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## APPLYING GENERAL EQUILIBRIUM ANALYSIS & GAME THEORY TO EXCHANGE NETWORKS

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### ABSTRACT

Under conditions holding in most experimental and simulation studies of exchange networks, the CORE as derived by game theory yields a solution set of exchange outcomes more appropriate than the equilibrium generated by general equilibrium analysis (see Yamaguchi 1996). This is because general equilibrium analysis assumes a large number of actors for each position in the exchange system--essentially that a network and its actors are afloat in a market of like networks and actors. I suggest a derivation of the core for isolated (i.e., not in a market) strong power networks alternative to that offered by Bienenstock and Bonacich (1992). Since the core for isolated networks is large, power distributions in such networks are necessarily the consequence of both network structure and actor strategies.

### INTRODUCTION

In this article I discuss analytic solutions to the distribution of power in exchange networks offered by two applications of microeconomic theory to exchange networks. These solutions are the competitive equilibrium provided by general equilibrium analysis (GEA), as used by Yamaguchi (1996; also Whitmeyer 1994, 1997); and the core provided by game theory. GEA arrives at a single (under usual conditions) outcome, the competitive equilibrium, through assuming a large number of actors of each type. This removes any opportunity for strategic behavior on the part of actors, and determines rates of exchange which they must follow. On the other hand, the core is appropriate for any number of actors, including the small number which has been a condition of extant research on exchange networks. For small numbers of actors, the core allows for strategic behavior and correspondingly typically consists of more than a single outcome. In this article, I discuss implications of these differences, and suggest modifications to the cores proposed by Bienenstock and Bonacich (1992) for exchange networks.

Note that here I am concerned only with EXCLUSIONARY networks--that is, where "actors with two or more relations cannot exchange in one or more of their relations" (Markovsky et al.

1993), thus power differences are created by (potential) exclusion of actors from exchange. This is also the principal focus of Yamaguchi (1996) and Bienenstock and Bonacich (1992).

## **GENERAL EQUILIBRIUM ANALYSIS**

General equilibrium analysis (GEA) is arguably THE fundamental tool of microeconomics (Weintraub 1985). In its simplest form it is applied to exchange systems involving only consumers and not producers. That is, it is applied to systems which consist only of a set of actors and a set of goods. Actors have preferences concerning those goods and begin with initial allocations of goods. GEA makes one behavioral assumption, that actors will choose more preferred bundles of goods over less preferred bundles. These characteristics are true also of exchange networks as they have been studied.

Yamaguchi (1996) makes a signal contribution to the study of exchange networks by showing a way to apply GEA to determine power distributions in certain types of exchange networks. He does this through two innovations. First, he assumes that the interests of actors--the elements of their utility functions--are in (exchange with) other actors, rather than in goods possessed by other actors. Second, he captures the distinction between negatively connected and positively connected networks by allowing alternative exchange partners to be substitutes and complements, respectively. He notes that his method may not be applicable to positively connected networks involving transfer of material resources. (See Note 1.) Yamaguchi compares the results of his use of GEA to experimental and simulation results for exchange networks, and finds that it does not do too badly.

GEA also makes assumptions about actor preferences, which are embodied in the form of a utility function. Most results of GEA rest on the assumption that utility functions are concave. That is, actors are characterized by declining marginal utility: the more of a good an actor has, the less valuable each additional unit of that good is to the actor (see Hildenbrand and Kirman 1988; Weintraub 1985). The Cobb-Douglas utility function used by Coleman (1990) is concave. Yamaguchi's (1996) modification of that utility function is concave (see equation (6) on p. 314) until the elasticity of substitution  $s = \text{infinity}$ , when the utility function becomes linear. This last limiting case ( $s = \text{infinity}$ ) is the most appropriate comparison for experimental and simulation studies of exclusionary networks, since no published study of exclusionary networks has embodied declining marginal utility.

