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"Production Function and Institutions"

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The classical economists were primarily engaged in analysis of the wonder that is production. The marginalists, while laying the foundation of mathematical economics, shifted its focus to rational behavior of a single consumer, presumably because there they found a problem readily amenable to the mathematical tools at hand. Production process was assimilated after a couple of decades, in a conveniently truncated form. In neoclassical writings production was a mirror image of the process of consumption, an optimal allocation problem by a decision maker. Later, economists got so accustomed to this formulation that no argument would be necessary for accepting this as the essence of actual production processes. A fundamental difference between the consumption and production processes is -one is a private action, the other is a joint one. A consumer can decide her best option all by oneself, but a single individual in a production process cannot. It was necessary, therefore, to reduce the production process to that of a single decision maker. Towards this end Walras conceived a hypothetical market of capital services, where landowners, workers and capitalists offer natural resources, personal skills, and capital proper. An entrepreneur, like an auctioneer, purchases productive services and combines them to activate the process of production (Ingrao and Israel, 1990,p. 107). The entrepreneur, the 'producer' in Debreu (1959), is the counterpart of individual consumer who could optimize individually while participating in a joint activity. Though Williamson's and the principal-agent formulations are far reserved about the omnipotent powers of one individual in a joint activity they still retain the primacy of one (or one group of) decision maker.

While consumption is an action, the production processes - certainly all those of our interest - are social processes or interactions. The reductionism, from interaction to action, can be identified as an influence of the then-ruling Cartesian paradigm (Parsons, 1968). No doubt, the neoclassicists excelled in Cartesian techniques of presentation like geometry. But despite mathematical elegance the paradigmatic impropriety remains. The trouble with the Walrasian modeling of production process is that the 'capital services' are not actually dispensed off by their owners as in an auction. In actual production processes, their availabilities are only commitments. Are these credible commitments? Within perfect competition no commitment of this nature can be credible. In a neoclassical economy a worker who shirks on the job pays no penalty for his conduct (Shapiro and Stiglitz, 1984). Even in imperfect markets, disciplining devices are no solutions unless the undisciplined behavior is of limited extent; the great majority must comply with their commitments. This is the distinctive feature of the production process. Here, many individuals have to cooperate, a necessity that does not exist in the consumption process. The early days of Robinson Crusoe fascinated the economists, for that is the only possible example of production process as mere allocation of resources. But once Friday

joined him, every object in the island of Robinson Crusoe became object of interest to both of them; every activity required active cooperation or tacit coordination between the two of them. Economists lost interest in them!

The purpose of the present paper is to redefine production process in this light. One may anticipate that the question of cooperation would lead the analysis to the phenomenon of institutional behavior. For mathematical formulation, we depend on the interactive decision theory, also known as game theory, and propose some changes for reformulation of Walrasian production process as games between participants. They may occupy various positions and may play different roles. But no one need to be principal. More formally, we retain the neoclassical assumption that economic relations within a firm are symmetrical, as against the asymmetric context assumed for principal-agent group of analyses. Transaction cost is nil by assumption. We will also retain the essential marginalist formulation. All these can be relaxed. But we make no such attempt, for we intend to remain as close as possible to the fundamental neoclassical formulation so as to bring out clearly in this paper, what was amiss in their formulation of the production process.

The first two Sections are intended to prepare the readers for the distinctive discourse necessary. Several notions in vogue are not applicable for this analysis, several new ones are needed. By using a comparative approach, in the first Section we will pinpoint how do we propose to modify the Walrasian production function. In the next Section we develop appropriate mathematical forms for quantitative representation of the institutional feature. Beginning with Section three we use formal approach and introduce a game formulation for production process. Section four brings in the systemic connection. Though we have presented it from the point of predictability this Section essentially introduces an approach to construct a general production function within its institutional context. In the final Section some major implications of this formulation is discussed showing particularly where they differ from the implications of conventional neoclassical paradigm.

1. FROM WALRASIAN TO GAME FORMULATION

The Walrasian model begins with resources as factors of production. Individuals are lost sight of because the model does not consider their control behind these resources. At the most, one of them is given such a right, as a profit maximizer. But a single decision maker does not help elevate the model into an interactive one. To bring the individual participants into focus one simply has to consider the resources not as technological inputs but as endowments E possessed by individuals. The endowments may be e.g. material and physical assets, skill etc., which are contributed as inputs of the production process.

Feasible actions for a participant is to contribute, at different rates, the endowments possessed by her.

Actually, this is a radical departure from the Walrasian model, but in complete accordance with many recent neoclassical models. The Walrasian model does not postulate any relationship between the quantity of inputs and the number of individuals contributing the same. It is rather by default that the marginalist depictions like 'one more unit of labor'¹ are often taken to mean 'one more laborer'. A possible alternative formulation is that different levels of effort may be obtained from the same laborer. Many recent theories, like that of involuntary unemployment or those belonging to principal-agent studies or the theories of organization, require that different levels of contributions are obtained from the same individual. In fact all the theories allowing for any kind of moral hazard implies differential levels of effort as possibilities, for, at the minimum, with and without the hazard are two levels of contributions obtainable from the same individual.

A corollary of this assumption is that the analysis may be conducted with a finite number of individuals. Various levels of efforts required for a particular production process or by an entrepreneur, may simply be obtained by varying intensities of contributions made by the same population. The assumption therefore, essentially accommodates fixed size (by number of participants) firms into the neoclassical theory

In a binary formulation one assumes only two types of feasible actions: contribute endowment and do not contribute. This is equivalent to the binary strategy of games: cooperate and defect. However, the nature of the conventional production function is better described by postulating a continuous case for feasible strategy options:

ASSUMPTION 1: *The set of actions $(A_i) = (a_{i1}, a_{i2}, a_{i3}, \dots)$ available to the i th participant is variable extent of contribution of her initial endowment \mathcal{E}_i i.e. $a_i(\mathcal{E}_i)$.*

The output levels are determined by the simultaneous availability of the different inputs from all participants.

