Can water allocation rules in irrigation schemes be used to share risk on resource availability between users ?

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Abstract

The research inquires to what extent can users of a CPR share the risk on resource availability between themselves or if the risk neutral State is bound to intervene to ensure an efficient risk taking and risk sharing. The study focuses on irrigation schemes and compares different traditional allocation rules, markets and insurance systems external to the CPR according to five goals that the collectivity may want to attain: water valorization, equity, risk sharing, an optimal collective risk taking and the diminishing of inter-annual income variability.
A model is developed, with farmers being heterogeneous with regards to their risk aversion, and with farmers taking into account the allocation rules and the potential taxes or subsidies in their cropping decisions. This heterogeneity creates an opportunity for risk sharing among farmers. Traditional allocation rules are categorized into *ex ante* rules, which do not create any interdependence between farmers' cropping choices, and *ex post* rules where farmers acquire returns that are proportional to what they put under crops. *Ex post* rules share risk efficiently between farmers but, because of farmers' strategic behaviors, it may lead to overcropping. In such an uncertain context, the "Tragedy of the Commons" may thus correspond to a too big collective bearing of risk which may be more profitable during average years but which will also be catastrophic during dry ones. An inquiry made in Tunisia showed this pattern on an irrigation scheme managed by farmers.
Finally, none of the studied rules can attain the five formerly defined goals all alone by itself. Nevertheless, *ex post* rules can be efficient if associated with a high water price, or if risk taking rotates among farmers: risk sharing between CPR users may hence be of interest.

Keywords: Risk pooling, Water allocation, Water market, Tunisia

The uncertainty about resource availability plays a large role in water management in many irrigation systems. Indeed, many schemes have been built in order to attain a maximal number of farmers, for example in the *warabandi* system set up by the British in India and Pakistan in the XIXth century (Jürriens and Mollinga, 1996). In these systems, the scheme is

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barely able to bring the necessary water: any failure causes large problems. Besides, twenty years ago, the main rule used by engineers to calculate the dimensions of a scheme was to ensure that the resource would be sufficient 4 years out of 5 only (Livingstone and Hazlewood, 1979). Moreover, this uncertainty can also stem from the rainfall when the latter constitutes an important complement to the water distributed by the irrigation scheme.
Furthermore, it becomes more and more difficult to create new resources, for two main reasons:
- the cost for creating new resources is increasing: dams are made at first on the best sites;
- there is a growing opposition to the building of new resources, for environmental reasons.

There is now a general need to set up mechanisms in order to share the existing water resource in this context of uncertainty on its availability.

In a very simplified way, it can be possible to draw two main situations in developed and in developing countries.
In developing countries, there is no formalized insurance system. For example, in Tunisia, many farmers using irrigation schemes do not have a property title because these titles are distributed only after a regrouping of lands inside the scheme is realized. These titles are necessary to obtain a credit from the National Agricultural Bank: farmers must hence self-finance their activities or obtain informal credits from friends. **This lack of risk sharing leads often to a low level of utilization of the water resource**.
In developed countries, insurance systems about water availability are topical for different reasons. First, fixed costs are more and more important: farmers need to stabilize their revenues. Exception made for some risks like hails or storms, the State must subsidize insurance premiums to permit the presence of private insurances. For example, the USA have been using since 1938 a program of subsidy to yield insurance (C.E.S., 1998). On the whole, countries like USA, Canada, Spain or Portugal devote between 15% to 35 % of their budget dedicated to agriculture to insurance programs for agriculture.

Of course, a direct control of the area put under crops allows to control the aggregate demand. In the Tambaparani system (India), the Irrigation Office decides the global cultivable area during Kar season (from June to September). Depending on water levels in the dams, the Office accepts a certain area in the first distributaries on the scheme. Then, at the beginning of July, it authorizes irrigation of new areas for distributaries in the tail part depending on water availability at that moment (Brewer et al., 1997).
This study deals here with the frequent case where, on the other hand, **farmers are autonomous regarding the area they put under crops**. **It is assumed that they have the same technical knowledge, i.e. the same production function, but they differ by their risk aversion: there is hence a potential gain to share the global risk on the CPR among them.**
This heterogeneity regarding the risk aversion can stem from different origins. First, a common assumption is to suppose that farmers have the same expected utility function but different initial endowments. It it then frequent to assume that the absolute risk aversion coefficient is decreasing with wealth. Moreover, farmers differ often regarding their possibilities to protect themselves against risks, using family networks or others.

If each farmer were alone, he or she would have to choose between maximizing the average revenue and minimizing its variance. The Collectivity constituted by all farmers cannot be seen as risk neutral since It faces a global risk on resource availability. As a matter of fact, It can try to achieve five different goals:
- ensure that the global tilled area corresponds to the collective risk aversion;
- set up a risk sharing mechanism;

- valorize water to the best for a given tilled area and an available water volume;
- diminish the inter-annual variability of revenues;
- attain equity objectives.

These five goals will be detailed hereafter: they will allow to compare different existing and possible allocation rules. Broadly speaking, the Collectivity will have to choose between a rigid allocation and risk sharing mechanisms that also often lead to strategic behaviors of over-cropping by less risk-averse farmers.
The questions addressed here are hence:

Depending on the objectives defined by the Collectivity (water valorization, risk-sharing, equity, etc.), how choosing the water allocation rule?
To what extent water allocation can be used to share a global risk among farmers?

The economic analysis of the link between Common Pool Resources and risk sharing

Almost all former studies on risk sharing in communities dealt with the sharing of risks both uncorrelated and imposed on farmers, without any risk-taking decision. Some studies measured the importance of informal risk sharing networks at village levels (Townsend, 1994, Lund and Fafschamps, 1997, Ligon, 1998). Arnott and Stiglitz (1991) showed that informal risk sharing between relatives is more efficient that a private insurance system only if the formers have better information on agent activities. If they have not, the use of informal systems is harmful because it eliminates private insurances without replacing them by something as efficient.
Another part of these studies tackled the question of the use of Common Pool Resources (CPR) to share individual risks. According to Nugent and Sanchez (1994), the more important the risk, the more farmers must diversify their fields geographically and there is a level of uncertainty above which it becomes easier for farmers to manage a CPR than to cope with the conflicts stemming from overlapped properties. Kimball (1988) calculated that, during Middle Age period, British farmers should have grouped themselves into cooperatives to share non correlated risks on the yields: given their risk aversions and their discount rates for the future, they would have had no interest to leave the cooperative if they had to become, for one year, a net contributor. Finally, Wilson and Thompson (1993) studied Mexican large productive farm (*ejidos*) in semi-arid climate: they showed that the main interest for such large farms is to allow cattle to move on a large area depending on rainfalls localization.

The approach used here differ from these previous studies on two main points:
- **the risk is related to the global water volume**, at the scale of the irrigation system;
- **farmers are autonomous regarding their cropping choices, i.e. the risk they take**.

Besides, McCarthy (2000), Provencher and Burt (1993), and Sandler and Sterbenz (1990) asked whether the addition of risk increases or reduces the Tragedy of the Commons. The answer appears to be that it increases the Tragedy if agents behavior changes the variance, and it decreases it on the contrary (McCarthy, 2000). Here, the question is the opposite: given a fixed level of risk about resource availability, what are the advantages and drawbacks of a risk-sharing allocation compared to one where there is no interdependence?

The study provides some elements of answer to the questions asked above.

Water allocation can be used to share the risk efficiently, but it will also usually lead to strategic interactions and overcropping. This “Tragedy of the Commons” pattern corresponds basically to a too big collective risk taking, but it may also lead to an increase in the average collective revenue compared to an allocation which does not share risk.
Overall, a rule that would distribute water *ex post* according to the needs, but that would be associated with a high water price, may both share risk efficiently and meet equity objectives. Nevertheless, on the whole, none of the studied rules (traditional rules, incomplete markets, insurance systems) is able to attain the 5 previously defined goals all alone by itself.
Besides, the article does not only study schematic rules; it tries also to present the various existing ways to allocate water in a situation of water shortage. Finally, technical difficulties to set up these mechanisms are not examined; they have already be extensively analyzed, especially regarding water markets implementation.

The paper is built as follows. A first part presents the theoretical framework, the choice made by a single risk-averse farmer in such a framework, and then the program that the Collectivity must address in order to maximize a given social welfare function. Next, different allocation rules without insurance system external to the irrigation scheme are determined (section 2). Section 3 presents different possible insurance systems. The following section provides an evaluation of these different rules according to the proposed criteria. Finally, an illustration is made on the case of two irrigation systems in Tunisia (section 5).

1 The model and the criteria for evaluation

1.1 The theoretical setting

[2].

When hydric stress is not too great, the yield r of a crop expressed in monetary units is a concave function C^1 of overall water used. We make the additional assumption that there is an amount of water V_m so that ($\forall V > V_m, \ r = r_m$) but this realistic hypothesis has no impact on the mathematical formulation as it is always optimal to remain on the growing part of the curve.
We suppose that the farmer can put under crops an area s_i, requiring irrigation, which is equivalent to supposing that he can choose between an irrigated crop and a non irrigated one providing a constant revenue. Furthermore, the area has a maximum s_m.
Each farmer will obtain a fraction V_i of the overall volume V and will distribute it evenly over the area s_i he has chosen[3]. The farmer pays also inputs costs that are here supposed to be proportional to the area under crops: $k.s_i$. These costs do not take into account water costs.
The farmer surplus per hectare will be hence:

$$surp._i = s_i[r(\frac{V_i}{s_i}) - k]$$

Moreover, each farmer make a monetary transfer t to the Manager or on the market place. This transfer most often represents a tax corresponding to water costs and hence is positive, but if it represents a monetary transaction made in a market context, it may be negative.

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[2] A companion paper has been written with the same theoretical setting (Faysse, 2002). This paper provides a similar comparison between *ex ante* and *ex post* rules, but in a deterministic context and with farmers differing in their capacity to valorize water

[3] Hereafter we use capital case V to indicate the global volume given to the overall individual area and lower case v to indicate the volume applied per hectare.

We assume that each farmer maximizes a von Neumann and Morgenstern (vNM) utility function of his revenue. The farmer hence chooses the area under crops in order to maximize:

$$\underset{s_i}{max} \quad EU_i(s_i) = \int_{V_i} u(\pi_i(s_i))h(V_i)dV_i \tag{1}$$

The function u is linear for a risk neutral agent and concave for a risk-averse one.
We suppose that the water volume availability has a density function $h(V)$, which is assumed to be known by each farmer. Let H be a support for this density function. Moreover, we suppose that the farmer's absolute risk tolerance T_i is constant[4]: either because the utility function has a form of a negative exponential or because variations of profit are considered to be small compared to the global wealth of the farmer.