Given these assumptions, GEA yields what are called COMPETITIVE EQUILIBRIA--redistributions of resources such that there is no excess demand for any good. Under the above assumptions the number of competitive equilibria is finite, and indeed there is typically but one competitive equilibrium (see Hildenbrand and Kirman 1988; Weintraub 1985). GEA also specifies a value (or price) for each good at competitive equilibrium. Actors' relative power therefore may be measured as their proportional control over value at equilibrium (Coleman 1990).

However, to derive a competitive equilibrium, GEA and therefore Yamaguchi (1996) make one critical assumption which is vastly different from the conditions of experimental and simulation studies of exchange networks. GEA assumes a large number of actors of each type--i.e., with a

given distribution of preferences and initial allocation of resources--in the system. The number must be large enough so that the behavior of any single actor makes no appreciable difference to the outcome. In other words, actors are PRICE-TAKERS not PRICE-MAKERS (Hildenbrand and Kirman 1988).

When the number of actors of one or more types is small, on the other hand, the situation is different. Some, perhaps all, actors are PRICE-MAKERS. That is, they can affect outcomes by their behavior. Correspondingly, there is no single equilibrium nor finite number of equilibria. Rather, there is a range of possible solutions or distributional outcomes, referred to as the CORE. The core consists of all those solutions that cannot be improved upon by any coalition of actors, including individual actors and the set of actors as a whole. As the number of actors of each type increases the core shrinks until it is simply the competitive equilibrium (Hildenbrand and Kirman 1988; Coleman 1990).

It may be useful to elaborate on the relationship between the core and the competitive equilibrium, since it has important implications for exchange networks. Imagine two people alone on a desert island: Agnes, who has widgets for sale, and Betsy, who wants to buy widgets. Agnes has some minimum price she will accept for so many widgets; otherwise she will keep them for herself. Betsy has a maximum price she will pay for so many widgets; otherwise she will do without them. In between those minimum and maximum prices, any exchange outcome is possible. The exchange that results will depend on the bargaining process, thus on strategic behavior by the actors. For example, Agnes may pretend her minimum price is higher than it really is, and so forth. The set of possible exchange outcomes is the core. Clearly, it consists of a large set of outcomes--infinite, if goods are measured continuously--within a certain range.

Now suppose we add more widget-sellers and more widget-seekers (identical to Agnes and Betsy, respectively). Each time we add one more seller and buyer each, the core shrinks. That is, the range of possible exchange outcomes decreases. This is because of the possibility of cooperative reallocations between widget-sellers and between widget-buyers. As the number of sellers and number of buyers approach infinity, under usual assumptions, the core shrinks to a single exchange outcome, the competitive equilibrium (see Hildenbrand and Kirman 1988 for extended discussion and mathematical proof). It is important to note that this single point competitive equilibrium is not an average over the many sellers and buyers. Rather, it is a set exchange outcome, typically conveyed by a set price, that every buyer who buys and every seller who sells is forced to accept. There remains no scope for strategic behavior.

The core contains the competitive equilibrium, but typically includes a much broader range of solutions, especially with a minimal set of actors. Moreover, there is no reason to expect the competitive equilibrium as a most likely solution or central tendency within the core. That will depend on the particular situation, the particular actors, and the particular behaviors or strategies the actors follow (Hildenbrand and Kirman 1988).

Applications of GEA to exchange networks such as Yamaguchi's (1996; see also Whitmeyer 1994, 1997) thus assume a multiple actor, multiple network situation. Specifically, they assume a large number of actors of each type--that is, dedicated to a particular network position, including having a specific configuration of preferences and initial allocation of resources. These actors are



For all networks, Bienenstock and Bonacich (1992) implicitly consider the game in question to be a single experimental round, an exercise in making a single set of deals. For some networks, this can constrain the core inappropriately. This is because links in an exchange network are conceptualized to be exchange RELATIONS--ongoing situations of possible exchange (Emerson 1972). This is embodied in experimental operationalizations by running experiments for more than one round, often for upwards of many rounds while not informing subjects of the number of rounds. In simulations it is embodied by the practice of using results of previous rounds to determine offers simulated actors make in the current round.

