A MODEL OF THE CONGRESSIONAL COMMITTEE ASSIGNMENT PROCESS: CONSTRAINED MAXIMIZATION IN AN INSTITUTIONAL SETTING*

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ABSTRACT

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The substantive focus of this paper is an institutionalized process in the U.S. House of Representatives known as the committee assignment process. There is, however, a wider class of problems of which this is a special case, namely the classification and selection of personnel. After reviewing the temporal sequence of events that constitute the committee assignment process, the principal actors and their goals are identified. This permits the process to be characterized by self-interested actors engaging in goal-seeking behavior. Institutional constraints, a consequence of formal rules and scarcity, restrict the form that goal-seeking takes. With the specification of goals and constraints the entire process is formalized as a special kind of linear programming problem, called (naturally enough) the assignment problem. Given this formal structure a number of theoretical properties are established in an effort to understand the operating characteristics of this important institutional process.

It has become rather common in the scholarly literature on the United States House of Representatives to focus on the activities of the chamber's subunits -- its standing committees. Woodrow Wilson's intuition that "Congress in session is Congress on public exhibition, while Congress in its committee rooms is Congress at work" has been sustained in numerous studies. Indeed, two of the best descriptive congressional studies (Fenno (1966) and Manley (1970)) focus almost exclusively on committee activities.[1]

A second set of studies, acknowledging the import of the committee structure, has sought to determine who is admitted to which committee rooms. Empirical studies of the committee assignment process, following Master's (1961) classic piece, have exhaustively analyzed the data of the public record as they pertain to matters of committee personnel. Descriptive studies by Bullock (1969, 1971, 1972, 1973a, 1973b), Clapp (1964), Congressional Quarterly (1973), and Gawthrop (1966), as well as quasi-theoretical pieces by Bullock and Sprague (1969), Rohde and Shepsle (1973), Uslaner (1971) and Westefield (1973), have detailed various aspects of the committee assignment process.

While I shall not review the fruits of this research here, I should, before beginning, detail several ways in which this paper differs from the research cited above. Although this study is very much an examination of the committee assignment process, it is, first and foremost, a formal theoretical piece. I have made little effort to reproduce the richness of detail found in other studies; description will be a decidedly secondary concern here. Rather I shall be concerned with analytical categories and their theoretical consequences.

Second, the underlying gestalt of this paper is economic rather than sociological. The committee assignment process is viewed as an allocation phenomenon in which scarce but valued committee slots are allocated among a well-defined clientele group according to carefully specified rules. The sociological nexus, especially between party leaders, members of the committees charged with making assignments, other congressmen, and outside clientele groups is given only indirect attention.[2]

Third, though the model presented here is economic and relatively abstract, I am fully aware that it is a *real* institutional process I am studying. I have, consequently, attempted to defend a middle ground between the highly abstract choice-theoretic literature and the often atheoretical descriptive literature on Congress. The resulting product undoubtedly does a disservice to both sets of studies, but may nonetheless provide an approach that possesses both theoretical power and empirical utility. That, in any event, is my intention.

The paper proceeds as follows: after characterizing the temporal sequence of the committee assignment process and bringing some empirical results to bear when appropriate, I shall, for theoretical purposes, partition the process into three distinct stages: the request stage, the stage of negotiated structure, and the assignment stage. In this paper the first two stages are only briefly considered. A formal model of the last stage is presented, its assumptions defended, its similiarity to general assignment problems developed, and some of its consequences traced. I conclude with some observations about institutional analysis and institutional design.

I. Temporal Sequence of the Committee Assignment Process

Committee assignments are a party responsibility. Each party has created a Committee on Committees (CC) to parcel out committee slots to party members in the chamber.[3] At the beginning of each Congress the party CCs are faced with the task of filling vacancies in the standing committees.

Directly following the November election, newly elected and returning congressmen submit to their respective CCs their requests for committee assignments. For newly elected congressmen, the request list is in the form of a preference ordering of variable length. That is, some freshmen reveal a preference for only a single committee, providing no further information about their preferences for any of the remaining committees. Other freshmen list as many as eleven items in their preference ordering. A three-item preference ordering is typical.[4]

For returning members, on the other hand, an informal property right is operative: nonfreshmen, whenever feasible, may retain committee assignments held in the previous Congress if they wish. If a change is desired, however, a returning member may request a transfer to another (presumably more preferable) committee, in which case he voluntarily yields his property claim on his previously held committee slot; or he may request a dual assignment, in which case he retains his previously held slot and is given an additional assignment as well.[5] Only under extreme conditions (so long as the property right norm operates) can a returning member be forced to resign a previously held committee slot.

After requests are made lobbying for assignments begins. For some the effort is rather casual and uninvolved. Clapp (1963, p. 208) reports that "some new congressmen become preoccupied with selecting staff, making arrangements to leave their business enterprises, and preparing to move their families to Washington, and do not concern themselves seriously with the matter of committee assignments. Sometimes inaction results neither from lack of interest nor a faulty assessment of priorities, but rather uncertainty about what to do." For others the effort is much more active. Congressmen write letters to members of their CC, setting forth arguments in behalf of their requests; pay personal visits to members of the CG, party leaders, committee chairmen and their delegation dean; and solicit letters of recommendation on their behalf from their delegations, from party leaders outside the House, and from relevant clientele groups.[6]

At the opening of a new Congress, party leaders negotiate a committee structure. At this point they determine the size of each of the twenty standing committees[7] and the distribution of slots on each committee between the majority and minority parties. On each of these decisions party leaders are given legislative guidance by the Legislative Reorganization Act (LRA) of 1946 and subsequent amendments. That act specifies committee sizes and recommends a division of slots between majority and minority closely in accord with the party ratio in the chamber.[8] The time sequence is important here, for the negotiations between party leaders are conditioned by three sets of data:

1. The distribution of vacancies in the previous committee structure created by election defeats;

2. The majority/minority ratio in the chamber; and

3. The requests submitted by newly elected and returning members.

The sizes and party ratios on committees negotiated often reflect an attempt by leaders to accommodate new demands for committee slots and to avoid "bumping" members from committees as a consequence of dramatic changes in the chamber party ratio (this is the extreme condition, alluded to above, in which the property right norm is inoperative). One piece of evidence supporting a leadership "accommodation strategy" is the rapid growth in the number of committee slots. Westefield (1973, p. 149) reports that in the first Congress following the LRA, in 1947, there were approximately 485 committee slots. In the 92nd Congress (1971-72) there were approximately 650 slots -- an expansion of more than 33%.[9] It is apparent, then, that the structure eventually negotiated by party leaders is not an automatic consequence of an accounting formula; this stage of the committee assignment process is very much a political process.

The final stage of the assignment process is the actual allocation of slots to new and returning congressmen by party CCs. Their final recommendations must be ratified by their respective party caucuses, but this is typically *pro forma*.[10]

For our purposes, then, the committee assignment process may be characterized by the following temporal sequence:

1. the committee configuration in the (t-1)st Congress;

2. an exogenous shock[11] --an election;

3. **the submission of requests;**

4. **the negotiation of a committee structure for the t^{th} Congress;**

5. **the creation of a committee configuration for the t^{th} Congress by the party CCs.**

This temporal sequence suggests that the committee assignment process is, in fact, composed of three distinct but interrelated processes, each involving different sets of actors (or the same actors in different roles). In order to provide a theoretical account of <u>request behavior</u>, <u>negotiated structure</u>, and <u>committee assignments</u>, I examine actor goals and motives in the next three sections. The focus here is on maximizing behavior in an environment of scarce, valued commodities and competing maximizers.

II. Actor Motives and Goals at the Request Stage

At the request stage, the committee preference ordering, as revealed by the applicant in the request list he submits to his CC, is the behavioral datum of interest.[12] To account for these revealed preferences of applicants, an inspection of the motives of the applicants, themselves, is useful. Fenno (1973, p. 1), after extensive interviewing, has discovered three important goals:

> Of all the goals espoused by members of the House, three are basic. They are: <u>re-election</u>, <u>influence</u> within the House, and good public policy. All congressmen probably hold all three goals. But each congressman has his own mix of priorities and intensities -- a mix which may, of course, change over time. If every House committee provided an equal opportunity to pursue re-election, influence, and policy, congressmen holding various mixes would appear randomly distributed across all committees. Such is definitely not the case. The opportunity to achieve the three goals varies widely

> among committees. House members, therefore, match their individual patterns of aspiration to the diverse patterns of opportunity presented by House committees. The matching process usually takes place as a congressman seeks an original assignment or a transfer to a committee he believes well suited to his goals.

Bullock (1973b) has charted the mix of motives observed by Fenno for freshmen in the 92nd Congress. He finds that 69% of those he interviewed (N=52) mention re-election goals, 83% mention policy-making goals, while only 25% mention prestige or influence goals.[13] More interesting is the fact that particular goals are rather strongly related to the type of committee requested. Of those who gave highest priority to re-election, the most frequently requested committees were Agriculture, Interior, and Public Works -- committees generally regarded as constituency-oriented. Similarly, applicants with strong policy-making preferences requested Banking and Currency, Education and Labor, Interstate and Foreign Commerce, and Judiciary most frequently. These are the committees charged with legislating in some of the most controversial policy areas.[14] Finally, for those seeking prestige and influence in the chamber, Appropriations and Ways and Means were most frequently requested.

The personal goals of the applicants, while of prima facie importance and, seemingly, of empirical significance, do not fully account for the pattern of requests. An observation by Masters (1961, p. 36) is germane:

> Applicants usually list their order of preference, taking into account not only their personal desires but also advice from other members and their own assessments of where they stand the best chance to land at least an acceptable assignment.

