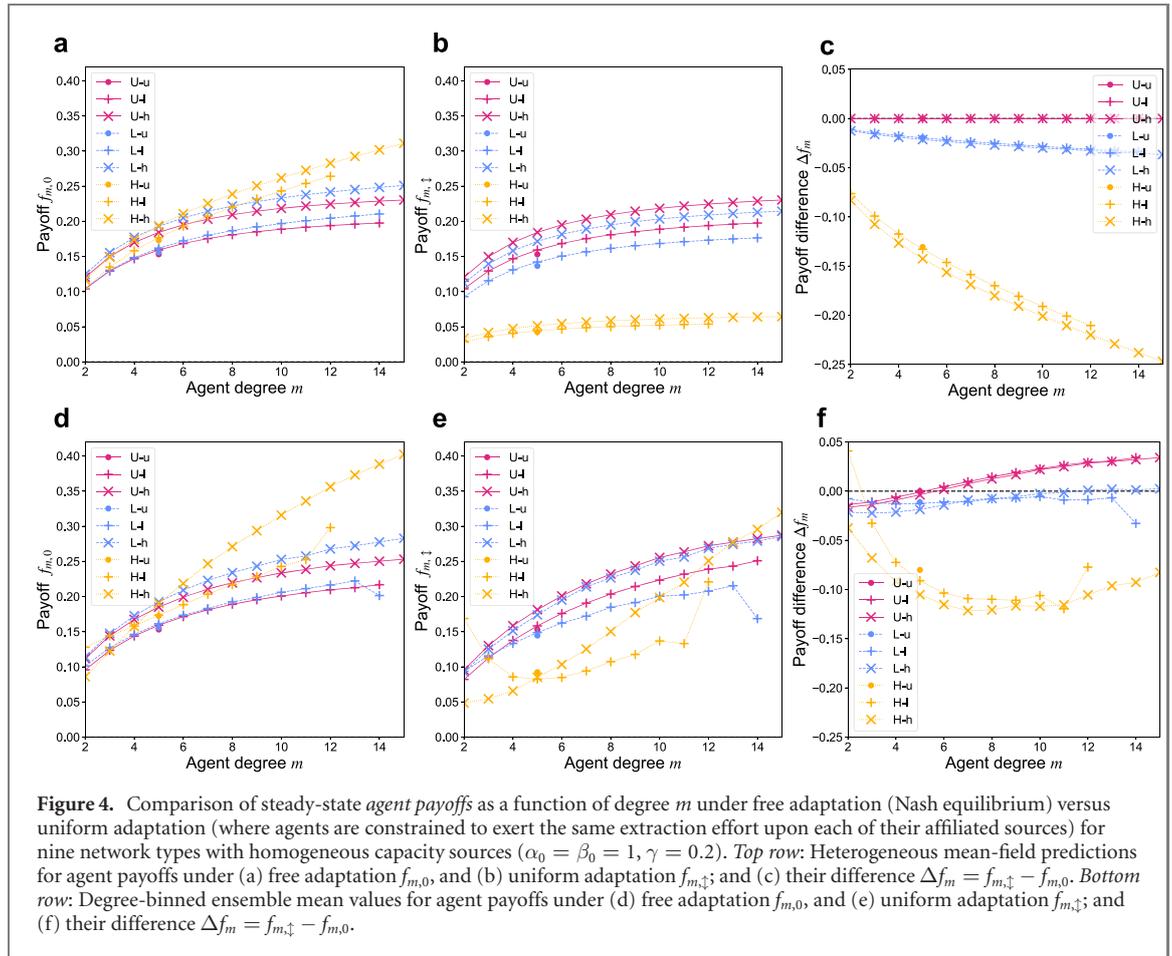
2.4. Adaptation dynamics

Free adaptation. As in a replicator rule [2, 26, 27], we assume that the rate at which an agent adjusts its behavior is proportional to the payoff increase it expects to gain thereby, as quantified by marginal utility. In the case that agents are free to separately adjust their extraction levels from each of their affiliated sources, which we will refer to as **free adaptation**, this gives

$$\frac{dq(a, s)}{dt} = k \frac{\partial f(a)}{\partial q(a, s)}, \quad (3)$$

where k is a positive constant. These dynamics approach a steady state ($\frac{dq(a, s)}{dt} = 0$) equal to a network's unique best-response, Nash equilibrium state ($\frac{\partial f(a)}{\partial q(a, s)} = 0$). Methods for computing these states for a given network [11, 12], and for estimating their properties from a heterogeneous mean-field perspective for networks represented by a given set of degree distributions [12], have been detailed in previous work, along with methods for computing Pareto efficient states.

Uniform adaptation. Next, we consider the case in which agents, instead of varying extraction from each source separately, are constrained to shift their extraction levels by the same amount at all affiliated sources. We will refer to this as **uniform adaptation**. The incremental payoff change (equations (1) and (2)) associated



with an adjustment in a single extraction level $q(a, s)$ is

$$\frac{\partial f(a)}{\partial q(a, s)} = \alpha(s) - \beta(s) \vec{q}(s) - \beta(s) q(a, s) - \gamma \overleftarrow{q}(a), \quad (4)$$

where we have considered parameters α and β as functions of source s . The total incremental payoff gain associated with a small change in individual extraction split evenly among the agent's affiliated sources $s \in S_a$ is then

$$\nabla_{\dagger} f(a) \equiv \sum_{s \in S_a} \frac{1}{m(a)} \frac{\partial f(a)}{\partial q(a, s)} = \frac{1}{m(a)} \left(\sum_{s \in S_a} \alpha(s) \right) - \frac{1}{m(a)} \left(\sum_{s \in S_a} \beta(s) \cdot [\vec{q}(s) + q(a, s)] \right) - \gamma \overleftarrow{q}(a). \quad (5)$$