The production function, consisting of n number of participants may then be written as:

$$(1) \quad Y(a_1(\mathcal{E}_1), a_2(\mathcal{E}_2), \dots, a_n(\mathcal{E}_n))$$

For comparison, where there are only two types endowments, hence two types of inputs, capital K and labor L, we get the school text version:

$$Y = f(K, L)$$

Let us now turn to the maximizing calculus. As in the case of the conventional production function, we assume that out of all feasible actions available to her a participant chooses one that maximizes her expected net benefit. To find the net benefit we should also know the compensations or benefits expected by her, and to find the maximum we need a relative price situation.

From this stage, the distinctive features of interactive formulation start appearing in the production function. The compensation, or the gross benefit received by one of the participants is some part of the output produced and therefore, determined by the levels of contributions made by all others. Obviously, the only way to formulate such a situation is as a simultaneous decision situation, as a game. For mathematical game

theory payoffs can be exogenous. But in a model of production process they must be endogenous. Indeed, the compensations received by all the concerned parties should equate to the total output to meet Pareto efficiency.

Fig. 1: DECISION ABOUT CONTRIBUTION

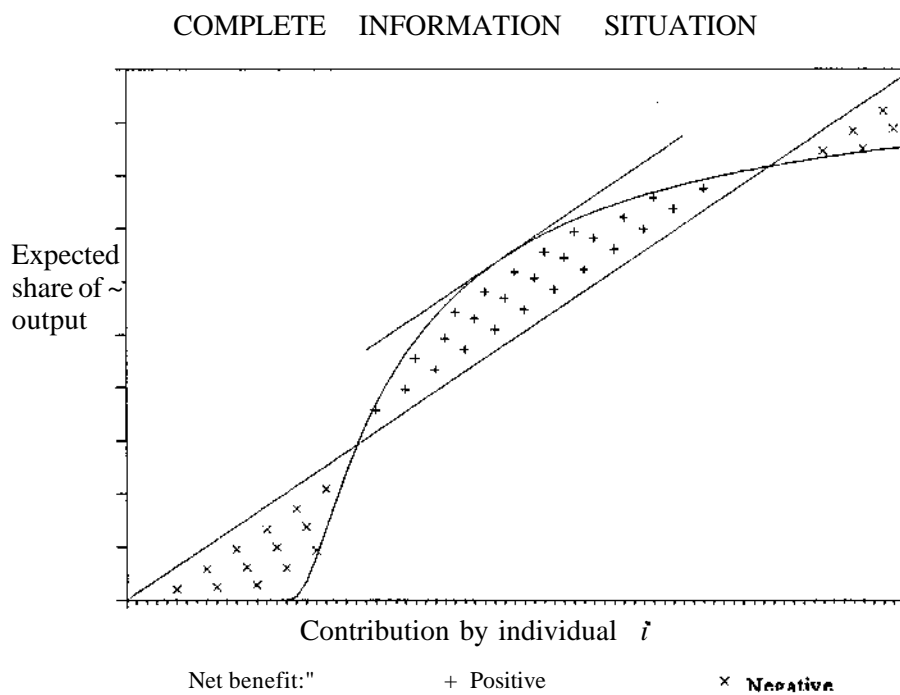
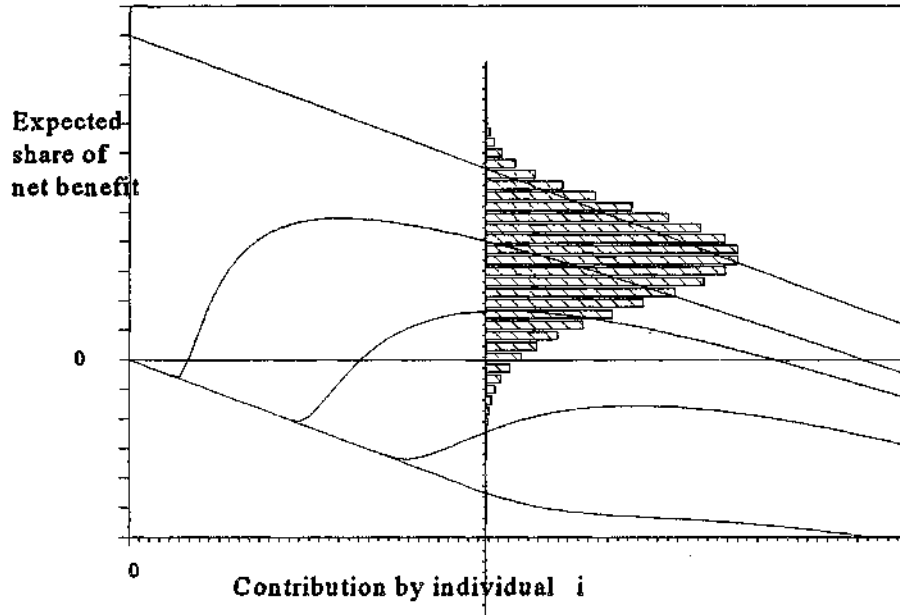


Fig. 2 : DECISION ABOUT CONTRIBUTION

INCOMPLETE INFORMATION SITUATION

Let us assume that one's due share of output is decided by a sharing scheme D . The benefit received by the i th individual may then be expressed as:

$$(2) \quad b_i (a_i , a_{-i}) = D_i (Y(a_1(\mathcal{E}_1) , a_2(\mathcal{E}_2) \dots a_n(\mathcal{E}_n)))$$

where a_{-i} stands for the profile of action taken by all other participants except the i th one.

