

# "Public Ownership and Public Goods,"

by

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## 1. Introduction

A powerful idea in economics is Adam Smith's "invisible hand", namely the notion that the pursue of the private self-interest by all members of society leads to a socially rational or efficient outcome. The idea applies to societies where all economic activity is based on the transformation of privately owned resources into private consumption goods by privately owned firms (and where some other conditions are satisfied as well). But real-life societies often differ from this paradigm. First, many resources are owned by society at large, and not by individuals. Second, the public sector supplies a variety of goods and services, many of which have the character of public goods. Third, the production or consumption of private goods often generate, via external effects, public good or bads. The relevance of the invisible hand can then be questioned. Indeed, such cases are not covered by today's precise version of the invisible hand idea, namely the first fundamental theorem of welfare economics.

Economic analysis has made an effort to accomodate public goods, partly because it is the physical properties of a good that makes it public or private, and there is no arguing with physical laws. Public ownership, on the contrary,

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\* This paper is based on the inaugural lecture for the Master Program in Economic Analysis, year 1991-92, given on September 10 at the *Universitat Autònoma de Barcelona*. Professor Ronald Coase, whose work was discussed in the lecture, was awarded the Nobel Prize in Economics a few weeks later.

belongs to the institutional sphere, and has not commanded a similar degree of respect. For instance, private property is often encouraged at the policy level, as illustrated by the pressure to privatize firms and resources that the international banking institutions exert on less developed countries. At the conceptual level, many academic authors make "private ownership" synonymous with "ownership," and "public ownership" with the absence of ownership: something is "publicly owned" if nobody owns it.<sup>1</sup> Moreover, they view "public ownership" as a major obstacle to the attainment of efficient outcomes.

Here, on the contrary, the public ownership of something means society's *or* everybody's ownership, typically exercised via the democratic process. These pages discuss the attainment of efficient states under public ownership, and argue that the identification of private ownership with efficiency is simplistic. The discussion is focused on three themes: "Coase's Theorem," the "tragedy of the commons" and the shareholders' decision in the generation of public bads.

## **2. "Coase Theorem"**

### **2.1. Pigou, Coase and the Coasians**

Arthur Pigou (1920) impugned the applicability of the "invisible hand" to cases where private activity generates public goods or bads: intervening in the economy would then be advisable. In his words (p. 172):

"... self-interest will [...] not tend to bring about equality in the values of the marginal social net products except when marginal private net product and marginal social net product are identical. When there is a divergence between these two sorts of marginal products, self-interest will not, therefore, tend to make the national dividend a maximum; and, consequently, certain specific acts of interference with

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<sup>1</sup> Barzel (1990) incorporates an extreme form of this view.

normal economic processes may be expected, not to diminish, but to increase the dividend."

Ronald Coase's 1960 paper "The Problem of Social Cost" frontally attacks the Pigovian view. The paper has been reprinted and commented in the 1988 the book The Firm, the Market and the Law. Both in its introduction and in Chapter 7, Coase shows disappointment about the prominence that the so called "Coase's Theorem" has attained, and about the manner how his writings have influenced the economics profession.<sup>2</sup> In his words:

"My point of view has not in general commanded assent, nor has my argument, for the most part, been understood." (1988, p. 1);

" 'The Problem of Social Cost,' in which these views were presented in a systematic view, has been widely cited and discussed in the economics literature. But his influence on economic analysis has been less beneficial than I had hoped. The discussion has been largely devoted to sections III and IV of the article and even here has concentrated on the so called 'Coase's Theorem,' neglecting other aspects of the analysis. " (1988, p. 13).

So it may be prudent to distinguish, paraphrasing Leijonhufvud, between Coasian economics and the economics of Coase. The Coasian discourse is exemplified in the treatment of externalities by numerous textbooks. Generally, it advocates *laissez faire*, expressing the notion that free contracts among the interested parties will lead to efficiency. Precise statements are not always available, but the one found in the widely used textbook by Harvey Rosen (1988) is representative:

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<sup>2</sup>I say "so called 'Coase Theorem'" because this is precisely Coase's own phrase (see quotation below). More recently (1992, p. 716), he writes: "This is the infamous Coase theorem, named and formulated by George Stigler, although it is based on work of mine." (See also Coase, 1988, p. 14. and p.157).

" [...] the efficient solution will be achieved *independently* of who is assigned the property rights as long as *someone* is assigned these rights. This result, known as the **Coase Theorem**, implies that, once property rights are established, no government intervention is needed to deal with externalities [Coase, 1960]" (p. 137, italics and boldface in the original).

This view leads to advocating the assignment of private property rights to someone. What is the logic behind the recommendation?

## 2.2 The farmer and the cattle-raiser.

The Coasian argument can be illustrated by the following example, inspired both in Coase (1960) and in the graduate lectures on welfare economics that Andreu Mas-Colell taught at the *Universitat Autònoma de Barcelona* back in the 198?-8?. There are a cattle-raiser and a farmer: the variable  $x$  is the number of steer raised by the cattle-raiser. Steers necessarily stray and destroy crops on the farmer's land. The larger  $x$ , the larger the damage caused. The utility functions are:  $\bar{u}_i(x, m_i) = v_i(x) + m_i$ ,  $i = C, F$  (C for cattle raiser, F for farmer), where  $m_i$  is the amount of numeraire that  $i$  ends up with. The initial amounts of numeraire that they own are  $\omega_1$  and  $\omega_2$  respectively: write  $\omega = \omega_1 + \omega_2$ . The functions  $v_i(x)$  have the shape of Figures 2.1 and 2.2: in particular, a larger  $x$  means a lower utility for the farmer. Except for the externality imposed, there are no costs in activity  $x$ .

Define the surplus function as  $s(x) = v_1(x) + v_2(x)$ , a function that attains a maximum at point  $x^*$  in Figures 2.1 and 2.2.

Consider first the case where, in the initial status quo, the cattle raiser has no duty to prevent steer from trampling: the farmer has no rights, and the cattle rancher can freely choose the level of  $x$ . If she chooses  $x$  without consideration for or agreement with the farmer, she will choose the amount that