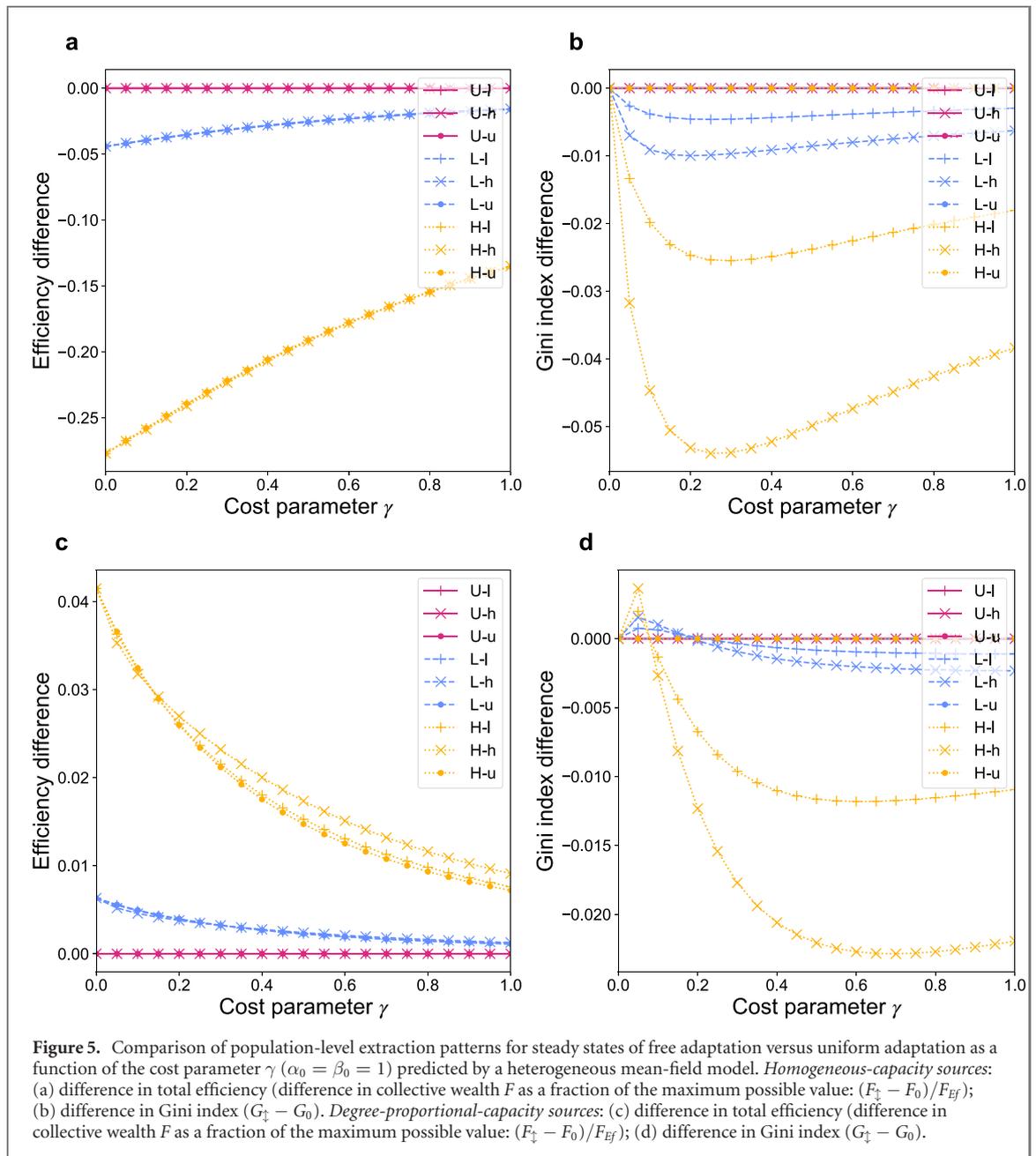
Similar to free adaptation (equation (3)), we then assume dynamics are described by

$$\frac{dq(a, s)}{dt} = k \nabla_{\dagger} f(a). \quad (6)$$

Along with the further assumption that agents begin with extraction effort allocated evenly among sources, $q(a, s) \equiv \frac{\overleftarrow{q}(a)}{m(a)}$, the condition defining the steady states of these dynamics ($\nabla_{\dagger} f(a) = 0$) becomes

$$\left[\gamma + \frac{1}{m(a)^2} \sum_{s \in S_a} \beta(s) \right] \overleftarrow{q}(a) + \frac{1}{m(a)} \sum_{s \in S_a} \left[\beta(s) \cdot \left(\sum_{a' \in A_s} \frac{\overleftarrow{q}(a')}{m(a')} \right) \right] = \frac{1}{m(a)} \sum_{s \in S_a} \alpha(s). \quad (7)$$

We use this condition to compute steady-state extraction states for specific networks under uniform adaptation dynamics (see appendix A), and then to estimate the degree dependence of various quantities based on degree distributions using a heterogeneous mean-field approach (see appendix B).



3. Results

3.1. Case 1: homogeneous-capacity sources

We first consider the case of homogeneous capacity sources, where all sources respond to collective extraction in an identical manner regardless of degree, comparing the steady states of uniform adaptation to those of free adaptation in terms of source quality (figure 2), individual extraction levels of agents (figure 3), and agent payoffs (figure 4). Under uniform adaptation, the convex curve describing the decline of expected source quality with degree at Nash equilibrium (figures 2(a) and (d)) gives way to an approximately linear decline in quality with degree (figures 2(b) and (e)). Since each agent is constrained to exert the same extraction effort across each link under uniform adaptation, a source's collective extraction tends to be proportional to its number of links. As a result, lower-degree sources experience lower collective extraction, and therefore greater quality, under uniform adaptation than under free adaptation; meanwhile, higher-degree sources suffer greater collective extraction and thus lower quality (figure 2). The net result is that uniform adaptation leads to lower overall extraction and improved resource quality. In networks with greater source degree heterogeneity, the presence of more over-exploited, higher-degree sources enhances the effect, further motivating agents to reduce their extraction levels from all sources and thus leading to even more elevated resource quality overall. For agents, however, the net result of that reduction in extraction by most agents (figure 3) is lower payoffs.