That is, the revealed preferences of applicants typically reflect the preferences of others and the anticipated opportunity structure in the House (to be negotiated after their requests are submitted). These elements often provide operational meaning to applicant goals, especially when the applicant is uninitiated in the ways of the House. Bullock, for example, (1971, p. 526) has called our attention to "delegation preferences," reporting that "delegation leaders often will attempt to insure that their representation on such committees [ones important to the state] are uninterrupted." That interested delegation leaders intervene at the request stage is corroborated by the remarks of James Corman (D Calif). Corman, the zone representative responsible for "processing" California requests in the Democratic CC, comments, "I try to encourage people to take committee assignments where we're weak in California representation. (Congressional Quarterly Service, 1971, p. 281)."

Fenno (1973, pp. 19-20) provides some very revealing data on the role played by others in the formulation of applicant requests. He partitions "applicants" into four categories: "Self-starters decide on their own to seek a committee assignment. Inner-circle choices have the idea suggested to them by someone else. Co-opted members are taken off the committee on which they sit [this applies to nonfreshman only], without their request, and assigned to another committee. Assigned members are placed on a committee they did not request; if nonfreshman, they do not lose any other assignment they may already hold." He finds that most committees are populated by self-starters. There is, however, a substantial minority of inner-circle choices (17 per cent of those interviewed). Co-optation tends to be reserved for the money committees -- Appropriations and Ways and Means -- whereas assigned members are commonly found on duty committees.[15]

The opportunity structure, the second aspect of Masters' observation above, also has an impact on the form taken by requests. Where there is a climate of strong competition for scarce committee slots, it is to be expected that revealed preferences and "actual" preferences differ. Bullock's (1973b, p. 20) interviews with 92nd Congress freshmen reveal, for example, that "most of the members expressing a preference for Appropriations or Ways and Means admitted that there was really no chance of obtaining so coveted a plum during the initial term. Moreover several of these members acknowledged that while they had discretely voiced desires for vacancies on a prestigious committee to party leaders, they had not even bothered to list such requests with the committee on committees (emphasis added)."

While the actual omission of committees from applicant request lists because of the improbability of assignment success is significant only for exclusive committees (Appropriations, Rules, Ways and Means), another form of "anticipated reaction" behavior was discovered by Rohde and Shepsle (1973) for other committees. Applicants often hedge their bets when the expected opportunities for assignment are unclear. This hedging behavior takes the form of expanding the length of the request preference ordering. Sixty-four per cent of the freshman applicants (over four Congresses) included three or more entries in their request list, while only eight per cent of the nonfreshman applicants had as many as three items on their list. With nonfreshman the property right norm assures them of no worse an assignment than the ones they held in the previous Congress. For freshmen, on the other hand, a large number of (personally) undesirable assignments are possible. If they assume that their requests will be honored whenever feasible, they will want to provide their CC with a relatively generous set of options. Nonfreshmen needn't fear this possibility.

Additional evidence provided by Rohde and Shepsle (1973) on this point is found in competition within state delegations for vacancies to which the state has a legitimate claim. When a state vacancy exists on a committee and there is no competition for that slot within the delegation, 75 per cent of the members expressing a preference for that slot listed at most one other committee on their request lists. If there is state competition, however, all members listed at least three preferences. As in the freshman/nonfreshman distinction, anticipated reactions are evident -- in this case because of the competition for state vacancies. It does appear, then, that the opportunity structure -- the distribution of vacancies, the nature of competition, expectations of appointment success -- affects the form if not the substance of request behavior.[16]

I have given a somewhat detailed, though selective, presentation of evidence as it relates to motives and behavior at the request stage in order to give the reader some feel for the complexity of things. As the reader may have gathered, scholarly attention to this aspect of the committee assignment process has resulted in a great deal of hypothesizing, but little theorizing -- a complaint that might well be lodged against most of the research on Congress. There are, however, some partially formulated theoretical notions about goal seeking, scarcity, and expectations which may yield in time to some general organizing principles.

In the next section I consider, in briefer fashion, the stage of negotiated structure and report on some theoretical lines of analysis found in the work of Westefield (1973).

III. Actor Motives and Goals at the State of Negotiated Structure

At the beginning of each Congress, it will be recalled, party leaders are charged with the task of organizing the chamber. While, in fact, the majority party has the votes to organize the chamber in any way it pleases, as a matter of practice there is negotiation between the party leaders of the majority and the minority on committee sizes and party ratios. Within bounds, each set of leaders appears to be willing to accommodate the needs of the other.[17]

The needs of party leaders, I assume, derive from two sources: policy preferences and institutional loyalty. Party leaders in the House are, first and foremost, party leaders. They are the liaison between their party in the chamber and the larger national party and the congeries of interests which are attracted to it. And, of course, for the party of the President, House leaders serve as chief advocates of the Administration program. It is enough, then, to observe that House leaders are motivated by policy concerns deriving from a number of sources (which I do not pursue here). These policy concerns are of greatest moment in the committee assignment process in the allocation of slots to the money committees (Appropriations, Ways and Means) and the agenda committee (Rules).[18]

Policy preferences, however, are modified to some extent by institutional loyalty. The clout of House leaders is, in a fundamental way, intertwined with the strength, i.e. "pivotalness," of the House in the larger political process, especially vis-a-vis the "other chamber," the bureaucracy, and the chief executive. This, in turn, is enhanced by the leaders' ability to get votes and other forms of cooperation from their party followers when they are needed. What follows, then, as far as committee assignments are concerned, is a leadership strategy of accommodation:

> The party leaders use their power over committee assignments variously, to reward members who have been loyal and cooperative, and to reinforce the strength of their own positions by rewarding members whose loyalty may be suspected but whose strength may no longer be safely disregarded (Masters, 1961, p. 43).

At the stage of negotiating structure, the practical problem for leaders is securing enough party slots on committees to satisfy the requests of party members. Inasmuch as leaders are privy, ahead of time, to the demand schedule for committee positions (see the temporal sequence in the first section), it is reasonable to suppose that their behavior in negotiating structure is heavily influenced by their need to accomodate member preferences.

Westefield (1973, Chap. 5) gives a rather natural economic interpretation to the leadership accommodation strategy. He constructs an argument in which "committee positions are given the status of a currency, a basis of exchange between the leaders and the followers." He continues:

> The leaders, we argue, perceive they can use the currency to accommodate the members and thereby induce the members to behave in ways the leaders desire. Indeed, the leaders can "manufacture" this currency and add to the resource base at their disposal.

Corroboration of this thesis is provided by Gawthrop (1966) and Westefield (1973): They document the relatively steady increase in the number of committee slots available,(see note 9 and the text to which it is appended). Surely this expansion has been negotiated by party leaders in order to provide themselves with a steady supply of bargaining levers.

Westefield's argument is more subtle -- and theoretically more powerful -- for he provides an equilibrating mechanism in his model. Committee assignments are valued precisely because they are scarce. If members had free access to any committee, then committees would play a far less consequential role in the life of the House. And, as a consequence, committee slots would be of less value to leaders as bargaining levers vis-a-vis their followers. Thus leaders are not free to "manufacture" the currency of committee slots at will. A second factor militating against the unrestricted creation of committee slots is currency inflation. Current holders of the currency, i.e. members already on committees in high demand, stand to lose as the currency they hold becomes less valuable. Thus the currently "wealthy" pose a countervailing force to the leadership tendency of currency inflation.

Westefield suggests that the most rapid "inflation" will occur in middle-level committees: low-level committees are not in enough demand to warrant very rapid expansion whereas very prestigious committees have strong countervailing forces associated with them.[19] "In short, the leaders try to buy cheap and sell dear." Although the evidence is mixed, there does seem to be a general trend of greater increases in middle-range committees than in either low-level or high-level committees (see Westefield, 1973, Figures 5.5 and 5.6).[20] --

To summarize, at the stage of negotiated structure party leaders are motivated by policy concerns and institutional loyalty. The former concern is most apparent in assignments to exclusive committees. The latter concern is manifested as an accommodation strategy in which the supply of party committee vacancies is adjusted, in part, to accommodate applicant demand.[21]

IV. Actor Motives and Goals at the Assignment Stage

At the assignment stage the party CCs assess applications for committee slots and make assignments. In addition to applicant preference orderings,"...both party groups are provided with materials (somewhat more detailed and elaborate for the Democrats) designed to facilitate their work. These materials include biographical information about first term members; party ratios, vacancies, and names of applicants for each committee; and in the Democratic Committee, an indication of who has endorsed the various candidates. (Clapp, 1964, pp. 213-214)."

While it is evident that party leaders, committee chairmen, state delegation leaders, clientele groups, and others outside the Chamber take an interest in the decisions of the party CCs, it is also clear that committee assignments are the party CCs' decisions to make. Indeed, in the case of the Democrats, authority over committee assignments is one of the chief attractions of the Ways and Means Committee.[22] Referring to a congressman who held no leadership position, possessed no policy expertise, but nonetheless was a power in the Chamber, one of the participants in Clapp's roundtable sessions reported:

> Much of his power rests with the fact he is on Ways and Means. Since that committee determines committee assignments, he is in a very important and strategic spot. He makes his deals with various groups as to which people he will support for certain spots. Naturally when the time comes that he wants something, he can make a request and people reciprocate (Clapp, 1964, pp. 29 - 30, emphasis added).