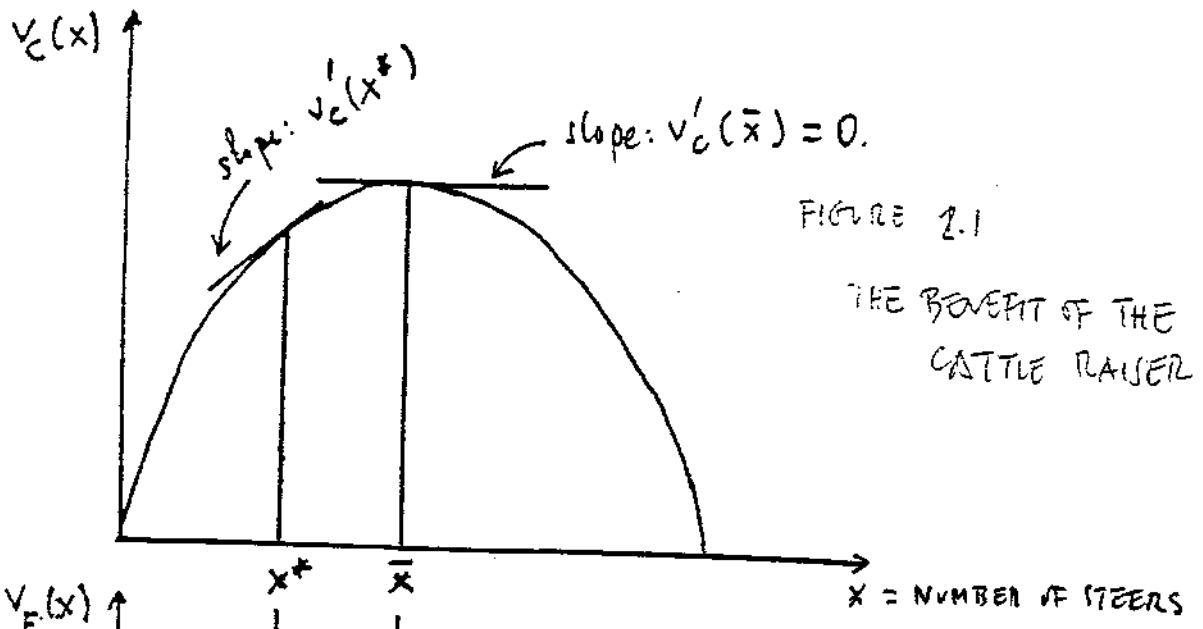


FIGURE 2.1

THE BENEFIT OF THE CATTLE RAISER

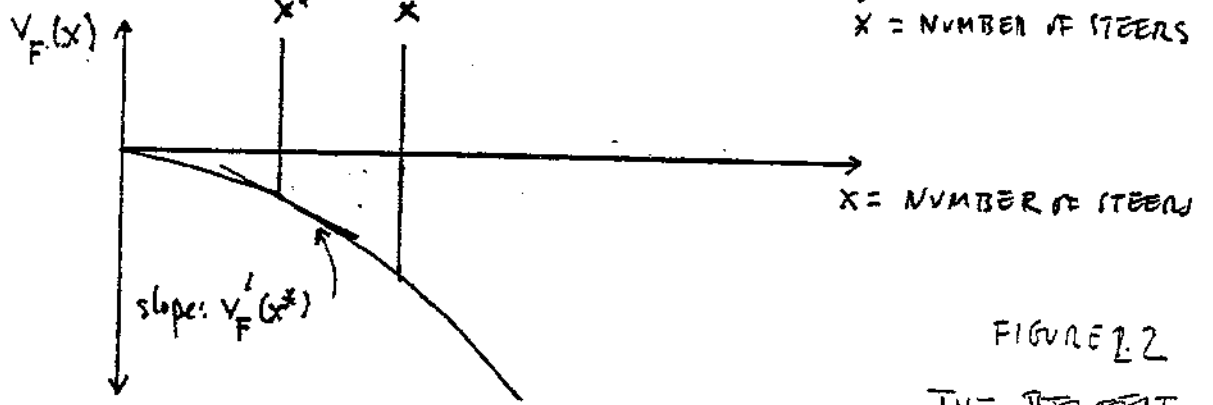


FIGURE 2.2

THE BENEFIT OF THE FARMER

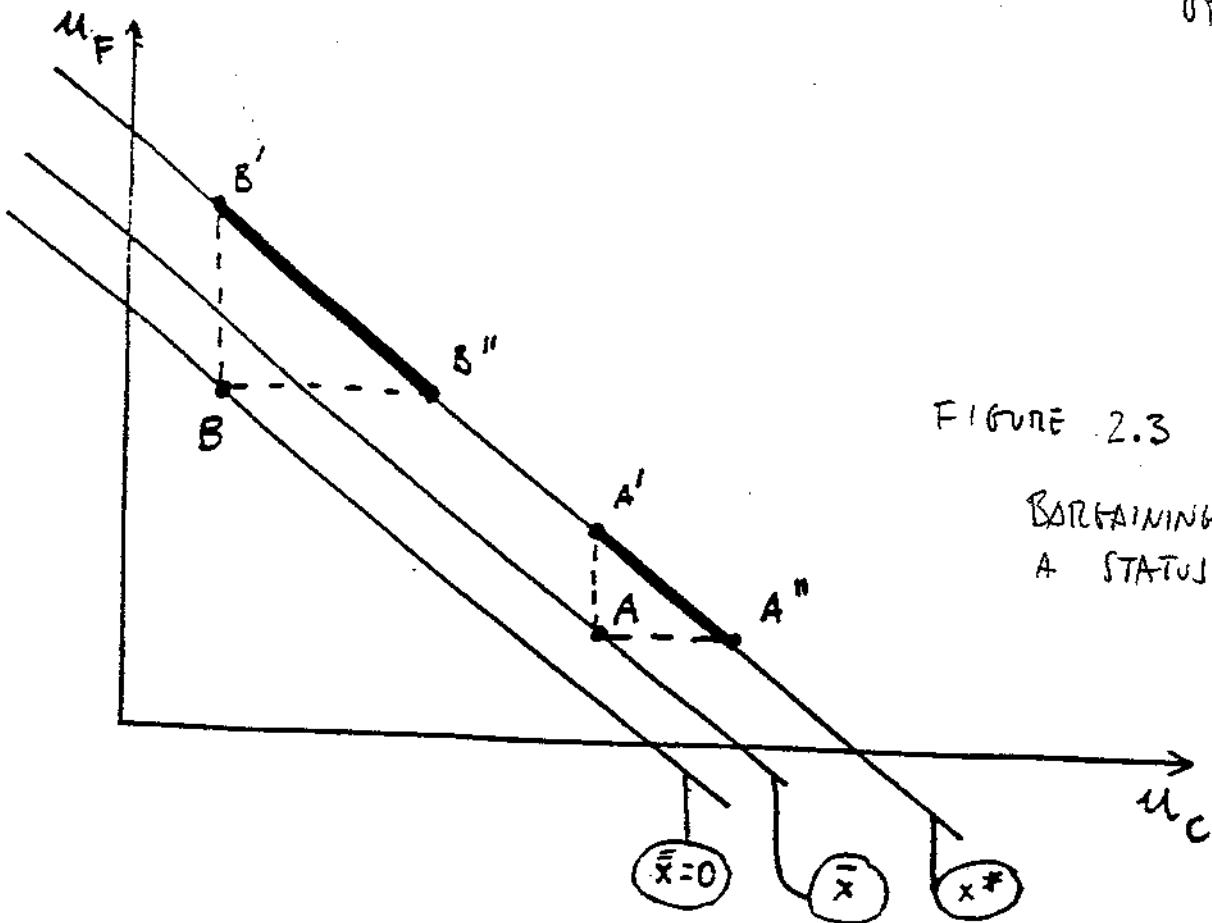


FIGURE 2.3

BARGAINING FROM A STATUS QUO

maximizes  $v_C(x)$ , namely  $\bar{x}$ , a suboptimal amount because  $s(\bar{x}) < s(x^*)$  (see Figure 2.1). The resulting utilities would then be:  $\bar{u}_C = v_C(\bar{x}) + \omega_C$  and  $\bar{u}_F = v_F(\bar{x}) + \omega_F$ , adding up to  $s(\bar{x}) + \omega$ . Figure 2.3 depicts the utility pair and shows that the situation is inefficient, because the farmer could "bribe" the cattle raiser to choose the lower level  $x^*$  and be both better off. Indeed, by setting  $x = x^*$  and transferring numeraire between them, any point on the line  $u_F = s(x^*) + \omega - u_C$  can be attained.<sup>3</sup> Because part of this line runs northeast of point  $(\bar{u}_C, \bar{u}_F)$ , Pareto improving bribes can be found. Actually, any transfer from the farmer to the cattle raiser in the nonempty interval:

$$(-v_C(x^*) - \omega_C + \bar{u}_C, -\bar{u}_C + v_C(x^*) + \omega_C) \quad (1)$$

will lead to a utility pair in the segment  $(A', A^*)$  of Figure 2.3, leaving both persons better off. After efficient bargaining, the final allocation of numeraire will be within the limits imposed by (1), but otherwise undetermined. In Coase's (1960) words:

"What payment will in fact be made would depend on the shrewdness of the farmer and the cattle raiser as bargainers."

Alternatively, let the cattle raiser have the duty to prevent trampling in the initial status quo. Now the farmer has the right *not* to be trampled, and, thus, the cattle raiser has the duty to prevent the steer from trampling. The farmer, therefore, can exercise his rights and dictate the level of  $x$ . If he chooses  $x$  without consideration for or agreement with the farmer, he will impose the number of steer that maximizes  $v_F(x)$ , namely  $x = 0$ , a suboptimal amount whenever  $s(0) < s(x^*)$  (see Figure 2.2). The resulting utilities would

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<sup>3</sup> Of course, I am disregarding the complications that arise when  $m_i$  is restricted to be nonnegative and, hence, person  $i$  cannot transfer more than  $\omega_i$  units of numeraire.

then be  $u^0_C = v_C(0) + \omega_C$  and  $u^0_F = v_F(0) + \omega_F$ . Again, the situation is inefficient. The cattle raiser could now bribe the farmer and reach a point on the segment  $(B',B'')$  of the line  $u_C = s(x^*) + \omega - u_F$ , where both are better off.

### 2.3. The role of quasi-linearity

It is at this point convenient to clarify an issue which is tangential to the main argument. In the previous example, efficiency requires that  $x^*$  steer be raised.<sup>4</sup> Thus, if, starting from either  $(\bar{u}_C, \bar{u}_F)$  or from  $(u^0_C, u^0_F)$ , an efficient outcome is reached by negotiation, then the number of steer will be  $x^*$ . One can talk of  $x = x^*$  as "the efficient solution," as in the above mentioned statement by Rosen. But this depends on the quasilinearity of the utility functions  $\bar{u}_i(x, m_i)$ . When the utility functions are not quasilinear, a different level of  $x$  typically corresponds to each point on the utility possibility frontier. The typical, non-quasi-linear case is illustrated in Figure 2.4, which maintains the notation of the previous one. The utility possibility frontier is the envelope of a family of utility possibility curves, each obtained by keeping  $x$  at a certain level and varying the distribution of numeraire. Not only the number of steers corresponding to a point in  $(A',A'')$  will differ from the number corresponding to a point in  $(B',B'')$ : two different points in  $(A',A'')$  will also entail different numbers of steers. In general, allowing for non-quasi-linear preferences requires saying "an optimal solution" instead of "the optimal solution."<sup>5</sup> Rosen's statement now becomes:

"an efficient solution will be achieved *independently* of who is assigned the property rights as long as *someone* is assigned these rights."

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<sup>4</sup> Subject, let me repeat, to the qualification of the previous footnote.

<sup>5</sup> This point is now well understood, yet Coase (1988, Chapter 7, Section IV) seems reluctant to accept it.

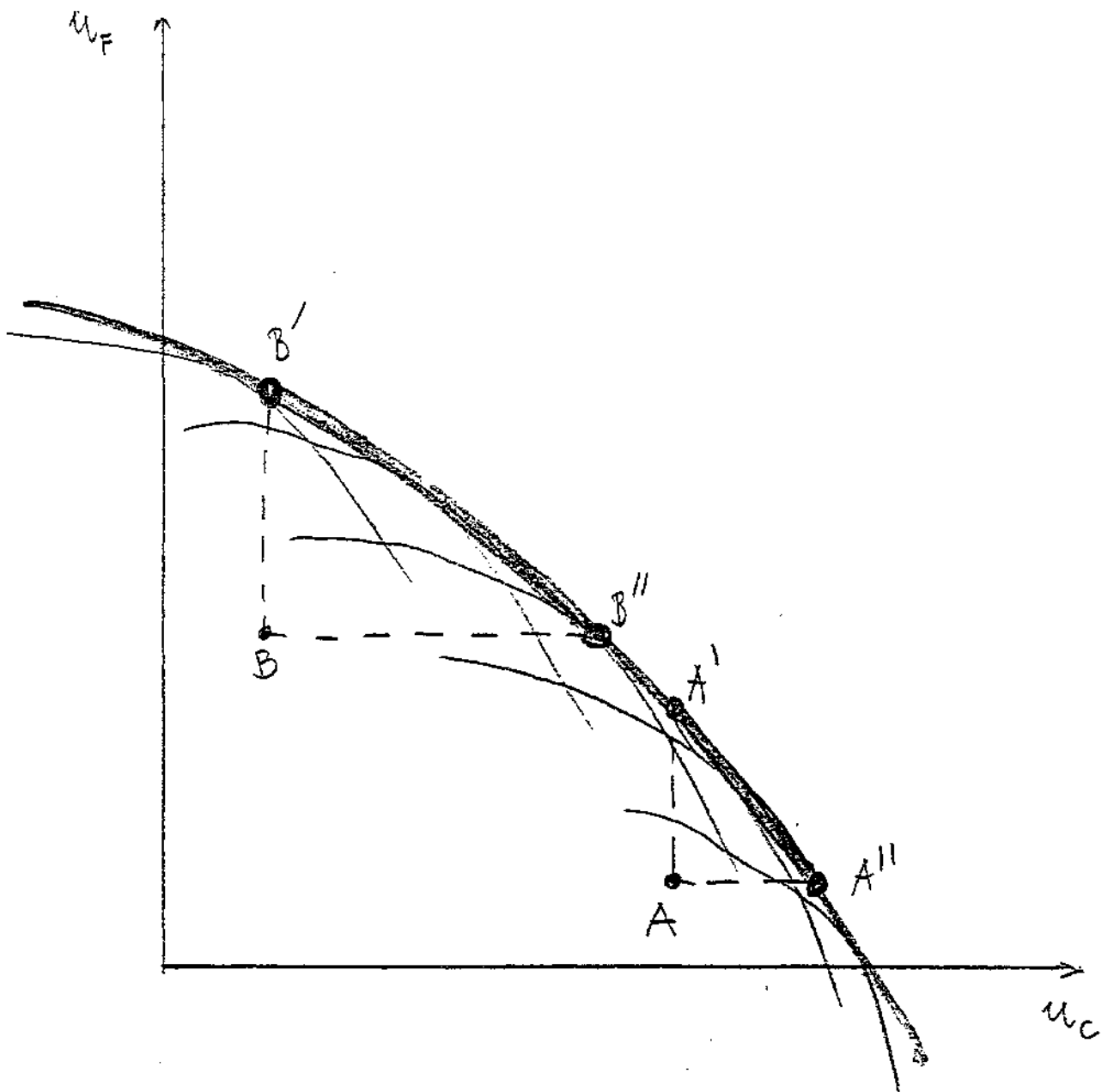


FIGURE 2.4

EFFICIENT BARGAINING WHEN UTILITIES ARE NOT QUASI-LINEAR



#### **2.4. Efficient bargaining and zero transaction costs**

The so called Coase Theorem asserts that bargaining will achieve an efficient solution. As a general statement, this is a certainly questionable : if one researches the history of conflicts between farmers and cattle raisers in enough detail one will find, no doubt, examples of efficient bargaining, but also some instances of grossly inefficient outcomes where cattle have been poisoned and fields have burned, even perhaps where a farmer has ended up with a cowboy's bullet in his skull. Indeed, explaining why the outcomes of human interaction are sometimes efficient and sometimes wasteful is a major task of the social sciences, which task remains by large unfinished. Sections 3.2-4 below comment on some relevant literature.

Coase himself views efficient bargaining not as the outcome to be expected in practice but as a conceptual point of reference, as something that would obtain only under the ideal conditions of "zero transaction costs." Indeed, it is precisely the fixation of the Coasians on this ideal situation that frustrates Coase. In his words:

"The world of zero transaction costs has often been described as a Coasian world. Nothing could be further from the truth." (1988, p. 174);

"It would not seem worthwhile to spend much time investigating the properties of such a world. What my argument does suggest is the need to introduce positive transaction costs explicitly into economic analysis so that we can study the world that exists. This has not been the effect of my article. The extensive discussion in the journals has concentrated almost entirely on the 'Coase Theorem,' a proposition about the world of zero transaction costs."(p. 15)

Because, in Coase's view, transaction costs include costs of search, information, bargaining, decision, policing and enforcement (see Coase. 1988, p.

6), the assert that, with zero transaction costs, bargaining is efficient is not very ambitious. Indeed, zero transaction costs imply efficiency if one defines "transaction cost" as any obstacle to efficient bargaining. In any event, the assumption of "zero transaction costs" must be included in the statement, to yield:

"an efficient solution will be achieved *independently* of who is assigned the property rights as long as transaction costs are zero and *someone* is assigned these rights." <sup>6</sup>

## 2.5. Efficient bargaining without property rights

How strong is the case for the assignment of private property rights? Well defined private property rights provide, in the farmer and cattleman example, a *status quo* (say, the pair  $(\bar{u}_C, \bar{u}_F)$  or the pair  $(u^0_C, u^0_F)$ ) for the bargaining process. Of course, rights could be well defined without being private: as will be discussed in Section 4, common ownership may entail precisely defined rights, providing a status quo for further negotiation within the political process or outside it. In any event, the main contribution of well defined rights, be they private or public, is the presence of a status quo which somewhat limits the possible outcomes of the negotiation, say to the segment  $(A', A'')$  of Figure 2.3 when the status quo is  $(\bar{u}_C, \bar{u}_F)$ . <sup>7</sup> Within this range, however, the outcome is indeterminate.

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<sup>6</sup> Most authors do mention the zero-transaction-cost assumption; Rosen, for instance, writes (1988, p. 137): "However, there are at least two reasons why society cannot always depend upon the Coase Theorem to 'solve' the externality problem. First, the theorem requires that the costs of bargaining do not deter the parties from finding their way to the efficient solution."