One's contribution, which is her chosen action, is given by $a_i(\mathcal{E}_i)$. Let Π_i be her relative valuation of the materials constituting her input and (the share of) output received by her. The net benefit is then given by:

$$(3) \quad b_i (a_i , a_{-i}) - \Pi_i a_i \quad \text{or} \quad D_i (Y) - \Pi_i a_i$$

which is maximized in a rational choice situation.

In a perfect information case, when the decision maker under inquiry knows the actions to be taken by all others, the only input of the production process which is a variable is her own contribution. Thus she can straightaway calculate the marginal productivity of her own contribution and knowing her share she may choose as the best action the particular input level that gives her the maximum net benefit. We have shown this in figure 1. (Since a wider group of production functions is admissible within the interactive case we have used a marginal productivity curve of a shape different from the conventional one. See Appendix for explanation of the shape.) A relative price line, shown explicitly in this figure demarcates areas above and below it, corresponding to positive and negative net benefit. Her best strategy is the point of tangency in the positive net benefit area. However, the price line here has a different meaning from that of the Walrasian model. This is her relative valuation of the materials constituting her input and output received by her. She may or may not value them in terms of market price. If in case her endowment is capital and she uses market prices to value her contribution and her benefits, we obtain the neoclassical profit maximization situation.

But the analysis can accommodate nonmarket valuation methods and actually is an indicator of preference pattern - 'tastes' in Debreu's terms (Debreu, 1959, p. 33).

In mathematical expression, her best action a_i^* in perfect information case is given by:

$$a_i^* = \underset{a_i}{\operatorname{argmax}} (b_i (a_i , \bar{a}_{-i}) - \Pi_i a_i)$$

where \bar{a}_{-i} signifies that the profile of action of all others is fixed.

But there is no way to know a priori that \bar{a}_{-i} will occur. In interactions individuals have to make decision in an incomplete information situation. The best that a decision maker can do is to assign a probability to the occurrence of the profile of action \bar{a}_{-i} . She understands that every other profile of actions (taken by the rest) has some probability of occurrence. Therefore, to obtain the marginal productivity of each particular level of her own contribution she has to aggregate the expected benefits for all probable profiles of actions by the rest. Thus her best action in incomplete information case can be written as:

$$(4) a_i^* = \underset{a_i}{\operatorname{argmax}} (\int \int \dots \int b_i (a_i , a_{-i}) dp_1 \dots dp_{i-1} dp_{i+1} \dots dp_n - \Pi_i a_i)$$

where p_j stands for the probability distribution of feasible actions by individual j , as expected by i . Therefore, $dp_1 \dots dp_{i-1} dp_{i+1} \dots dp_n$ pertains to the expected (by i) joint probability distribution of a particular profile of actions by all others excepting the decision maker i herself.

In figure 2 we have explained the decision procedure under incomplete information case. The net benefits have been shown directly. By moving the probability distribution along the x-axis one can estimate the expected benefit for each action taken by individual i . The individual in question then decides whichever action gives her the maximum expected net benefit under the prior probability distribution.

Every participant decides her best action in this manner.

This should be the appropriate representation of the production process. In the past, the mathematical tools available were insufficient to handle such simultaneous decision situations. There was therefore, an excuse for the analysts using a reductionist approach leading to a single decision maker and only profit maximization objective. But to assume profit maximization implies that the decision making in production rests only with the profit earner. There is no decision making process for the other contributors. This is not just unethical, but inappropriate. Even if the workers are brought to work under threat, whenever they work, they cooperate.

One must have realized by now that the incomplete information situation can be represented as a Bayesian game. But we will wait some more for, the probability distributions of different profiles of actions have some additional meaning in terms of institutions. "Institutions reduce uncertainty by providing a structure to everyday life..... (they) define and limit the set of choices of individuals." (North, 1990,p. 3-4). In an excellent institutional atmosphere there is very little uncertainty about others' behavior. The decision situation therefore, is close to the perfect information case. In absence of institutions, the dispersion of possible actions is large. It is desirable therefore, that the place of institutions in the present analysis is explained first. The Bayesian production function would then appear with a better meaning. Which particular institution is in place is immaterial for this analysis. While structures differ, from institution to institution, their states of existence range universally from complete disorder to perfect order. Since uncertainties are reduced in proportion to the state of order, it is not the institution per se but the state variables which determine the possible variations in the behaviors of individuals in a production process.

2. INSTITUTIONAL VARIABLES

What is an institution? Crawford and Ostrom (1995,p. 582) identifies three kinds of definitions found in abstract analyses: institutions as equilibria, as norms and as rules. Nelson (1995,p. 80-81) distinguishes two strands. One is, as in sociology, and includes shared values, norms, beliefs, meanings, symbols, customs, and all standards that delineate the range of expected and accepted behavior, that is all those which can be brought under the general fabric of 'culture'. Another strand, he calls 'new economic', focuses on the self enforcing character of institutionalized behavior. For the time we set aside the qualifying characteristics like

equilibrium or self enforcement. The definitions, as rules, norms or as cultural stands are macro regularities and cannot be integrated readily into the current constitution of micro economic theory. We believe this has been the major factor preventing the desired incorporation. A possible way out is to use instead, another institutional phenomenon found in sociology: the social roles. Roles, defined as bundles of norms and expectations (viz. Biddle and Thomas ed., 1966; Baker and Faulkner, 1991, p. 280) are the embodiment of all institutional norms in a society. Being defined on individuals, they can be easily integrated into the existing frameworks of economic analysis. Institutional behavior can then be expressed conveniently as role complying behavior by individuals as well as expectations of the same from the other individuals.