The existence of important differences in outcomes between one-time and indefinitely repeated exchange situations has been well established (see, e.g., Taylor 1987). We can see this for the core by considering the simplest possible exchange network, the Line3 (see Figure 1a), where linked actors negotiate the division of 24 points and only one exchange may be completed per round. According to Bienenstock and Bonacich, for all outcomes in the core, the total outcome for a coalition of A and B will be at least 24 ( $A + B \geq 24$ ), likewise for B and C, and for A, B, and C all together ( $B + C \geq 24$ ,  $A + B + C \geq 24$ ). All this is saying is that, if A and B want to, they can ensure that their combined earnings are 24 points. The same is true for B and C, and in fact for all three actors together. This implies that the core consists of the solution whereby B gets 24 points per round and the others get 0.

However, where exchange is repeated, a coalition of A and B whereby on average  $A + B \geq 24$  implies that A and B complete an exchange each and every round. That is, to say that  $A + B \geq 24$  is to say that the A - B coalition has the option of never failing to complete a deal. But in a situation of repeated exchange, that inequality can hold only by ruling out C as a possible exchange partner, thus effectively making the A - B relation a two-person network, a bilateral monopoly. The core of such a network consists of all outcomes whereby  $A + B = 24$ --that is, a far larger set of outcomes than simply the extreme outcome  $B = 24$  (see Coleman 1990). In order for B to keep a THREE-actor network and prevent a bilateral monopoly, B must occasionally complete a deal with C. But this will mean that as an average per round,  $A + B < 24$ . Thus the core derived by Bienenstock and Bonacich (1992) seems inappropriate for at least some networks involving repeated exchange.

Unfortunately, it is difficult to ascertain the core for a situation of repeated exchange. A leading game theorist says, "Cooperative solution concepts such as the core...do not adapt with ease to...infinite-horizon games. This is not necessarily a fault of these solutions. They address general questions involving possible cooperation without going into details concerning the dynamics of communication and bargaining or the form of cooperative agreements" (Shubik 1982:286).

Nevertheless, I suggest a possible derivation of the core for an isolated network here, alternative for some networks to that proposed by Bienenstock and Bonacich (1992). It should be noted that outcomes I give are not in terms of exchange ratios, as is common in network research, but as average outcomes over a number of rounds.

To begin with, I assume that a two-actor coalition as such is impossible in a network of repeated exchange. The two-actor coalition is the crucial coalition since the exchange rules specify that an

exchange is an agreement between two actors. For a two-actor coalition to exist would mean that two actors have an arrangement which covers the entire existence, the life-span, of the network. Such an arrangement is not possible in most exchange networks as they have been operationalized. Typically, there is no mechanism by which such INSTITUTIONS can persist, no means of formalization or enforcement, for example.

What is crucial then is the strategy of individual actors. Let us define a STRONG actor as an actor who always completes a deal in any round in which no more deals can be completed. A WEAK actor is one that is not strong, that is, one that may be excluded from exchange in a round in which no more deals can be completed. Whether an actor is strong or weak will be determined by a combination of network structure and exchange rules. For example, any actor linked to an end actor, that is, any actor that has a partner for whom it is the only partner, is necessarily strong. The end actor will be weak if its partner has more exchange partners than it is allowed exchanges. Thus in all networks in Figure 1, actor B is strong, since at a minimum B is never excluded from exchange with A. A is always weak. Included in the core is always the outcome in which strong actors receive the entire negotiated resource (i.e., 24 points) each round.