<sup>7</sup> An implicit assumption in determining the endpoints  $A'$  and  $A''$  is that, during the bargaining process, the actors never take actions that temporarily put them below their *status quo* levels of utility (as burning your bridges behind you); this is not always the case in actual negotiations, for instance,

It is not clear why one needs a well-defined status quo derived from property rights to engage in useful negotiation. Whether the status quo is determined by well-defined private property rights or not, the actual outcome of bargaining may well be indetermined and depend on the "shrewdness of the farmer and the cattle raiser as bargainers." The importance of having precise private property rights for efficient bargaining should be seen as an empirical question, on which not much evidence is available: some work mentioned in Section 3.4 below actually suggest that precise property rights are not very important. The Coasian emphasis on them seems misplaced.

How does Coase himself view the importance of private property rights? Coase did think in 1960 that private property rights were important. For instance, we can read in "The Problem of Social Cost." p. 119: <sup>8</sup>

"Of course, if market transactions were costless, all that matters (questions of equity apart) is that the rights of the various parties be well defined and the results of legal actions easy to forecast."

But, to his credit, he has since conceded that they are logically superfluous.

Quoting Steven N. S. Cheung (1982, p. 37), Coase (1988, p. 14-15) writes:

"Cheung has even argued that, if transaction costs are zero, 'the assumption of private property rights can be dropped without in the least negating the Coase Theorem and he is no doubt right.'"

It follows that, by Coase's own admission, the "Theorem" does not justify the advocacy of private property rights adopted by the Coasians.

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labor disputes; of course, the assumption of zero bargaining costs may be violated there, in particular concerning the information that an actor has about the characteristics of the other one.

<sup>8</sup> See also page 104: "It is necessary to know whether the damaging business is liable or not for damage caused, since without the establishment of this initial delimitation of rights there cannot be market transactions to transfer or recombine them."

### 3. A "tragedy of the commons"?

#### 3.1. The invisible hand and the rational herder

Coase's paper "The Problem of Social Cost" begins with the words "This paper is concerned with those actions of business firms which have harmful effects on others."<sup>9</sup> The paper is indeed confined to externalities that are unidirectional (the level of activity of the cattle-raiser affect the productivity of the farmer's labor, but no vice-versa) and that involve few firms.<sup>10</sup> Yet the paper aims at more ambitious target, namely, "exposing the weaknesses of Pigou's analysis of the divergence between private and social products."<sup>11</sup> The Coasian discourse does take the more general view: indeed, "Coase theorem" is often presented as a defense of laissez-faire, perhaps accompanied by the privatization of some resources, as the correct policy towards any type of externality.

Common-pool resources, like fisheries, pastures, forests, irrigation systems, groundwater basins or oilfields, present another type of negative production externality. This externality displays more symmetry, or less unidirectionality, than the example of the cattle raiser and farmer.<sup>12</sup> For instance, as one herder increases her herd, it depletes the edible grass in the

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<sup>9</sup> The cattle raiser example is, of course, hypothetical. But the other ten examples discussed are actual court decisions on damage-causing firms.

<sup>10</sup> Cattle trample the crop, but they do not actually eat it. If the cattle fed on the farmer's crop, then there would also be a positive external effect from the farmer to the cattle raiser.

<sup>11</sup> Coase (1992, p. 717).

<sup>12</sup> But irrigation fields are asymmetric: the upstream farmer causes an external effect, but receives none. Groundwater systems may also be asymmetric, see, for instance, Ostrom's (1990, Ch. 4) discussion of the Central and West groundwater basins in the Los Angeles area, where the West basin is downstream from the Central basin.

pasture and decreases the productivity of every other herder. Pigou (1920) did not mention this type of commons (although he explicitly discusses air pollution, afforestation and urban zoning), but his approach, based on the need to gap the difference between the private cost (to a particular herder) and the social cost (to all herders) of adding another animal to a particular herd, applies without modification.<sup>13</sup>

Forty eight years after Pigou's Economics of Welfare, a biologist, Garrett Hardin, popularized the argument in the colorfully worded short article "The Tragedy of the Commons." Hardin considers two types of commons: the "commons as a food basket" (as pastures or fisheries) and "the commons as a cesspool" (as pollution, visual blight or noise). He argues that we must, in his words, "exorcize the spirit of Adam Smith" and abandon the policies of laissez-faire. Taking the pasture as an illustration, he writes:

"As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, he asks, "What is the utility *to me* of adding one more animal to my herd?" [...] (T)he rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another; and another.... But this is the conclusion reached by each and every rational herdsman sharing the commons. Therein is the tragedy. [...] Freedom in a commons brings ruin to all." (p.162)

What solutions to the tragedy does Hardin propose? In the case of the "commons as a breadbasket," the options are:

"We might sell them off as private property. We might keep them as public property, but allocate the right to enter them." (p. 1245)

Whereas for the case of the commons as a cesspool:

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<sup>13</sup> See H. Scott Gordon (1954). A. D. Scott (1955) and Vernon Smith (1969) for the development of the analysis in the case of fisheries.

"[...] the tragedy of the commons as a cesspool must be prevented by different means, by coercive laws or taxing devices that make it cheaper to the polluter to treat his pollutants than to discharge them untreated." (p.1245)

Hardin mentions three solutions altogether: privatization, public property with entry limitations and "coercive laws or taxing devices." Privatization, in turn, admits two forms. First, granting a particular person or firm the *sole ownership* of the resource. The externality would then be internalized.<sup>14</sup> Some, but not all, common pool resources admit a second form of privatization, namely, parceling out the resource. For instance, the common pasture grounds may be divided among the herders: each herder's grounds can then be fenced in.

### 3.2. Governing the commons

Hardin's paper has been read as asserting that, in the absence of privatization and external coercion, tragedy will strike. The reading is perhaps inaccurate, because Hardin does mention public property with limited access as a solution, yet he views this as "something formally like" private property. (p. 1245).

A growing literature, in part coordinated by the International Association for the Study of Common Property (IASCP) and exemplified by Elinor Ostrom's recent book Governing the Commons, has contested the view that only privatization or outside intervention can prevent the tragedy.<sup>15</sup> It has produced a

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<sup>14</sup> See, for instance, A. D. Scott (1955) and Vernon Smith (1969).

<sup>15</sup> The interested reader may contact IASCP at 332e Classroom-Office Building, 1994 Buford Avenue, St. Paul, Minnesota 55108, USA.

variety of examples where the local users of a common pool resources have developed efficient institutions for its exploitation.<sup>16</sup>

The cases "Coase vs. Pigou" and "Ostrom vs. Hardin" are somewhat parallel. Both contest an established view that negates the invisible hand. Both question the need for the intervention of government: they show that, in the presence of negative production externalities, the involved parties often manage to find efficient arrangements.

But there is an important difference between the two cases. As argued in the above discussion of "Coase theorem," the Coasian discourse advocates laissez-faire with private property rights, whereas the IASCP literature tends to oppose both government's intervention and the establishment of private property rights on common pool resources.

The cooperation between the parties also assumes different forms. In the Coasian tradition, efficiency is viewed as resulting from bilateral bargaining. Governing the Commons, on the contrary, is rooted in institutional analysis and focuses on problems of collective action and the design of robust, stable institutions. At a methodological level, even though "The Problem of Social Cost" is indeed more empirically oriented than the work that it contests, the empirical analysis of the efficient bargaining in the case of unidirectional externalities involving firms is, by no means, as extensive and developed as the study of

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<sup>16</sup> Of course, many early authors were aware of the fact that communities often develop efficient rules for the exploitation of common pool resources. Gordon (1954), for instance, explicitly affirms that inefficient exploitation is rare in "primitive cultures," and mentions several examples where coordination has been successful and where elaborate rules regulating the use of the resource have developed.

common pool resources. Fenton Martin's periodically updated bibliography lists 1500 entries in its 1989 edition.<sup>17</sup>

### 3.3. The costs and the benefits from cooperation

It should be emphasized that both Coase and the common property literature strongly qualify their advocacy of nonintervention. Coase's original discussion in his 1960 paper goes as follows. Efficiency is achieved when the bargaining between the parties is costless, or, in his synonymous if somewhat idiosyncratic expression, when there are "no costs involved in carrying out market transactions." In Coase's words (1960, p. )

"This is, of course, a very unrealistic assumption. In order to carry out market transactions, it is necessary to discover who it is that one wishes to deal with, to inform people that one wishes to deal and on what terms, to conduct negotiations leading up to a bargain, to draw up the contract, to undertake the inspection needed to make sure that the terms of the contract are being observed, and so on. These operations are often extremely costly, sufficiently costly at any rate to prevent many transactions [...]"

Once the transaction costs are taken into account, it is not necessarily true that socially desirable bargains that modify or rearrange the initial property will always be negotiated. In Coase's words:

"Once the costs of carrying out market transactions are taken into account, it is clear that such rearrangement of rights will only be undertaken when the increase in the value of production consequent upon the rearrangement is greater than the costs which would be involved in bringing it about."

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<sup>17</sup> Governing the Commons discusses a dozen cases. James Acheson and Bonnie McCay (1990) for a representative collection of papers, and Glenn G. Stevenson (1991) offers a detailed discussion of two grazing commons.



Coase suggests that the comparison of the aggregate costs and benefits from efficient bargaining will explain its occurrence. Game 5, p. 16, of Governing the Commons, reproduced in Figure 3.2 reflects a similar idea. There are two herders sharing some common pasture. Hardin's tragedy of the commons is described by the prisoner-dilemma game of Figure 3.1, but in the game of Figure 3.2 the herders may engage in a binding agreement enforced by an external party. Reaching and enforcing the agreement entails an aggregate cost of  $e$ , to be distributed equally between the herders, and the agreement itself yields a gain of 10 per capita with respect to the nonagreement outcome. Governing the Commons argues that the outcome of the game will be the efficient solution as long as  $e/2 < 10$ . Indeed, even though the game has two Nash (and subgame-perfect) equilibria, only the first one, where both play (A,D), is a trembling-hand perfect equilibrium (the other one, where both play ( $\tilde{A}$ , D), is not trembling-hand perfect because the strategies ( $\tilde{A}$ , D) are weakly dominated by (A,D)).

Realistic situations may well be more complex. The game of Figure 3.2 simplifies the bargaining process by assuming that, once the herders agree to cooperate, there is a unique outcome  $(10 - e/2, 10 - e/2)$ , where they divide the transaction costs exactly by two. The assumption is justified in Governing the Commons by appealing to the symmetry of the underlying prisoner's dilemma game. But even if the payoffs are symmetric, the herders may differ in other features, say age or "shrewdness as bargainers," so that the actual point reached may be one out of a continuum that Pareto dominates the disagreement point (0,0), as happened in the case illustrated by Figure 2.3.