It follows, then, that members of the CC are in the quid pro quo business. Their influence in the Chamber derives from the favors and access they are able to extract from members whom they have assisted. Although they may respond to the party leadership on the filling of exclusive committee vacancies, they are free agents otherwise.[23]

In what follows, I assume that the CC acts so as to maximize (in all but exclusive committee assignments) the satisfaction of its party applicants. That is, the members of the CC cooperate with one another, in the sense that they make the internal trades and come to agreement on how the "spoils" shall be divided, in order to maximize the net value of the CC as an allocator of scarce but valued resources. In sum, goal seeking for the CC involves maximizing the number of instances in which a quid pro quo may be demanded by a member of the CC (assuming that the members of the CC make the requisite internal bargains regarding how the spols will be shared), which, in turn, leads to a serious effort to match, as frequently as feasible, assignments with requests.

On the face of it this assumption is, I believe, plausible though an obvious oversimplification.[24] A careful specification of the management goal is provided in the next section to avoid confusion and reduce ambiguity. For now, however,one piece of evidence will enhance the assumption's plausibility. Rohde and Shepsle (1973) found, after investigating the request data of the 86th, 87th, 88th and 90th Congresses, that 46 per cent of all Democratic freshmen received their first preference; another 26 per cent received some other assignment listed in their preference ordering. Thus, nearly three-quarters of all applicants received something they requested. Bullock (1973b, p. 21) found an even more impressive success rate among 92nd Congress freshmen:

> Forty-seven of the 52 freshmen for whom data were collected received at least one assignment which they preferred and 13 lucky members obtained two of their preferences. Thus 90 per cent of the class [86 per cent of the Democrats] enjoyed some success in the distribution of committee seats, with 80 per cent of all appointments made coming from among the set of stated member preferences.

With success rates as high as those found by Bullock and Rohde and Shepsle, the assumption that members of the CC attempt to match requests and assignments appears reasonable and will be retained (see note 30 for a consideration of alternatives to this specification of CC goals).

V. Formal Rules as Constraints

In the last three sections I have attempted to specify a context in which actors seek to realize goals or objectives. Applicants seek "good" committee assignments; party leaders seek "responsive" followers with, in some instances, "correct" policy preferences; members of the CC pursue "profitable" trades, bargains, and _quid pro quos_; and other actors, e.g. state delegation leaders, committee chairmen, policy coalition leaders, and interest group representatives, through their interactions with the principal actors, attempt to influence the latter's interpretations, respectively, of "good," "responsive," "correct," and "profitable." Thus goal-seeking is the principal mode of behavior and a specific temporal sequence provides the behavioral context.

If this were all we had -- well-defined goals and a specific context -- we would be limited to a considerable extent in what we could say with confidence. One of the fortunate things about institutional analysis, however, is that the feasible range of behavior in pursuit of goals is greatly delimited by institutional constraints, or what might be called rules of the game. Some of these constraints are "natural" in the sense that they follow directly from a definition of scarcity. Others are simply agreed-upon rules of behavior, e.g. the property-right norm. Together these constraints define feasible outcomes. The domain of goal-seeking, then, is restricted to a feasible set defined by institutional constraints. Our task in this section is to identify these institutional constraints and to give them a formal characterization. Before that, some notational conventions are stated.

Let $M = \{1,2,\ldots, m\}$ be a set of m applicants for committee assignments in the i^{th} Congress.[25] Let $C = \{c_1,c_2,\ldots, c_n\}$ be the set of n committees with the following subsets:

> $E = \{c_1,c_2, \ldots, c_e\}$ is the subset of exclusive committees;
>
> $S = \{c_{e+1}, c_{e+2},\ldots, c_s\}$ is the subset of semiexclusive committees; and
>
> $N = \{c_{s+1},c_{s+2},\ldots, c_n\}$ is the subset of nonexclusive committees.[26]

Finally let $v = (v_1,v_2,\ldots, v_n)$ be the committee vacancy vector. The v_i are a function of the election returns and the decisions made at the stage of negotiated structure. Obviously $v_i \geq 0$ for all i.

Define an mxn assignment matrix A with typical element a_{ij}. The element a_{ij} gives the disposition of the i^{th} congressman vis-a-vis the j^{th} committee. If $a_{ij} = 1$ then i is assigned to committee j; if

$a_{ij} = 0$ he is not (we have not, as yet, interpreted values of a_{ij} other than zero or unity). The assignment matrix A is a formal characterization of a <u>decision</u> by the CC. Its m rows are associated with the m applicants; its n columns are associated with the n committees. Each cell in a given row determines whether the applicant associated with the row is assigned to the committee associated with the column.

The CC is not unrestricted in the assignments it can make. Some of these restrictions, as I observed above, are "natural"; others are formal rules imposed by the Legislative Reorganization Act and its amendments or by the party caucus. Restrictions fall neatly into two categories: <u>apportionment constraints</u> and <u>service restrictions</u>.

<u>Apportionment Constraints</u>

[I] The number of assignments to the j^{th} committee may not exceed v_j.

$$\sum_{i=1}^{m} a_{ij} \leq v_j \qquad j = 1,\ldots,n$$

There are n constraints of this variety. If they are satisfied as equalities then all vacancies are filled; otherwise some vacancies remain unfilled.[27]

[II[Every congressman must serve on at least one committee.

$$-\sum_{j=1}^{n} a_{ij} \leq -1 \qquad i = 1,\ldots, m$$

[III] No congressman is permitted to serve on more than two committees.[28]

$$1 \leq \sum_{j=1}^{n} a_{ij} \leq 2 \qquad i = 1,\ldots, m.$$

Service Restrictions

[IV] A congressman may serve on at most one exclusive committee; if he does he may serve on no other committee. A congressman serving on a semiexclusive committee may serve on at most one nonexclusive committee.

$$3 \sum_{j=1}^{e} a_{ij} + 2 \sum_{j=e+1}^{s} a_{ij} + \sum_{j=s+1}^{n} a_{ij} \leq 3 \qquad i=1,\ldots,m$$

It should be noted at this point that fractional assignments have not been dismissed. For example, none of the constraints thus far prohibit a congressman from receiving one and a half slots on a semiexclusive committee (constraint [IV] is satisfied as an equality). Or, to give another bizarre example, an applicant may receive one-third of a slot on each of three exclusive committees without violating any of the constraints. Of course, these illustrations are substantively nonsensical. Shortly, however, I prove a rather nonobvious result, namely that fractional assignments are optimal only under very special conditions. In many cases they pose no problem.

[V[A congressman may serve on no more than one semiexclusive committee.[29]

$$\sum_{j=e+1}^{s} a_{ij} \leq 1 \qquad i = 1,\ldots,m$$

[VI] A congressman may not receive a multiple assignment to the same nonexclusive committee.

$$a_{ij} \leq 1 \qquad i = 1,\ldots, m \qquad j = s+1,\ldots,n.$$

These constraints are, for the most part, self-explanatory. Notice that they go part of the way toward eliminating some of the bizarre examples above (though not entirely).

Constraint classes [I]-[VI] define the set of feasible assignments as specified by the formal rules of the game. For technical as well as obvious substantive reasons, one additional class of constraints is included:

[VII] nonnegativity

$$-a_{ij} \leq 0 \qquad i=1,\ldots,m \qquad j=1,\ldots,n.$$

Having characterized the domain of feasible A- matrix values formally, I conclude this section with a mathematical statement of the CC objective: the management goal.

Objective Function

Recall that the management goal has the CC attempting to maximize the satisfaction of its applicant clientele by matching, to the extent feasible, assignments with requests. Let us, then, define a preference matrix $P = [p_{ij}]$, of the same order as the assignment matrix A, where

$$p_{ij} = 1 \quad \text{if applicant } i \text{ lists committee } j \text{ in his preference ordering}$$
$$= 0 \quad \text{otherwise.}$$ [30]

CC Objective Function: $\max\limits_{A} \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} p_{ij} a_{ij}$

That is, the CC's objective is to select a configuration of assignments -- an A matrix -- that maximizes the "correlation" between expressed preferences and actual assignments. This is accomplished by the A matrix that maximizes the product of its elements and the respective elements of P.

The task, then, of the CC is to select a matrix consisting of nm variables so as to maximize an objective function linear in those variables, subject to 2mn - ms + 4 m + n constraints linear in those variables.[31] What we have is a linear programming problem of a very special sort: it is a variant of the general assignment problem.

VI. Assignment Problems[32]

Assignment problems constitute a general class of problems concerned with the efficient allocation of indivisible resources. While they are surveyed in most linear programming texts, e.g. Gale (1960), Karlin (1959), Vajda (1961), they have had very little impact on the way indivisibilities are treated in political allocation problems.[33] Assignment problems have evoked some interest, however, in a number of quarters. A general, detailed survey is provided by Motzkin (1956); algorithmic discussions (especially as it relates to the classic transportation problem) are found in Balinski (1968), Dantzig (1959, 1960), Kuhn (1955, 1956), Markowitz and Manne (1957), and Tornqvist (1953); relevant theoretical treatments appear in Gale (1956a), Goldman (1956), Heller and Tompkins (1956), Hoffman and Kruskal (1956), Hoffman and Kuhn (1956), Hoffman and Markowitz (1963), Shapley (1962), and von Neumann (1953); finally, applications of assignment problems include suburban housing and community development (Barr, 1973), optimal college admissions policies (Gale and Shapley, 1962), optimal pairing of marriage partners (Gale and Shapley, 1962), optimal location of production facilities (Koopmans and Beckmann, 1957), and trading on the Bohm-Bawerk horse market (Shapley and Shubik, 1972).[34]

In the remainder of this section I present the simple assignment problem in order to motivate the results pertaining to contraints [I]-[VII] and the objective function given in the previous section. The reader is cautioned to observe that the simple assignment differs in significant ways from the committee assignment problem of the last section. This difference is spelled out below. Nevertheless, the results reported in this section apply to the committee assignment problem as well.