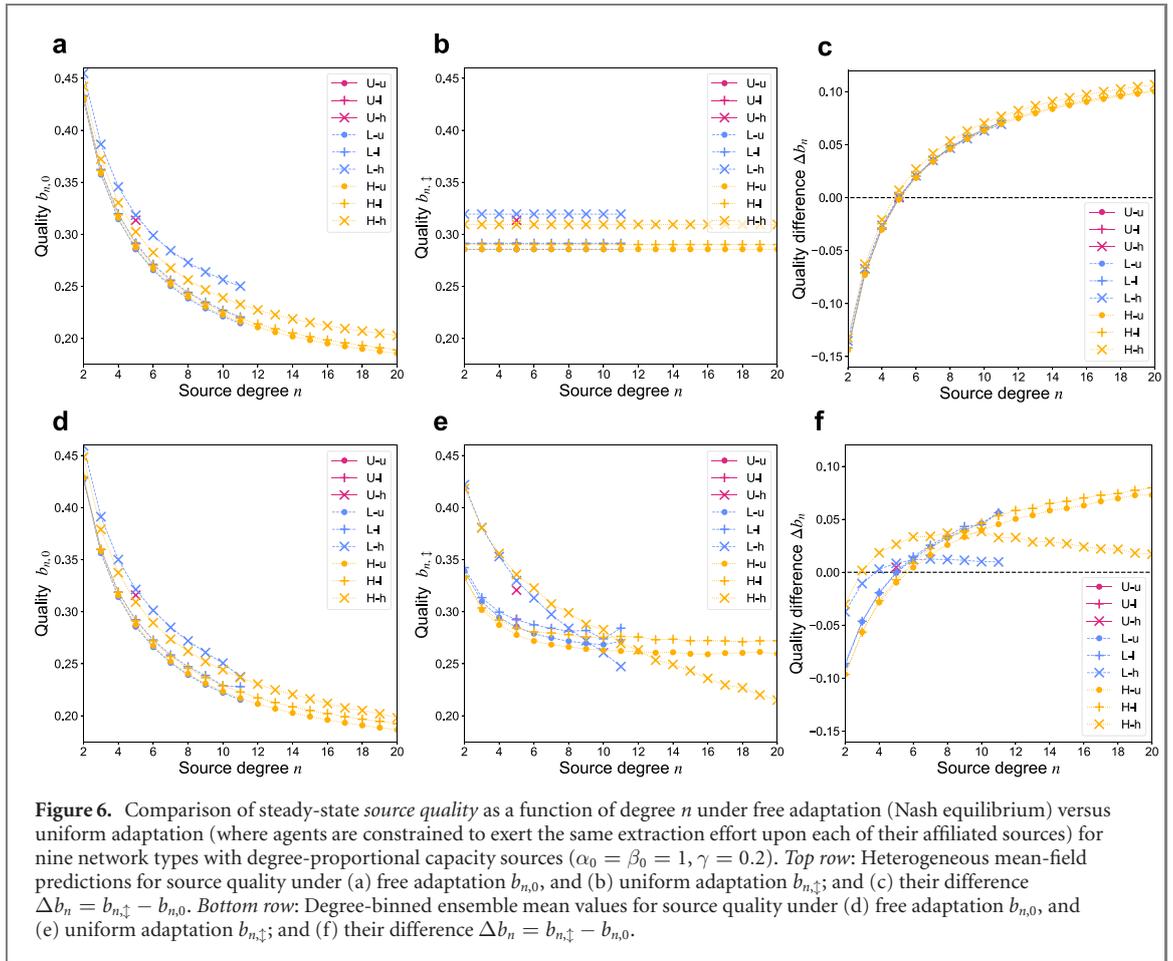
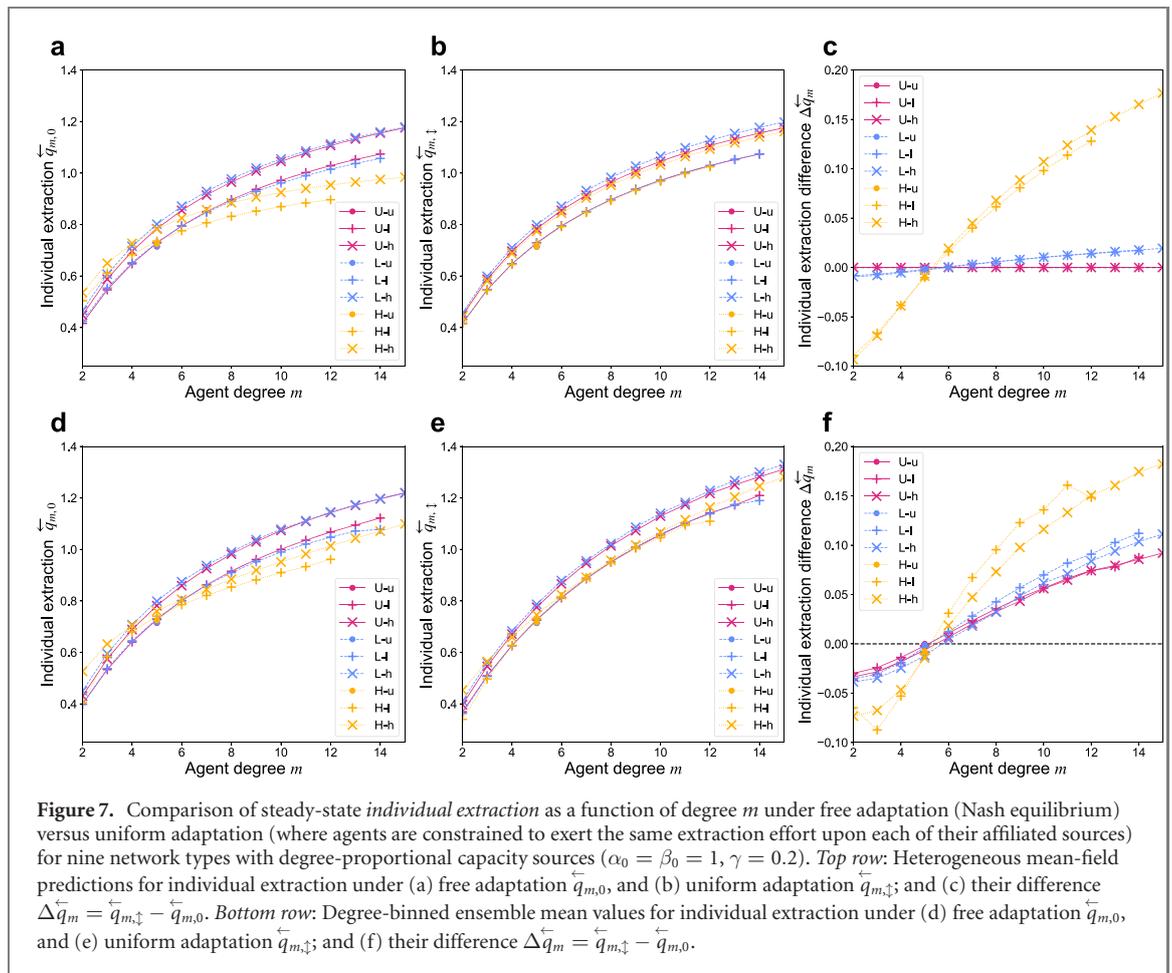


Figure 6. Comparison of steady-state source quality as a function of degree n under free adaptation (Nash equilibrium) versus uniform adaptation (where agents are constrained to exert the same extraction effort upon each of their affiliated sources) for nine network types with degree-proportional capacity sources ($\alpha_0 = \beta_0 = 1, \gamma = 0.2$). *Top row:* Heterogeneous mean-field predictions for source quality under (a) free adaptation $b_{n,0}$, and (b) uniform adaptation $b_{n,1}$; and (c) their difference $\Delta b_n = b_{n,1} - b_{n,0}$. *Bottom row:* Degree-binned ensemble mean values for source quality under (d) free adaptation $b_{n,0}$, and (e) uniform adaptation $b_{n,1}$; and (f) their difference $\Delta b_n = b_{n,1} - b_{n,0}$.