### 3.4. Some empirical evidence on initial property rights

Section 2.5 above refers to the lack of solid empirical evidence connecting the definition of initial property rights with the achievement of

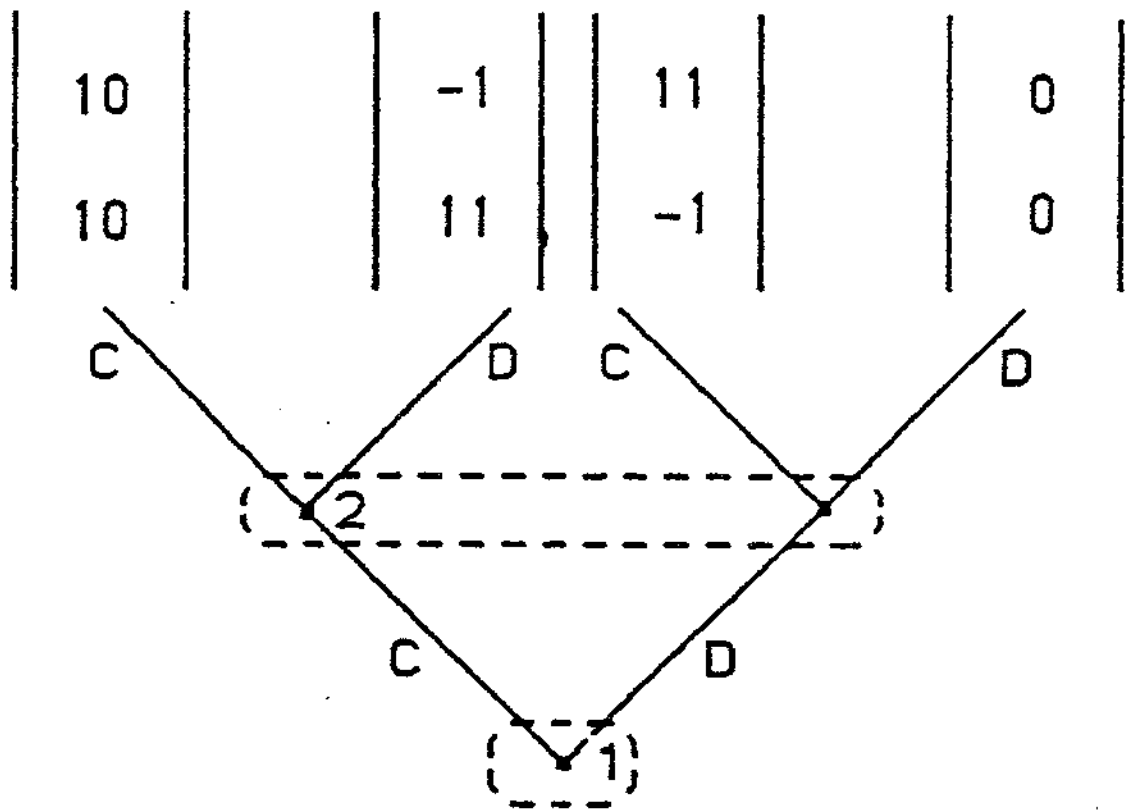


FIGURE 3.1

A PRISONER'S DILEMMA GAME  
 (OSTROM, 1990, p. 4)



efficiency. The analysis of groundwater basins in Chapters 4 and 5 of Governing the Commons suggests that well-defined initial property rights are not crucial for the achievement of efficiency. Chapter 4 discusses the Raymond, Central and West groundwater basins in the Los Angeles area, whereas Chapter 5 refers to those in San Bernardino County. In the Los Angeles basins, the users have managed to negotiate an efficient agreement, whereas in the San Bernardino basins they have not. The two groups of basins differ in some significant ways, but not on the definition of the initial property rights. Indeed, such property rights were rather confuse in the California legal system and the results of legal actions rather unpredictable. We read in Governing the Commons, page 108:

"The simultaneous existence of the doctrines of correlative and appropriative rights in the same state introduced considerable uncertainty about the relative rights of one groundwater producer against the others. The uncertainty was compounded by the presence of a third common-law doctrine that enabled groundwater producers to gain rights through 'adverse use' or prescription."

[...]

"The situations in these basins can be characterized as an open-access CPR for which clear limits have not been established regarding who can withdraw how much water."

The success stories of the Raymond, Central and West groundwater basins indicate that well-defined initial property rights are not necessary conditions for the successful negotiation of an efficient agreement.

### 3.5. Distribution and the ownership of the resource

Consider a fishery where the fishers have cooperatively agreed to limit their catch in an efficient manner. Each fisher obtains a given return from her

fishing effort. I wish to argue that the return partly reflects the appropriation of a fraction of the fishery's value.

In order to define the value of a common pool resource, imagine for a moment that it has been privatized and turned over to a profit-maximizing firm. The firm then exploits the resource in a regime of sole ownership: it hires fishers, paying them wages, and it sells the fish. I define the value of the resource to be the profits of such hypothetical firm. Essentially the same value could be defined as the proceeds of auctioning off the right to fish to a single beneficiary. As a third interpretation, one could visualize a regime where an efficient number of tradeable permits have been issued. The market valuation of the total amount of permits issued would again express the value of the resource.

When the fishers cooperatively agree on limiting production, the return to a fisher can be then seen as composed of two parts. First, a wage income, equal to what the fisher would earn were she employed by the hypothetical sole-ownership firm, or, more precisely, equal to the marginal value product of the fisher's time multiplied by the hours spend fishing. The second component, defined as the amount that she actually earns minus the first component, reflects the appropriation of a fraction, proportional to the time spent fishing, of the value of the resource.

More generally, efficiency schemes based on quantity limits or quotas, whether agreed upon by the exploiters themselves or imposed by an outside agency, imply a particular distribution of the value of the resource.<sup>18</sup> Schemes based on Pigovian fees and subsidies, on the contrary, offer several levels of freedom in the distribution of that value. On the one hand, they allow for alternative distributions among the producers who exploit the resource. On the

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<sup>18</sup> See Dasgupta (1982, Chapter 2).

other, they allow for channelling part of the value of the resource outside the group of producers, for instance, to the consumers, or to the general public.

True, in many cases it is natural to confine the distribution of the value of the resource to the producers, say, to the irrigators or to the fishers. They may have exclusive property rights to the resource that have been recognized for centuries. Another reason may be that the irrigators or the fishers are poorer, as a group, than the general population, or poorer than the consumers of the vegetables or fish that they supply. Keeping the value of the resource in the hands of the producers helps equalize the distribution of income.

But there are other instances of common pool resources, say oilfields on public lands, which should be seen as owned by the general public rather than by the exploiters of the resource. The exploiters may also be relatively wealthy. It may then be desirable to channel part of the value of the resources to other groups. A Pigovian fee-subsidy scheme may be superior to a free agreement between the exploiters in cases where the benefits from the commons should be shared among a larger group, despite the disadvantages of centralization.<sup>19</sup>

A related idea, present in Martin Weitzman (1974), in John Roemer (1989) and in my recent paper (1992), is based on the observation that a move towards efficiency may well hurt some people. A flexible scheme that permits to redirect the benefits as to compensate the losers may be useful. An example will illustrate. Imagine a fishery which, in the initial status quo, is inefficiently operated at the open-access or noncooperative equilibrium level. Now the fishers organize and efficiently limit their fishing effort. They are now better off. But

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<sup>19</sup> But one should keep in mind that these disadvantages can be substantial, particularly in less developed countries. Ostrom (1990, p. 23) gives several examples of disastrous nationalizations of formerly communal forests in Thailand, Nepal and India.

what about the consumers of the fish? The effects on their welfare depend on whether the aggregate catch is, in the new steady state, higher or lower than before: the inefficiency of the open access solution often shows itself as a lower catch per unit of effort, but the total catch may actually be larger in the inefficient open access solution than in the cooperative, efficient solution. In this case, the cooperation among fishers does solve the tragedy of the commons, but, to some extent, at the expense of consumers. A Pigovian fee-subsidy scheme may distribute the gains more equitably.

#### 4 Shareholding and public bads

##### 4.1. Two scenarios: foul odors and owner-consumers

A familiar example of common property can be found at the heart of a capitalistic economy: a corporation is jointly owned by its shareholders. Economists often adopt three simplifying hypotheses concerning the decisions of a corporation. First, corporate decisions should further the interests of the shareholders. Second, there is no uncertainty on the future consequences of the decisions. These two hypotheses will be maintained here, despite the fact that they deserve to be questioned, and are indeed questioned in important lines of recent research. But I will deviate from a third, usual assumption -- namely, that all shareholders agree to maximize profits.

One can consider two scenarios where some shareholders may prefer a decision that does not maximize profits. First, let the firm create a public bad, say a foul smell that invades the city, in the production process. Let  $x$  be the level of smell. (The firm can generate as much smell as it wishes.) Assume that the level  $x$  (which in turn determines output and profits) is the only decision to be made. A shareholder's interests as a profit earner clash with his interests as a citizen who suffers from stink. A stink-sensitive shareholder who owns only

one share among thousands will prefer a low level of  $x$ , whereas a large shareholder who is insensitive to odor will like a large  $x$ . Ultimately, the utility of shareholder  $i$  ( $i = 1, \dots, M$ ) depends on the decision  $x$  of the firm and on his profit share  $\theta_i$ . This case is inspired in the recent paper by John Roemer (1991).

There is no odor in the second scenario, but, in contrast, the firm is a monopoly and its shareholders are also consumers of its product, which they must buy in the market. The firm faces a given demand curve and the quantity to be produced and sold is determined once the price is chosen. The only choice to be made is the price  $p$ . An owner-consumer is affected by the price in two opposite ways: as a profit earner, interested in high profits and, thus, in relatively high prices, and as a consumer, interested in low prices. Say that the firm monopolizes telephone service. An owner-consumer who receives a large fraction of the firm's profits but who, on the other hand, dislikes talking on the phone will prefer a price close to the monopoly price, whereas a person who owns only one share and spends many hours calling long distance will prefer a price close to zero. Again, the ultimate utility of owner-consumer  $i$  depends on the decision of the firm, now called  $p$ , and on his profit share  $\theta_i$ . This is the case studied in Joseph Farrell (1985), Richard Manning (1986) and in my 1991 joint work with Andreu Mas-Colell.

#### 4.2. Unanimity and efficiency

People with a high profit share  $\theta_i$  will prefer a high level of the public bad, or a high price, where the opposite is true of people with low profit share.<sup>20</sup> Is there a distribution of shares  $(\theta_1, \dots, \theta_M)$  that will make everybody

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<sup>20</sup>This is made precise in Section 4.4 below.



agree on the same level of the public bad  $x^*$  or on the same price  $p^*$ ? If so, what properties does it have?

The second question is easier than the first one: the existence of such a vector  $(\theta_1, \dots, \theta_M)$  is by no means guaranteed. But, interestingly, it turns out that if such a vector exists, then the resulting outcome is Pareto efficient.<sup>21</sup>

Let us start with the stink case. The primitive data of the model are as follows (see Roemer, 1991). There are three goods (or bads), stink, denoted by  $x$ , output, denoted by  $y$  or  $Y$ , and a private input, denoted by  $z$  or  $Z$ , which is useful in production but which does not enter the utility function. Each of the  $M$  persons of the economy is defined by a utility function  $u_i(x, y_i)$ , increasing in her consumption of output  $y_i$ , and by an endowment  $\omega_i$  of private input. We write  $\omega = \sum_i \omega_i$ . There is a single firm which produces output by using the private input,  $Z$ , and generating stink,  $x$ . Its technology is given by the production function  $Y = f(x, Z)$ . (A capital  $Y$  or  $Z$  denotes aggregate amounts of output or input). All functions are assumed differentiable. Let  $w$  denote the real price of the input (i.e.,  $w$  is the price of the input divided by the price of output). The firm is a price taker, i.e., once  $x$  has been decided, it chooses  $Z$  (and, consequently,  $Y$ ) in order to maximize  $f(x, Z) - wZ$ , i.e.,  $L$  satisfies " $w = \partial f / \partial Z$ ." Equilibrium in the labor market requires the full employment of  $\omega$ . This motivates the definition:

$$\hat{w}(x) = \frac{\partial f}{\partial Z} \Big|_{(x, Z) = (x, \omega)} \quad (4.1)$$

Given  $x$ , person  $i$  receives the amount of output  $\hat{w}(x)\omega_i$  in exchange for his contribution  $\omega_i$  of input, plus the amount  $\theta_i[f(x, \omega) - \hat{w}(x)\omega]$  as his share in the

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<sup>21</sup>See Andreu Mas-Colell and Joaquim Silvestre (1991) for a discussion of the existence issue in owner-consumer scenario. The argument for efficiency is similar to the one for the sharing the cost of a public good presented in Mas-Colell and Silvestre (1989); see Silvestre (forthcoming) for a comparison.

firm's profits. Given  $\theta_i$ ,  $i$ 's utility ultimately depends on  $x$ , and can be expressed as:

$$\bar{v}_i(x) = \bar{u}_i(x, \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]). \quad (4.2)$$

It is easy to show that if the vector  $(\theta_1, \dots, \theta_M)$  induces a unanimous agreement, then the resulting outcome is Pareto efficient.<sup>22</sup>