Suppose there is a legislature consisting of n members and n committee slots.[35] Assignments to committees are governed by the following simple rules, where a_{ij} is the extent to which the i^{th} individual is assigned to the j^{th} slot:

(C.1) Each legislator is "completely assigned"

$$\sum_{j=1}^{n} a_{ij} = 1 \qquad i = 1,\ldots,n$$

This precludes multiple assignments and less-than-exhaustive assignments (with regard to individuals). It does not, however, prohibit fractional assignments of members to committees (so long as the fractions, for each i, sum to unity).

(C.2) Every vacancy is filled

$$\sum_{i=1}^{n} a_{ij} = 1 \qquad j = 1,\ldots,n$$

This requires an assignment exhaustive with regard to committee slots.

(C.3) Nonnegativity

$$a_{ij} \geq 0 \quad i=1,\ldots,n \quad j = 1,\ldots,n.$$

(C.1) through (C.8) define $n^2 + 2n$ constraints, although only $n^2 + 2n - 1$ of them are independent. Since $\sum_i(\sum_j a_{ij}) = \sum_j(\sum_i a_{ij})$, one of the constraints from (C.1) or (C.2) can be derived from the 2n-1 others.

Definition: An assignment is said to be feasible if it satisfies (C.1) through (C.3).[36]

The set of feasible points in n^2 - space (since there are n^2 variables a_{ij}) is characterized by the intersection of 2n-1 hyperplanes (C.1 and C.2) with n^2 half-spaces (C.3). Since (C.1) through (C.3) require $0 \leq a_{ij} \leq 1$,[37] for all i and j, i.e. the a_{ij}'s are bounded, and since the constraints are linear in the a_{ij}'s, the set of feasible points is a convex polyhedron.

THEOREM 1 (Koopmans and Beckmann, 1957): A linear objective function defined on a convex polyhedron reaches its maximum at a vertex. If it does not reach a maximum at a vertex, then it reaches a maximum at no other point in the polyhedron. If it reaches a maximum at more than one vertex, then it achieves a maximum at every point on the face of the polyhedron defined by those vertices.

An extremely rigorous proof of this theorem is given in Koopmans (1951, p. 88, note 17). A less rigorous proof is given below, but first some geometric intuition is brought to bear. Consider the simple linear system, consisting of five constraints in two-space, depicted in Figure 1:

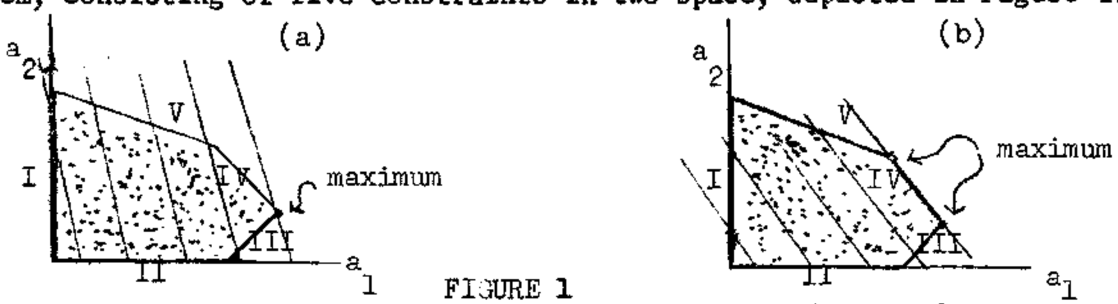

FIGURE 1

I and II are nonnegativity constraints and III, IV, and V are linear constraints. The dotted region -- a convex polyhedron -- is the feasible set of values of a_1 and a_2. Projections of the objective function, which is linear in a_1 and a_2, are also given in Figure 1. For any two points on the same projection, the values of the objective function are identical. Any point on a higher projection (one to the "northeast") determines a larger value for the objective function than a point on a lower projection (one to the "southwest"). If the contours of the objective function are not parallel to any of the constraint hyperplanes, as in (a), then a unique vertex of the convex polyhedron maximizes the objective function. If, on the other hand, the contours are parallel to a constraint hyperplane (other than the nonnegativity constraints), as in (b), then several vertices and the face defined by them maximize the objective function.

Proof of Theorem 1:[37a]

Let C(a) be a convex polyhedron in n-space with vertices $a^1, a^2, \ldots, a^q$ (q finite), i.e.
$a^i = (a^i_1, a^i_2, \ldots, a^i_n)$. Let $v(a) = \beta_1 a_1 + \beta_2 a_2 + \ldots + \beta_n a_n$ be the objective function to be maximized.

Consider the point

$$a^* = \lambda_1 a^1 + \lambda_2 a^2 + \ldots + \lambda_q a^q$$

where $\sum_i \lambda_i = 1$, $\lambda_i > 0$ for at least two values of i, and $\lambda_i \geq 0$ for all i, i.e. a* is not itself a vertex. By the linearity of v,

$$v(a^*) = \lambda_1 v(a^1) + \lambda_2 v(a^2) + \ldots + \lambda_q v(a^q).$$

If a* maximizes v, then

$$v(a^*) = v(a^i) \text{ for all i for which } \lambda_i > 0,$$

in which case a vertex maximizes v. If this were not the case then $v(a^j) < v(a^*) < v(a^k)$ (where $\lambda_j, \lambda_k > 0$) for some j and k, in which case a* is not a maximum of v (since $v(a^*) < v(a^k)$). Therefore, if v has a maximum in C(a) it occurs at a vertex, though it may occur elsewhere as well. Alternatively, if the vertices $a^1, a^2, \ldots, a^p$ $(p \leq q)$ are maxima of v in C(a), then all points of the form $a^* = \lambda_1 a^1 + \lambda_2 a^2 + \ldots + \lambda_p a^p$, where $\sum_i \lambda_i = 1$, $\lambda_i \geq 0$, are also maxima. This follows from the linearity of v and the fact that $v(a^1) = v(a^2) = \ldots = v(a^p)$ since they are all maxima: $v(a^*) = \lambda_1 v(a^1) + \lambda_2 v(a^2) + \ldots + \lambda_p v(a^p) = v(a^1) = v(a^2) = \ldots = v(a^p)$. Therefore, if v has a maximum in C(a) it occurs at a vertex, and if several vertices are maxima then their convex combinations are too.

Q.E.D.

Theorem 1 tells us that an optimal assignment, for a given linear objective function, occurs at (at least) one of the vertices of the feasible set. The next task, then, is to identify the vertices of (C.1) through (C.3) and to examine their characteristics. The following definition will ease the discourse:

Definition: A feasible assignment is called optimal (or a solution to the assignment problem) if and only if it maximizes a given objective function.

Theorem 1 asserts that the solution of an assignment problem is a vertex. The vertices of (C.1) through (C.3) are given in a well-known theorem of Birkhoff (1946).

Definition: A feasible assignment is called a permutation if and only if $a_{ij} = 0$ or 1 for all i and j.

The permutation matrices (themselves A matrices), for any n, are quite literally the permutations of individuals among slots (or slots among individuals).

For n=2, they are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} .$$

For any n, there are n! permutations.

Theorem 2 (Birkhoff, 1946): A feasible assignment can be written as a weighted average, with nonnegative weights, of the n! permutations:

$$a_{ij} = \sum_{r=1}^{n!} \lambda_r (a^*_{ij})_r$$

$\lambda_r \geq 0$, $\sum_{r=1}^{n!} \lambda_r = 1$, and $(a^*_{ij})_r$ is the r^{th} permutation.

A proof of this theorem is provided by Koopmans and Beckmann (1957, Appendix A).

Theorem 2 identifies the vertices of the convex polyhedron defined by (C.1) through (C.3) as the set of permutations. Together with Theorem 1, this has a nonobvious consequence: the maxima (if it exists at all) of any objective function, linear in assignments, results in an integral assignment. Thus even if fractional assignments were feasible in a substantive sense, there would never be any need to resort to them.

The economic consequences of these theorems, especially as they relate to decentralized markets and price systems, are traced by Koopmans and Beckmann (1957) and will not concern us here. The method, however, will prove most useful.

Before returning to the "real world" -- I take constraints [I] through [VII] and the objective function of the previous section to be a close approximation of the "real world" -- some major differences between the simple assignment problem of (C.1) through (C.3) and the committee assignment problem of the last section should be made explicit. First, while it is apparent that the simple assignment problem has a feasible set, the same may not be said about the committee assignment problem. The simple assignment problem, at the outset, provides n slots for n members. The constraints require the n slots to be allocated so that each member receives, in total, exactly a "full" assignment. The committee assignment problem, on the other hand, gives no assurances of feasibility. There is no guarantee, that is, that the vacancy vector v provides enough slots to satisfy the other constraints. In the next section, then, it is necessary to examine the question of feasibility in detail.

Constraints (C.1) to (C.3) of the simple assignment problem suggest an extremely simple structure. Because of the simple structure, e.g. the coefficients in all constraints equations are zeros and ones, Birkhoff's Theorem readily identifies the vertices as the simple permutations. The structure of the committee assignment problem is much less simple and elegant -- one of the high costs of dealing with *real* institutional processes. It will take considerably more analytical effort to achieve interesting results.