Proposition 1 *For a single farmer, there exists one and only one surface put under crops that maximizes expected utility. Furthermore, the more the agent is risk-averse, the less he puts under crops.*

All demonstrations are made in appendix.
With this result it it possible to define a function $s = OA(h(V), T)$ which links an optimal area under crops to the probability function on V and the risk tolerance of the farmer.
For example, if we suppose that the available water volume can take only two values $\overline{V}$ and $\underline{V}$, figure (1) presents the mechanism for choosing the tilled area. The axes of the figure give the profits made if the volume available is important (X-axis) or small (Y-axis). The curve C, in continuous line, describes the location of possible profits for a given area s put under crops. The curve begins of course from the origin for $s = 0$, then attains point A where the yield per hectare equals the fixed cost k in case of drought. The corresponding profit is then null. Dashed lined are iso-utility curves: the optimal choice corresponds hence to the point on C where the dashed line is tangent to C: we obtain s_1 for a low risk-averse farmer and s_2 for a very risk-averse one.
Furthermore, the area put under crops diminishes with an increase on water volume uncertainty. Perry and Narayanamurthy (1998) showed that irrigation at lower than agronomic requirement can be both economically profitable and risky because, under a certain level of water supply, yield falls brutally. The authors verified that, in *warabandi* systems, farmers use this deficit irrigation but also that the more the water distribution is uncertain, the more farmers limit their area under crops in order to reduce the risk of falling under the critical level of water supply.
In Tunisia, water costs in irrigation schemes are an important part of production costs, and this all the more if there are seepage losses on the scheme. Hence, on an irrigation scheme named El Melalsa, close to the city of Kairouan, some farmers on the tail part of the scheme choose to take risks on rainfall levels during winter and irrigate little their wheat fields (Faÿsse, 2001).

1.2 At the collective level

1.2.1 A social welfare function

We consider here an irrigation system made of a set $I = \{1..n\}$ of farmers, each being free to choose the area he puts under crops at the beginning of the campaign. Moreover, we assume that farmers choices are made simultaneously.

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[4] The absolute tolerance of an agent is the inverse of the absolute risk aversion coefficient: $T_i(\pi) = \frac{1}{I_{ai}(\pi)} = \frac{-u'(\pi)}{u''(\pi)}$.

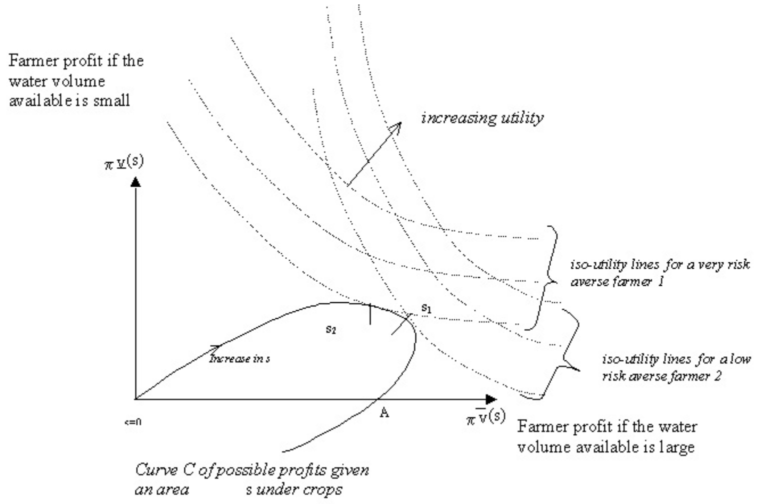


Figure 1: cropping choice for a farmer when there are only two possible water volumes

We suppose a social welfare function of the following form:

$$F\left[EU_1\left(\pi_1\right), ...EU_n\left(\pi_n\right)\right] \tag{2}$$

We also assume that one farmer's cropping choice only influences other farmers by affecting the total area under crops $S = \sum s_i$. The water allocation rules are then defined by a couple made of an allocation function $V_i = f(s_i, S, V)$ so that $V = \sum\limits_{i\in I} f(s_i, S, V)$ and a tax function $t(s_i, S, V)$ with $\sum\limits_{i\in I} t_i \geq 0$.

1.2.2 Optimal allocation

One way to obtain optimal allocation is obvious: it consists in complete markets on contingent assets both on V and S, the aggregate area under crops. These two parameters are a priori continuous but it is possible to transform them into sets of discrete values as large as wanted. Hence, each farmer buys at the beginning of the campaign a volume contingent to a future state defined by certain values for V and S, at a price $p(V, S)$. A farmer i maximizes hence both a surface s_i and a set of contingent volumes $V_i(S, V)$:

$$\underset{s_i, V_i(V,S)}{Max} \qquad EU_i\left[s_i[r(\frac{V_i(S,\widetilde{V})}{s_i}) - k] - p(S,\widetilde{V})V_i(S,\widetilde{V})\right]$$

The set of these n maximization programs and the water volume constraint $V = \sum V_i$ permit to calculate equilibrium prices $p(S, V)$.

Whatever the form of the social welfare function, the optimal allocation leads to a Pareto-efficient risk sharing (Gollier, 2001). Let $z(S, V)$ be the global value created on the whole irrigation system. A necessary condition for efficient risk sharing is the Arrow-Borsch condition (Gollier, 2001):

$$\forall i \in I \quad \frac{\partial \pi_i}{\partial z} = \frac{T_i}{T}$$

Gollier and Eeckhoudt (1993) showed that there are four cases for which optimal risk sharing allocations are represented by linear allocations of the form $\pi_i = a_i + b_i z(S, V)$, with $\sum a_i = 0$ and $\sum b_i = 1$:

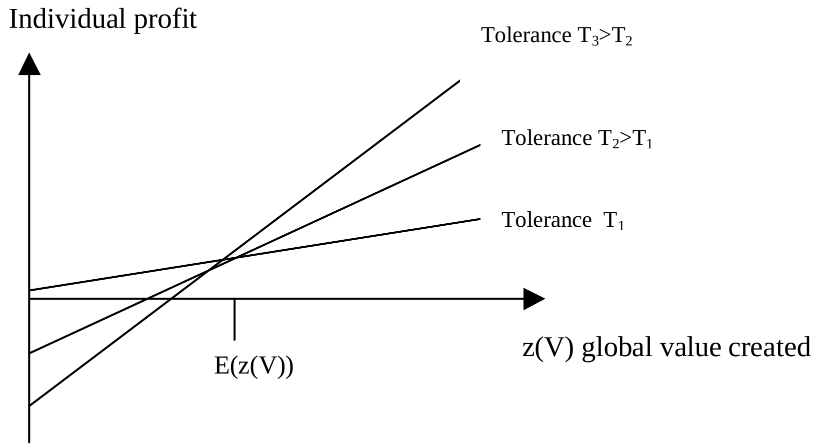


Figure 2: individual revenue depending of the global value created, in the case of a linear risk sharing and a constant average revenue for all farmers

○ when absolute tolerances are constant, hence $\pi_i = a_i + \frac{T_i}{T} z(S,V)$;
○ when relative tolerances are constant $\pi_i = b_i z(S,V)$;
○ when one member is risk neutral, consequently he bears the whole risk alone and other agents get a constant profit;
○ when utility functions are quadratic $u(\pi) = \pi - \beta\pi^2$, hence $a_i = \frac{1}{2\beta_i} - b_i \sum \frac{1}{2\beta_k}$.

Moreover, if we require that each farmer receives the same average revenue, the expected individual profit is:

$$\forall i \in I \quad E(\pi_i) = a_i + b_i E(z(V)) = C$$

Adding on all farmers: $\sum a_i + E(z(V)) \sum b_i = nC$, hence $C = \frac{E(z(V))}{n}$ and $a_i = \frac{E(z(V))}{n} - b_i E(z(V))$. The individual profit is consequently of the form:

$$\pi_i = \frac{E(z(S,V))}{n} + b_i(z(S,V) - E(z(S),V))$$

The sharing can hence be represented as on figure (2).

Finally, **the Collectivity can be seen generally as an agent with an absolute tolerance $T = \sum_{i \in I} T_i$** (Gollier, 2001). Therefore, since all farmers are risk averse (i.e. they all have a finite tolerance towards risk), the Collectivity is also risk averse.
The optimal global area under crop is the one that would be calculated by the program for a single farmer but with a representative agent with an absolute tolerance T.

1.2.3 When farmers are autonomous regarding their cropping choices

We assume that the Collectivity does not know the individual risk aversions and hence cannot use them in order to allocate water. Given the high difficulties to measure risk aversion, this is a very realistic assumption.
The collective allocation is given by two functions: a water allocation function $f(s_i, S, \widetilde{V})$ so that $\widetilde{V} = \sum\limits_{i \in I} f(s_i, S, \widetilde{V})$ and a taxation function $t(s_i, S, \widetilde{V})$ with $\sum\limits_{i \in I} t(s_i, S, \widetilde{V}) \geq KT(\widetilde{V})$, where $KT(\widetilde{V})$ stands for water production costs.

If the allocation rules create an interdependence between farmers's choices, we will describe these choices at the Nash equilibrium (while there is no general proof of existence and unicity of the Nash equilibrium in this theoretical setting, it appears to be the case for all studied allocation rules).
The Collectivity tries hence to solve the following program:

$$\underset{f,t}{Max} \quad F\left[EU_1, ...EU_i\left(s_i[r(\frac{f(s_i,S,\widetilde{V})}{s_i}) - k] - t(s_i,S,\widetilde{V})\right), ...EU_n\right] \tag{3}$$

$$_{s.t.}\begin{cases} \forall i \in \{1,n\} \quad s_i = Arg \underset{s_i}{Max} \; EU_i\left[s_i[r(\frac{f(s_i,S,\widetilde{V})}{s_i}) - k] - t(s_i,S,\widetilde{V})\right] \\ \forall \widetilde{V} \in H \quad \sum_i f(s_i,S,\widetilde{V}) = \widetilde{V} \\ \forall \widetilde{V} \in H \quad \sum_i t(s_i,S,\widetilde{V}) \geq KT(\widetilde{V}) \end{cases}$$

Hereafter, in order to get lighter mathematical expressions, the density function will be omitted in the integrals.

1.3 Criteria for evaluation

Complete markets based on contingent volumes V and contingent global tilled areas S do not exist on the field, probably because the contracts would be too complex and expensive. Therefore, this study tries to compare different existing and potential allocation rules when these complete markets cannot be set up. There is hence no global optimality property; it is nevertheless possible to define 5 criteria for evaluation.
The first 3 criteria can be seen as first order conditions of the former program (3).

1) Equalization of marginal valorizations for given tilled area and volume
This criteria corresponds to a necessary condition for *ex post* efficiency.

2) A global tilled area corresponding to the collective risk tolerance.
This is the *ex ante* efficiency condition.

3) The set up of a risk sharing mechanism.
Since farmers differ regarding their risk aversion, there is a possible gain to organize a transfer of risk-taking between farmers.

Finally, it is possible to add two others criteria not directly linked to the former problem.