The dependence of agents' payoffs upon their degrees is similar within steady states of free adaptation (figures 4(a) and (d)) and uniform adaptation (figures 4(a) and (d) versus 4(b) and (e)), but the practice of uniform adaptation leads to reduced collective wealth, especially on networks with higher degree heterogeneity (figures 4(c) and (f) and 5(a)).

Shifting to a population-level perspective, and considering the role played by increasing marginal costs in regulating agents' extraction levels, we first examine the estimated differences between steady states of *free adaptation* and *uniform adaptation* in terms of collective wealth (figure 5(a)) and wealth equality as quantified by Gini index (figure 5(b)). When sources all have identical capacity, uniform adaptation dynamics lead to decreased efficiency as compared to free adaptation; the magnitude of the difference decreases monotonically as the cost parameter γ increases. This decrease is more pronounced in networks with higher source degree heterogeneity; agent degree heterogeneity seems to play no perceptible role. The steady states of uniform adaptation are predicted to consistently show a lower Gini index than their free adaptation counterparts, indicating a more egalitarian wealth distribution (figure 5(b)); higher-degree (and thus higher-extraction) agents stand to lose more than lower-degree agents (figures 4(c) and (f)) from the loss of flexibility in allocation when they are constrained to uniform adaptation. By increasing wealth inequality compared to the corresponding Nash equilibrium states, *uniform-allocation* adaptation (which removes agents' flexibility regarding allocation entirely) interestingly yields a similar result as does the extreme opposite behavior of adaptation by *reallocation* alone [12]. Higher-degree agents enjoy access to a greater diversity of sources, and the flexibility of *free adaptation* allows them to take full advantage of this access and extract optimal payoffs from each source in accord with its unique conditions. As such, they stand to lose relatively more by the introduction of constraints that reduce this flexibility, whether these constraints fix agents' extraction efforts in terms of allocation or magnitude. The reduction in Gini index is most pronounced at intermediate values of the cost parameter ($.2 < \gamma < .25$).

For networks with homogeneous-capacity sources, we conclude that populations practicing uniform adaptation will tend to achieve *lower collective wealth* than if they practiced the free adaptation behavior that leads to Nash equilibrium. The reduction occurs since the uniform-allocation constraint serves to redistribute the burden of over-exploitation onto higher-degree sources. Since a relatively large number of agents are affiliated with these over-exploited sources, the presence of each of these highly degraded hub CPRs influences a large number of the network's agents to reach steady states characterized by lower overall extraction. Although this

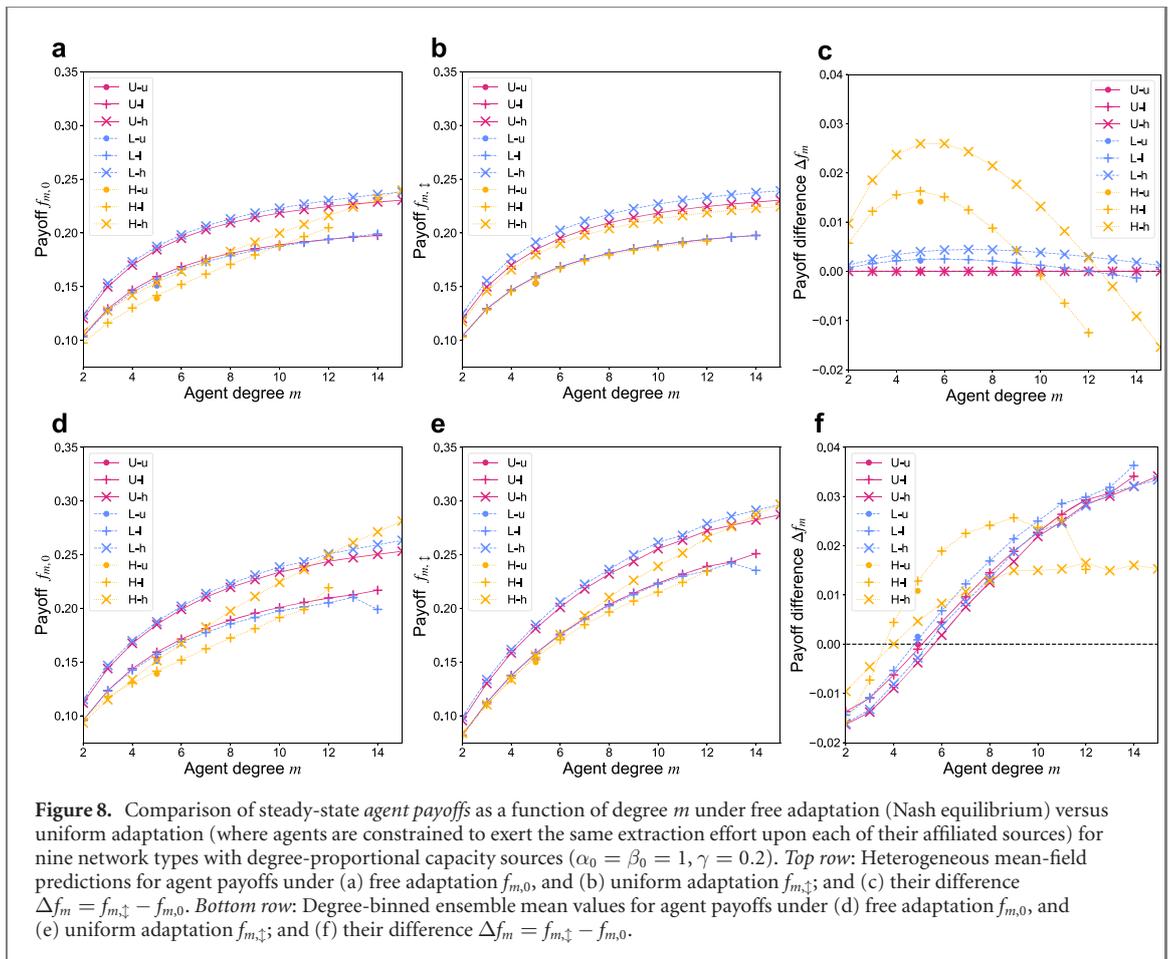


reduced extraction does serve to improve quality at lower-degree sources, leading to improved overall resource quality, the net result of this reduced extraction for most agents is that they extract lower payoffs. However, the resulting wealth distributions are more egalitarian than those achieved under free adaptation, since higher-degree agents incur greater reductions in payoffs as a result. The differences are most pronounced in networks with high source degree heterogeneity.