The question whether the human agents are rational or role players is at the root of the deep divide between the economists and the sociologists. But there is some need to reconsider whether the gulf is as wide as believed. Role playing, in the forms of status and class respectively, had occupied considerable importance in the analysis of production process by Adam Smith and Karl Marx. But this was abandoned in the wake of neoclassical economics. Lately, the new institutional economics has again come to accommodate social behavior along with rational, so much so that the sociologists may consider (e.g. Granovetter, 1985) the economists' view 'oversocialised'. They may, for the sociological role theories have moved far away from the original romantic and rigid notions. By assimilating the two already converging notions, one arrives at a concept enjoying the merits of both the disciplines.

A production process of size n may be described as consisting of n positions which should be occupied by n number of individuals. Each position requires some specific kinds of contributions, as demanded by the physical characteristics of the tasks. The manner of assignment of participants to these positions need not bother us, for whatever be the manner, the commitment to cooperate is determined by one's net benefit. Net benefits depend on the shares one receive from the total output. Since the total output depend on the actions taken by oneself as well as by others, one must know, a priori, others' actions in order to pursue one's objective. But one can only predict, one cannot know what others' future actions will be. One manner of prediction possibility has already been accommodated in economics through the theories of reputation-building (Kreps and Wilson, 1982; Milgrom and Roberts, 1982; Fudenberg and Tirole, 1991). We recommend use of impersonal features towards this end, as concepts of role structures formed on the basis of past experiences of one's (institutional) environment. Following a sociological approach, one may say:

from the requirements of the task structure in question and the role expectations of the interactants, one predicts the type and range of responses likely to be obtained from each of the other interactants assigned at particular positions.¹ On the basis of this, she predicts the output levels, and knowing her share she can calculate her net benefit for each particular action taken by herself. She chooses the best action that she expects will give her the maximum net benefit.

This argument is not in conflict with the economic rationality. Since one's current action is under one's own control, one can pursue a rational decision there. But others' current actions are not under her control and one can at best try to predict those by using some other source of information. In effect, everyone behaves as if she herself is a rational decision maker while all others are role followers. Unfortunately, the two poles of this interactive decision setting are studied independently by the two disciplines. Without using the two formulations together the totality cannot be analyzed. To pursue this research program an economist must cease believing that "economics is all about how people make choices; sociology is all about how they don't have any choices to make." (Duesenberry, cited in Granovetter, 1985, p. 485). One should note that this indeed, is the characteristic circumstance under which individuals function; the decision maker finds she herself has many choices for her own action, but no choice in the matter of actions to be taken by others. By hypothesizing role complying behavior on others, every decision maker only facilitates her own decision process. She is not actually able to impose or enforce such behavior on others. Rather, she remains aware of this deficiency and tries to make rational decisions within this incomplete information limitation.

A crucial issue in the description of an interaction is how exactly the n number of individuals are assigned to specific positions in a production process of size n . Whoever has the authority is a principal in the setting. Sociologists have given primacy to the group (e.g. Turner, 1978) as well as to individual (e.g. Baker and Faulkner, 1991). In a more abstract manner this debate has been waged on the question whether a person receives a position first and then behaves in a role-appropriate manner or the vice versa. This issue is irrelevant for us. All that we need is that imperfections in assignments are always possible and the individuals participating in an interaction are aware of this possibility. When a decision maker finds another person assigned, in some manner, to a position in a task, she has an expectation that a certain type and range

of response will ensue from the assignee. This expectation is formed on the basis of an aggregative past experience, both institutional and personal. We single out the institutional ones and express it as role structure. The early role theories would not have reconciled with the notion that roles are cumulative past experiences. Modern role theories would not have any qualm. Roles are not considered anymore as static and stable but as ones made and remade, in the day to day process of social existence (Turner, 1978). No more are they granted an existence independent of the people involved in institutions. Although we will not address the dynamic issues in this paper we may enrich our understanding about the contributions of past experiences in modern concepts of roles by familiarizing ourselves a little with the arguments of the process analysis of role theory (Turner, 1987). Tasks are not identical and implications of each to an assignee vary. Minor variations, between one's self-conception of role and the materialized one, occur more often than not. Accordingly, the response made by the assignee may vary from lukewarm to very enthusiastic, differing in degrees. The reluctant response of one in an assignment may also act as a signal to whoever is principal. If the signal is understood and if the physical situation permits, the assigned role may be revised. Otherwise, in course of time, the assignee changes his self-understanding to accept the new role. Thus, the role structure changes all the time even within a structurally stable institutional system.²

In incomplete information settings, the economic and sociological (role theoretic) decision approaches are not poles apart; they are only approaches from different sides to the private information possessed by each individual. The more correct one is in her predictions about others' actions, no matter that the predictions are in the form of role following responses, the more objectively rational will be her decision about her own action. In turn, others are able to predict her actions more accurately, which improves the correctness of their own decisions. In an ideal situation everyone knows enough about others to make correct predictions and base self action decisions on the same. When a priori decisions made by everyone turn out to be a posteriori rational, then these become objectively rational and everyone's actions are perfectly predictable. Thus rational and role-following responses converge in complete information situation.

Let us now provide a suitable definition for the purpose of mathematical analysis. One would recall that goods have many attributes, like color, shape, composition etc. The only attribute which is relevant for

economic analysis is quantity. Likewise, we need not bother about the whole bundle of norms defining any particular role. The types of responses that have some significance to the production process, and are therefore factors of the same, can be distinguished as norms relevant for economic analysis. The 'responses' are identical with 'actions' a_j defined earlier, and they too specify the contributions $a_j(\mathcal{E}_j)$ made by the j th participant. But we will continue to use both the terms for convenience of understanding. When we talk of one's own decision about her contribution we call it 'action' (and use 'she'). When we have to discuss a prediction about some others' contribution we call it a 'response' and denote it by x_k (and the agent as 'he'). We may therefore, define likewise that the set of all possible responses $(x_j) = (x_{j1}, x_{j2}, x_{j3}, \dots)$ available from the j th participant assigned to a particular position is variable extent of contribution of her initial endowment \mathcal{E}_j i.e. $x_j(\mathcal{E}_j)$. Like in case of actions, we postulate that the possible types and range of responses from an individual is a continuous random variable. Some more mathematical conventions are useful for analysis.