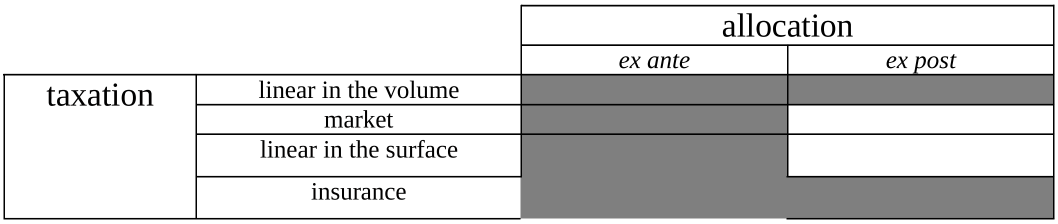
4) The reduction of inter-annual revenue variability.
This criterion corresponds to a possibility of putting aside a capital from one year to the following. The budget constraint becomes hence: $E\left(\sum t(s_i,S,\widetilde{V})\right) \geq E(KT(\widetilde{V}))$.

5) Equity in water allocation.
Risk aversion is an individual characteristic: hence, an allocation will often be considered as fair if it does not take into account such a parameter. Therefore, an allocation will be equitable if it creates a Pareto-increase from an allocation where farmers receives the same amount of water.

1.4 The different allocation rules examined

The reference allocation is often the *ex ante* allocation, where each farmer receives a fixed fraction λ_i of the collective volume V, whatever the value this volume attains.
It is then possible to define three main water allocation rules families in order to manage the uncertainty on water availability.
First, a possibility consists in **sharing risk through water allocation**. The main example is a rule that we will call *ex post* rule: it distributes to each farmer a volume proportionally to the surface put under crops, i.e. $V_i = \frac{s_i}{S}V$. This is a very simplified version of a commonly used rule that gives priority to water demanding cultures, for example market crops, compared to less water requiring ones, such as cereals.
A second family is based on **markets**. We have already proposed an explanation for the absence of complete contingent markets. There exist nonetheless two simplified forms of market: an *ex ante* market where farmers exchange parts of the collective volume at the beginning of the campaign (and hence before the cropping choice), and an *ex post* market where the exchange is made after the collective volume to be shared and the aggregate tilled area are known by farmers.
Finally, there exist **insurance mechanisms external to the irrigation system**.

These different possible allocation rules can be built from two water allocation rules and four general forms of taxation functions.
- Water allocation can be done proportionally to the global resource ($\lambda_i V$) or by distributing water according to the needs ($\frac{s_i V}{S}$).
- The tax can be calculated from the volume allocated (with a form pV_i if it is linear), by a market, on a surface basis $t(s_i)$, or with an insurance system.
The different possible couples are presented on figure (3) (parts colored in gray tint show the rules really used on the field).
The next section determines the allocation rules when there is no external insurance system are characterized: rules based on *ex ante* allocation (with volume taxation or with a market), then rules using an *ex post* allocation (also with a volume taxation or a market). The following section presents different possible external insurance systems.



		allocation	
		ex ante	*ex post*
taxation	linear in the volume	▨	▨
	market	▨	
	linear in the surface	▨	
	insurance	▨	▨

Figure 3: the different allocation rules studied (in gray tint: existing allocation)

2 Allocation rules without external insurance systems

2.1 With an *ex ante* water allocation

It is possible to define three different rules that are based on a water allocation of the *ex ante* type: a rule using a taxation based on volume, an *ex ante* market or an option market.

2.1.1 Tax based on volume allocation and *ex ante* water allocation

The *ex ante* rule allocates to each farmer a fixed portion λ_i of the global volume V; each farmer may pay a price p by unit of volume, known *ex ante*, but in most cases the farmer will pay a lump sum price depending on λ_i. With such an allocation rule, the global risk is transferred linearly to every user.

The *ex ante* allocation rule is very commonly used, and at all different scales of water management.

First, at the irrigation scheme level: for example, the irrigation schemes set up by the British in India during the XIXth century were designed in order to share equitably both water scarcity and the uncertainty on water availability. At the tertiary level, the *warabandi* system allocates hence the whole water flowing in the canal to each farmer for a given period, whatever the actual value of the flow.

At the upper level, the secondary canals (*distributaries*) are opened or closed according to a given rotation of 8 days during the drought period. Here also, the probability to receive a small flow is the same for all canals (Perry and Narayanamurthy, 1998).

In South-West of France, historical permits to divert water for water mills uses were calculated often as a share of the total flow (Faÿsse, 1998). In the irrigation scheme *huerta* in Valence, Spain, the river flow is shared in *filas* which are portions of the global flow (Fernandez, 2001). Another example is the Pasten system in Indonesia, where engineers calculate two times a month the global water needs of each tertiary block and afterward share the volume available proportionally to these needs, while taking also into account seepage losses (Howe, 1990).

Again, at a superior level, the Ganges treaty signed in 1996, stipulates that, if the flow of the Ganges at the frontier between India and Bengladesh passes under 1980 m^3/s, then the flow is proportionally shared between these two countries (Dinar, 1997). Moreover, 8 of the 21 contracts that govern water allocation between different States of the USA are based on a proportional rule (Bennett and Howe, 1998).

Finally, the *ex ante* allocation rule is also used, in times of water scarcity, when water rights are defined without any priority rights as in the system of riparian rights used in England and in the USA (Scott and Coustalin, 1995).

With such an *ex ante* allocation rule, farmers make their choices independently from one to another, **there is no risk sharing**. The choice made by each farmer is described by the one of a single farmer, as showed above.

2.1.2 *Ex ante* market

At the beginning of the campaign, farmers can exchange rights on the water that will be available. They choose afterwards the extent of the surface put under crops.

These water markets have existed in some traditional oasis in the South of Tunisia until the XXth century (Bédoucha, 1984). Farmers exchanged then a certain duration of irrigation from the canal, whatever the flow that would occur.

This kind of market has been set up recently on more large areas. A first example is the Chile during the 70s, when the administration distributed initial water rights and then organized the legal possibility for their exchange (Bauer, 1997). These rights are usually termed as a given flow, but if the River flow becomes too weak, the rights are transformed into portion of the total flow (Rosegrant and Binwanger, 1994). Another example is the Big-Thomson dam in the USA: users can exchange at the beginning of the campaign parts that are a share of 1/310000 of the total water stocked in the dam (a part corresponds to 864 m^3/year on average)(Howe et al., 1995).

Dudley (1992) proposed that the uncertain water volume stocked in a dam be divided into different virtual parts that could be owned by users downstream and that could be exchanged.

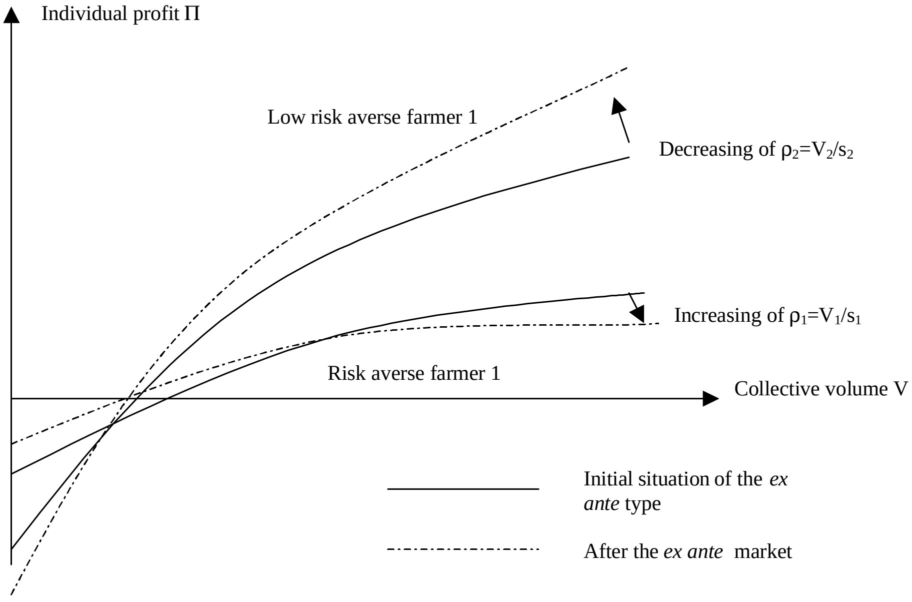


Figure 4: Evolution of farmers profits depending on the global available water volume with an *ex ante* water allocation possibly completed with an *ex ante* market.

Since there is hence no interaction between farmers, the probability function associated to each part is constant and easily known for both the seller and the buyer.

Proposition 2 *The ex ante market allows each farmer to refine his risk taking: with reference to an initial ex ante water distribution, more risk averse farmers will increase their average water amount irrigated per hectare while less risk averse farmers will diminish it (fig. 4). Nevertheless, there is neither optimal risk sharing nor optimal ex post water valorization.*

For example, during the allocation using auctions of water rights in the state of Victoria, Australia, some farmers bought additional rights in order to secure their resource during dry years (Simon and Anderson, 1990).
Nevertheless, the *ex ante* market does not always lead to a better valorization of resources, if farmers differ regarding their technical skills. Zimmerman and Carter (1999) showed this result on the example of a land market. They studied land property rights in the Sahel region, where the land productivity decreases generally with the farm size. They calculated the option value associated with the possibility of a land market between farmers differing in their initial land endowment. Zimmerman and Carter showed that this option value is high for poor farmers because it allows them to reduce their revenue variability. Hence, a market may lead to a decrease in land average productivity because **more risk averse small farmers, who valorize well their land, will sell their land to big farmers, less risk averse but with a lesser capacity to valorize land.**
With a more practical point of view, Bauer (1997) thought that one of the main reasons for the very low number of water rights transaction in Chile comes from the fact that, since river flows are very variable and the exchange must be made on fixed portions of the flow, it proves to be difficult to built systems that allow such a division and that also enable an evolution of this sharing.

Option markets

Option markets for water allocation are used mainly in the USA. They can be considered also as a simplification of complete contingent markets. An option consists in a right to buy a certain amount of water, either chosen by the buyer during the irrigation campaign or defined in the initial contract; it is paid at an exercise price when conditions initially defined are met. The buyer must also pay an option price *ex ante*, whether he really uses the right to buy water afterward or not. In the USA, a water Bank defines sometimes the River flow under which the option right can be used and also a price that is either the exercise price or the option price. If one of these two parameter is determined by the Bank, the other is established on a market.

This type of markets are for example used in California: cities on the Coast buy such option rights to farmers owning lands riparian to the Colorado River, in order to guarantee a water availability in case of drought (Michelsen and Young, 1993, Shupe et al., 1989).
For instance, in 1995, the Californian Water Bank bought dry year option water rights, for a total amount of 36 millions of m^3. The Bank acquired this option at a price of 3\$ per 1000m^3, in order to be able to buy them afterwards at an exercise price of 32\$ per 1000m^3 (Jercich, 1997).

2.2 An *ex post* water allocation coupled with a volume based taxation or a market

2.2.1 Volume based taxation

Here, a farmer i receives an allocation $\frac{s_i V}{S}$, which he pays at a price p per unit of volume. Therefore, his profit is $\pi(s_i) = s_i \left[(r(\frac{V}{S}) - k - \frac{pV}{S}\right]$. We suppose that, since there are many farmers, the total area under crops S can be regarded as little dependent of a farmer choice s$_i$.