Let us turn to our second scenario, where there is no stink but where the shareholders must agree on the price of output. The technology is defined by an inverse production function  $C(Y)$ , i.e.,  $C(Y)$  is the amount of the input  $Z$  needed to produce  $Y$  units of output. We now assume that the input good is an argument in the utility functions, now written  $u_i(y_i, z_i)$ , a function increasing in  $i$ 's consumption  $z_i$  of the input. I assume that all functions are differentiable, and that  $u_i$  is strictly quasi-concave,  $i = 1, \dots, M$ . As before,  $\omega_i$  is  $i$ 's endowment of input. I now use the private input as numeraire, and denote the normalized price of output by  $p$  (i.e.,  $p$  is the price of the output divided by the price of the input). Given  $p$  and wealth  $I_i$  (normalized in the same way), person  $i$  chooses  $y_i$  and  $z_i$  in order to maximize utility subject to the budget constraint:  $py_i + z_i = I_i$ . This yields  $i$ 's demand functions  $\tilde{y}_i(p, I_i)$  and  $\tilde{z}_i(p, I_i)$ . Write:  $\tilde{Y}(p, I_1, \dots, I_M) = \sum_i \tilde{y}_i(p, I_i)$ . Attention is restricted to prices  $p$  for which the equation in  $Y$ :

$$Y - \sum_i \tilde{y}_i(p, \omega_i + \theta_i[pY - C(Y)]) = 0. \quad (4.3)$$

<sup>22</sup> Proof: Let  $x^*$  satisfy:  $\bar{v}_i(x^*) \geq \bar{v}_i(x)$  for all  $x$ ,  $i = 1, \dots, M$ . I claim that, if  $(x; y_1, \dots, y_M)$  satisfies:  $\bar{u}_i(x, y_i) \geq \bar{v}_i(x^*)$ , for all  $i$ , with strict inequality for some  $i$ , then  $y_i \geq \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]$  for all  $i$ , with strict inequality for some  $i$ . Suppose not, i.e., let  $\bar{u}_i(x, y_i) \geq \bar{v}_i(x^*)$  but  $y_i < \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]$ . Then:  $\bar{v}_i(x) = \bar{u}_i(x, \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]) > \bar{u}_i(x, y_i) \geq \bar{v}_i(x^*)$ , contradicting the assumption on  $x^*$ . Thus,  $y_i \geq \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]$  for all  $i$ . The same argument shows that, if  $\bar{u}_i(x, y_i) > \bar{v}_i(x^*)$ , then  $y_i > \hat{w}(x)\omega_i + \theta_i[f(x,\omega) - \hat{w}(x)\omega]$ . Because  $\sum_i \theta_i = 1$ , summing over  $i$ , we obtain:  $\sum_i y_i > \hat{w}(x)\omega + \sum_i \theta_i [f(x,\omega) - \hat{w}(x)\omega]$ , i.e.,  $\sum_i y_i > f(x,\omega)$ , which implies that the state  $(x; y_1, \dots, y_M)$  is not feasible.

has a unique solution  $\hat{Y}(p)$ . Moreover, I assume that  $\hat{Y}$  is differentiable. Person  $i$ 's utility depends on  $p$  and on his wealth  $I_i$ , which in turn also depends on  $p$ .

Denoting by  $v_i(p, I_i)$   $i$ 's indirect utility function, i.e.,  $v_i(p, I_i) = u_i(\tilde{y}_i(p, I_i), \tilde{z}_i(p, I_i))$ ,  $i$ 's utility ultimately depends on  $p$  in accordance with the expression:

$$v_i(p, \omega_i + \theta_i[p\hat{Y}(p) - C(\hat{Y}(p))]), \quad i = 1, \dots, M. \quad (4.4)$$

Now let the vector  $(\theta_1, \dots, \theta_M)$  induce a unanimous agreement on a given price  $p^* > 0$ , i.e., let  $p^*$  maximize expression (4.4) for  $i = 1, \dots, M$ . By the necessity of the first order condition, we have that:

$$\frac{\partial v_i}{\partial p} + \frac{\partial v_i}{\partial I_i} \theta_i [\hat{Y}(p) + p\hat{Y}' - C' \cdot \hat{Y}] = 0, \quad i = 1, \dots, M,$$

or, dividing by  $[\partial v_i / \partial I_i]$  and applying Roy's identity:<sup>23</sup>

$$- \tilde{y}_i(p, \omega_i + \theta_i[p\hat{Y}(p) - C(\hat{Y}(p))]) + \theta_i [\hat{Y}(p) + p\hat{Y}' - C' \cdot \hat{Y}] = 0, \quad i = 1, \dots, M. \quad (4.5)$$

Summing over consumers and using (4.3), we obtain:

$$-\hat{Y} + \hat{Y} + [p - C'] \hat{Y}' = 0,$$

which, as long as  $\hat{Y}' \neq 0$ , implies that  $p = C'$  i.e., that the price equals the marginal cost. Because the optimality conditions involving the consumers' marginal rates of substitution are satisfied at  $(\tilde{y}_i(p, \omega_i + \theta_i[p\hat{Y}(p) - C(\hat{Y}(p))]), \tilde{z}_i(p, \omega_i + \theta_i[p\hat{Y}(p) - C(\hat{Y}(p))]))$ , the equality of the price and the marginal cost implies that the resulting allocation is Pareto efficient when either  $C$  is convex or when there is only one allocation satisfying the first order necessary conditions for efficiency. Once again, unanimity implies efficiency.

It should also be noted that the equality of price and marginal cost, together with (4.3), allows us to rewrite (4.5) as,

$$- \tilde{y}_i + \theta_i \sum_h \tilde{y}_h = 0, \quad i = 1, \dots, M,$$

i.e.,

<sup>23</sup> See, e.g., Hal Varian (1984, p. 126)

$$\theta_i = \frac{\tilde{y}_i}{\sum_h \tilde{y}_h} \quad i = 1, \dots, M.$$

In words, if the vector  $(\theta_1, \dots, \theta_M)$  induces a unanimous agreement,  $i$ 's share in profits,  $\theta_i$ , equals his share  $\tilde{y}_i / \sum_h \tilde{y}_h$  in the consumption of the produced good. Hence, a unanimous agreement on  $p$  induces what my forthcoming paper with John Roemer calls the "proportional solution."

### 4.3. Restricted unanimity

The case, just discussed, where the decision on the amount of the public bad  $x$  or on the price  $p$  is unanimous depicts an ideal degree of participation. The remainder of this section considers less-than-ideal situations. What is the departure from efficiency caused by departures from perfect unanimity?

Manning (1986) studied a first departure from full unanimity for the owner-consumer scenario. The set of  $M$  persons is divided between two groups: the owners of the firm, indexed 1 to  $M_0$ , and the nonowners, indexed  $M_0+1$  to  $M$ : we postulate that, for  $i = M_0 + 1, \dots, M$ ,  $\theta_i = 0$ . Nonowners have no say, but the decision on  $p$  requires the unanimous agreement of the owners. The limit case  $M_0 = M$  corresponds to full unanimity, whereas, as  $M_0$  decreases, the decision process becomes less democratic.

The previous analysis is modified in the following way. At a price  $\bar{p}$  that receives the unanimous support of owners, equation (4.5) is satisfied for  $i = 1, \dots, M_0$ . Let us simplify the analysis by assuming that preferences are quasilinear, i.e.,  $u_i(y_i, z_i) = \beta(y_i) + z_i$ ,  $i = 1, \dots, M$ . Now, for  $i = 1, \dots, M$ ,  $\bar{y}_i(p, I_i) = \bar{y}_i(p)$ , a function of price only.<sup>24</sup> Indirect utility is  $v_i(p, I_i) = \beta_i(\bar{y}_i(p)) + I_i -$

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<sup>24</sup> Assuming interiority, i.e., disregarding situations where a nonnegativity constraint on  $z_i$  is binding.

$p\bar{y}_i(p)$ . Write  $\bar{Y}(p) = \sum_{i=1}^{M_0} \bar{y}_i(p)$  and  $\bar{Y}_{-0} = \sum_{i=M_0+1}^M \bar{y}_i(p)$ . It is clear from (4.3) that  $\hat{Y}(p) = \bar{Y}(p)$ . The first order condition of utility maximization is now:

$$\bar{y}_i(p) + \theta_i[\bar{Y} + (p - C'(\bar{Y}(p))) \bar{Y}'(p)] = 0, \quad i = 1, \dots, M_0. \quad (4.6)$$

and the second order condition is:

$$\bar{y}_i' + \theta_i[\bar{Y}' + (1 - C'' \cdot \bar{Y}') \bar{Y}' + (p - C') \bar{Y}'] \leq 0, \quad i = 1, \dots, M_0. \quad (4.7)$$

Aggregating the equalities (4.6) we obtain:

$$\bar{Y}_{-0}(p) + [p - C'(\bar{Y}(p))] \bar{Y}'(p) = 0.$$

which implies, because  $\bar{Y}'(p) < 0$ , that  $p > C'$ . In words, under restricted participation, the price is too high and there is inefficiency, see Figure 4.1,

whereas under perfect unanimity the price equals the marginal cost and there is efficiency. It is intuitively plausible that the price will be lower the higher the degree of participation. For the sake of arguing this point while using calculus techniques, let me assume that the demand of owners is a fraction  $\alpha \leq 1$  of total demand, i.e.,  $\bar{Y}_0(p) = \alpha \bar{Y}(p)$ . What happens when  $\alpha$  increases and, thus, more people participate in the unanimous decision? Writing  $\bar{Y}_{-0}(p) = (1-\alpha)\bar{y}(p)$  and implicitly differentiating (4.8), one obtains:

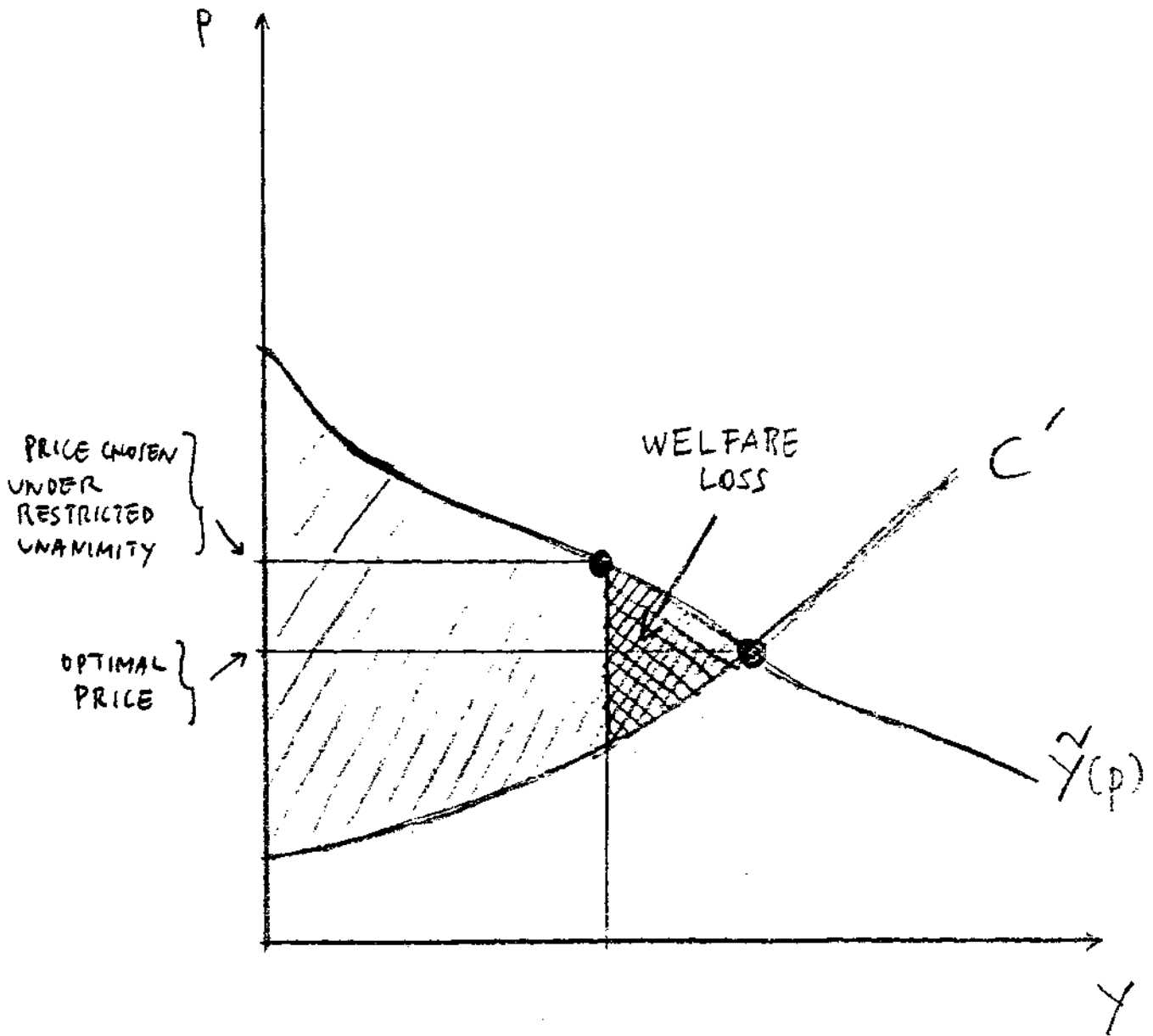
$$\frac{dp}{d\alpha} = - \frac{\bar{Y}}{(1-\alpha)\bar{Y}' + (1 - C'' \cdot \bar{Y}') \bar{Y}' + (p - C') \bar{Y}'}. \quad (4.8)$$

Aggregating the inequalities (4.7), assuming that at least one is strict and writing  $\bar{Y}_{-0}' = (1-\alpha)\bar{Y}'$ , we have that:

$$(1-\alpha)\bar{Y}' + (1 - C'' \cdot \bar{Y}') \bar{Y}' + (p - C') \bar{Y}' < 0,$$

i.e., the denominator of (4.8) is negative, which implies that  $dp/d\alpha < 0$ , i.e.,

(see Figure 4.1) an increase in the number of people participating in the decision reduces the price and increases the social surplus.