A third important difference between (C.1) and (C.3) and [I] through [VI] (excluding the common nonnegativity conditions) is seen at a glance. The former are *equality* constraints while the latter are *inequalities*. As a result, the *committee* assignment problem opens up an additional possibility: incomplete assignments. And empirically (see note 27) there are numerous instances of committee slots left vacant by the party CCs.

Finally, there is the problem of multiple assignments. Constraints [IV]-[VI] provide for the possibility of multiple assignments in the committee assignment problem. This feature, perhaps more than any other, distinguishes the committee and simple assignment problems.[38]

In the next section I begin the task of mapping out the theoretical consequences, i.e. the operating characteristics, of the committee assignment problem. The first matter to be dealt with is feasibility.

VII. Committee Assignment Problem: Results

Feasibility Theorem; The condition both necessary and sufficient for feasibility is

$$\sum_{j=1}^{n} v_j \geq m.$$

Proof:

(1) Necessity:

From [I], feasibility $\Rightarrow \sum_{i=1}^{m} a_{ij} \leq v_j \qquad j=1,\ldots,n$

$$\Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij} \leq \sum_{j=1}^{n} v_j$$

From [II], feasibility $\Rightarrow \sum_{j=1}^{n} a_{ij} \geq 1 \qquad i=1,\ldots,m$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \geq m$$

By transitivity the condition follows and necessity is established.

(2) Sufficiency:

Suppose $\sum_{i=1}^{n} v_j \geq m$. Then arbitrarily select m of the vacancies, assign them (again arbitrarily) one to each i, and leave the remaining vacancies (if any) unfilled. [I] and [II] are clearly satisfied, and [III] through [VI], which apply to multiple assignments, are trivially satisfied. [VII], too, is satisfied since $a_{ij} = 1$ or 0. Sufficiency is established.

Q.E.D.

I assume in the remainder of this paper that the condition in the Feasibility Theorem is satisfied. That is, at the stage of negotiated structure, allowance is made for the number of applicants in the creation of vacancies. For a similar theorem as applied to transportation problems, see Gale (1960, p. 5).

In order to underscore the complexity of our problem, it is useful, now that feasibility problems have been disposed of, to focus on some of the other features that distinguish it from the simple assignment problem. The first is completeness.

Definition: An assignment is said to be complete if and only if

$$\sum_{i=1}^{m} a_{ij} = v_j$$

for every j.

That is, a complete assignment is one in which each class [I] constraint is satisfied as an equality.

THEOREM 3: For some configurations of vacancies, every feasible assignment satisfies some constraint in [I] as an inequality. That is, for some vacancy vectors, no feasible assignment is complete.

Proof:

A simple example establishes the result. Whenever $\sum_{j=1}^{e} v_j > m$, no complete assignment is feasible (since [IV] is violated otherwise). Other conditions rendering complete assignments infeasible are:

(1) $\sum_{j=e+1}^{s} v_j > m$

(2) $\sum_{j=e+1}^{n} v_j > 2 \cdot m$

and so on.

Q.E.D.

Even if we were to suppose that party leaders, at the stage of negotiated structure, attempt to provide the CCs with a vector of vacancies that can be completely allocated, the CC objective function may not be compatible with a complete assignment.[39]

<u>THEOREM 4: Even when complete assignments are feasible, they may not be optimal.</u>

<u>Proof</u>:

An example illustrates this result. Let us assume that only three committees have vacancies: a semiexclusive committee with three vacancies ($v_s = 3$) and two nonexclusive committees with one and two vacancies, respectively ($v_{n-1}=1$, $v_n=2$). Suppose further that there are three applicants, X, Y and Z, whose preferences are as follows:

<u>X</u>	<u>Y</u>	<u>Z</u>
c_{n-1} c_n	c_s	c_s
c_s	c_n	c_n
	c_{n-1}	c_{n-1}

That is, X prefers c_{n-1} and c_Q to c_s, but is indifferent between the two nonexclusive committees. Y and Z, on the other hand, strictly prefer

c_s to c_n to c_{n-1}. Suppose the CC, in weighting these requests (see note 39), imputes interpersonally comparable utilities to the revealed preferences[40] and seeks to maximize "social welfare." They employ the following P matrix:

		committees		
		c_s	c_{n-1}	c_n
	X	1	10	10
applicants	Y	10	1	5
	Z	10	1	2

A feasible complete assignments, A^o, is:

		committees		
		c_s	c_{n-1}	c_n
applicants	X	1	1	0
	Y	1	0	1
	Z	1	0	1

A quick inspection of [I] establishes completeness, while the condition of the feasibility theorem establishes feasibility. The value of A^o is

$$\begin{aligned} v(A^o) &= \sum_i \sum_j p_{ij}\, a_{ij} \\ &= (1 + 10 + 0) + (10 + 0 + 5) + (10 + 0 + 2) \\ &= 38. \end{aligned}$$

Among feasible complete assignments, A^o maximizes v(A). No other complete assignment yields as high a value of v(A). This follows because completeness requires that each member be assigned to c_s. Consider now the incomplete assignment A*:

		committees		
		c_s	c_{n-1}	c_n
	X	0	1	1
applicants	Y	1	0	1
	Z	1	0	0

Note that $\sum_i a_{is} < v_s$, so A* is incomplete. However,

$$v(A^*) = (0 + 10 + 10) + (10 + 0 + 5) + (10 + 0 + 0)$$
$$= 45$$
$$> v(\dot{A}).$$ [41]

Q.E.D.

A summary to this point is in order. Having given the necessary and sufficient condition for feasibility (Feasibility Theorem), I have demonstrated that for some vacancy configurations no complete assignment is feasible.(Theorem 3). For others, complete assignments may be feasible, but they are not optimal. (Theorem 4). These results have underscored three important characteristics of the assignment process: feasibility, completeness, and optimality. One last characteristic -- that of an integral assignment -- is considered in Theorem 5. But first two important lemmas.

LEMMA 1: Any convex combination of complete assignments is itself a complete assignment.

Proof:

Let $A^* = \lambda_1 A^1 + \lambda_2 A^2 + \ldots + \lambda_r A^r$,
where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$, and the A^i's are complete. Consider the j^{th} committee in A* (the j^{th} column of A*):

		committees		
		c_s	c_{n-1}	c_n
	X	0	1	1
applicants	Y	1	0	1
	Z	1	0	0

Note that $\sum_i a_{is} < v_s$, so A* is incomplete. However,

$$\begin{aligned} v(A^*) &= (0 + 10 + 10) + (10 + 0 + 5) + (10 + 0 + 0) \\ &= 45 \\ &> v(\dot{A}). \end{aligned}$$ [41]

Q.E.D.

A summary to this point is in order. Having given the necessary and sufficient condition for feasibility (Feasibility Theorem), I have demonstrated that for some vacancy configurations no complete assignment is feasible.(Theorem 3). For others, complete assignments may be feasible, but they are not optimal. (Theorem 4). These results have underscored three important characteristics of the assignment process: <u>feasibility</u>, <u>completeness</u>, and <u>optimality</u>. One last characteristic -- that of an <u>integral assignment</u> -- is considered in Theorem 5. But first two important lemmas.

<u>LEMMA 1: Any convex combination of complete assignments is itself a complete assignment.</u>

<u>Proof:</u>

Let $A^* = \lambda_1 A^1 + \lambda_2 A^2 + \ldots + \lambda_r A^r$,

where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$, and the A^i's are complete. Consider the j^{th} committee in A* (the j^{th} column of A*):

$$a^*_{1j} = \lambda_1 a^1_{1j} + \lambda_2 a^2_{1j} + \ldots + \lambda_r a^r_{1j}$$

$$a^*_{2j} = \lambda_1 a^1_{2j} + \lambda_2 a^2_{2j} + \ldots + \lambda_r a^r_{2j}$$

$$\vdots$$

$$a^*_{mj} = \lambda_1 a^1_{mj} + \lambda_2 a^2_{mj} + \ldots + \lambda_r a^r_{mj}$$

$$\sum_{i=1}^{m} a^*_{ij} = \lambda_1 \sum_{i=1}^{m} a^1_{ij} + \lambda_2 \sum_{i=1}^{m} a^2_{ij} + \ldots + \lambda_r \sum_{i=1}^{m} a^r_{ij}$$

$$= \lambda_1 v_j + \lambda_2 v_j + \ldots + \lambda_r v_j$$

since each A^i is complete

$$= v_j \qquad \text{since} \quad \sum_i \lambda_i = 1$$

Q.E.D.

<u>LEMMA 2: A complete assignment is either a convex combination of complete assignments or it is an extreme point</u>.

<u>Proof</u>:

Suppose A is complete and feasible, but is neither a (nondegenerate) convex combination of complete assignments nor itself extreme. Since A is not extreme it must be a convex combination of extreme points, some of which are complete and some of which are not. Suppose

$$A = \lambda_1 A^1 + \lambda_2 A^2 \qquad (\lambda_1 + \lambda_2 = 1)$$

Without loss of generality, suppose A^2 is incomplete in the j^{th} column: $\sum_i a^2_{ij} < v_j$.

Then

$$a_{ij} = \lambda_1 a^1_{ij} + \lambda_2 a^2_{ij} \qquad i = 1,\ldots,m$$

$$\sum_i a_{ij} = \lambda_1 \sum a^1_{ij} + \lambda_2 \sum_i a^2_{ij}$$

The left hand side is v_j, since A is complete, but the right-hand side is less than v_j since $\sum_i a^2_{ij} < v_j$ by construction. The contradiction establishes the result.