Proposition 3 *The ex post allocation coupled with a volume based taxation respects the necessary conditions for optimal risk sharing at the first order. Moreover, this allocation rule leads to optimal ex post valorization. Nevertheless, it does not generally lead to an optimal risk taking for the Collectivity. Equity objectives can be met by using a high water price, with taxes redistributed according to the chosen equity criterion.*

In appendix, it is also shown that a farmer solves then a dilemma between Average value and Variability depending on his risk tolerance:

$$\boxed{s_i \simeq Min\left(s_m, Tolerance * \frac{\text{Average profit}}{\text{Profit Variance+Average profit}^2}\right)}$$

It gives the relation defining the aggregate area under crops S:

$$S \simeq \sum \min\left(s_m; T_i \frac{\left[\int_V r(\frac{V}{S}) - \frac{pV}{S}\right] - k}{\int_V (r(\frac{V}{S}) - k - \frac{pV}{S})^2}\right)$$

If no farmer puts under crop his total arable area, S is, to the first order, linear in the total collective tolerance $T = \sum T_i$. Moreover, if this tolerance becomes important, S tends towards the area that corresponds to a null average profit, i.e.: $\int_V r(\frac{V}{S}) - \frac{pV}{S} = k$: **when the**

aggregate tolerance becomes important, the Tragedy of the Commons leads to a null average profit in case of no limitation on areas put under crops.
Moreover, there exists also a certain level of aggregate tolerance T^* above which the global area put under crops will be greater than the area put under crop by a single risk neutral agent. Hence, this area put under crops will be also bigger than the one chosen by an agent having a risk tolerance of T^*: **there exists hence a certain level of aggregate risk tolerance above which there is a Tragedy of the Commons corresponding to a too big risk taking**.
The very generic form of the model does not allow here to compare *ex ante* and*ex post* allocations: therefore it is not possible to characterize more precisely this over risk taking. This is why the next subsection uses a more precise yield function in order to make the comparison.
Finally, if the farmer does not put his maximal area under crops:

$$E\pi_i \simeq T_i \frac{([\int_V r(\frac{V}{S}) - \frac{pV}{S}] - k)^2}{\int_V (r(\frac{V}{S}) - k - \frac{pV}{S})^2}$$

The average profits are hence proportional to risk tolerances. This is why such a distribution will often be considered as unfair. Two solutions may nevertheless exist.
First, it is possible to associate the *ex post* allocation with a high water price. The taxes are afterwards distributed according to the equity criteria, for example it can be evenly distributed among farmers.
Second, it may exist a rotation in risk taking. Hence in Tunisia, the rule used in many irrigation schemes, that considers water stress sensitive crops having priority to less sensible crops, can be sometimes regarded as fair when, due to cultural constraints, each farmer is complied to sow some year cash crops and other years cereals.
In the French Adour River Basin, some farmers get contracts with agronomic industry to crop vegetables and maize for seeds production. These cultures are profitable and sensitive to water stress: they were not subjected to irrigation limitation when the river flow became low during summer. However, since these contracts are given to the same farmers from one year to another, this exception rule was considered as unfair by many farmers: it has now been cancelled (Faÿsse, 2001).

2.2.2 *Ex post* market

The *ex post* water price q is here endogenous and determined on a market place depending on the actual global volume V and the total area put under crops: it can be denoted $q(V, S)$. *Ex post* markets were used in the oasis in South of Tunisia for centuries (Bédoucha, 1984), but also in the Valencia region (Ostrom, 1992).
More recently, such markets were expanded to broader areas, as for *ex ante* markets; the California Drought Emergency Water Bank was set up in 1991 and 1992 in order to coordinate the exchange between potential sellers and buyers. In 1998, 300 water purchases were made on the Rio Grande River (Texas): they were mainly constituted of water transfers from farm use to other uses (Yoskowitz, 1999). This kind of market can also be informal: Bandaragoda (1998) thought that water transactions exist on approximately 5% of *warabandi* schemes.
Let $W_{i0}(V)$ be the initial endowment of a farmer i when the collective volume is V and W_i his volume after the market. The individual profit is hence:

$$\pi(s_i, W_i) = s_i \left[r(\frac{W_i}{s_i}) - k \right] - q(V, S)(W_i - W_{i0}(V))$$

Proposition 4 *The ex post market verifies the necessary conditions for optimal risk sharing to the first order and valorizes ex post water to the best. As for ex post allocation paired with a volume based taxation, it does not lead necessarily to an optimal collective ex ante risk taking.*
When the collective risk tolerance is important and water price is low, ex post allocation paired with volume based taxation leads to a Tragedy of the Common of over risk taking. In such conditions, the total area put under crops with an ex post market is lower than the one with a volume based taxation: an ex post market permits then to diminish the over-cropping problem.

The *ex post* market will always bring to each farmer a profit at least equal to the one authorized with *ex ante* allocation since no farmer is complied to sell or buy water. Finally, *ex post* markets will be all the more of interest that farmers anticipate correctly the total area put under crops.

2.3 Use of a hyperbolic yield function

All preceding calculations using *ex post* allocation were made exactly because the yield function was always evaluated at $\frac{V}{S}$. However, to evaluate a rule using the *ex ante* allocation, it is necessary to keep in the integral terms like $r(\frac{\lambda_i V}{s_i})$, which prevents of getting exact solutions. This is why we propose a more precise yield function. Therefore, we set here that, for every $v \succeq 1$: $r(v) = 1 - \frac{1}{v}$, which gives:

$$r(\frac{V}{s}) = 1 - \frac{s}{V}$$

The profit is hence:

$$\pi(s) = s.r(\frac{V}{s}) - ks - pV = s(1 - \frac{s}{V}) - ks - pV = (1-k)s - \frac{s^2}{V} - pV$$

For a risk neutral agent, the choice of area under crops is made according to the following program:

$$\begin{aligned} \underset{s}{Max} \quad \int_V s.r(\frac{V}{s}) - ks - pV &= \underset{s}{Max} \quad \int_V s(1 - \frac{s}{V}) - ks - pV \\ \frac{d}{ds}E\pi &= 0 = (1-k) - 2(\int_V \frac{1}{V})s \end{aligned}$$

The chosen surface is then $S_N = \frac{1-k}{2\int_V \frac{1}{V}}$ and the profit $\Pi(S_N) = \frac{(1-k)^2}{4\int_V \frac{1}{V}}$.
The use of a hyperbolic yield function allows to obtain a relationship S(T) for different allocation rules (figures 5 and 6). Calculus is omitted[5].
In this framework, the *ex post* market leads to a collective area under crops close to the *ex post* one with volume based taxation for a low collective tolerance and tends towards S_N when T tends towards infinity.
Remark that, beyond a certain level of tolerance (T_B on figure 5), *ex post* allocation paired with volume based taxation leads to an area under crops greater than the one chosen by a risk neutral agent. Since the latter maximizes the average profit, for a collective tolerance greater

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[5] They can be sent upon request, or see Faysse (2001)

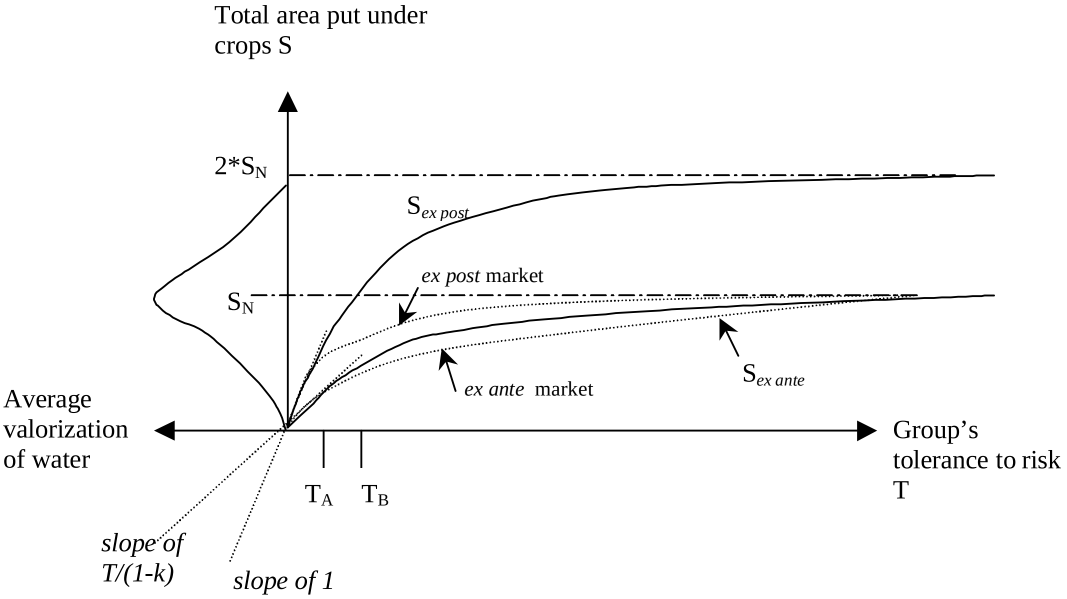

Figure 5: Evolution of aggregate area under crops depending on the collective risk tolerance with *ex ante* allocation or *ex post* allocation and a low water price

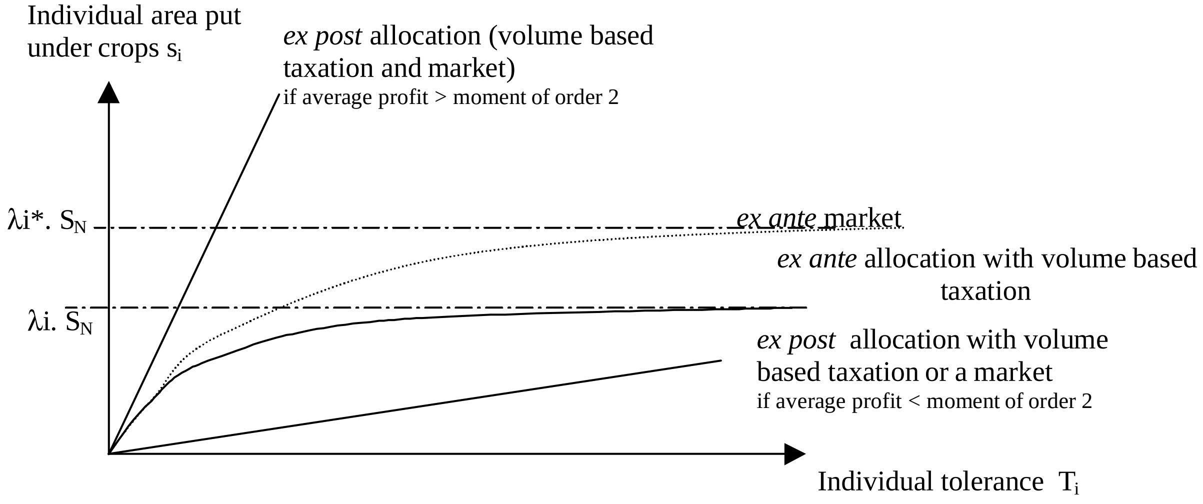

Figure 6: Evolution of individual area under crops depending on the collective risk tolerance with *ex ante* allocation or *ex post* allocation and a low water price

than T_B, the situation is the classical one of a Tragedy of the Commons where a decrease in the aggregate area put under crops would lead to an increase in the average profit.