ASSUMPTION 2: The set of feasible actions or that of the possible responses form a complete, transitive, continuous, and regular order.

We will conclude this Section by introducing the concept of role structure. Roles are concepts specific to interactions and therefore, must not be defined in a Cartesian approach. Individual roles exist only in relation to other roles and therefore, are derived from the totality, the role structure.

Let $\mathfrak{R} = \{x : x_1, x_2, \dots, x_m ; m \geq n\}$ denote the set of all possible roles within a system boundary separated from its environment in space and time; the subscripts here indicate distinct roles not responses from a role. If H is the set of all institutional situations $(1, 2, \dots, s)$, the role structure of the system contained within the boundary is the Cartesian product $H \times \mathfrak{R}$. Each component of the structure-matrix x_{hj} describes the norm of action - the ideal expected response - appropriate for the corresponding role in that situation. Each distinct role j consists of a vector of norms x_j defined on the s -dimensional Euclidean space. The difference of a possible response from its normative expectation can therefore, be measured in Euclidean distance. Different institutions in a society define norms for different social phenomena. Thus, a column of the structure matrix includes all institutional prescriptions for the corresponding single role. The ensemble of institutions, therefore, can be represented by a role structure. In

this manner, by identifying the different roles and possible institutional situations, role structures can be defined for any subsystem within a social system, just as well as for a whole system. Subsystem role structures are members of the power sets of the set of roles constituting the role structure of the whole systems

Let us complement it with some dynamic understanding. Infinitely many different role structures, with minor variations in norms, may come into existence in the course of evolution of a single structurally stable system. The space they belong to, can be identified as a compact metric space. The structurally equivalent column vectors of this set of role structures can also be grouped into subsets and then indexed over a set of names. These names then become the role-names, which may be the popular identities, like father, mother, employer, supervisor, serfs, tenants, free or attached labor. But one would note that the codomain of each index is a family of elements of the role set Hence popular role-names indicate not unique, but generic norms within a structurally stable system.

3. PRODUCTION PROCESS AS A GAME

The starting point of a mathematical analysis is not the definitions and axioms, but the primitive terms. Once those are chosen, further developments, including the admissible list of concepts and definitions, are regulated by them. In other words, they come to exert decisive influence on the course of development of the analysis. Failure to specify a relevant primitive term may completely deform a whole body of theory. The primitive terms in neoclassical economics are: commodity, price, consumption, production, preference etc. Missing from this list is one that can account for the institutional state. Our proposal is to consider social roles as another primitive term. No doubt, there are caveats in the existing definitions of social roles. But 'primitives' are decided primarily by their objective and universally accepted existence. Behavioral norms, and their collections as social roles, are known to exist even in primate societies. The conceptual understanding of each of the primitive terms of economics has been refined again and again in the light of analytical developments that occurred till then. It is legitimate to expect the same with respect to the proposed new addition.

Social Production is an interaction process involving more than one individual. Each participating individual is characterized by his/her (a) initial endowments, (b) possible actions, and (c) an objective functions (utility maximization). This parallels the definition of consumption unit in Arrow-Debreu model. We propose to add a fourth: each participating individual also possesses (d) some information about other interactants. Some of the information is general, some others particular. Institutional information is perceived in the form of a role structure; non-institutional information are like person to person familiarity. Like endowments, possible actions, and objective functions, information about other interactants too is a personal possession.

Each interactant occupies a particular position in a task of production; each of them expects that particular types of responses would be obtained from each other assignee. At the same time every interactant is also aware that she is unable to make a definite prediction; at best she can tell with a margin of probability a range of responses that may be obtained from another interactant in an assigned role. Each decision maker assigns some probability of occurrence to every possible response from another interactant in an interaction. Refusal to participate is also a response. The possible types and range of responses from any role x_j therefore, constitute a Boolean field. To accommodate continuum of responses we extend the Boolean field to a suitable Borel field and then define the probability function $p = p(x_j)$ on it. It must satisfy the probability axioms of non-negativity and unit aggregate.

Let $p_{ij} = p_i(x_j)$ be the probability distribution over the type and range of response the i th individual expects from another interactant required to play the x_j th role by being assigned to a particular position in the task. In this manner, in an interaction of scale n , when the rest of the $n-1$ interactants occupy $x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n$ roles, she can still assign joint probabilities of occurrence of different ranges of variation for this role profile. Let us denote this joint probability distribution assigned by the i th interactant as P_i . This is a subjective probability. In turn, every other participant sets subjective probabilities of her responses. These need not be the same although they are attributed to the same action by the same individual. The person herself, while deciding in a rational manner, arrives at a specific action which, as a response, indicates her self-conception of her role. Thus, what each assignee thinks of her appropriate role is her private information. As we will find, an appropriate game formulation requires that these subjective probabilities are common knowledge, the private information are not.

Decisions are mediated through individual preferences. For this purpose, we assume existence of some individual utility functions U_i 's, in von Neumann-Morgenstern utility scale. Depending on the actual output, her share of the output, the effort given by her, and her valuation method of output and her input, individual i expects to receive a payoff $U_i(a, x)$ from the resulting profile. The decision function of each individual may then be expressed as maximization of their respective U_i .