THE PRICE UNDER RESTRICTED UNANIMITY IS TOO HIGH

FIGURE 4.1

A similar conclusion can be reached in the alternative scenario of the foul-smelling factory. I start by deriving in the usual way the marginal conditions for efficiency as the first order conditions of the program:

$$\max_{(x, y_1, \dots, y_M)} \bar{u}_1(x, y_1) \text{ subject to } \bar{u}_i(x, y_i) = k_i, i = 2, \dots, M, \text{ and } f(x, \omega) = \sum_{i=1}^M y_i.$$

where  $k_2, \dots, k_M$  are arbitrarily predetermined utility levels. The Lagrangean is:

$$\bar{u}_1(x, y_1) + \sum_{i=2}^M \lambda_i (\bar{u}_i(x, y_i) - k_i) + \mu (f(x, \omega) - \sum_{i=1}^M y_i).$$

Setting to zero the derivatives of the Lagrangean with respect to  $y_1, \dots, y_M$  we obtain:  $\mu \partial u_i / \partial y_i = \lambda_i$  (where  $\lambda_1 = 1$ ), which, in turn, can be substituted into the derivative with respect to  $x$  to yield the Samuelson condition:

$$- \sum_{i=1}^M \frac{\partial \bar{u}_i / \partial x}{\partial \bar{u}_i / \partial y_i} = \frac{\partial f}{\partial x}. \quad (4.9)$$

If we again assume quasilinearity, i.e.,  $\bar{u}_i(x, y_i) = \bar{\beta}_i(x) + y_i$ , then (4.9) becomes:  $-\sum_i \bar{\beta}_i(x) = \partial f / \partial x$ . Because  $x$  is a public bad,  $-\bar{\beta}_i(x) < 0$ . Hence  $-\bar{\beta}_i(x)$  is the marginal damage, measured in units of output, caused by  $x$ , whereas  $\partial f / \partial x$  is the marginal product of  $x$ . Equation (4.9) has the familiar interpretation: "the marginal social damage caused by the public bad equals its marginal product", see Figure 4.2.

Let us now assume that a unanimous decision among the set of owners of the firm, persons 1 to  $M_0$ , determines the level of the public good. Owner  $i$  maximizes (see (4.2) above):

$$\bar{\beta}_i(x) + \hat{w}(x)\omega_i + \theta_i [f(x, \omega) - \hat{w}(x)\omega],$$

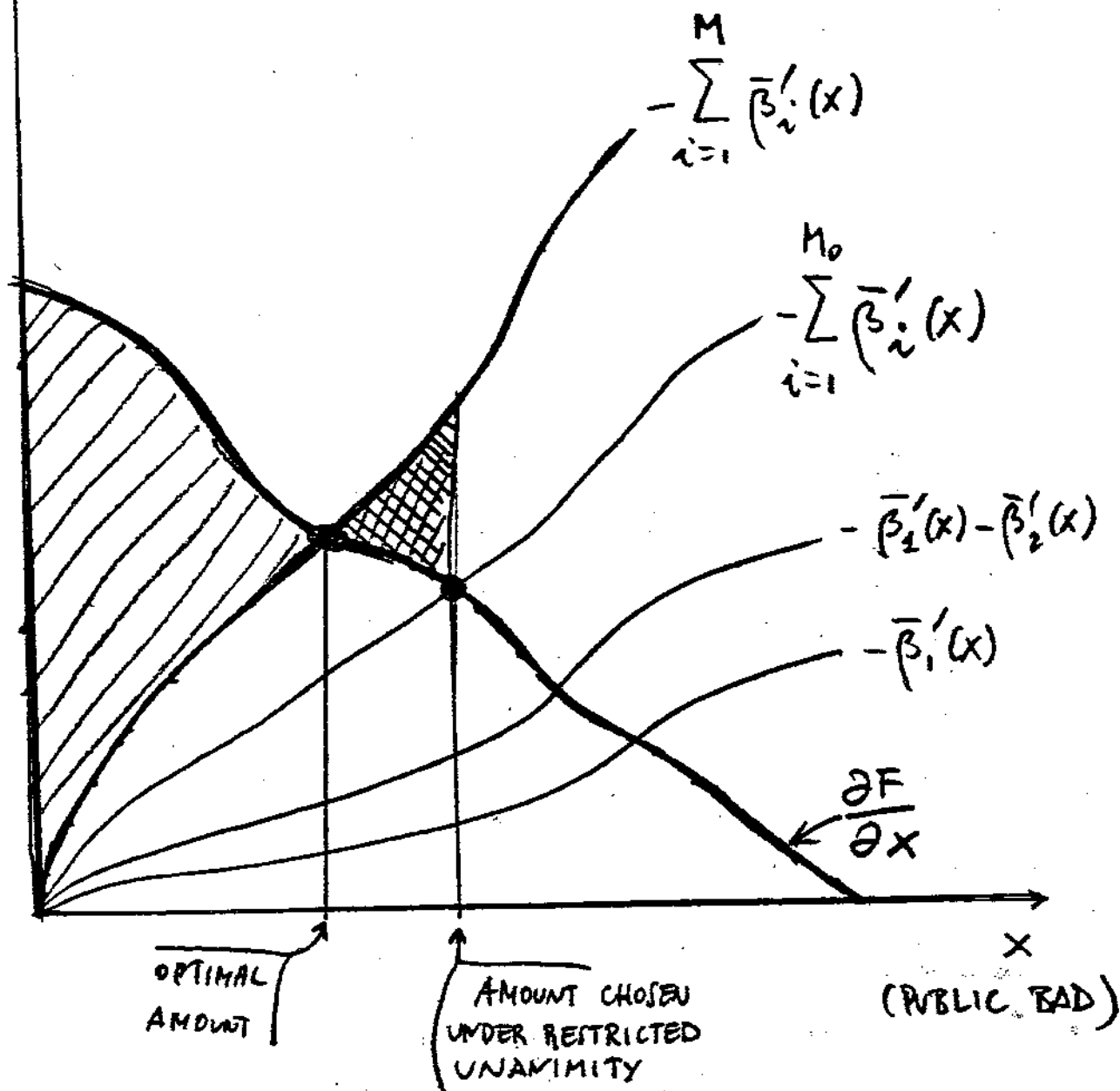
with first order condition:

$$\bar{\beta}_i'(x) + \hat{w}'(x)\omega_i + \theta_i [(\partial f / \partial x) - \hat{w}'(x)\omega], \quad i = 1, \dots, M_0,$$

which, after aggregation, yields:

$$- \sum_{i=1}^{M_0} \bar{\beta}_i'(x) - [\sum_{i=1}^{M_0} \omega_i - \omega] \hat{w}'(x) = \partial f / \partial x. \quad (4.10)$$

MARGINAL  
DAMAGE,  
MARGINAL  
PRODUCT



THERE IS TOO MUCH PUBLIC BAD UNDER RESTRICTED UNANIMITY

FIGURE 4.2



Again, when  $M_0 = M$ , the efficiency condition (4.9) obtains. Otherwise the result will typically be inefficient. Suppose, to simplify the analysis, that the production function  $f$  is separable in the inputs  $x$  and  $Z$ , i.e.,  $\partial^2 f / \partial x \partial Z = 0$ . Then (see (4.1)),  $\hat{W}'(x)$  is zero and (4.10) becomes:  $-\sum_{i=1}^{M_0} \bar{\beta}_i'(x) = \partial f / \partial x$ . Graphically, the sum of only the first  $M_0$  marginal damages is equalized to the marginal product of  $x$ , resulting in a too high level of the public bad. Social welfare, represented by the area below the marginal product curve and above the social marginal damage curve, is too low. It is graphically clear that, when  $M_0$  is increased, additional curves  $-\bar{\beta}_i'(x)$  are included in the sum  $-\sum_{i=1}^{M_0} \bar{\beta}_i'(x)$ , so that its intersection with the curve  $\partial f / \partial x$  moves northwest.<sup>25</sup> In words, an increase in the number of people participating in the decision reduces the level of the public bad and increases the social surplus.

Summarizing, the two scenarios give similar results for the case of restricted unanimity, namely (A) the outcome is inefficient, yielding a level of the price or the public bad that is too high; (B) the lower the degree of participation, the higher the level of the public bad or price, and, hence, (C) the lower the degree of participation, the higher the social loss.

#### 4.4. Identical, quasilinear preferences

From now on I assume that all persons are identical, except possibly in their profit shares  $\theta_i$ 's. It is easy to see, using the previous analysis, that, if  $\theta_i = 1/M$ ,  $i = 1, \dots, M$ , then all consumers unanimously agree on an efficient level

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<sup>25</sup> By the same argument, a positive term  $-\left[\sum_{i=1}^{M_0} \omega_i - \omega\right] \hat{W}'(x)$  would entail higher efficiency. Because  $\sum_{i=1}^{M_0} \omega_i - \omega < 0$ , this requires that  $\hat{W}'(x) > 0$ , i.e.,  $\partial^2 f / \partial x \partial z > 0$  is positive. In words, the public bad is complementary to the private input in production, increasing its marginal product.

of the public bad or the price.<sup>26</sup> In particular, they all prefer a price equal to the marginal cost in the owner-consumer scenario. If, on the other hand,  $\theta_i \neq 1/M$  for some  $i$  (or  $\theta_i \neq 1/M_0$  in the restricted unanimity case) then a unanimous agreement is, typically, impossible.<sup>27</sup> The most preferred level of the public bad, or price, will vary with  $\theta$ .

It is intuitively plausible that, the higher  $\theta$ , the higher the price, or the level of the public bad, desired by the consumer. Indeed this is the case under quasilinear utilities and, in the foul smell scenario, under the additional condition that  $\partial^2 f / \partial x \partial z \leq 0$ . The arguments are similar in both scenarios. Consider the owner-consumer case. Denote by  $\beta(y) + z$  the individual utility function, by  $\tilde{y}(p)$  the individual demand function, by  $s(p)$  the individual consumer surplus at  $p$ , i.e.,  $s(p) = \beta(\tilde{y}(p)) - p\tilde{y}(p)$ , and by  $\Pi(p)$  the profit function, i.e.,  $\Pi(p) = pM\tilde{y}(p) - C(M\tilde{y}(p))$ . The ultimate utility of the person with profit share  $\theta$  is  $\tilde{v}(p|\theta) = s(p) + \theta\Pi(p)$ . We say that the price  $p$  is most preferred by the person with share  $\theta$  if it maximizes  $\tilde{v}(p|\theta)$ . The Appendix proves the following propositions.

Proposition 1: Let  $\theta > \theta'$  and assume that  $p$  (resp.  $p'$ ) is most preferred by the person with share  $\theta$  (resp.  $\theta'$ ). Then  $p \geq p'$ .

Proposition 2: Let  $\theta > \theta'$ , let  $p > 0$  and assume that  $p$  (resp.  $p'$ ) is most preferred by the person with share  $\theta$  (resp.  $\theta'$ ). Then  $p > p'$ .

<sup>26</sup> Clearly, maximization of individual utility for  $\theta_i = 1/M$  is now sufficient for unanimity and, hence, for efficiency.

<sup>27</sup> The previous analysis shows that, under differentiability and interiority (i.e.,  $x > 0$  or  $p > 0$ ), unanimity implies that  $\theta_i = 1/M$ . Moreover, in the consumer-owner scenario with quasilinear utilities,  $p = 0$  may be a unanimous agreement only if, on the one hand,  $C' = 0$ , (because of optimality) and moreover,  $s'(0) + \theta_i(M\tilde{y}'(0) + (p - C')M\tilde{y}''(0)) \leq 0$ , i.e.,  $(\theta_i M - 1)\tilde{y}'(0) \leq 0$ , which implies that  $\theta_i M \leq 1$ , for all  $i$ . i.e.,  $\theta_i = 1/M$ . (The individual demand function is denoted by  $\tilde{y}(p)$ ).