Moreover, we can consider that *ex post* market is the rule closest to complete contingent markets: it is hence the mechanism that gives the best estimation of the optimal aggregate area put under crop. The over-risk taking associated with *ex post* allocation exists then as soon as the tolerance is equal to T_A (figure 5). Nevertheless, for risk tolerance comprised between T_A and T_B, the too important risk taking leads to an increase in the expected profit.

Proposition 5 *The Tragedy of the Commons associated to ex post allocation with volume based taxation can lead to an increase in the average valorization.*

This important result will be illustrated afterwards one the example of an irrigation scheme in Tunisia (see section 5). It has also been shown on the case of pastures in Ethiopia (McCarthy, 2001). McCarthy (2001, 2001) studied the cattle density chosen by pastors: she demonstrated that, in common pastures, the revenue attained during average rainfall year was superior to the one obtained on private pastures, but that during drought years, the revenue on common pastures fall sharply, much more than on private ones.

2.4 Priority rights between users

Another form of allocation of the *ex post* type is the definition of a priority right between users.

In California, the right used in the Eastern part of the USA was used at first, i.e. a riparian right from the British custom. Later, during the XIXth century, the appropriative right appeared, divided into *priority rights* ("first in time, first in right") and *proportional rights.* All these rights generate interactions between users.

In Chile as well, the administration distinguishes between permanent rights and *eventuales*, which are rights granted only if the resource is still sufficient after other demands have been satisfied.

Whatever their particular form, the rights based on a priority rule do not lead to an efficient *ex post* water valorization (Burness and Quirk, 1979, Howe et al., 1995). Moreover, in many cases, farmers try to obtain rights greater to their needs, either because they want to built an insurance or because they want to set a long term patrimony. In order to prevent these strategic behaviors, the Californian State requires that the rights are put to *beneficial use* (Burness and Quirk, 1979).

Moreover, Burness and Quirk (1979) thought that a **market for priority rights allows to attain a Pareto-optimum on investments and also is easier to set up than a market for proportional rights because new rights do not put formers rights again into question.** In California, on the Truckee River, a hydroelectric company hence bought water rights at prices that depended on their degree of priority, with a final difference up to 25% between juniors and seniors prices (Saliba and Bush, 1987).

Alaouze (1991), Brennan and Scoccimarro (1999) proposed to set up market with differentiated rights: each right would correspond to a share of the volume that would evolve with the total amount, i.e. $\lambda_i(V)$ with $\forall V$, $\sum_i \lambda_i(V) = 1$. The farmers would hence be able to build a **portfolio of water rights with volume and security levels corresponding to their needs.**

3 Insurances external to the irrigation system

Insurance mechanisms are based on a fund used to share risk between different regions or between different periods. They can be a priori associated with both *ex ante* and *ex post* water allocation. As a matter of fact, these insurance systems reduce the gap between the size of areas put under crops by low and very risk averse farmers and hence they reduce the gap between valorization obtained with *ex post* and *ex ante* water allocation: this may explain why **insurance systems are always paired with *ex ante* mechanisms**.
Two types of insurance are presented here:
- **yield insurance**, which guarantees the equivalent of a minimum yield per hectare;
- **revenue insurance**, which guarantees a given income.

We try here to see how these insurance systems can influence farmers cropping decisions.
It is important to note that, with reference to the previous situation, another very low risk averse agent, i.e. the insurer, is added to the system: the group tolerance towards risk is often much bigger.

3.1 Crop insurance

Crop insurance in case of drought is used in many countries. In the USA, it has been functioning since 1938, and was recently reformed in 1994 by the Federal Crop Insurance Reform Act. Farmers can now choose a guaranteed yield (between 50 % and 75 % of a reference yield) and the compensation in case of an actual yield under the guaranteed one (between 60% and 100% of the anticipated price) (C.E.S, 1998).

Proposition 6 *The area under crops chosen by the farmer increases with the level of guaranteed crop yield.*

The difference with income yield comes from the fact that, here, the guaranteed total income increases with the area put under crops.
The impact of such crop insurance programs in the USA was measured by an econometric analysis by Young et al., 1999 (cited by Young and Westcott, 2000). This study showed that, following the insurance program implementation, the areas put under crop indeed increased, but in a slight way.

3.2 Revenue insurance

The use of revenue insurance is rather recent: it has been set up in Canada and tested in some States of the USA since 1997 (C.E.S., 1998). In the two latter examples, it can be used in case of drought.

Proposition 7 *The area put under crops increases with the guaranteed income.*

Babcock and Hennessy (1996) found the same result. They used yield probabilities (here $sr(\frac{V}{s})$) conditional to the level of input (here s): the key factor in both approaches is that **an increase in the input leads to a reduce of income for low values of the uncertain variable.**
The main interest of income insurance is that it does not disturb much the market. This is why it has been put in the "green" box of public aids[6], contrary to crop yield insurance

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[6] Another necessary condition to put such a system in the green box is that the indemnification must take place only if the loss is at least 30% and, in that case, it cannot be bigger than 70% of total losses.

that are put in the "orange" one which provokes perturbations (C.E.S., 1998). Crop yield insurance is hence of less interest because it will be subject to restrictions of public aids negotiated in the World Trade Organization.
Besides, the previous results have been obtained with an *ex ante* allocation: the conclusions would have been the same with an *ex post* one.
Such results lead to a proposition for policy making: **it is necessary to add to any insurance scheme that diminishes losses during drought periods in a CPR context, an institutional approach to help communities to set up their own mechanisms in order to prevent an increase in the collective risk taking** (see also McCarthy, 2001).

4 Evaluation of the rules according to the given criteria

The results drawn from the previous study can be presented as follows.

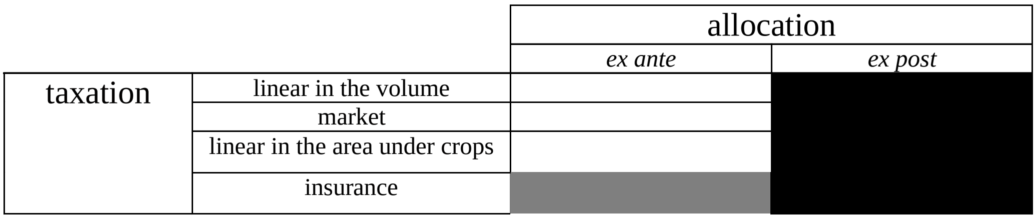
Criterion 2) A global area under crop corresponding to collective risk tolerance

For all the studied allocation rules, it is possible to choose parameters (water price, insurance premium, etc.) in order to get a collective area under crops corresponding to the collective risk tolerance.

Criteria 1) and 3) The set up of risk sharing mechanisms and the maximal *ex post* valorization for a given area under crops and a given water volume available

Only *ex post* allocation rules lead to a maximal water valorization. However, insurance mechanisms, by limiting risks, reduce the heterogeneity between farmers and by this reduce the differences between area choices and water valorization.

The criterion for risk sharing leads to the same ranking: *ex post* allocation rules verify first order conditions for optimal risk sharing whereas insurance mechanisms paired with an *ex ante* allocation share risks, but not in an optimal manner (figure 7).



		allocation	
		ex ante	*ex post*
taxation	linear in the volume		■
	market		■
	linear in the area under crops		■
	insurance	▨	■

Figure 7: criteria for optimal valorization and risk sharing (black = optimal, gray tint = positive effect)

4) The reduce of inter-annual income variability

The mechanisms reducing the inter-annual income variability are those which create a global surplus when the available volume is big and use this surplus during dry years. Previously studied forms of market cannot play this role. Volume based taxation and insurance mechanisms can reduce this variability (fig. 8).

		allocation	
		ex ante	*ex post*
taxation	linear in the volume	■	■
	market		
	linear in the area under crops		
	insurance	■	■

Figure 8: criteria for reducing the inter-annual income variability

5) **Equity regarding water allocation**

With an *ex post* allocation rule paired with volume based taxation and a small water price, less risk averse farmers have put large areas under crops and hence have caught rents compared to more risk averse farmers. These rents can be considered as unequitable with reference to the initial *ex ante* situation. There are two ways to define an equitable *ex post* allocation associated with volume based taxation.

- Farmers can organize a rotation in risk taking. This happens sometimes in Tunisian irrigation schemes because of agronomic constraints of rotation between different crops. This lack of rotation caused the end of crop exceptions in the Adour River Basin.

- Farmers can receive a share of the collected tax, which corresponds to their initial right, their profit being then $\pi(s) = s\left[(r(\frac{V}{S}) - k - \frac{pV}{S}\right] + p\lambda_{i0}V$, or by using an *ex post* market.

5 Collective risk-taking on Tunisian irrigation schemes

A comprehensive study has been made on 2 small irrigation schemes settled in Central Tunisia, near Kairouan. These schemes are run by Water Users Associations (WUAs) in a semi-arid region: irrigation is necessary for all crops. It is possible to cultivate non irrigated crops during winter, but the yields are then risky.

Because of the high variability of rainfalls and of the recurrent problems on the pumps, risk management of the collective water resource is of major importance for these irrigation schemes.

The aim is here to understand the global risk taken at the scheme level, by comparing the actual cropping choices and allocation rules to other cropping possibilities and potential rules. Given an allocation rule, does a collective cropping choice correspond to a high or a low risk taking ?

In our opinion, it would have been very difficult to estimate the individual risk tolerances and, with them, the collective risk tolerance because the farms diversity appeared to be very high and there are complex informal risk sharing networks. That is why we did not try to determine the optimal risk taking

The study remains at the scale of the scheme: we consider, for example in El Melalsa scheme, three global fields put under crops, one with wheat, another with pepper-bean and the last one with melon. A model calculates the different yields taking into account the water stresses impact, and hence the global profit, for a given rainfall and a given allocation rule (see appendix).