Every interactant i therefore, faces a decision task requiring considerations of the following variables:

- n : interaction of scale n
- A_i : set of actions available to her
- x_i : her self-conception of her role, which is a private information
- P_i : the joint probability distribution subjectively assigned by her over the possible combinations (profiles) of the self-conception of roles of the rest of the interactants
- U_i : her utility function

If it is assumed that all the data the analyst requires for finding the solution of this simultaneous decision making problem, is also common knowledge among the interactants, the production possibility can be represented in terms of game theory. This however, belongs to the class of incomplete information games, Since the self-conception of roles of individuals is private information possessed by them before they face the interaction situation, those correspond to (Harsanyi's) types in game theory literature. The simultaneous decisions situation is therefore, represented by a Bayesian game:

$$(5) \quad \Gamma^b = (n , (A_i)_{i \in N} , (x_i)_{i \in N} , (P_i)_{i \in N} , (U_i)_{i \in N})$$

This is the legitimate representation of the process of production as a system of interactions.

The solution of this game is an action profile (a^*, x) , where $a^* = (a_1^*, a_2^*, \dots, a_n^*)$ indicates the best actions, corresponding to the participants identifiable by their self conception of roles (types) $x = (x_1, x_2, \dots, x_n)$. In brief, we write the solution as (a^*, x) . No interactant i can increase

her expected utility by unilaterally deviating from the equilibrium action a_i^* . Since Bayesian games are reducible to strategic forms, the only problem in the way of establishing existence of solutions is the assumption of infinite number of alternative actions. Intuitively, a compact continuum can be approximated to a very large number of finite grids whereupon there is no difficulty in applying Nash equilibrium concepts. Formal proofs along this line have been developed and can be found in standard books on game theory (Myerson, 1991; Fudenberg and Tirole, 1991).

One should note that the general solution depends on the subjective beliefs of the participants. For the purpose of prediction, every decision maker, as well as the analyst, must know the subjective beliefs of probability distributions of each other. Thus, even though, under common knowledge assumption, the game has a solution for arbitrary beliefs, it holds little appeal for actual analysis of production process. Only if individuals form their beliefs in relation to some objective variables, and the interactants as well as the analysts know that fact and can correctly predict these beliefs, some useful predictions and analyses can be made. Here we will introduce one possibility.

4. INSTITUTIONAL ORDER AS COMMON PRIOR

The existence of priors for drawing subjective probabilities is not unusual in the real world. People form their beliefs about others' behavior from past experiences, either directly with them or from general experiences of the surrounding. The existing probability distribution of types and ranges of responses under different roles in a reference population Ξ_j may act as the prior distribution forming the beliefs of interactant i . The reference may be to the whole population in the economic system or to a particular ethnic group or a firm, relevant for the situation under consideration. The subjective beliefs $p_{ij} = p_i(x_j)$ can then be replaced by the objective probabilities of the type and range of responses from the x_j th role in reference population Ξ_j used by the i th individual.

But this assumption confers on the decision maker only a source of belief; she must still know each of the distinct probabilities p_{ij} . Since infinite number of variations are possible in responses, knowing each p_{ij} is impossible. Instead, we assume:

ASSUMPTION 3: *The decision maker is risk averse. While predicting another participant's action she presumes maximum uncertainty except for whatever definite information she has about the source population on the basis of which she forms her beliefs.*

To operationalize this maximization problem we need some measure of uncertainty. The amount of uncertainty I in a reference population Ξ_j is a function of the probabilities of the possible responses $p(x_j)$'s. A suitable measure should have the following properties:

- (i) It is continuous in the probability space.
- (ii) If all elementary responses are equally likely, then I increases monotonically with each response.
- (iii) If a response has been partitioned in two responses and its probability has been partitioned between them as ζ and $\bar{\zeta}$, ($\zeta + \bar{\zeta} = 1$) then the original information is augmented because there is an additional uncertainty about which of the two partitions has occurred. The augmentation should be given by the same measure: $I(\zeta, \bar{\zeta})$.

The only function that meets these three conditions is (Shannon, viz Shannon and Weaver, 1949; Khinchine, 1957):

$$(6) \quad I_{\Xi_j}(x_j) = - \int p(x_j | \Xi_j) \log p(x_j) dx_j$$

This is the celebrated Shannon measure of information, or, as von Neumann had pointed out, 'information entropy'. As in thermodynamics, here too, entropy is a measure of disorder in the system. One can easily verify that the measure attains its maximum value when all possible responses have equal probability and the minimum when exactly one response has probability one. The name, originating in communications theory, is a little misleading. In a general context, it is aptly interpreted as a measure of the amount of uncertainty.

While maximizing this function for the purpose of predictions about the reference population Ξ_j , the decision maker should take at least one constraint in consideration, which is $\int p(x) dx = 1$.

Maximization of I under this constraint leads to uniform distribution over the permissible range.

If, in addition, the decision maker knows the mean (μ_{ij}) and the variance (σ_{ij}^2) of the responses from a role (x_j) in the reference population Ξ_j at a particular point of time, those should also be taken in for constrained maximization. By defining the range suitably as ($-\infty, +\infty$), it can be shown (Rao, 1965, p. 162) that the maximum of the measure (I) subject to the three constraints is the Normal (Gaussian) probability distribution $N(x | \mu_{ij}, \sigma_{ij}^2)$ with parameters μ_{ij} and σ_{ij}^2 . For Normal distribution, the mean and the mode coincide. Thus if the locational observation is in terms of the modal behavior, which the dynamic role theory explains as the current identity of the role, the same conclusion about the probability distribution still applies.

We find that under some suitable assumptions, it is possible to envisage the belief structures of the decision makers. Since in their experiences they usually learn about the average or modal behavior of a social role and the existing extent of deviations from it, it is reasonable to postulate that most of the time the rational decision makers base their decisions expecting a normal distribution of responses from others. But this is not always the case. Sometimes the decision makers may have much less information and may therefore predict that all possible responses have a uniform distribution. Some other times one may know more than just the mean and variance. Any additional information about the reference population Ξ_j that a decision maker may have, should be treated as additional constraints for maximization and for obtaining the appropriate probability distribution thereof. In such a case distributions other than Normal distribution may describe the pattern.