Q.E.D.

We have not, to this point, restricted the a_{ij}'s to zero or one (though substantively this is all that makes sense). In fact, in the <u>simple</u> assignment problem, Theorem 2 demonstrates that integral assignments emerge as a consequence of goal-seeking behavior. It is, however, somewhat more problematic in the committee assignment problem.

<u>Definition</u>: An assignment is said to be <u>integral</u> if $a_{ij} = 0$ or 1 for all i and j.

Clearly if nonintegral A's are extreme points (vertices) of the polyhedron C(A), defined by [I] through [VII], then they will be optimal for <u>some</u> objective functions (see Figure 1). Unfortunately,

<u>THEOREM 5: For some vacancy configurations, nonintegral points are extreme.</u>

<u>Proof</u>:

Suppose there are two committees with three vacancies, an exclusive committee with one slot (v_e=1) and a nonexclusive committee with two slots (v_n=2). Let m = 2. By the Feasibility Theorem, feasible assignments exist. With these parameters, however, no <u>integral</u> assignment is complete: either one applicant receives the exclusive slot and the other one of the nonexclusive slots ([VI] prohibits him from receiving both) or both receive nonexclusive slots ([IV] prohibits an applicant from serving on any other committee if he receives an exclusive committee slot). <u>Yet there do exist nonintegral complete assignments</u>:

$$\begin{array}{ccc} & & \text{committee} \\ & & \begin{array}{cc} c_e & \quad c_n \end{array} \\ \text{applicant} & \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} X & 1 \\ 1-X & 1 \end{bmatrix} \end{array}$$

where $1/3 \leq X \leq 2/3$. Note that [IV] is satisfied by both applicants. From Lemma 2 a complete assignment is either a convex combination of complete assignments or is an extreme point. In either case, some non-integral points are extreme. In fact,

$$\begin{bmatrix} X & 1 \\ 1-X & 1 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1/3 & 1 \\ 2/3 & 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2/3 & 1 \\ 1/3 & 1 \end{bmatrix}$$

where $\lambda_1 = 2-3X$ and $\lambda_2 = 3X-1$, so that the latter two A-matrices are extreme points.

Q.E.D.

We must conclude, then, that nothing in the structure of the problem precludes fractional assignments. The example in Theorem 5 is our first sure illustration that nonintegral assignments may qualify as extreme points.

At this point there are several possible courses of action. One is to move away from the general linear programming model, opting instead for a more restrictive approach: <u>integer programming</u>.[42] This possibility is of interest, but will not be reported on here. A second, and I think more appealing, course of action is to draw on a descriptive feature alluded to earlier. It will be recalled that freshmen are rarely assigned to exclusive committees. In the 86th, 87th, 88th and 90th Congresses, no freshman (Democrat) was assigned to any of the exclusive committees. Moreover, when freshmen are assigned to these committees, it is typically at the behest of the party leadership.[43] Since more senior members are often co-opted to serve on exclusive committees, since the leadership plays an active role in recruiting for these committees, and since freshmen are rarely involved in exclusive committee recruitment, it makes sense in a study of freshmen assignments to separate out the process by which exclusive committee vacancies are filled and focus, instead, on the remaining committees. Hence:

<u>ASSUMPTION 1: $\sum_{i=1}^{e} v_i = 0.$</u>

This assumption has a very salutary effect as seen in the following consequence. With Assumption 1, constraint [IV] becomes

$$2 \sum_{j=e+1}^{s} a_{ij} + \sum_{j=s+1}^{n} a_{ij} \leq 3 \qquad i=1,\ldots,m$$

which, in turn, may be written as

$$\sum_{j=e+1}^{s} a_{ij} + \sum_{j=e+1}^{n} a_{ij} \leq 3 \qquad i = 1,\ldots,m$$

Moreover, since $\sum_{j=1}^{e} a_{ij} = 0$ by Assumption 1, the second term on the left-hand side is recast:

$$[\text{IV*}] \qquad \sum_{j=e+1}^{s} a_{ij} + \sum_{j=1}^{n} a_{ij} \leq 3 \qquad i = 1,\ldots,m.$$

<u>But this is simply the sum, for each $i=1,\ldots,m$, of constraints [III] and [V]</u>. This result and its consequence are given in LEMMA 3 and THEOREM 6.

LEMMA 3: Let X be the set of points satisfying the system of constraints

(1) $\sum_{j=1}^{n} \alpha_{ij} x_j \leq \gamma_i \qquad i = 1,\ldots,m.$

Then X satisfies

(2) $\sum_{j=1}^{n} \beta_j x_j \leq b$

where $\beta_j = \sum_{i=1}^{m} \lambda_i \alpha_{ij}$, $b = \sum_{i=1}^{m} \lambda_i \gamma_i$, and

$\lambda_i \geq 0.$

Lemma 3 simply asserts that any point satisfying a system of constraints (1) also satisfies any linear combination of those constraints. (This lemma is reminiscent of Farkas' Theorem -- see Goldman and Tucker, 1956, Theorem 3).

Proof:

Suppose $x = (x_1,\ldots,x_n)$ is a point in X satisfying (1).

Then $\sum_j \alpha_{ij} x_j \leq \gamma_i \qquad i = 1,\ldots,m$

Multiplying by λ_i: $\sum_j \lambda_i \alpha_{ij} x_j \leq \lambda_i \gamma_i \qquad i=1,\ldots,m$

$$\Longrightarrow \sum_i \sum_j \lambda_i \alpha_{ij} x_j \leq \sum_i \lambda_i \gamma_i$$

$$\Longrightarrow \sum_j x_j \left(\sum_i \lambda_i \alpha_{ij} \right) \leq \sum_i \lambda_i \gamma_i$$

$$\Longrightarrow \sum_j \beta_j x_j \leq b$$

and (2) is satisfied.

Q.E.D.

<u>THEOREM 6: The convex polyhedra C(a) and C'(a), defined by [I] through [VII] and [I] through [III] -- [V] through [VII], respectively, are identical.</u>

<u>Proof</u>:

By Assumption 1, [IV] becomes [IV*]. In particular, for the k^{th} applicant (i=k), [IV] becomes:

$$\sum_{j=e+1}^{s} a_{kj} + \sum_{j=1}^{n} a_{kj} \leq 3.$$

But, for the k^{th} applicant constraints [III] and [V] are, respectively:

$$\sum_{j=1}^{n} a_{kj} \leq 2$$

and $$\sum_{j=e+1}^{s} a_{kj} \leq 1$$

Letting [III] and [V] represent the constraint system (1), [IV*] represent constraint (2), and the relevant λ's be unity, Lemma 3 establishes that [IV*] is satisfied by all points in C'(a); hence it is identical to C(a).

Q.E.D.

The salutary effect of Assumption 1 is the following: it is possible to demonstrate the redundancy of class [IV] constraints (Theorem 6). Not only is the magnitude of the problem reduced with the elimination of these m constraints; as is seen below, precisely the "right" constraints have been eliminated, i.e. class [IV] turns out to be the culprit which made fractional extreme points possible.

<u>LEMMA 4 (Hoffman - Kruskal, 1956; Hoffman - Kuhn, 1956): If, in a system of linear inequalities with integral coefficients and constant terms, every non-singular square submatrix of the coefficient matrix has determinant $\pm$ 1, then every extreme solution is integral.</u>

Lemma 4 provides a useful sufficient condition for the determination of integral extreme points. Notice that [I] - [III], [V] - [VII] is a system of linear inequalities with integral coefficient and constant terms. Also note that a necessary condition for a system to satisfy the premises of the lemma is that every coefficient be 0, + 1, or -1 (since each coefficient is a <u>minimal</u> i.e. lxl, submatrix of the coefficient matrix). This is true of the committee assignment problem by virtue of Assumption 1 and Theorem 6; otherwise the coefficients of [IV] would have violated the premises of Lemma 4.

The property that every submatrix of the constraint coefficient matrix have a determinant of 0, + 1, or -1 is known as the <u>unimodular property</u> (Hoffman - Kruskal, 1956) or the <u>Dantzig property</u> (Heller - Tompkins, 1956).[44]

We have, in effect, replaced one problem with another. Lemma 4 tells us when a constraint system has integral extreme points. We need, however, some means of determining if the premises of Lemma 4 are satisfied. The next two lemmas and a definition provide such a strategy.

<u>LEMMA 5 (Heller - Tompkins, 1956): Let A be an mxn matrix whose rows can be partitioned into two disjoint sets B and C, with the following properties:</u>

(1) <u>every column of A contains at most two non-zero entries</u>;

(2) <u>every entry in A is 0, + 1, or -1</u>;

(3) <u>if two non-zero entries in a column of A have the same sign, then the row of one is in B, and the other in C</u>;

(4) <u>if two non-zero entries in a column of A have opposite signs, then the rows of both are in B, or both in C</u>.

<u>Then every minor determinant of A is 0, + 1, or -1.</u>

<u>Definition</u>: A submatrix B of A is said to be <u>Dantzig sufficient</u> for A if A is unimodular whenever B is.

That is, B is Dantzig sufficient for A if, after removing appropriate rows and columns of A to produce B and determining that B is unimodular, it is necessarily the case that A is unimodular too.

<u>LEMMA 6: Dantzig sufficiency in transitive</u>.

This Lemma follows directly from the previous definition. It asserts that if B is Dantzig sufficient for A, and C Dantzig sufficient for B, then C is Dantzig sufficient for A.