5.0.1 A big risk taking on El Melalsa irrigation scheme

The El Melalsa irrigation scheme is one of the two studied schemes. It irrigates 160 ha owned by 54 farmers, using a pump that delivers a flow of 24 l/s. The cropping pattern is based on the following rotation: wheat then pepper associated with bean and finally melon.
There is no control over the area put under crops. Besides, there is a water turn but when a farmer gets his turn, he can irrigate as long as he wants: the water turn length lasts up to three weeks during spring, when both beans and melon have to be irrigated. These two facts lead also to an over-cropping situation that can be viewed as a Nash equilibrium (Faÿsse, 2001).
Two allocation rules were simulated on this scheme (a brief description of the model is given in appendix): first, the actual one, with a free individual irrigation length, and then an *ex ante* allocation rule where the water turn length is fixed and each crop receives during its water turn in proportion to its needs.
By definition, a "safe" cropping choice is one for which crops are sufficiently irrigated during median rainfall year and with an irrigation set up when the soil reservoir becomes lower than 0.85 than the Usable Water Reserve. The calculated cropping choice is made of 40 ha of wheat, 20 of pepper-bean and 15 of melon. For a risky cropping choice, we used the actual one on year 98-99, i.e. 60 ha of wheat, 21 of pepper-bean and 27 of melon.
The choice between two cropping repartitions and two allocation rules leads to four scenarios. On figure 9, the left table presents the scenario description while the right table gives the total valorization made on the whole scheme for different rainfalls: quinquennial drought year, quinquennial wet year, the median year and the year 98-99 (which stands between a median year and a quinquennial dry one). Moreover, the study made on the field for the 98-99 campaign showed that the total profit made had not exceeded 50 000 DT[7].

scenario	cropping pattern	wheat	pepper-bean	melon	water turn
0 (réel)	risky	61	21	27	unlimited irrigation period
1	risky	61	21	27	unlimited irrigation period
2					daily repartition
3	safe	40	20	15	unlimited irrigation period
4					daily repartition

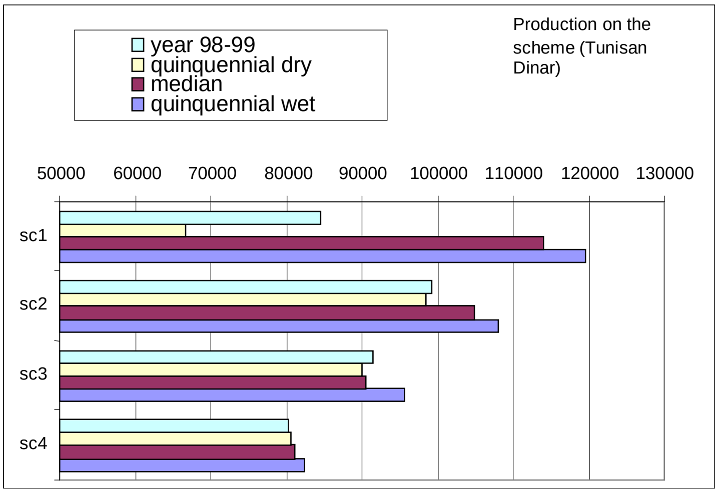


Figure 9: Total profit made on the El Melalsa irrigation scheme given a global cropping pattern, an allocation rule and a rainfall

With this figure, it is possible to draw four conclusions.
○ **A "risky" cropping pattern gives always a profit with an average and a variance more important than a "safe" one**, which is of course expected.
○ The allocation rule with free individual length brings a profit at least equal to the one with a water turn length under control for median and humid rainfalls. Indeed, the former rule

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[7] in 2000, 1 DT ≃ 0.8 USD

enables a better use of the soil reservoir than the *ex ante* rule: the *ex ante* rule was simulated with a threshold for the beginning of irrigation of 0.85, which does not take well profit of the rainfalls. On the other hand, **during dry years and with a risky cropping pattern, the allocation rule with free individual irrigation length leads to catastrophic results for two reasons: first, because the water turn length becomes very long and second because this rule does not organize any priority rules between the different crops.**

○ With the current allocation rule, the actual cropping pattern made during the year 98-99 corresponds to a high risk taking. The comparison between scenarios 1 and 3 shows that a reduce in the area under crops would have limited significantly the losses during dry years.

○ Finally, given an *ex ante* allocation rule, scenarios 2 and 4 show that, nevertheless, the risky cropping pattern is always more profitable than the "safe" one.

Hence, the actual cropping pattern was not a priori too big since it values water more during median year than a less risky pattern. Nevertheless, given the actual allocation rule, this pattern leads to an important collective risk taking. **As showed by McCarthy (2000), the Tragedy of the Commons can sometimes lead to an increase in the average profit but, on the other hand, it is associated with a too big risk taking.**

Some farmers of El Melalsa own a well outside the scheme. One could suppose that, since these farmers have a profit guaranteed outside the scheme, they are less risk averse and that they are the ones who had imposed such a big cropping pattern inside the scheme. As a matter of fact, these farmers are constrained by their work force and they do not put large areas under crops inside the scheme. The risk taking is mainly due to farmers close to the pumping site, who are not subjected to seepage losses.

5.0.2 A good risk taking on the Souaidia irrigation scheme

Souaidia is another small-scale irrigation scheme that irrigates 120 ha; the cropping pattern is based on an alternation between pepper and wheat.

The areas put under crops are under control: a farmer cannot sow more than one third of his arable area for summer crops (exception made if his surface amounts to less than one hectare). Hence, the water turn length remains no longer than 10 days.

Three global cropping patterns were simulated on this scheme. First, the actual cropping pattern chosen during year 98-99, made of 48 ha of wheat and 57 of pepper. Also, a safer cropping pattern (48 ha of wheat, 50 ha of pepper) and a more risky one (60 ha of wheat, 60 ha of pepper) were simulated. Besides, the rainfall during the year 98-99 can be considered as median on the whole.

Given a cropping pattern of 48 ha of wheat and 57 ha of pepper and the rainfall of the year 98-99, a simulation gave a ratio between real yield and maximal one of 95 % for wheat and of 51 % for pepper. Figure 10 shows that **the actual cropping pattern corresponds to a good collective risk taking**.

Moreover, there is an explicit management of fraud attempts. Sometimes, a farmer wants to put under crops more than one third of his area. At the beginning of the campaign, the water distributor gives around 450 m^3/ha to each farmer during the first irrigations. Since the second and third irrigations often necessitate less than this volume, the farmer can use then this water in surplus to initiate another sowing irrigation (some farmers try even to sow during night). If his cheating is rapidly discovered, the Association refuses his extension, whereas if not, it is accepted. This explicit and progressive way to manage frauds corresponds to one of the principles for long-enduring systems proposed by Ostrom (1992).

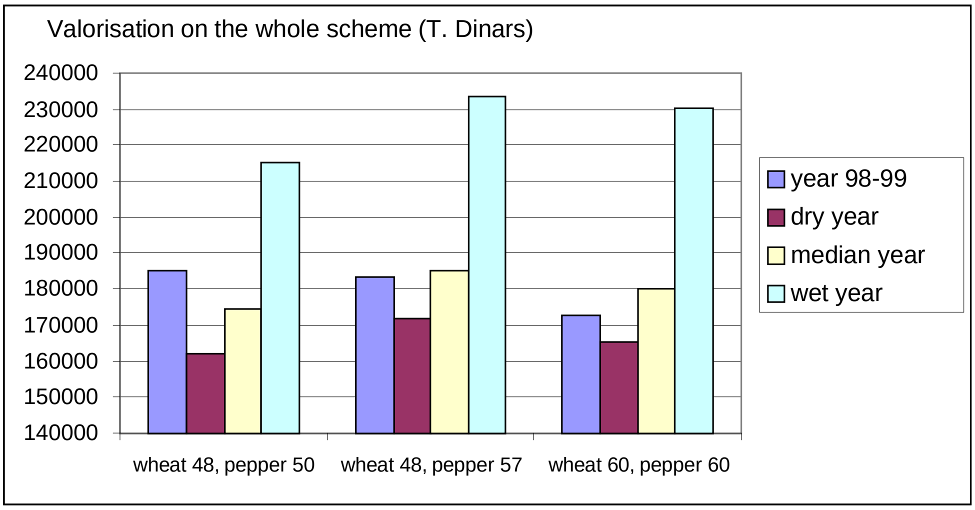

Figure 10: total valorization on the Souaidia irrigation scheme, depending of the global collective cropping pattern and the rainfall

6 Conclusion

The study compared different rules to allocate water when the collective volume to be shared is uncertain at the cropping decision moment, and in a context where farmers are autonomous regarding their cropping decision. Five criteria were proposed to establish a comparison between existing and potential water allocation rules.
It appears that no rule can attain these five goals all alone by itself: depending on the objectives chosen by the Collectivity, different allocation rules may be of interest.
Every allocation rule using an *ex post* water allocation allows to attain the maximal water valorization for a given area under crops, and to realize an optimal risk sharing. The *ex post* allocation rule incites farmers into reveal their types, i,e, here their tolerance to risk. **The *ex post* rule functions hence as an indirect revelation mechanism**. However, such an allocation, if non created by an *ex post* market, may lead to a Tragedy of the Commons of over-cropping, and hence to a too big collective risk taking, even if it may also lead to a better average global income. **When there exists a rotation among farmers for risk taking or when the allocation is associated with a high tax (redistributed afterwards), the allocation may both share risk efficiently and meet equity objectives**.

Exception made for the goal of reducing the inter-annual income variability, markets are the most efficient mechanisms. An *ex post* market can be used when farmers are not well aware of the functioning of the total irrigation system or when productions highly vary from one year to another. Markets based on priority rights are of interest when there are different kinds of use, for example farmers and municipalities. Finally, an *ex post* market is rather close to the optimal complete contingent market on both collective volume available and total area under crop if all farmers anticipate correctly the total area which will be put under crop. The implementation of water markets is currently eased by the reduction in transaction costs: these markets can now be set up on much larger regions.

The main interest for using insurance mechanism is to reduce the inter-annual variability. These mechanisms lead however systematically to an increase in the areas put under crops.

As for their implementation, there are moral hazard problems on the way culture is undergone during the campaign and on the declared losses; these problems can partly be settled by an area yield crop insurance (Mahul, 1999). Moreover, since the risk may be correlated on large areas, big insurance companies are required, or the use of financial markets. The other interest of such mechanisms is to add a very low risk averse agent to the system.

One possibility may be to trying to use jointly different rules. For instance, farm insurances in Japan can be described as hybrid systems. Approximately 2500 cooperatives exist in this country: they share risk first inside themselves but also at the level of Prefectures, these latter being finally reassured by a public institution (C.E.S., 1998).

In order to complete the comparison between internal and external risk sharing systems, it is necessary to take into account the costs of setting up a control of areas put under crops (c.f. the approach used by Arnott and Stiglitz, 1991).

Other mechanisms for risk sharing
In many irrigation systems, risk sharing by water allocation is in fact a complement to other existing mechanisms.
First, a credit may be granted by the WUA treasurer if a farmer does not succeed in financing water costs during the irrigation campaign. Sometimes, in Tunisia, this credit possibility is also used to prevent the rich farmers to get the whole water turn by asking and paying for water much before those who encounter problems to finance their irrigation campaign (Faÿsse, 2001).
Of course, the people in charge of the WUA must be cautious to recover the money lent after the campaign. Many WUAs in Central Tunisia were forced to be disbanded because of a lack of reimbursement. For the president of Souaidia irrigation scheme, the rule of a maximal one third of the arable area which can be put under crops serves also to guarantee that the farmer does not take a too big risk and will be able to finance his whole irrigation campaign.
The risk sharing can also be organized through different location contracts. In the Souaidia irrigation scheme, if the land owner participates in the investments for the campaign, he will receive half of the total yield whereas if he does not, he will only receive one third.

References

[1] ALAOUZE, C. (1991). Transferable water entitlements which satisfy heterogeneous risk preferences. *Australian Journal of Agricultural Economics 35 (2).*

[2] ARNOTT, R. AND STIGLITZ, J. (1991). Moral hazard and non market institutions: dysfunctional crowding out or peer monitoring? *American Economic Review* 81 (1), 179-190.