3.2. Case 2: degree-proportional-capacity sources

In networks with sources that have degree-proportional capacity, a source's quality decreases proportional to the average collective extraction it receives *per link*; for example, if a degree-2 source and a degree-4 source both receive the same collective extraction effort, then the quality of the degree-4 source is degraded only half as much as is the degree-2 source. For this case, we again compare the steady states of uniform adaptation to those of free adaptation in terms of source quality (figure 6), agents' individual extraction levels (figure 7), and agent payoffs (figure 8). In this case, we find that on networks with higher source degree heterogeneity, populations extract *more* efficiently under uniform adaptation (figure 5(c)), in contrast to the homogeneous capacity scenario. Under uniform adaptation, the quality of lower-degree sources is reduced, while the quality of higher-degree sources is improved in comparison to free adaptation (figures 6(c) and (f)). Unlike the steady states of free adaptation, under uniform adaptation, agents apply an equal extraction effort across each of their links; when source capacity is proportional to degree, then all sources come to share a uniform *expected* quality value regardless of their degrees (figure 6(b)), though the trend is somewhat more complicated on actual networks (figure 6(e)). As shown in previous work in the context of reallocation-based adaptation dynamics [12], greater collective wealth is achieved when the burden of over-exploitation is spread as evenly as possible among a network's sources than if the same overall extraction effort is allocated otherwise. However, the payoff gains thus achieved via uniform allocation here are of a lower order of magnitude than the losses experienced in the case of homogeneous-capacity sources discussed above (figures 5(c) and 8(c) and (f)). This puts the gains on a scale similar to the gains achieved via reallocation from Nash equilibrium observed in previous work.

Source degree heterogeneity also plays a reduced role here (figures 6(b) and (e)). As for agents' payoffs, similar trends of payoff versus agent degree are observed for both free adaptation (figures 8(a) and (d)) and uniform adaptation (figures 8(b) and (e)), except in the case of networks with high source degree heterogeneity.



The heterogeneous mean-field approach predicts that uniform adaptation could benefit some intermediate-degree agents, while reducing payoffs for higher-degree agents (figure 8(c)), thus resulting in a reduced Gini index (figure 5(d)); however, computations on individual networks suggest that this may not necessarily be the case, as higher-degree agents continue to receive greater payoffs than those predicted by the heterogeneous mean-field approach (figure 8(f)). For networks with degree-proportional capacity sources, we conclude that populations practicing uniform adaptation can manage to extract somewhat more efficiently than under free adaptation, since uniform adaptation leads to states where sources experience similar values of total collective extraction *per link*, and therefore similar quality values. Although fixed-allocation *uniform adaptation* and fixed-magnitude *reallocation* seem to represent extreme opposite limiting cases of adaptation behavior, this result seems to demonstrate that in certain circumstances, they can actually manage to achieve similar ends.

4. Discussion

Empirical studies of collective CPR extraction have shown that some real-world CPR appropriators, when faced with choices between multiple distinct resources, react in ways that appear to reveal an innate predisposition to de-emphasize selective allocation among sources. In the investigations presented above, we proposed that these individual habits can be modelled by introducing constraints into the adaptive behavior of CPR appropriators, of which the uniform-allocation constraint considered here represents one extreme example. Within a networked model of resource extraction, we explored the effects of these uniform-allocation constraints upon different classes of agents and sources within different types of networks. We found that on networks with *homogeneous capacity* sources, where resources all degrade in an identical way regardless of degree, a preference for fixed-allocation *uniform adaptation* by agents tends to shift the burden of over-exploitation away from lower-degree resources (those with fewer appropriators) and onto higher-degree sources (those with more appropriators), as compared to the unconstrained *free adaptation* behavior that leads towards Nash equilibrium. The presence of higher-degree, more over-exploited resources leads their many affiliated agents to reduce their overall extraction levels due to increasing marginal costs, which improves the quality of lower-degree resources. Although this shift actually does cause the system to sustain higher levels of resource quality, the net result is reduced collective wealth, and the reduction is more pronounced in networks with greater

source degree heterogeneity. However, in a *degree-proportional capacity* case, where higher-degree resources are assumed to be able to maintain higher quality while supporting greater extraction than lower-degree sources can, then *uniform adaptation* can improve payoffs somewhat since it serves to redistribute the burden of over-extraction more evenly across the network's multiple CPRs.

Taken together, these results strongly reinforce the finding that flexibility of allocation plays a crucial role in CPR appropriators' ability to maximize payoffs within heterogeneous environments. Constraints that limit this flexibility can help a population sustain higher resource quality and prevent environmental degradation. Nonetheless, on the linearly degrading CPRs considered here, when the system's multiple resources all share the same capacity regardless of their different numbers of appropriators, then uniform-allocation adaptation behavior leads to greatly reduced collective wealth. These effects are enhanced on networks with greater source degree heterogeneity. By way of comparison, previous work showed that a population can expect to improve its extraction efficiency by about 3.5% (in the most extreme case of **H-h** networks with maximum degree heterogeneity) by performing myopic gain-seeking reallocation moves from Nash equilibrium [12]. Here, we have shown that if reallocation is instead *de-emphasized* by constraining agents to allocate their efforts uniformly among resources, then the same kinds of populations can suffer an efficiency decrease of more than 25%. Thus, although empirical evidence suggests that some CPR users tend to apply similar behavior towards all of their available resources, rather than adapting by way of more targeted reallocation behavior, the current model even more dramatically demonstrates the importance of selective, targeted allocation of resource usage to individuals. In the specific scenario that each CPR's capacity is proportional to its network degree, where we did find that the uniform-allocation constraint provides an advantage, it is beneficial precisely because it serves the same function as did adaptive, fixed-magnitude reallocation: it serves to distribute the load of over-exploitation more evenly among the system's multiple resources than does unconstrained, rational extraction.