Quasilinearity allows us to speak of the optimal price or optimal level of the public bad, defined as the maximizer of the total surplus function. This function is, in the case of the owner-consumers:

$$M\beta(\tilde{y}(p)) - C(M\tilde{y}(p)), \quad (4.11)$$

and in the public-bad case:

$$M\bar{\beta}(x) + f(x, \omega),$$

where  $\bar{\beta}$  describes the utility function by :  $\bar{u}_i(x, y_i) = \bar{\beta}(x) + y_i$ ,  $i = 1, \dots, M$ .

Consider an owner-consumer whose profit share is precisely equal to  $1/M$ . What is his most preferred price? It is the one that maximizes  $\tilde{v}(p | 1/M)$ , i.e.,

$$s(p) + [1/M][pM\tilde{y}(p) - C(M\tilde{y}(p))] = \beta(\tilde{y}(p)) - p\tilde{y}(p) + p\tilde{y}(p) + [1/M]C(M\tilde{y}(p)).$$

But this is nothing but the total surplus function (4.11) divided by  $M$ . Hence the following proposition is proved. (The argument is identical for the public-bad case).

Proposition 3: A price (or public-bad) level is most preferred by a person with profit share  $\theta = 1/M$  if and only if it maximizes total surplus.

The next proposition is the result of putting together the two previous ones.

Proposition 4: Let  $p^* > 0$  be a surplus-maximizing price, and let  $p$  be most preferred by the person with share  $\theta$ .

- (i). If  $\theta > 1/M$ , then  $p > p^*$ .
- (ii). If  $\theta < 1/M$ , then  $p < p^*$ .

#### 4.5. One share, one vote

If the profit shares are not uniform, then decisions must be made in accordance with rules other than unanimity. Now I consider, as in Roemer (1991), pairwise majority voting with the rule "one share, one vote." A level of

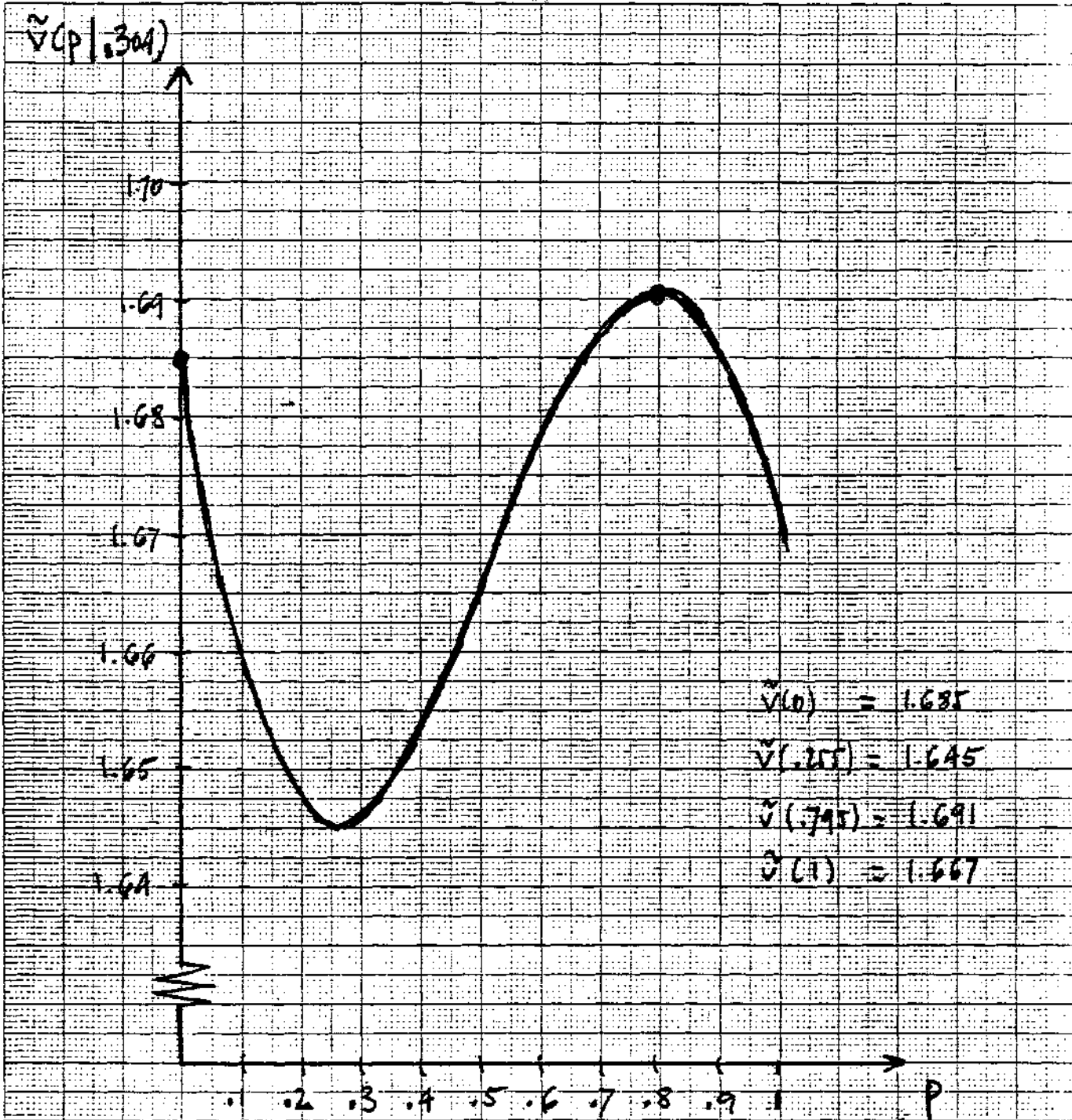
public bad  $x^*$ , or a price  $p^*$ , is the social decision if it is a Condorcet winner, i.e., if there is no other  $x$  (or  $p$ ) that is preferred to  $x^*$  by a group of shareholders holding more than one half of all shares. Unless preferences are single peaked, there is no guarantee, in principle, that a Condorcet winner exists. It is not hard, in our context, to construct examples where single peakedness is violated. For instance, Figure 4.3 shows the graph of  $\tilde{v}(p|\theta)$  for the following specification:  $\beta(y) = (16/3) - (2/3)(4 - y)^{3/2}$ , yielding the individual demand function:  $\tilde{y}(p) = 4 - p^2$ ;  $M = 3$ ; the cost function is:  $C(Y) = Y$ , and the profit share is  $\theta = .304$ . The function has two peaks, one at  $p = 0$  and another one at  $p = .795$ . Similar examples can, no doubt, be found for the public-bad scenario. Yet multi-peakedness does not necessarily imply the nonexistence of a Condorcet winner: whether one necessarily exists or not without additional assumptions is, as far as I know, an open question.

An additional assumption that does guarantee single-peakedness in the owner-consumer scenario is the linearity of the demand and cost functions. Let  $\beta(y) = ay - (1/2)y^2$ , i.e.,  $\tilde{y}(p) = a - p$ , and let  $C(Y) = cY$ ,  $a > c$ . One can compute:

$$\begin{aligned}\tilde{v}(p|\theta) &= (\theta M - 1)(a - p) - \theta(p - c)M, \\ \tilde{v}'(p|\theta) &= -(\theta M - 1) - \theta M = 1 - 2\theta M.\end{aligned}\tag{4.12}$$

By (4.12),  $\tilde{v}(p|\theta)$  is either convex (for low  $\theta$ , namely  $\theta \leq 1/2M$ ) or strictly concave. In the case where  $\tilde{v}$  is convex we have that  $\tilde{v}(0|\theta) = (1/2)a^2 - \theta cMa > (1/2)a^2 - \theta Ma^2$  (because  $a > c$ ), i.e.,  $\tilde{v}(0|\theta) > (1 - 2\theta M)a^2/2 \geq 0$  (because  $\theta \leq 1/2M$ ), i.e.,  $\tilde{v}(0|\theta) > 0$ . Moreover,  $\tilde{v}(a|\theta) = 0$  and  $\tilde{v}(p|\theta) \geq 0$  for  $p \in [0, a]$ . Thus, the convex function  $\tilde{v}$  is decreasing on  $[0, a]$ , attaining its single peak at  $p = 0$ . Single-peakedness also obtains, clearly, when  $\tilde{v}$  is strictly concave.<sup>28</sup>

<sup>28</sup> The single peak is  $p = 0$  as long as  $\tilde{v}'(0|\theta) \leq 0$ , i.e., from (4.15), as long as  $(\theta M - 1)a + \theta cM \leq 0$ , i.e.,  $\theta(Ma + Mc) \leq a$ , or  $\theta \leq a/M(a + c)$ , where  $a/M(a + c) >$



A TWIN-PEAKED EXAMPLE

$$\begin{cases} \beta(y) = \frac{16}{3} - \frac{2}{3}(4-y)^{3/2} \\ C(y) = y \end{cases}$$

FIGURE 4.3

The reader is referred to Roemer (1991) for a discussion of sufficient conditions for single-peakedness in the public-bad scenario.

Given  $(\theta_1, \dots, \theta_M)$ ,  $i$  is a median shareholder if writing:

$$\theta_i = \sum_{\{h \mid \theta_h = \theta_i\}} \theta_h ,$$

$$\theta_{<i} = \sum_{\{h \mid \theta_h < \theta_i\}} \theta_h ,$$

$$\theta_{>i} = \sum_{\{h \mid \theta_h > \theta_i\}} \theta_h ,$$

one has:  $\theta_{<i} + \theta_i \geq 1/2$  and  $\theta_{>i} + \theta_i \geq 1/2$ . The median shareholder can be visualized in a Lorenz-Gini square, see Figure 4.4 . Given a share distribution, rank shareholders by increasing share size (in the case of shareholders with the same  $\theta$ , the order does not matter): the poorest one is said to be in Position 1, and the richest one in position M. Permute the subscripts such that  $\theta_i$  now means "the share of the person in Position  $i$ " rather than "the share of the person named  $i$ ." The base of the square (of unit length) is divided in M intervals, with endpoints labelled "Position 1" to "position M." The ordinate corresponding to Position  $j$  is the sum, over  $h \leq j$ , of  $\theta_h$ . The position with the lowest ordinate greater than or equal to  $1/2$  is the median-share position (Position  $i$  in Figure 4.4.). Any person whose share equals that of the person in the median-share position is a median shareholder.

The curve obtained by joining the points in the graph is the Lorenz curve of the share distribution, and twice the area between the Lorenz curve and the diagonal of the square (the square has unit-length sides) is the Gini coefficient of the inequality of the distribution: if  $\theta = 1/M$ ,  $i = 1, \dots, M$ , then the Lorenz curve coincides with the diagonal and the Gini coefficient is zero.

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$1/2M$ . If  $\theta > a/M(a + c)$ , then the single peak is obtained at the positive price given by  $a - (\theta M / [2\theta M - 1])(a - c)$ .

It is easy to prove that, in accordance with a fundamental result in voting theory, under single peakedness, the most preferred point of a median shareholder is the Condorcet winner of the one-share-one-vote rule.

Let us compare the results of this decision rule with the limited-unanimity one discussed in Section 4.3 above. First, the price (or the level of the public bad) was in that case always too high (or just right under complete unanimity). Here (see Proposition 4) it may be too high or too low, depending on whether the median share (i.e., the profit share  $\theta$  of a median shareholder) is greater than or less than  $1/M$ . For an illustration, let  $M = 3$ . If the shares are  $(\theta_1, \theta_2, \theta_3) = (0.26, 0.26, 0.48)$ , then the median share is  $.26 < 1/3 = 1/M$ , and the price will be lower than the marginal cost (or the level of the public bad will be too low). If, on the other hand,  $(\theta_1, \theta_2, \theta_3) = (0.10, 0.41, 0.49)$ , then the median share is  $0.41 > 1/M$ , and the price will be higher than the marginal cost.

If  $(\theta_1, \theta_2, \theta_3) = (1/3, 1/3, 1/3)$ , then optimality will obtain: i.e., perfect equality, which is also a property of total unanimity when consumers have the same preferences, is a sufficient condition for optimality. But, contrary to the limited-unanimity case, it is no longer necessary. If, for example,  $(\theta_1, \theta_2, \theta_3) = (51/300, 100/300, 149/300)$ , then the median share is still  $1/3$ , and, thus, the outcome is still optimal, even though the distribution of shares is no longer equal. The solid line of Figure 4.5 depicts the Lorenz curve of a share distribution very similar to this one.