The probability distribution of the set of all possible profiles of responses from the rest of the interactants is then given by the joint probability distribution over the responses from each relevant role. If each of these are expected to follow normal distribution the expected distribution of the profile of actions from all the other $n-1$ participants is given by a multivariate normal distribution $N_{n-1}(\mu, \Sigma | \Xi_j)$ where μ is the vector of means and Σ , the dispersion matrix of response

variables. Since there is no obvious reason for the components of role structures to be correlated, the dispersion matrix is a non-negative definite, diagonal matrix.

With this, the production function is completely specified. Given, in addition to the technological input output relations - a distribution scheme D_i , the relative valuation Π_i , and the reference population Ξ_i to draw the belief structure, the payoff functions are completely determined for each of the i th participant. It is then possible to specify the Bayesian game and obtain the equilibrium, which, as we have discussed, exists. More specifically, the solution is given by:

$$(7) \quad a_i^* = \underset{a_i \in A_i}{\operatorname{argmax}} U_i \left(\int \int \dots \int b_i(a_i, a_{-i}) dp_1 \dots dp_{i-1} dp_{i+1} \dots dp_n - \Pi_i a_i \right)$$

which is a modification of equation (4) replacing net benefits by utilities. The $b_i(a_i, a_{-i})$ terms are given by the technological relation shown in equation (2). Given a relative valuations Π_i and the expected probability distribution of different profiles of actions, the equation (7) can be put in a computable form. Some more simplifying conditions may still be required, for, the computation techniques available at present may still prove to be insufficient. In the following Section we have made one such exercise. But let us put it in categorical terms that the paper essentially concludes here. We have suggested an approach which is applicable in a very general context. Except that the production process under inquiry is an interaction, we have not imposed any other restriction. The information measure in its nonparametric form, equation (6), is applicable to all situations. The analysis is applicable to individual firms, specific industry or to the economy as a whole. Alternatively, particular distribution schemes, relative preferences or information flow and processing methods can be treated within the model. By specifying the parameters appropriate for the situation under inquiry and by directing the analysis along a desired course, one may draw many significant conclusions. The analytical approach deserves fullest attention, and hence is the proposal to draw curtain here.

One particular characteristic of the analysis must be noted. Transaction cost has not been included anywhere though information is at the core of the present model. This is because we have introduced only that information which is available in a costless manner - information about the existing institutional situation

which the individuals learn in course of their day to day chores. It does not require any additional effort; the information is obtained through the experiences of their social form of existence. In fact, the neoclassical individual is the one who has to put an extra effort, for he has to keep his eyes shut, ears closed, and other natural faculties stunted, to remain oblivious of what exists in his surroundings. One can actually estimate the informational contribution of institutions. In absence of institutions every possible responses (or actions) have equal probability. This is identical with the neoclassical setting. In this case we get the maximum possible uncertainty which, by using Shannon measure (standardized) as in equation (6), is 1. Also, as discussed, one can measure the uncertainty I_E in the presence of an institution, at a particular state. According to the third axiom of Shannon the maximum can be split between a 'message' and a residual as:

$$(8) \quad 1 = I_E + I_R$$

The institutional state thus eliminates $I_R = 1 - I_E$ amount of uncertainty from the system or carries I_R amount of information. This information is available in a costless manner, through the very existence of an individual within the institutional setting. The remaining uncertainty I_E is the source of risk and leaves room for individual efficiency. An important property of this measure is, it is content free - which particular institution is in existence does not influence the measure. The institutional information I_R increases and decreases with role compliance and role conflict, irrespective of which institution is in force (Sengupta, 1998).

5. SOME IMPLICATIONS

Since all the three variables involved D_i , Π_i , and Ξ_i may be objective variables, there is a determinate solution under common knowledge assumption. But this is only theoretically so; the common knowledge repository has to be vast so as to include all the pertinent information for each of the i th participant. Different individuals involved in an interaction may actually use different reference population as prior distribution or different relative price Π_i . If the common knowledge assumption applies the system can be studied. We will use here a simplifying assumption that all individuals use the same set of information for these three variables. This too, is of great interest for it is directed to studying holistic characteristic of an economic system. The system may be denoted by Ξ , the same for all the individuals

of our concern. Any unit, which qualifies for being identified as a system, like having a clear boundary separating it from its environment, may be identified as Ξ .

If all the interactants use a common reference, such a situation is called consistent in belief. In consistent belief situations, the prior probabilities are functions of the current statistics on the common reference population Ξ . We assume that the individuals use only the first two moments to form their beliefs. Then the common prior is the n dimensional normal distribution $N_n(\mu, \Sigma | \Xi)$ and the individuals i 's belief structure P_i is the conditional probability given that the i th individual has assumed x_i th role and the possible responses from all others is the appropriate $n-1$ dimensional multivariate normal distribution. Let us, in brief, express the posterior distributions as:

$$(9) \quad P_i = P_i(\mu_{\Xi}, \Sigma_{\Xi} | i); \quad \forall i$$

The sharing scheme (D_i), which is followed (equation 2) in dividing the output among the participating n individuals, may be a specific characteristic of each particular unit of a production process. But there may also exist a social norm of sharing between the different roles in a reference population. In an economy, all the production units use certain norms of distribution. We will be concerned with this general norm of sharing between roles D_{Ξ} . The set of objective prices Π_{Ξ} reflect some common preference pattern of all the participants in the reference Ξ . Every society develops its preferences for certain resources and products. At present we are witnessing a changing global preference in favor of sustainable production and conservation of natural resources.