The uniform-allocation constraint typically leads to less-optimal individual and collective outcomes over the longer term, and in the case where it was observed to improve payoffs, these gains were relatively small. The model at hand does not yet appear to offer any evolutionary explanation for how agents could be incentivized to adopt uniform-allocation tendencies, nor does it demonstrate that these tendencies ultimately serve some collectively beneficial function to a community, aside from improving environmental quality for its own sake. Some other mechanism would seemingly be required to explain how and why these tendencies could emerge. We speculate that in more realistic systems, the emergence of these constraints might be attributed to some cognitive or logistical limitations on the part of CPR appropriators, which would prevent the agents from gathering and/or acting upon accurate, detailed information about many distinct resources. Extended models that consider the consequences of incomplete information, or more explicitly account for the effective costs associated with each of these different adaptation strategies, could provide further evolutionary insights into how these constraints emerge, and perhaps even into how they might eventually be formalized in terms of rules.

The current work also prompts further questions about the types of network structures upon which these games are played. Our consideration of network topology was limited to different types of degree heterogeneity, but in many real-world situations, the underlying factors that lead some agents to access greater numbers of distinct resources could also be intertwined with the factors that determine the exclusivity of these resources. In other words, the factors that determine agent degree may be related to those that determine source degree, such that some degree-based preferential attachment is involved in their formation. Further investigations could clarify the role of degree-degree correlations within agent-resource affiliation networks, since our analyses so far indicate that these could significantly alter results. At an even more fundamental level, these generative processes could be explored more directly in dynamic network models, where an individual's access to resources could itself co-evolve along with resource quality and extraction levels.

Finally, an important qualitative difference between the current CPR model and the 'laboratory-in-field' games mentioned in the introduction further complicates our attempts to relate the conclusions of the former to the latter. The model at hand considers linearly degrading CPRs, for which resource quality changes *gradually* as collective extraction is varied. Some of the experimental results that motivated the current study, on the other hand, involved CPRs whose quality would shift abruptly if extraction levels surpassed certain critical tipping points. This may indeed be a more appropriate model for many realistic socio-ecological systems [28]. Also, the concept of 'shifting baselines' [29, 30], originally introduced in discussions of common-pool fisheries, suggests that CPR users' perceptions of risks can be attenuated when environmental changes occur gradually rather than suddenly. Recent experimental CPR research has focused directly on the differences in individuals' responses to gradual versus sudden-onset changes in resource conditions [31]. These ongoing discussions emphasize that differences in the specific ways that the rivalrous property of CPRs is manifest within a given situation—either through gradual degradation, or through more abrupt, catastrophic regime shifts—can evoke drastically different adaptive responses in CPR appropriators. In light of this, further work

could apply the present networked-CPR framework to study how its conclusions would be altered if CPRs undergo sudden-onset depletion events, rather than varying gradually in quality.

Discussions about CPRs sometimes allude to the ‘(spatial) diversity and flexibility’ of the commons as key factors that ‘enable the best use of natural resources’ and prevent over-exploitation [32]. The model studied here provides a framework in which agents may have diverse degrees of access to an environment, and where this environment itself may also exhibit a complex internal structure with potentially high heterogeneity among its component parts. Nonetheless, few—if any—real-world commons will strictly conform to all the assumptions of the model investigated here. The relationships between resources and appropriators in real-world commons may seldom conform to such a rigid, static networked structure. However, rather than trying to provide a predictive model that makes accurate forecasts about a particular category of real-world commons, the assumptions of the current model serve to make these notions of ‘spatial diversity’ and ‘flexibility’ more concrete and amenable to systematic study. In doing so, the model gives some indications of the complex interrelationships that can arise between the adaptive habits of individuals, the systemic factors that constrain their access to resources, and the collective well-being of a community and its environment. We hope that by illustrating one instance of these interrelationships, these results encourage further investigations into how individual and collective behavior are connected via structures of access in real-world commons.

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Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: https://osf.io/d3nh5/?view_only=b9507a247ce648c2847af9f72bcff55d.

Appendix A. Computing steady states of uniform adaptation on agent-resource affiliation networks

Uniform adaptation dynamics, which represent the gain-seeking behavior of agents constrained to allocate their extraction effort evenly among their affiliated sources, lead to steady states described by equation (7). Grouping together the terms involving $\overleftarrow{q}(a)$ in equation (7) yields

$$\left[\gamma + \frac{2}{m(a)^2} \sum_{s \in \mathbf{S}_a} \beta(s) \right] \overleftarrow{q}(a) + \frac{1}{m(a)} \sum_{s \in \mathbf{S}_a} \left[\beta(s) \cdot \left(\sum_{a' \in \mathbf{A}_s \setminus \{a\}} \frac{\overleftarrow{q}(a')}{m(a')} \right) \right] = \frac{1}{m(a)} \sum_{s \in \mathbf{S}_a} \alpha(s), \quad (8)$$

where $\mathbf{A}_s \setminus \{a\}$ is the set of affiliated sources of s excluding a . Rearranging the summations, we obtain

$$\left[\gamma + \frac{2}{m(a)^2} \sum_{s \in \mathbf{S}_a} \beta(s) \right] \overleftarrow{q}(a) + \sum_{a' \in \mathbf{A}_s \setminus \{a\}} \left[\frac{1}{m(a)m(a')} \sum_{s \in \mathbf{S}_a \cap \mathbf{S}_{a'}} \beta(s) \right] \overleftarrow{q}(a') = \frac{1}{m(a)} \sum_{s \in \mathbf{S}_a} \alpha(s), \quad (9)$$

where $\mathbf{S}_a \cap \mathbf{S}_{a'}$ is the set of sources shared by agents a and a' . This set of conditions forms a linear system,

$$\mathbf{X} \overleftarrow{\mathbf{q}} = \boldsymbol{\alpha}, \quad (10)$$

where if the length- M vector $\overleftarrow{\mathbf{q}}$ is indexed such that $\overleftarrow{q}_a = \overleftarrow{q}(a)$, then the entries of $\boldsymbol{\alpha}$ are $\alpha_a = \frac{1}{m(a)} \sum_{s \in \mathbf{S}_a} \alpha(s)$, and the entries of the M -by- M matrix \mathbf{X} are

$$X_{a,a'} = \gamma \delta_{a,a'} + \frac{1 + \delta_{a,a'}}{m(a)m(a')} \sum_{s \in \mathbf{S}_a \cap \mathbf{S}_{a'}} \beta(s), \quad (11)$$