However, weak actors can follow strategies too. In the Line3, Line5, and Branch31 networks, let  $n$  be the number of weak actors and  $m$  the number of strong actors. Suppose weak actors follow a strategy of never making a deal in more than  $m$  of any  $n$  consecutive rounds. Thus in the Line3 network, A and C sit out alternative rounds; in the Line5 network, A, C, and E each sit out one round out of three; in the Branch31 network, A, C, and D each sit out two out of three consecutive rounds. This means that in the rounds in which weak actors do participate, their receiving the entire negotiated resource is in the core. Thus the core of these networks consists essentially of all outcomes between the extremes of the strong actor(s) receiving the entire resource every round and the weak actors receiving the entire resource  $m$  out of  $n$  rounds.

Let us assume that a deal consists of a division of 24 points. Then, for Line3 the core is:  $A + B + C = 24$ ;  $B \leq 24$ ;  $A \leq 24$  and  $C = 0$  one of every two consecutive rounds; and  $C \leq 24$  and  $A = 0$  one of every two consecutive rounds. The works out to the following. B can average as much as 24 points (proportionally, 1) every round, if both A and C get 0, which is possible. It is possible that every round B's partner, alternately A and C, gets 24 points, thus that B gets 0. On the other hand, while A and C can get a maximum of 24 points, it is for only one out of every two rounds, giving a maximum average of 12 points (proportionally, 1/2). Their minimum, again, is 0.

For Branch31 the core follows from:  $A + B + C + D = 24$ ;  $B \leq 24$ ;  $A \leq 24$  and  $C = D = 0$  one of every three consecutive rounds;  $D \leq 24$  and  $A = C = 0$  one of every three consecutive rounds; and  $C \leq 24$  and  $A = D = 0$  one of every three consecutive rounds. For Line5, there are some complications with intermediate outcomes when either end actor (A or E) abstains. However, these can be elaborated easily (which I do not do here) using the core for Line4 (see table below).

The key to the core for these three networks is the weak actor strategy of sitting out rounds. This may appear not to be individually rational--but it is, for the following reason. Suppose that more than  $m$  weak actors participate in the rounds. Then, it is possible that a given weak actor will be

excluded more than its fair share--i.e., more than  $(n - m)$  times every  $n$  rounds. This will lead it to make concessions to its strong partners, thus weakening and ultimately destroying the bargaining position of all weak actors, including itself. A strategy of abstention, as described here, forestalls this problem.

Let us turn to the other two networks. Both the Line4 and the Stem networks are "weak power networks" (Markovsky et al. 1993), which can be defined as networks that either have no strong actors, no weak actors, or in which it is not inevitable that a weak actor be excluded from exchange. Thus no actor should follow a strategy of sitting out some rounds, meaning in turn that repetition of exchange does not affect the core. Consequently I agree with Bienenstock and Bonacich's (1992) determination of the core for these weak power networks. In the Line4, the core is given by the solution:  $A + B = C + D = 24$  and  $B + C \geq 24$ . In the Stem network, the core is given by the solution:  $A + B = C + D = 24$ ,  $B \geq C$ , and  $B \geq D$ .

To summarize, Table 1 shows the maximum and minimum average gain per round--that is, maximum and minimum relative power--for each actor in the core for each network as a proportion of total possible points earned in the network. This can be taken as a measure of relative or proportional power in the network, or of "systemic power" (Coleman 1990). Maximum and minimum average points per round, out of 24, are shown in parentheses. Positions identical due to symmetry are easily inferred, therefore not shown. For example, actor C in Line3 is identical to A, with a proportional maximum of 1/2 and minimum of 0.

**Table 1. Actors' Maximum and Minimum Relative Power in Core for Exclusionary Networks of Figure 1 (Average Points Per Round in Parentheses)**

	Actor A		Actor B		Actor C	
Network:	Max	Min	Max	Min	Max	Min
<b>Line3</b>	1/2	0	1	0		
	(12)	(0)	(24)	(0)		
<b>Line4</b>	1/2	0	1/2	0		
	(24)	(0)	(24)	(0)		
<b>Line5</b>	1/3	0	1/2	0	1/3	0
	(16)	(0)	(24)	(0)	(16)	(0)
<b>Branch31</b>	1/3	0	1	0		
	(8)	(0)	(24)	(0)		
<b>Stem</b>	1/4	0	1/2	1/4	1/2	0
	(12)	(0)	(24)	(12)	(24)	(0)