[3] BABCOCK, B. AND HENNESSY, D. (1996) Input demand under Yield and Revenue Insurance. *American Journal of Agricultural Economics*, 78 (may).

[4] BANDARAGODA, D. (1998). Design and practice of water allocation rules: lessons from warabandi in Pakistan's Punjab. Tech. rep., IWMI Research Report n 17.

[5] BAUER, C. (1997) Bringing water markets down to Earth: the political economy of water rights in Chile, 1976-1995 *World Development*, 25 (5), 639-656.

[6] BEDOUCHA, G. (1984) *L'eau, l'ami du puissant.* Ed. des archives contemporaines, CNRS, Paris.

[7] BENNETT, L., AND HOWE, C. (1998). The interstate river compact: Incentives for noncompliance. *Water Resources Research 34 (3)*, 485–495.

[8] BRENNAN, D., AND SCOCCIMARRO, M. (1999). Issues in defining rights to improve Australian water markets. *Australian Journal of Agricultural and Resources Economics 43 (1).*

[9] BREWER, D., SAKTHIVADIVEL, R., AND RAJU, K. V. (1997). Water distribution rules and water distribution performance: a case study in the Tambraparani irrigation system. Research report no. 12, International Water Management Institute.

[10] BURNESS, S. AND QUIRK, J.P. (1979) Appropriative Water Rights and the Efficient Allocation of Resources. *The American Economic Review*, 69 (1).

[11] CONSEIL ECONOMIQUE ET SOCIAL (C.E.S.) (1998). Avis sur l'agriculture face aux risques climatiques.

[12] DINAR, A. (1997). Droughts, crisis management and water rights: An international perspective. In *4th Workshop on Joint Management of Shared Aquifers* (Istanbul, Turkey).

[13] DUDLEY, N. (1992). Water allocation by markets, common property and capacity sharing: Companions or competitors ? *Natural Resource Journal 32, fall.*

[14] FAYSSE, N. (1998). L'eau entre Etat et usager dans le bassin de l'Adour : de la gestion réglementaire à la gestion concertée ? Montpellier II.

[15] FAYSSE, N. (2001). L'influence des règles collectives d'allocation de l'eau sur les choix stratégiques des agriculteurs. Des petits périmètres irrigués tunisiens aux prélèvements en rivière dans le bassin de l'Adour. PhD thesis. Downloadable at http://www.montpellier.cemagref.fr/doc/publications/theses/nicolas-faysse.html

[16] FAYSSE, N. (2002). Allocating irrigation water: the impact of strategic interactions on the efficiency of the rules Name of the review?

[17] FERNANDEZ, S. (2001). Analyse économique et institutionnelle d'un système de gestion en propriété commune de la ressource en eau de surface destinée à l'irrigation : le cas de la huerta valencienne au sud de l'Espagne. Tech. rep., ENGREF.

[18] GOLLIER, C. (2001). *The economics of risk and time.* MIT Press.

[19] GOLLIER, C., AND EECKHOUDT, L. (1993). *Les risques financiers : évaluation, gestion et partage.* Mc Graw Hills, Paris.

[20] HOWE, C. W. (1990). Equity versus efficiency in Indonesian irrigation: An economic evaluation of the PASTEN method. In *Social, Economic and Institutional Issues in Third World Irrigation Management*, Westview Press, Oxford.

[21] HOWE, C. W., SCHURMEIER, D.R. AND SHAW, W.D. (1995) Innovative approaches for water allocation: The potential for Water Markets. *Water Resources Research*, 22 (4), 439-445.

[22] JERCICH, S. (1997). California's 1995 water bank program: Purchasing water supply option. *Journal of Water Resources Planning and Management jan-feb 1997*.

[23] JURRIENS, R., AND MOLLINGA, P. (1996). Scarcity by design: protective irrigation in India and Pakistan. *ICID Journal 45 (2)*, 31–53.

[24] KIMBALL, M. (1988). Farmers' cooperatives as behavior toward risk. *American Economic Review, 78 (1)*.

[25] LIGON, E. (1998). Risk sharing and information in village economies. *Review of Economic Studies, 65*.

[26] LIVINGSTONE, I., AND HAZLEWOOD, A. (1979). The analysis of risk in irrigation projects in developing countries. *Oxford Bulletin of Economics and Statistics 41 (1), feb*.

[27] LUND, S., AND FAFSCHAMPS, M. (1997) Risk sharing networks in rural Philippines. Mc Kinsey Consulting Group and Stanford University, Working paper.

[28] MAHUL, O. (1999) Optimum Area Yield Crop Insurance *Amercian Journal of Agricultural Economics*, 81.

[29] McCARTHY, N. (2000). Production risk and common pool resources: Impact on externalities, profits and welfare. Tech. rep., International Food Policy Research Institute.

[30] McCARTHY, N. (2001). Rainfall variability, traditional and commercial rangelands management and the drought cycle: some theoretical considerations and empirical evidence from Ethiopia. In *4th Conference on Environmental Economics of Toulouse, France*.

[31] MICHELSEN, A., AND YOUNG, R. (1993). Optioning agricultural water rights for urban water supplies during drought. *American Journal of Agricultural Economics 75, nov*.

[32] NUGENT, J., AND SANCHEZ, N. (1998). Common property rights as an endogeneous response to risk. *American Journal of Agricultural Economics 80 (aug)*.

[33] OSTROM, E. (1992). *Crafting institutions for self-governing irrigation systems*. Institute for Contemporaries Studies Press.

[34] PERRY, C., AND NARAYANAMURTHY, S. (1998). Farmer response to rationed and uncertain irrigation supplies. Tech. Rep. Research Report 24, IWMI.

[35] POPE, R., AND JUST, R. (1991). On testing the structure of risk preferences in agricultural supply analysis. *American Journal of Agricultural Economics, aug*.

[36] PROVENCHER, B AND BURT, O. (1993). The Externalities Associated With a Common Property Exploitation of Groundwater. *Journal of Environmental Economics and Management, 24*, 139-158.

[37] ROSEGRANT, M., AND BINSWANGER, H. Markets in tradable water rights: Potential for efficiency gains in developing country water resource allocation. *World Development, 22 (11)*, 1613–1626.

[38] SALIBA, B., AND BUSH, D. (1987). *Water markets in theory and practice*. Westview Press, Boulder, Colorado.

[39] SANDLER, T. AND STERBENZ, F.A. (1990). Harvest Uncertainty and the Tragedy of the Commons. *Journal of Environmental Economics and Management* 18, 155-167.

[40] SCOTT, A. AND COUSTALLIN, G. (1995). The Evolution of Water Rights. *Natural Resource Journal*, 35 (fall).

[41] SHUPE, S., WEATHERFORD, G., AND CHECCHIO, E. (1989). Western water rights: The era of reallocation. *Natural Resources Journal, 29, spring.*

[42] SIMON, B., AND ANDERSON, D. (1990). Water auction as an allocation mechanism in Victoria, Australia. *Water Resources Bulletin 26(3)*, 387–395.

[43] TOWNSEND, R. (1994). Risk and insurance in village India. *Econometrica 62, n 3.*

[44] WILSON, P., AND THOMSON, G. (1993). Common property and uncertainty: compensating coalitions by Mexico's pastoral ejidarios. *Economic Development and Cultural Change 41 (2).*

[45] YOSKOWITZ, D. (1999) Spot markets for water along the Texas Rio Grande: Opportunities for water management. *Natural Resources Journal, 39, spring.*

[46] YOUNG, E., AND WESTCOTT, P. (2000). How decoupled is U.S. agricultural support for major crops ? *American Journal of Agricultural Economics 82 (3)*, 762–767.

[47] ZIMMERMAN, F., AND CARTER, M. (1999). A dynamic option value for institutional change: Marketable property rights in the Sahel. *American Journal of Agricultural Economics 81, may*, 467–478.

7 Appendixes

Demonstration of proposal 1: choice for a single farmer

The farmer chooses his area s under irrigated crops in order to maximize the following function W:

$$W(s) = EU\left[s\left(r(\frac{V}{s}) - k\right) - pV\right]$$

Hence $W'(s) = E\left[U'(\pi)(r(\frac{V}{s}) - k - \frac{V}{s}r'(\frac{V}{s})\right]$.
$W''(s) = E\left[U'(\pi)(\frac{V}{s^2})r''(\frac{V}{s}) + U''(\pi)(r(\frac{V}{s}) - k - r'(\frac{V}{s})^2\right] < 0$ because both functions r and U are concave. There is therefore one and only one area that maximizes the expected utility.
The following demonstration about risk aversion is based on the fact that the second cross derivative $\frac{\partial^2\pi}{\partial s \partial V}$ is positive.
Let U and V be two agents so that V is more risk averse than U. Hence, the von Neumann Morgenstern utility functions of these two agents are such as: $V = g(U)$ where g is a concave function. The area s_u^* put under crops by agent U verifies:

$$E\left[U'(\pi)(r(\frac{V}{s_u^*}) - k - \frac{V}{s_u^*}r'(\frac{V}{s_u^*})\right] = 0 \tag{4}$$

The evaluation of the first order condition for farmer V with area s_u^* yields:

$$A = E\left[g'(U(\pi))U'(\pi)(r(\frac{V}{s_u^*}) - k - \frac{V}{s_u^*}r'(\frac{V}{s_u^*}))\right] \tag{5}$$

Moreover, $\frac{\partial\pi}{\partial V} = r'(\frac{V}{s}) > 0$ therefore π increases in V so $U(\pi)$ increases in V hence $g'(U(\pi))$ decreases in V. On the other hand, $U'(\pi)$ is positive since

$$\frac{\partial}{\partial V}(r(\frac{V}{s}) - k - \frac{V}{s}r'(\frac{V}{s})) = -\frac{V}{s^2}r''(\frac{V}{s}) > 0$$

The negative terms in the null integral of (4) are on the "left" part of the integral, i.e. for the low values of volume V. Finally, since the difference between equations (4) and (5) consists in the fact that, with $g'(U(\pi))$, more "weight" is given to low volumes, we obtain that $A < 0$. Consequently, since W is concave, $s_v^* > s_u^*$.

Demonstration of proposal 2: *ex ante* market

The farmer receives here a profit: $\pi(s_i) = s_i\left[(r(\frac{\lambda_i V}{s_i}) - k\right] - p\lambda_i$
Let be $\varrho_i = \frac{\lambda_i}{s_i}$. To determine the optimal choice, it is equivalent to consider that farmer i maximizes on both ϱ_i and s_i the following function:

$$EU_i\left[s_i\left(r(\varrho_i V) - k_p\varrho_i\right)\right]$$

The first order in ϱ_i yields:

$$E\left[s_i\left(Vr'(\varrho_i V) - p\right)U_i'(\pi_i)\right] = 0 \tag{6}$$

And the first order in s_i gives:

$$E\left[\left(r(\varrho_i V) - k - p\varrho_i\right)U_i'(\pi_i)\right] = 0 \tag{7}$$

If we multiply equation (6) by ϱ_i, then divide it by s_i and finally we subtract the resulting equation to (7), we obtain:

$$E\left[\left(r(\varrho_i V) - k - V\varrho_i Vr'(\varrho_i V)\right)U_i'(\pi_i)\right] = 0 \tag{8}$$

This equation allows to consider that we are back to the situation of the former demonstration, replacing ϱ_i by $\frac{1}{s_i}$, which proves that the more a farmer i is risk averse, the more the factor ϱ_i is important.
Moreover:

$$d\pi = \lambda_i r'(\varrho_i V)dV$$
$$dz = \sum_j = \lambda_j r'(\varrho_j V)dV$$

Hence $\frac{d\pi_i}{dz}$ is not linear: there is no optimal risk sharing.
Finally, we do not attain optimal *ex post* valorization because farmers have chosen different volumes per hectare $\varrho_i V$ depending on their risk aversion (figure 3). The *ex ante* market does not eliminate the need for an *ex post* market.