where $\delta_{a,a'}$ is the Kronecker delta: $\delta_{a,a'} = 1$ if $a = a'$ and $\delta_{a,a'} = 0$ if $a \neq a'$. Along with the condition $\overleftarrow{q}(a) \geq 0$, this represents a *linear complementarity problem* [33] that can be solved by minimizing the objective function

$$J(\overleftarrow{\mathbf{q}}) = \overleftarrow{\mathbf{q}}^T (\mathbf{X} \overleftarrow{\mathbf{q}} - \boldsymbol{\alpha}), \quad (12)$$

subject to the constraints

$$\mathbf{X}\overleftarrow{\mathbf{q}} - \boldsymbol{\alpha} \geq 0, \quad (13)$$

and

$$\overleftarrow{\mathbf{q}} \geq 0. \quad (14)$$

Given a particular network that defines the set of affiliated sources \mathbf{S}_a for each agent a used in equation (11), a quadratic programming algorithm can be used to solve this minimization problem (here, we use Python 3.7.3 with Scipy 1.6.2 [34]), and so to find the extraction intensity values $\overleftarrow{q}(a)$ that represent steady states of uniform adaptation dynamics [35].

Appendix B. Estimating steady-state extraction patterns using a heterogeneous mean-field approach

Shifting to a heterogeneous mean-field perspective, we bin all nodes of common type and degree into a single class, and then estimate the mean behavior of each of these classes. We thus use the expected individual extraction value of the class of degree- m agents, denoted as \overleftarrow{q}_m , to represent all degree- m agents, and estimate the degree distribution of a node's neighbors using the conditional degree distributions $P_S(n|m)$ and $P_A(m|n)$. As in previous work, we consider the parameters α and β to be functions only of source degree (which we indicate by subscripts: $\alpha(s) \equiv \alpha_{n(s)}$ and $\beta(s) \equiv \beta_{n(s)}$), for all $s \in \mathbf{S}$ such that $n(s) = n$. Substituting the sums taken over local node neighborhoods in equation (7) with the corresponding degree-statistical expected values (e.g., for a degree- m agent a , $\sum_{s \in \mathbf{S}_a} \beta(s) \rightarrow n \cdot \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \beta_n$), we estimate

$$\begin{aligned} & \left[\gamma + \frac{1}{m} \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \beta_n \right] \overleftarrow{q}_m + \left[\sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \beta_n \cdot n \cdot \left(\sum_{m'=n_{\min}}^{m_{\max}} P_A(m'|n) \cdot \left[\frac{\overleftarrow{q}_{m'}}{m'} \right] \right) \right] \\ & = \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \alpha_n. \end{aligned} \quad (15)$$

Rearranging, we arrive at

$$\begin{aligned} & \sum_{m'=n_{\min}}^{m_{\max}} \left[\frac{1}{m'} \left(\sum_{n=n_{\min}}^{n_{\max}} \beta_n \cdot n \cdot P_S(n|m) P_A(m'|n) \right) + \left(\gamma + \frac{1}{m} \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \beta_n n \right) \delta_{m,m'} \right] \overleftarrow{q}_{m'} \\ & = \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \alpha_n. \end{aligned} \quad (16)$$

This set of conditions (one for each agent degree m represented in the system) forms a linear system

$$\mathbf{X}\overleftarrow{\mathbf{q}} - \boldsymbol{\alpha} = 0, \quad (17)$$

where if the entries of are indexed by agent degree m , then the entries of $\boldsymbol{\alpha}$ are $\alpha_m = \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \alpha_n$, and the entries of the matrix \mathbf{X} are

$$X_{m,m'} = \frac{1}{m'} \left(\sum_{n=n_{\min}}^{n_{\max}} \beta_n \cdot n \cdot P_S(n|m) P_A(m'|n) \right) + \left(\gamma + \frac{1}{m} \sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \beta_n n \right) \delta_{m,m'}. \quad (18)$$

Given the conditional degree distributions $P_S(n|m)$ and $P_A(m|n)$ representing a given type of network, then the system can be solved numerically as $\overleftarrow{\mathbf{q}} = \mathbf{X}^{-1}\boldsymbol{\alpha}$ to yield estimates for the individual extraction values \overleftarrow{q}_m . These are then used to compute the expected collective extraction \overleftarrow{q}_n of sources as $\overleftarrow{q}_n = n \cdot \sum_{m=m_{\min}}^{m_{\max}} P_A(m|n) \cdot \left[\frac{\overleftarrow{q}_m}{m} \right]$, expected source quality b_n as $b_n = \alpha_n - \beta_n \overleftarrow{q}_n$, and expected agent payoffs f_m as $f_m = m \cdot \left(\sum_{n=n_{\min}}^{n_{\max}} P_S(n|m) \cdot \left[\frac{\overleftarrow{q}_m}{m} \right] \cdot b_n \right) - \frac{\gamma}{2} \overleftarrow{q}_m^2$. The Gini index is computed as $G = \frac{1}{2\overline{f_m}} \left[\sum_{m_1=m_{\min}}^{m_{\max}} P_A(m_1) \cdot \left(\sum_{m_2=m_{\min}}^{m_{\max}} P_A(m_2) \cdot |f_{m_1} - f_{m_2}| \right) \right]$, where $\overline{f_m} = \sum_{m=m_{\min}}^{m_{\max}} P_A(m) \cdot f_m$.

In the results presented above, we further assume that links are formed without any degree-based preferential attachment, and so apply the approximations $P_S(n|m) = P_S(n) \frac{n}{\langle n \rangle}$ and $P_A(m|n) = P_A(m) \frac{m}{\langle m \rangle}$. When sources have homogeneous capacity, then equation (18) simplifies to become

$$X_{m,m'} = \frac{\beta_0 \langle n^2 \rangle}{\langle m \rangle \langle n \rangle} P_A(m') + \left(\gamma + \frac{\beta_0}{m} \right) \delta_{m,m'}, \quad (19)$$

while all entries of α are $\alpha_m \equiv \alpha_0$. When sources have degree-proportional capacity, then equation (18) becomes

$$X_{m,m'} = \frac{\beta_0 \langle n \rangle}{\langle m \rangle} P_A(m') + \left(\gamma + \frac{\beta_0}{m} \right) \delta_{m,m'}. \quad (20)$$

The presence in equation (19) of a factor $\langle n^2 \rangle$, which is larger for networks with larger variance among source degrees, and its absence from equation (20), foreshadows that source degree heterogeneity will play a more prominent role in the networks with homogeneous-capacity sources than for those with degree-proportional source capacity. Here, we solve the resulting systems using Python 3.7.3 with Scipy 1.6.2 [34] using degree distributions $P_S(n)$ and $P_A(m)$ extracted from network ensembles (figure 1) [35].