Does increasing equality increase efficiency under the one-share-one vote rule? It follows from the previous analysis that the answer is: if and only if it puts the median share closer to  $1/M$ . If for instance, the median share is lower than  $1/M$ , a small increase in it will increase efficiency (total surplus) by increasing the price, whereas if the median share is greater than  $1/M$ , a reduction in the median share will increase efficiency by reducing the price.

But, intuitively, an increase in equality could lead the median share away from  $1/M$ . Suppose that we take an "increase in equality" to mean Lorenz domination, i.e., a move of the Lorenz curve towards the diagonal. Formally:

Definition: The share distribution  $(\theta_1, \dots, \theta_M)$  Lorenz-dominates  $(\theta'_1, \dots, \theta'_M)$  denoted  $(\theta'_1, \dots, \theta'_M) L (\theta_1, \dots, \theta_M)$ , if, for  $i = 1, \dots, M-1$ ,

$$\sum_{h=1}^i \theta'_h > \sum_{h=1}^i \theta_h.$$

It is intuitively and graphically clear (see Figures 4.4 and 4.5) that, as the Lorenz curve moves upwards, the median shareholder cannot move to the right. Indeed, the observation that the median position is the lowest  $i$  for which  $\sum_{h=1}^i \theta_h \geq 1/2$  proves the following proposition.

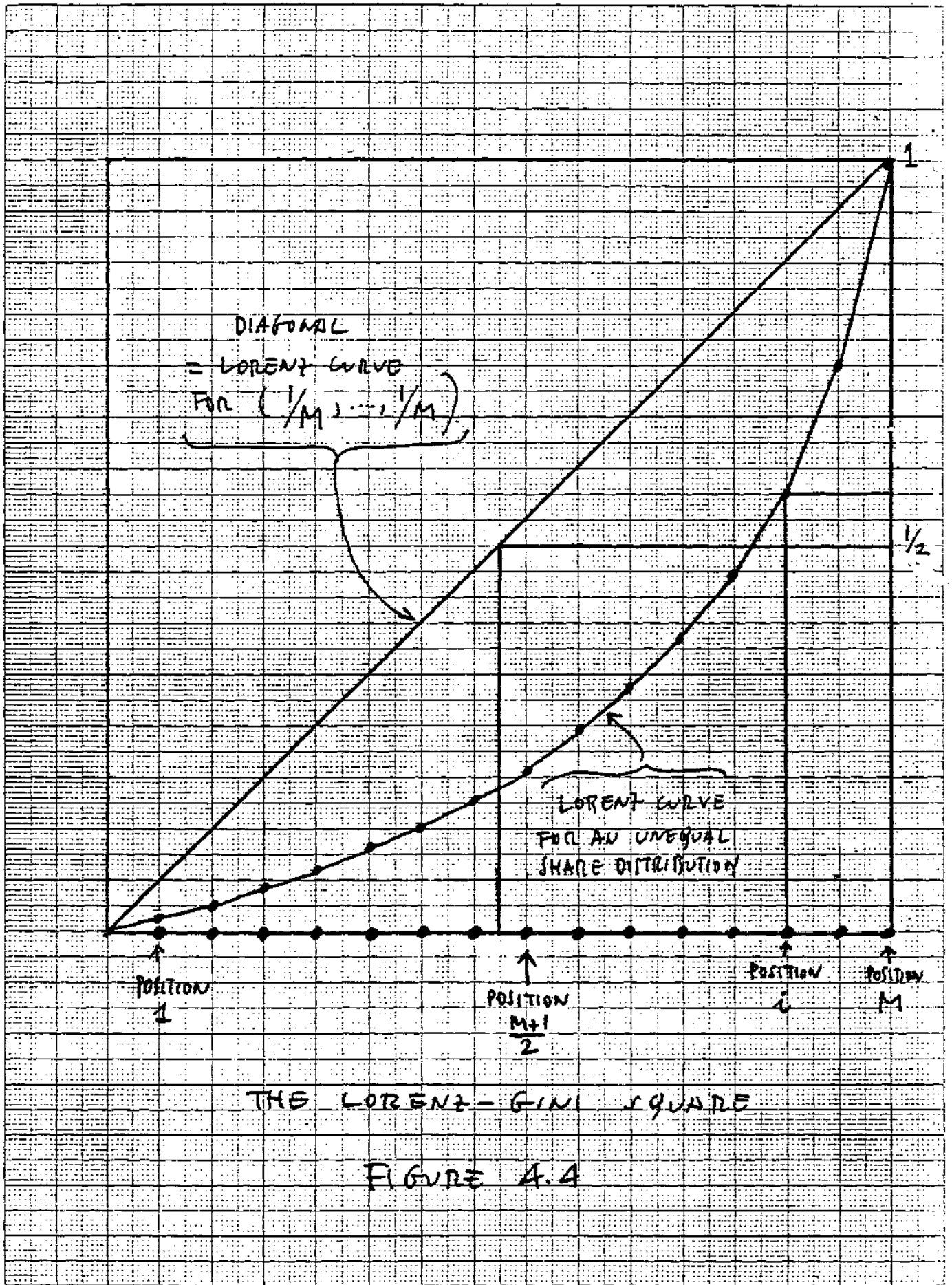
Proposition 5: If  $i$  (resp.  $i'$ ) is the median position for the distribution  $(\theta_1, \dots, \theta_M)$  (resp.  $(\theta'_1, \dots, \theta'_M)$ ) and  $(\theta'_1, \dots, \theta'_M) L (\theta_1, \dots, \theta_M)$ , then  $i' \leq i$ .

But the Lorenz curve may move towards the diagonal, yet the median share may move away from  $1/M$ . Start, for instance, at  $(\theta_1, \theta_2, \theta_3) = (51/300, 101/300, 148/300)$ , a slight modification of the previous example: the Lorenz curve is the solid line in Figure 4.5. Consider a move to  $(\theta'_1, \theta'_2, \theta'_3) = (90/300, 105/300, 105/300)$ : see the new Lorenz curve as a dashed line in Figure 4.5. The Lorenz curve has moved towards the  $45^\circ$  line, which is the Lorenz curve for  $(1/M, \dots, 1/M)$  yet the median share has moved away from  $1/M$  to  $105/300$ , hence total surplus has decreased. A stricter notion of "more egalitarian" is needed if one wants to associate an increase in equality with a move of the median share towards  $1/M$ . One such notion is provided by John Roemer (1991, p. 13), formally defined as follows. (Roemer says "is more egalitarian.")

Definition: Let  $(\theta_1, \dots, \theta_M) = (1/M, \dots, 1/M)$ . The share distribution  $(\theta'_1, \dots, \theta'_M)$  Roemer-dominates  $(\theta_1, \dots, \theta_M)$ , denoted  $(\theta'_1, \dots, \theta'_M) R (\theta_1, \dots, \theta_M)$ , if there exist numbers  $t_i \in (0, 1]$  such that, for  $i = 1, \dots, M$ ,

$$\theta'_i = t_i [1/M] + (1 - t_i) \theta_i. \quad (4.13)$$







In words, the share of each position moves closer to  $1/M$ . As noted, the example of Figure 4.5 does not satisfy this condition, because  $\theta_2$  moves away from  $1/M$ .

It is intuitively plausible that, if each share moves towards  $1/M$ , the Lorenz curve should move towards the diagonal. This is made precise in the following proposition, proved in the Appendix.

Proposition 6. If  $(\theta'_1, \dots, \theta'_M) R (\theta_1, \dots, \theta_M)$ , then  $(\theta'_1, \dots, \theta'_M) L (\theta_1, \dots, \theta_M)$ .

Roemer (1991) analyzes the effects of a more egalitarian distribution on the median share, and, thus, on efficiency. Of course, if one starts at a distribution  $(\theta_1, \dots, \theta_M)$  with median share different from  $1/M$ , and one moves to  $(\theta'_1, \dots, \theta'_M)$ , where  $\theta'_i = t_i [1/N] + (1 - t_i) \theta_i$ , and if  $t_i$  is very close to one for all  $i$ , then each share, including the median one, moves very close to  $1/M$ . Otherwise, the conclusion depends on whether the median share, at both distributions, is above or below  $1/M$ . If it is above  $1/M$ , then it will decrease because, by Proposition 5, the median is attained at a (weakly) lower position, and all positions with shares greater than  $1/M$  get lower shares (see argument in the proof of Proposition 6). Thus, the median share decreases, i.e., it gets closer to  $1/M$ . In other words, if the median share is above  $1/M$ , then a more egalitarian distribution in the sense of Roemer will lower the median share, bringing it closer to  $1/M$  and, hence, improving efficiency. If, on the other hand, it is below  $1/M$  the outcome is indeterminate: the median is still attained at a (weakly) lower position, but positions with shares lower than  $1/M$  see their shares increased. The two effects are, thus, of opposite sign.

The comparison with the case of restricted unanimity can be summarized as follows. (A) there, incomplete unanimity necessarily yields inefficiency; here, efficiency may in principle result under incomplete equality; (B) there, the inefficiency always takes the form of too high a price (or too much public bad);

here, it can also be too low.(C) there, an improvement in the degree of participation always improves welfare; here, the conclusion holds only for particular forms of improvement in the degree of equality.

#### 4.6. One person, one vote

An alternative decision rule is pairwise majority voting with the rule "one person, one vote." Now, under single-peakedness, it is the most preferred point of the median voter that is a Condorcet winner. Formally, given  $(\theta_1, \dots, \theta_M)$ ,  $j$  is a median voter if, denoting by  $\#S$  the number of elements in the finite set  $S$  and writing:

$$M_j = \# \{h \mid \theta_h = \theta_j\},$$

$$M_{<j} = \# \{h \mid \theta_h < \theta_j\}$$

$$M_{>j} = \# \{h \mid \theta_h > \theta_j\}$$

one has:  $M_{<j} + M_j \geq M/2$  and  $M_{>j} + M_j \geq M/2$ . In a parallel manner to the case of the median shareholder, define the median-vote position to be  $M/2$ , if  $M$  is even, and  $(M + 1)/2$ , if  $M$  is odd. Any person whose share is that of the median-vote position is a median voter.

It is graphically clear that the median voter's share is never larger than the median share, with equality only if  $\theta_i = 1/M$ ,  $i = 1, \dots, M$ . Thus, not surprisingly, moving from one-share-one-vote to one-person-one-vote will typically decrease the price or the amount of the public good. This will entail greater efficiency if the median share is lower than  $1/M$ , but lower efficiency if it is larger than  $1/M$ .

Otherwise, the properties of the one-person-one-vote rule are similar to the previous one. As before, if the median voter's share is  $1/M$ , then the outcome is efficient, whereas otherwise is not. If the median voter's share is less than  $1/M$ , then the price (or the level of the public good) is too low. Indeed, in many

realistic distributions of corporate shares the median voter's share is close to zero, yielding a very low price or level of the public bad. If the median voter's share is less than  $1/M$ , then an increase in it will increase efficiency by increasing the price (or the level of the public bad).

Conversely, it is conceivable that the median voter's share is greater than  $1/M$ . (In the examples of Figure 4.5, the share of the median voter is  $101/3$  or  $105/3$ , greater than  $1/3$ .) Then the resulting price (or level of the public bad) is too high. A change that lowers the share of the median voter will increase efficiency by lowering the price.

Again, Lorenz domination is not sufficient for the share of the median voter to move towards  $1/M$ , as the example illustrated in Figure 4.5 shows: the median voter is in position 2 in both cases, and her share moves away from  $1/M$ .

The main difference from the one-share-one-vote case is that, now, Roemer-dominance implies increased welfare (as long as welfare is not maximized initially). The argument is very simple: the median-vote position,  $M/2$  or  $(M+1)/2$ , is unchanged. Without loss of generality, let it be  $M/2$ . Assume that  $(\theta'_1, \dots, \theta'_M) R (\theta_1, \dots, \theta_M)$ , which implies that, for some  $t_{M/2} \in (0, 1]$ :

$$\theta'_{M/2} = t_{M/2} [1/M] + (1 - t_{M/2}) \theta_{M/2} .$$

i.e., the median-voter share always moves towards  $1/M$ . Except for this difference, the comparison with the limited-unanimity rule summarized at the end of Section 4.4 is valid here.

## 5. Concluding comment

The example of the stench-generating firm is based in the presence of a resource that can be put into different uses: the air can be used for the breathing of humans and other forms of life, or as an input in a production process. Actually, many natural resources are of the "multiple use" variety. They may be