After due modification of the net benefit function (3) for inclusion of utility and the system specific relative valuation and distribution, we get the payoff function of individual i as:

$$(10) \quad U_i(D_{\Xi}(y(a_1(\mathcal{E}_1), a_2(\mathcal{E}_2), \dots, a_n(\mathcal{E}_n)))) - \Pi_{\Xi} a_i$$

This function is then maximized in incomplete information situation as in equation (4), after the inclusion of prior probabilities under consistent belief as given by equation (9). It may be noted that the solution of the maximization problem takes the following form:

$$(11) \quad (a^*, x) = a^* ((x_i)_{i \in n}, \mu_{\Xi}, \Sigma_{\Xi}, D_{\Xi}, \Pi_{\Xi})$$

where $a^* = (a_1^*, a_2^*, \dots, a_n^*)$.

The equilibrium output is then given by a function of equilibrium inputs:

$$(12) \quad Y(a, x) = Y(a_1^*, a_2^*, \dots, a_n^*)$$

Each a_i^* stands for the contribution of his endowment by the i th participant. In terms of the initial endowments \mathcal{E}_i 's of each i , the production function (12) may then be rewritten, along with a correction factor g , as:

$$(13) \quad Y(a, x) = f(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_n) \cdot g(\mu_{\Xi}, \Sigma_{\Xi}, D_{\Xi}, \Pi_{\Xi})$$

For comparison, let us consider a situation where there are only two types endowments, hence two types of inputs, capital K and labor L. Then equation (13) reduces to:

$$(14) \quad Y = f(K, L) \cdot g(\mu_{\Xi}, \Sigma_{\Xi}, D_{\Xi}, \Pi_{\Xi})$$

Many other important implications may be obtained by pursuing the suggested interactive approach in many different directions. But probably the most important one is given by the above equation. Production functions everywhere, are not the same as neoclassical analysis suggests. Even if there is no difference² in

$u=$, the modal rates of contributions obtained from the owners of each type of endowment, the same amount of output may not be obtained from two economies similar in their initial endowments and technological levels. Three other factors regulate productivity. These are:

- (a) state of order in the environment in which the production activity is being conducted
- (b) distribution norms
- (c) relative prices as reflected by common (social) preferences.

By ignoring the interactive features of the production process and thereby, dismissing institutions, neoclassical economics had banished from the study of production performances the contributions of three significant variables. While theories continued to show that these factors have no implications on productive efficiency, experiences went otherwise (Stiglitz, 1987; Bardhan, 1989). Economic performances were never independent of role conflicts and consequent social disorder or of changing distribution norms. Nor has it been established in the real world that prices rule supreme, social preferences are subordinate to it.

The correction factor g is identifiable as a performance measure. Though the economic theory claimed otherwise, managers knew all the while that performances can often be improved even without any addition to endowments. Thanks to Leibenstein (1966), such real world phenomena have some place now in economic theory (Frantz, 1992). Institutions, North (1990, p. 107) had thought, determine the long run performance of economies. As we show here, institutions affect even short run performances. The dynamic analysis is outside the scope of the present paper. But one must note that the uncertainties are endogenous and therefore, the model is necessarily dynamic.

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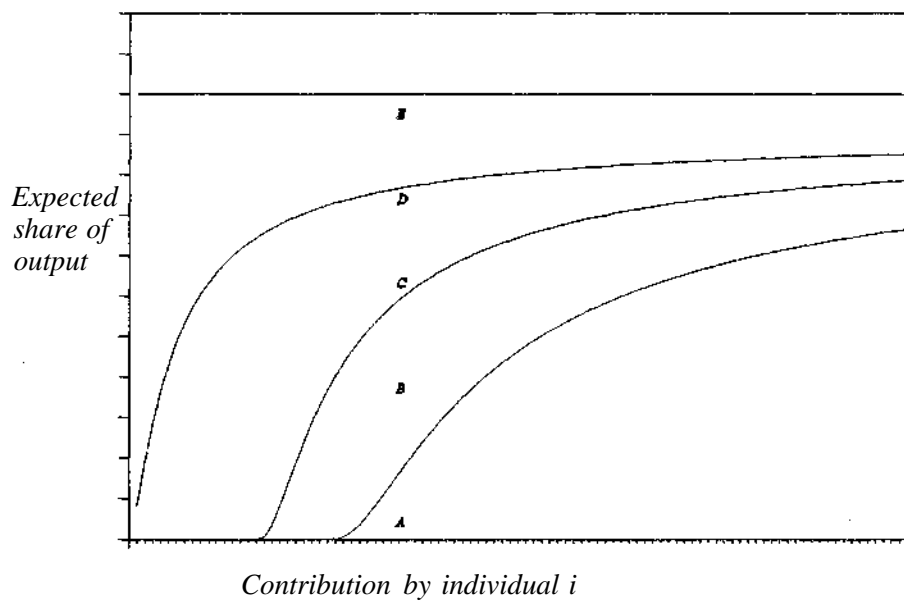
APPENDIX

When production is an interactive process the factor productivities need not always follow the diminishing marginal productivity relation. In the accompanying figure 3 we show the different possible shapes of the production possibility frontier.

Each curve represents a fixed level of contribution by the rest of the interactants and shows the rate of change of output with variations in contributions by individual i . The curve D is the classical marginal productivity curve. The curves C and B, and A are obtained by a minor modification over the conventional marginal productivity relations to account for some possible cases of non-substitutability in interactive production. For very low level of contribution by one or more of the other participants, no output may be produced.

The curve E shows another possibility: gross benefit is independent of individual i 's contribution. This may happen when others have made sufficient contribution on all inputs and so much so that marginal product does not even increase by some additional contribution.

Fig 3 : PRODUCTION POSSIBILITY FRONTIER



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