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References

- [1] Ostrom E 1990 *Governing the Commons: The Evolution of Institutions for Collective Action* (Cambridge: Cambridge University Press)
- [2] Santos F C, Santos M D and Pacheco J M 2008 Social diversity promotes the emergence of cooperation in public goods games *Nature* **454** 213–6
- [3] Gómez-Gardeñes J, Vilone D and Sánchez A 2011 Disentangling social and group heterogeneities: public goods games on complex networks *Europhys. Lett.* **95** 68003
- [4] Gómez-Gardeñes J, Romance M, Criado R, Vilone D and Sánchez A 2011 Evolutionary games defined at the network mesoscale: the public goods game *Chaos* **21** 016113
- [5] Perc M, Gómez-Gardeñes J, Szolnoki A, Floría L M and Moreno Y 2013 Evolutionary dynamics of group interactions on structured populations: a review *J. R. Soc., Interface* **10** 20120997
- [6] Lei C, Wu T, Jia J-Y, Cong R and Wang L 2010 Heterogeneity of allocation promotes cooperation in public goods games *Physica A* **389** 4708–14
- [7] Li J, Wu T, Zeng G and Wang L 2012 Selective investment promotes cooperation in public goods game *Physica A* **391** 3924–9
- [8] Zhang H, Shi D, Liu R and Wang B 2012 Dynamic allocation of investments promotes cooperation in spatial public goods game *Physica A* **391** 2617–22
- [9] Fan R, Zhang Y, Luo M and Zhang H 2017 Promotion of cooperation induced by heterogeneity of both investment and payoff allocation in spatial public goods game *Physica A* **465** 454–63
- [10] Wang Q, Wang H, Zhang Z, Li Y, Liu Y and Perc M 2018 Heterogeneous investments promote cooperation in evolutionary public goods games *Physica A* **502** 570–5
- [11] İlkılıç R 2011 Networks of common property resources *Econ. Theor.* **47** 105–34
- [12] Schauf A and Oh P 2021 Myopic reallocation of extraction improves collective outcomes in networked common-pool resource games *Sci. Rep.* **11** 886
- [13] Prediger S, Vollan B and Frölich M 2011 The impact of culture and ecology on cooperation in a common-pool resource experiment *Ecol. Econ.* **70** 1599–608
- [14] Castillo D, Bousquet F, Janssen M A, Worrapimphong K and Cardenas J C 2011 Context matters to explain field experiments: results from Colombian and Thai fishing villages *Ecol. Econ.* **70** 1609–20
- [15] Cardenas J-C, Janssen M and Bousquet F 2013 Dynamics of rules and resources: three new field experiments on water, forests and fisheries *Handbook on Experimental Economics and the Environment* ed J A List and M K Price (Cheltenham, UK: Edward Elgar) pp 319–45
- [16] Noailly J, Withagen C A and van den Bergh J C J M 2007 Spatial evolution of social norms in a common-pool resource game *Environ. Resour. Econ.* **36** 113–41
- [17] Janssen M A, Goldstone R L, Menczer F and Ostrom E 2008 Effect of rule choice in dynamic interactive spatial commons *Int. J. Commons* **2** 288
- [18] Sugiarto H S, Lansing J S, Chung N N, Lai C H, Cheong S A and Chew L Y 2017 Social cooperation and disharmony in communities mediated through common pool resource exploitation *Phys. Rev. Lett.* **118** 208301
- [19] Deck C and Jahedi S 2015 The effect of cognitive load on economic decision making: a survey and new experiments *Eur. Econ. Rev.* **78** 97–119
- [20] Drichoutis A C and Nayga R M 2020 Economic rationality under cognitive load *Econ. J.* **130** 2382–409
- [21] Sood V and Redner S 2005 Voter model on heterogeneous graphs *Phys. Rev. Lett.* **94** 178701
- [22] Cimini G 2017 Evolutionary network games: equilibria from imitation and best response dynamics *Complexity* **2017** 1–14
- [23] Ohkubo J, Tanaka K and Horiguchi T 2005 Generation of complex bipartite graphs by using a preferential rewiring process *Phys. Rev. E* **72** 036120
- [24] Bousquet F, Cambier C, Mullon C, Morand P, Quensiere J and Pavé A 1993 Simulating the interaction between a society and a renewable resource *J. Biol. Syst.* **01** 199–214
- [25] Baland J-M and Platteau J P 1996 *Halting Degradation of Natural Resources: Is There a Role for Rural Communities?* (Rome: Food and Agriculture Organization of the United Nations)
- [26] Ohtsuki H and Nowak M A 2006 The replicator equation on graphs *J. Theor. Biol.* **243** 86–97
- [27] Hauert C and Doebeli M 2004 Spatial structure often inhibits the evolution of cooperation in the snowdrift game *Nature* **428** 643–6
- [28] Scheffer M, Carpenter S, Foley J A, Folke C and Walker B 2001 Catastrophic shifts in ecosystems *Nature* **413** 591–6
- [29] Pauly D 1995 Anecdotes and the shifting baseline syndrome of fisheries *Trends Ecol. Evol.* **10** 430

- [30] Pauly D and Jacquet J (David Suzuki Institute) 2019 Vanishing fish: shifting baselines and the future of global fisheries <https://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db=nlabk&AN=2093883> (accessed 23 June 2021)
- [31] Cerutti N and Schlüter A 2019 Resource changes: exogenous or endogenous, gradual or abrupt. Experimental evidence *Int. J. Environ. Stud.* **76** 1004–18
- [32] Ricoveri G 2015 *Nature for Sale: The Commons versus Commodities* (London: Pluto Press)
- [33] Cottle R, Pang J-S and Stone R E 2009 *The Linear Complementarity Problem [Classics edn]* (Philadelphia, PA: SIAM)
- [34] SciPy 1.0 Contributors *et al* 2020 SciPy 1.0: fundamental algorithms for scientific computing in Python *Nat. Methods* **17** 261–72
- [35] Schauf A 2021 Effects of uniform-allocation constraints in networked common-pool resource extraction games (Demonstration code and files) Repository name: Open Science Framework https://osf.io/d3nh5/?view_only=b9507a247ce648c2847af9f72bcff55d