## DISCUSSION AND CONCLUSIONS

The core specifies the set of all possible exchange outcomes for rational actors, defined minimally as actors who choose better outcomes over worse outcomes. Clearly, for all isolated networks considered here the core is a large set of solutions, in which even the order of actors in terms of outcomes varies. Unlike the core proposed by Bienenstock and Bonacich (1992), the core as I have proposed it does contain both Yamaguchi's GEA results (which it should according to microeconomic theory) as well as results from experimental exchange networks.

What the size and range of the core means in terms of power and power use is the following. Arguably, network exchange studies make use of a Weberian conception of power: power is the ability to achieve one's interests (Weber 1968:926). (See Note 2.) Network structure then affects the distribution of power between network members, which is measured in empirical studies by power use. This measure is appropriate under the assumption that actors use their power somewhat efficaciously. The large size of the core tells us then that **FOR ISOLATED NETWORKS, NETWORK STRUCTURE DOES NOT DETERMINE A POWER DISTRIBUTION THAT IS UNIQUE OR EVEN LIES WITHIN A NARROW RANGE.** This is supported also by simulation results in Markovsky (1987).

To put it differently, the core shows that for an isolated network under a minimal assumption of actor rationality (i.e., that an actor will choose better outcome over a worse one), network structure only partially determines power. A large set of power outcomes are consistent with the assumptions. Power--the ability to achieve one's interests--is produced by a combination of network structure and the distribution of actor strategies. (See Note 3.) These strategies may be forward-looking or backward-looking (Lovaglia et al. 1995).

If empirical studies find a narrower range of outcomes for isolated networks than the core, it cannot be because of network structure directly. Rather, it is due to particular actor strategies. Thus recent studies (Lovaglia et al. 1995; Skvoretz and Zhang 1997; Thye et al. 1997) are right to focus on actor strategies. Nevertheless, results for the core for isolated networks are somewhat disquieting, for it suggests that strategic skill of subjects is a key scope condition of experiments. For example, simulations I have run suggest that the strategy of periodic participation suggested above for weak actors in strong power networks will bring them higher payoffs than subjects in those positions earn in empirical studies. This is because experimental subjects rarely or never try those strategies. If appropriately trained, of course, they would. This means there is an educational, cultural factor determining power in isolated exchange networks.

GEA solutions are appropriate for a different structural situation, one in which an exchange network and its actors are part of a larger market of networks and network actors. While Yamaguchi's GEA analysis does match many empirical results, it does so by using seemingly arbitrary values of  $s$ , rather than the theoretically appropriate value of infinity. This discrepancy is not surprising, given the different structural situation. That GEA solutions match empirical outcomes for some values of  $s$  may indicate something about what strategies are used in isolated networks.

However, these considerations suggest further that exchange network studies may be extended profitably from isolated networks to scenarios of many networks and available actors. Such scenarios may correspond better to many natural situations than does an isolated network. For

example, both people and organizations often have a choice of with whom they enter into long-term relations, which networks they join; and they can change them. Firms decide on which banks they use, which suppliers, and so forth--and change them on occasion. Students choose their professors. Graduate students choose and change their advisors, and their advisors choose their graduate students.

Moreover, as discussed above, economic theory tells us that when the numbers of all types of actors increase, the efficacy of actor strategies diminishes; actors go from being price-makers to price-takers. Thus, it is in a market-like situation that actor strategies will play no part in determining their relative power. In a market-like situation (ONLY!), network structure will determine power, with no place for skill or cultural factors.

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## NOTES

1. See Whitmeyer (1997) for such an application.
2. This definition differs from that of Markovsky et al. (1993:198) only in that it does not stipulate that the source of power be structural.
3. Note that actor characteristics such as interests and their network configurations into interest structures also will affect power distributions (Whitmeyer 1994).

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