Demonstration of proposal 3: *ex post* allocation

Since by hypothesis each farmer has a constant absolute risk aversion:

$$u'(w_0+\pi) = u'(w_0) - I_a \pi u'(w_0) + \pi^2 \epsilon(\pi)$$

with $\epsilon(\pi)$ tending towards 0 with π. The first order condition is then:

$$\int_V \frac{\partial \pi}{\partial s}(1 - I_a\pi + \frac{\pi^2\epsilon(\pi)}{u'(w_0)}) = 0$$

Besides $\frac{\partial \pi}{\partial s} = \int_V \left[r(\frac{V}{S}) - k - \frac{pV}{S}\right]$ does not depend on s_i and if, at the first order, we neglect the term, $\frac{\epsilon(\pi)}{u'(w_0)}$:

$$\int_V \left[r(\frac{V}{S}) - k - \frac{pV}{S}\right]\left[I_a s_i(r(\frac{V}{S}) - k - \frac{pV}{S}) - 1\right] \simeq 0$$

Hence

$$s_i \simeq \min\left(s_m; T_i \frac{\left[\int_V r(\frac{V}{S}) - \frac{pV}{S}\right] - k}{\int_V (r(\frac{V}{S}) - k - \frac{pV}{S})^2}\right)$$

The numerator $\left[\int_V r(\frac{V}{S}) - k - \frac{pV}{S}\right]$ is the average profit per hectare (constant for all with this kind of allocation) and $\int_V (r(\frac{V}{S}) - k - \frac{pV}{S})^2$ is the second order moment of the profit. Moreover, with $Esp(X^2) = Var(X) + (Esp(X))^2$, there is indeed dilemma between Expected revenue and its variance:

$$\boxed{s_i \simeq Min\left(s_m, Tolerance * \frac{\text{Average profit}}{\text{Profit variance+Average profit}^2}\right)}$$

Finally, let be $A = \frac{\left[\int_V r(\frac{V}{S}) - \frac{pV}{S}\right] - k}{\int_V (r(\frac{V}{S}) - k - \frac{pV}{S})^2}$.
The *ex post* allocation leads to an optimal risk sharing since, for a given aggregate area under crops, $\frac{d\pi}{dZ} = \frac{T_i dA}{T dA} = \frac{T_i}{T}$ which corresponds to the necessary Arrow-Borsch conditions.

Demonstration of proposal 4: Tragedy of the Commons

The first order condition on W_i is: $q(V,S) = r'(\frac{W_i}{s_i})$ hence
$\frac{W_i}{s_i} = \frac{V}{S}$ and $q(V,S) = r'(\frac{V}{S})$.
The profit can therefore be rewritten as:

$$\pi(s_i) = s\left[r(\frac{V}{S}) - k\right] - r'(\frac{V}{S})(\frac{V}{S}s_i - W_{iO}(V)) = s_i\left[r(\frac{V}{S}) - k - r'(\frac{V}{S})\frac{V}{S}\right] + r'(\frac{V}{S})W_{iO}(V)$$

We are then back to a case similar to the one of the *ex post* allocation with volume based taxation:

$$s_i = \min\left(s_m; T_i \frac{\int_V \left[r(\frac{V}{S}) - k - r'(\frac{V}{S})\frac{V}{S}\right](1 - \frac{V}{S}W_{i0})}{\int_V (r(\frac{V}{S}) - k - r'(\frac{V}{S})\frac{V}{S})^2}\right)$$

If the initial endowment is the same for all farmers, then the distribution of revenues is proportional to farmers tolerances T_i. If there is no limit on surfaces put under crops, when

the aggregate tolerance T becomes big, the total area under crops tends towards the one of a risk neutral farmer.
As for *ex post* allocation paired with volume base taxation, the *ex post* market verifies the Arrow-Borsch conditions for optimal risk sharing.

The last step consists in showing that, when the aggregate tolerance is big enough, *ex post* market allows to reduce the total area put under crops with reference to *ex post* allocation with volume based taxation. It can be sent upon request (or, see Faysse, 2001).

Demonstration of proposal 6: crop insurance

With a crop insurance system, the farmer receives a volume $V_i = \lambda_i V$ with the assurance of a minimal revenue per hectare equivalent to an irrigation of a volume v_0. The farmer expected profit is:

$$EU(\pi) = \int_0^{\frac{s_i v_0}{\lambda_i}} u\left[s_i(r(v_0) - k)\right] h(V)dV + \int_{\frac{s_i v_0}{\lambda_i}}^{\infty} u\left[s_i(r(\frac{V}{s_i}) - k)\right] h(V)dV$$

Using the continuity at the frontier:

$$\frac{\partial EU}{\partial s_i} = \int_0^{\frac{s_i v_0}{\lambda_i}} \left[(r(v_0) - k)\right] u'(\pi)h(V)dV + \int_{\frac{s_i v_0}{\lambda_i}}^{\infty} \left[(g(\frac{\lambda_i V}{s_i}) - k)\right] u'(\pi)h(V)dV = 0$$

We consider that $\frac{\partial^2 EU}{\partial s_i^2} = \Delta \leq 0$ hence, by differentiating:

$$\frac{ds_i}{dv_o} = -\frac{1}{\Delta}\left[r'(v_0)\int_0^{\frac{s_i v_0}{\lambda_i}} u'(\pi)h(V)dV + \frac{s_i}{\lambda_i}\left(r(v_0) - k\right)u'(r(v_0 - k)) - \frac{s_i}{\lambda_i}\left(\left(r(v_0) - k - v_0 r'(v_0)\right)u'(r(v_0 - k))\right)\right]$$

$$\frac{ds_i}{dv_o} = -\frac{1}{\Delta}\left[r'(v_0)\int_0^{\frac{s_i v_0}{\lambda_i}} u'(\pi)h(V)dV + \frac{s_i}{\lambda_i} v_0 r'(v_0) u'(r(v_0 - k))\right] > 0$$

Demonstration of proposal 7: revenue insurance

The guaranteed revenue is here $\pi \geq \pi_0$. The insurance is being used for $s_i(r(\frac{\lambda_i V}{s_i}) - k) \geq \pi_0$ i.e. $V_{0i} = \frac{s_i}{\lambda_i} r^{-1}(\frac{\pi_0}{s_i} + k)$.
The farmer's expected utility is hence:

$$\begin{aligned} EU(\pi) &= \int_0^{Vi_0} u\left[\pi_0\right] h(V)dV + \int_{V_{0i}}^{\infty} u\left[s_i(r(\frac{\lambda_i V}{s_i}) - k)\right] h(V)dV \\ &= u\left[\pi_0\right] u(V_{i0}) + \int_{V_{0i}}^{\infty} u\left[s_i(r(\frac{\lambda_i V}{s_i}) - k)\right] h(V)dV \end{aligned}$$

Furthermore $\frac{\partial EU}{\partial s_i} = \int_{\frac{s_i}{\lambda_i} r^{-1}(\frac{\pi_0}{s_i} + k)}^{\infty} \left(r(\frac{\lambda_i V}{s_i}) - k - \frac{\lambda_i V}{s_i} r'(\frac{\lambda_i V}{s_i})\right) u'(\pi) f(V)dV = 0$. We suppose also here that $\frac{\partial^2 EU}{\partial s_i^2} = \Delta \leq 0$ hence, by differentiating, with $v_{i0} = \frac{\lambda_i V_{i0}}{s_i} = r^{-1}(\frac{\pi_0}{s_i} + k)$, we get

$$\frac{ds_i}{d\pi_o} = \frac{1}{\Delta}\frac{1}{\lambda_i}(r^{-1})'(\frac{\pi_0}{s_i} + k)\left[r(v_{i0}) - k - v_{i0} r'(v_{i0})\right] u'(\pi_0) h(V_{i0})$$

From the first order equation and since $\left(r(\frac{\lambda_i V}{s_i}) - k - \frac{\lambda_i V}{s_i} r'(\frac{\lambda_i V}{s_i})\right)$ is increasing, this function evaluated at $\frac{s_i}{\lambda_i} r^{-1}(\frac{\pi_0}{s_i} + k)$ is negative. Hence, $\frac{ds_i}{d\pi_o} > 0$.

A model for simulated allocation rules

A model has been built to evaluate a soil water balance and to predict the yield reduce caused by water stresses, on a daily basis. It allocates water between different fields according to a given allocation rule.
First, the allocation rule with unlimited irrigation period is simulated as follows: at the beginning of a water turn, the model calculates the amount of water required to irrigate the different fields in order to maximize the soil water reserve. This gives the duration of the water turn. The more this duration is important, the more the soil water reserve will be depleted at the end of the turn because of evapotranspiration and the more the next water turn will be long (fig. 11).

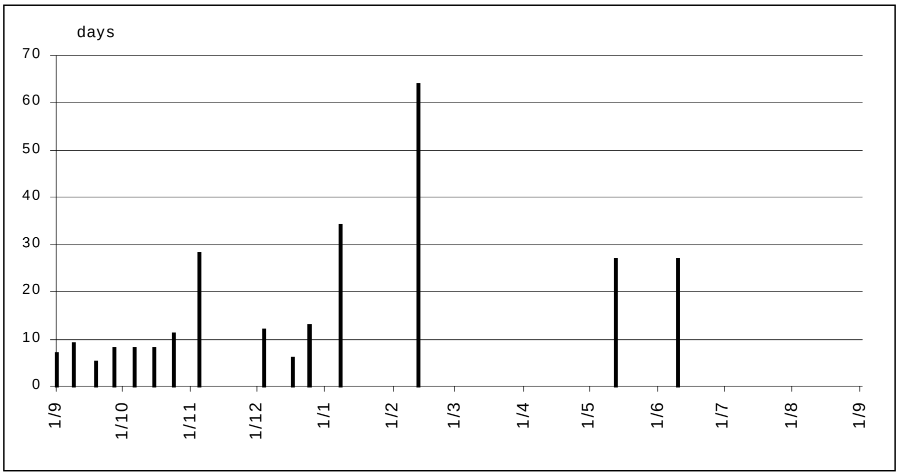

Figure 11: simulation of the evolution of water turn length for allocation rule with an unlimited irrigation period and with the 1998-1999 rainfall at El Melalsa

Second, to simulate an *ex ante* rule, water is everyday distributed between the different fields. In case of water scarcity, water is shared in proportion to the needs.