My strategy is as follows: beginning with the constraint coefficient matrix defined by [I]-[III], [V]-[VII], a series of Dantzig sufficiencies are employed to produce a matrix to which Lemma 5 is applied directly. With its unimodularity established, it follows from Lemma 6 that the original constraint matrix is unimodular and, from Lemma 4, that every extreme solution is integral. Before beginning this task, it is convenient to transform the system to matrix notation.

Instead of writing assignments as a matrix $A= [a_{ij}]$, it is convenient to write it as a column vector (which I call $\underline{a}$) by attaching the rows in A end-to-end in sequence:

$$\underline{a} = \begin{bmatrix} a_{11} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{1n} \\ a_{21} \\ \cdot \\ \cdot \\ \cdot \\ a_{2n} \\ \cdot \\ \cdot \\ \cdot \\ a_{m1} \\ \cdot \\ \cdot \\ \cdot \\ a_{mn} \end{bmatrix}$$

Similarly, the weight matrix $P = [p_{ij}]$ is written as a row vector:

$\underline{p} = [p_{11}, p_{12}, \ldots, p_{1n}, p_{21}, p_{22}, \ldots, p_{2n}, \ldots, p_{ij}, \ldots p_{m1}, p_{m2}, \ldots, p_{mn}]$

Write the coefficients from the constraint inequalities as rows of a matrix C and, finally, write the constant terms of the constraint system as a column vector

$$\underline{d} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_m \\ -1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 2 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_m \end{matrix}}\right\} [\mathrm{I}] \\ \left.\vphantom{\begin{matrix} -1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -1 \end{matrix}}\right\} [\mathrm{II}] \\ \left.\vphantom{\begin{matrix} 2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 2 \end{matrix}}\right\} [\mathrm{III}] \\ \left.\vphantom{\begin{matrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{matrix}}\right\} [\mathrm{V}] \\ \left.\vphantom{\begin{matrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{matrix}}\right\} [\mathrm{VI}] \\ \left.\vphantom{\begin{matrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{matrix}}\right\} [\mathrm{VII}] \end{matrix}$$

Notice that class [IV] has been eliminated. For convenience semiexclusive committees are now indexed $c_1, \ldots, c_s$, and nonexclusive committees $c_{s+1}, \ldots, c_n$ (i.e. there are still n committees indexed).

The committee assignment problem may now be written as a primal linear programming problem in matrix form:

$$\max_{a} \ \underline{p} \cdot \underline{a} \qquad \text{subject to}$$

$$C \cdot \underline{a} \leq \underline{d}.$$

The constraint system for five applicants and seven committees (three semiexclusive and four nonexclusive) is illustrated in Figure 2.

LEMMA 7: The matrix composed of coefficients from [I] and [V] is Dantzig sufficient for the entire constraint coefficient matrix, C.

Proof:

(a) Let C_1 be the submatrix of C with the mn rows defined by [VII] deleted. C_1 is Dantzig sufficient for C: the coefficients of [VII] appear as mn rows, each of which has (mn-1) entries of zero and one entry of -1. Consider an arbitrary (k x k) submatrix, K, of C, one (or more) of whose rows is from [VII]:

> case 1: The column of this row with the -1 entry is in K. Then compute Det (K) by expanding along this row. Det (K) = Det(K_1) where K_1 (in C_1) is the (k-1) x (k-1) submatrix of K with the type [VII] row and the column in which the -1 appears deleted. Since type [VII] constraints affect the determinant by a factor of -1, if K_1 is unimodular, so is K.
>
> case 2: Now the column of the type [VII] constraint row with the -1 entry is not in K. The row, then, is composed entirely of zeros. Clearly if K_1 (as defined above) is unimodular, so is K.

Together these two cases establish the proposition and the mn type [VII] rows may be deleted. (b) By Theorem 6, the submatrix C_2 of C_1 with the m type [IV] constraints deleted is Dantzig sufficient for C_1.

(c) Let C_3 be the submatrix of C_2 with the m rows of [II] deleted. C_3 is Dantzig sufficient for C_2: since any row of [II], say the r^{th} $(1 \leq r \leq m)$, is the negative of the r^{th} row of [III], any submatrix

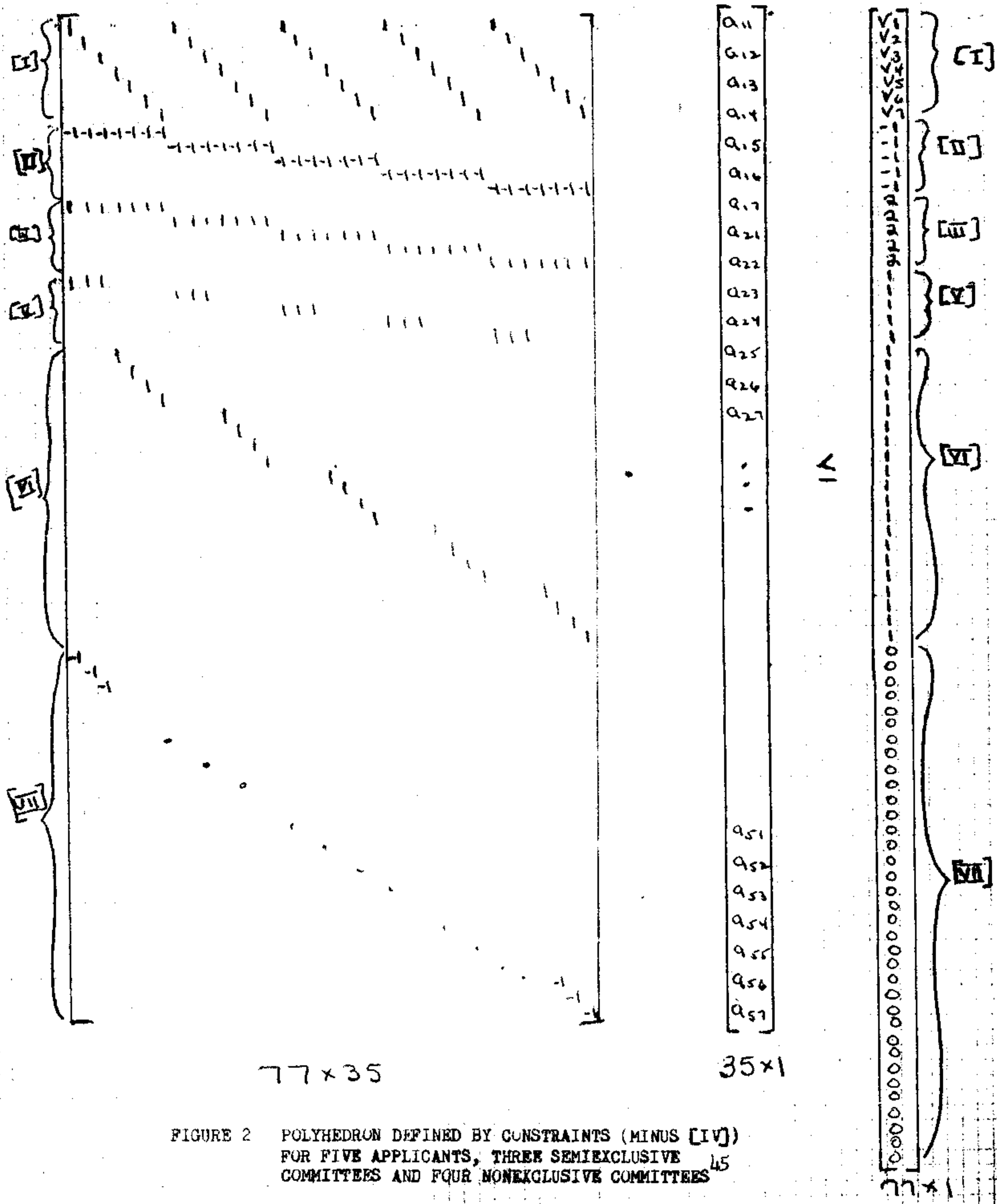

FIGURE 2 POLYHEDRON DEFINED BY CONSTRAINTS (MINUS [IV]) FOR FIVE APPLICANTS, THREE SEMIEXCLUSIVE COMMITTEES AND FOUR NONEXCLUSIVE COMMITTEES

of C_2 containing both has a zero determinant. Moreover, any submatrix of C_2 containing the r^{th} row of [II] and the s^{th} row ($s \neq r$) of [III] has a determinant differing only in sign from the submatrix containing the r^{th} and s^{th} rows of [III]. Therefore, if C_3 is unimodular so is C_2, and we may delete the type [II] coefficients.

(d) Let C_4 be the submatrix of C_3 with the m rows of [III] deleted. It is easy to see (consult Figure 2) that any row of [III] is simply the sum of the appropriate row of [V] and the (n-s) appropriate rows of [VI]. By virtue of this linear dependency (and the detailed reasoning set forth in Hoffman and Kuhn, 1956, pp. 205-206), <u>C_4 is Dantzig sufficient for C_3</u>.

(e) Let C_5 be the submatrix of C_4 with the m (n-s) rows of [VI] deleted. By reasoning identical to (a), <u>C_5 is Dantzig sufficient for C_4</u>. C_5 is an (m+n)xmn matrix whose elements are the coefficients of classes [I] and [V] exclusively. Since Dantzig sufficiency is transitive by Lemma 6, <u>C_5 is Dantzig sufficient for C.</u>

Q.E.D.

It is now possible to prove

<u>THEOREM 7: The constraint matrix C is unimodular.</u>

<u>Proof</u>:

By Lemma 7 attention is focused exclusively on the coefficients in [I] and [V]. It is shown that this matrix (C_5 of Lemma 7) satisfies the Heller - Tompkins conditions of Lemma 5. Condition (4) is trivially satisfied since no coefficients in [I] and [V] are negative. Condition (2) is satisfied since all coefficients are 0 or +1. Observe in [I] -- the reader may wish to consult Figure 2 -- that each column has <u>exactly</u> one non-zero element. In [V] each column has <u>at most</u> one non-zero element.

Therefore in C_5 no column has more than two non-zero elements. Hence, (1) is satisfied. Finally, to satisfy (3) simply partition C_5 into C_5^* and C_5^{**}. The former is the n x mn matrix defined by [I] alone; the latter is the m x mn matrix defined by [V] alone. With (1) - (4) of Lemma 5 satisfied, C_5 is unimodular. By Lemma 7, C is unimodular.

Q.E.D.

The next two theorems summarize developments to this point.

THEOREM 8: The extreme points of C are integral.

Proof:

This follows from Assumption 1, Theorem 7, and Lemma 4.

THEOREM 9: Any linear function of assignments is maximized at an integral assignment.

Proof:

This is a direct implication of Theorems 1 and 8.

Theorems 8 and 9 are important. Along with the Feasibility Theorem, they characterize three of the four important features of the committee assignment problem: feasibility, integral assignments, optimality. These results came at a cost -- namely Assumption 1 --but they are nonetheless important partial equilibrium results. These theorems notwithstanding, however, Theorem 4 should not be forgotten. Some of the integral extreme points represent incomplete assignments. And, as the example in Theorem 4 is intended to show, incompleteness is a fact of life whether Assumption 1 is employed or not.

In fact, a simple corollary of Theorems 1, 4, 8 and 9 is:

Corollary: The extreme points of the convex polyhedron C, defined by [I] through [VII], are a proper subset of all "permutations" of complete and incomplete assignments. In particular, every integral assignment, in which at least m vacancies are filled and distributed in accord with the remaining constraints, is optimal for some objective function.

Thus, for even moderate m, n, and v_i's, the number of extreme points is large indeed.[46]

Completeness, then, is still a characteristic of the committee assignment process about which we know little. As an empirical matter, the table in note 27 suggests that instances of incomplete assignments are not infrequent. Some theoretical mileage is purchased by returning to the original linear programming model.

Recall that the primal problem sought to

$$\max_{a} \ \underline{p} \cdot \underline{a} \quad \text{subject to}$$

$$C \cdot \underline{a} \leq \underline{d} \qquad\qquad [\,A\,]$$
$$\underline{a} \geq \underline{0}.$$ [47]

It is well known that associated with every maximum problem is a minimum problem known as the dual linear program:

$$\min_{b} \ \underline{b} \cdot \underline{d} \quad \text{subject to}$$

$$C^T \cdot \underline{b} \geq \underline{p} \qquad\qquad [\,B\,]$$
$$\underline{b} \geq 0.$$

The dual variables $b_1, b_2, \ldots, b_k$ (where $k = n + 3m + m\,(n-s)$ -- the number of constraints in [A]) are sometimes referred to as shadow prices by economists.[48] The principal theorem relating primal to dual is:

THEOREM 10 (Fundamental Duality Theorem): If $a_1, a_2, \ldots, a_{mn}$ and $b_1, b_2, \ldots, b_k$ are feasible solutions to [A] and [B], respectively, they are optimal if and only if

$$\sum_{i=1}^{mn} p_i a_i = \sum_{j=1}^{k} b_j d_j.$$

If either [A] or [B] is infeasible, then neither has an optimal solution.

A proof is found in Gale (1960, pp. 10-12).

In a substantive sense, the dual variables -- there is one associated with each constraint in the primal problem -- signify the marginal cost of satisfying constraints and the marginal value of resources (committee slots of each type and applicants of each "type").[49]

<u>THEOREM 11: The dual of the committee assignment problem is minimized at integral prices if the p- vector is integral.</u>

<u>Proof</u>:

Since p is integral, if C^T is unimodular then its extreme points will be integral (Lemma 4). A theorem analogous to Theorem 1 assures that there are integral minima. We show that C is Dantzig sufficient for C^T. By Theorem 7, then, C^T is unimodular. Let K be an arbitrary k x k submatrix of C. Det (K) = 0, + 1 or -1 from the unimodularity of C. But Det (K) = Det (K^T). Hence K^T, an arbitrary k x k submatrix of C^T is singular or has a determinant of $\pm$ 1 whenever C is unimodular. C, then, is Dantzig sufficient for C^T and, by Theorem 7, C^T is unimodular. With integral p's the result follows.

Q.E.D.

In solving the dual problem, we obtain integral prices, b_j. The next two results shed light on the question of optimal <u>complete</u> assignments.

<u>THEOREM 12 (Equilibrium Theorem): The feasible solutions $a_1, a_2, \ldots, a_{mn}$ and $b_1, b_2, \ldots, b_k$ of [A] and [B], respectively, are optimal solutions if and only if</u>

$$\underline{b_j = 0 \quad \text{whenever } C_j \cdot a < d_j \; (j = 1, \ldots k)}$$

<u>and</u>

<u>$a_i = 0$ whenever $C_i^T \cdot b > p_i$ $(i=1,\ldots,m)$ where C_j and C_i^T are the j^{th} and i^{th} rows of C and C^T, respectively.</u>

Gale (1960, pp. 19-20) provides a proof of this well-known theorem. The theorem asserts that optimal solutions to the primal and dual problem have the characteristic that a variable is zero whenever the associated constraint is satisfied as an inequality.

THEOREM 13: Only complete assignments are optimal if the first n dual variables, $b_1, b_2, \ldots, b_n$, are strictly positive.

Proof:

Observe that the first n constraints of the primal problem constitute class [I], i.e., those constraints that refer to the availability of vacancies. Let $a_1, a_2, \ldots, a_{mn}$ be optimal and suppose it is incomplete. Then, given the nature of the class [I] constraints, it must be the case that

$$C_j \cdot \underline{a} < d_j \quad (= v_j)$$

for some $j = 1, \ldots, n$. In fact, suppose it is true for $j = r$. By the "only if" part of Theorem 12, the optimality of $a_1, \ldots, a_{mn}$ implies $b_r = 0$.

Incompleteness of an optimal assignment, then, implies some zero dual solution components. The contrapositive of this statement gives the desired result: a strictly positive dual solution implies complete optimal assignments.

Q.E.D.

Although I have been unable to prove it, I believe that whenever the weights of the primal objective function (the $\underline{p}$ vector) is strictly positive, the strict positiveness of the first n dual variables is necessary, as well as sufficient (Theorem 13), for an optimal assignment to be complete.

VIII. Concluding Remarks

I stop here not because the picture is complete in any sense, but rather because I fear the reader is exhausted. In an effort to map some of the general characteristics of the committee assignment process, I have omitted many details and failed to follow all the leads this analysis has generated. Of obvious interest is the empirical analysis and associated problems. In particular, what are the optimal assignments (for the various Congresses on which data are available) and how do they compare to actual assignments? I hope to report on this elsewhere.

A second interesting question might be dubbed the <u>weighted assignment problem</u>. Assuming that the actual assignments, A*, in a given Congress, differ from the allegedly optimal assignments, A^o (although the empirical analysis has only just begun, I think this outcome will generally hold), what set of weights, λ_{ij} (if any), will bring the optimal solution of a <u>weighted</u> objective function into line with actual assignments? That is, find $\lambda_{ij} \geq 0$ such that

$$\sum_i \sum_j \lambda_{ij} (p_{ij} a^*_{ij}) \geq \sum_i \sum_j \lambda_{ij} (p_{ij} a_{ij}),$$ where the a_{ij}'s are points in the convex polyhedron of [I]-[III], [V]-[VII] and the a^*_{ij}'s are actual assignments. Geometrically, this amounts to finding a transformation of the objective function that picks out, as the optimal extreme point, the assignments actually made. This is displayed in Figure 3.[50]

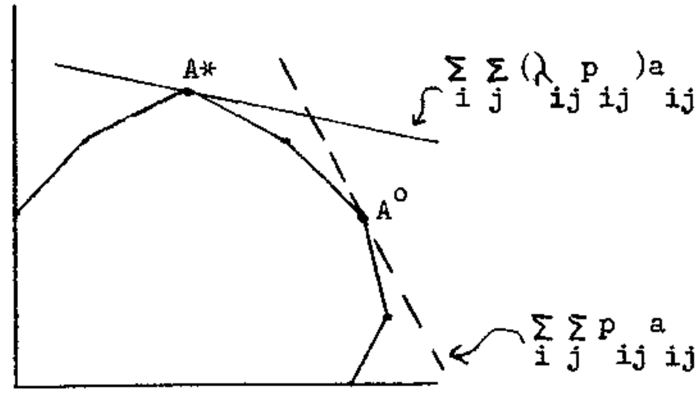

FIGURE 3

Let me also note that a more complete equilibrium model, in which transfer activity for nonfreshmen is incorporated and property-right constraints added, is of ultimate interest.

In the latter part of this essay I have abstracted away much empirical detail (though trying to persuade the reader, on the basis of evidence, that the formulation is reasonable). I shall not review the results here; the reader may wish to re-read the principal theorems. The strategy of analysis employed here -- conceptualizing goals and constraints in a formal structure and deducing consequences -- is limited and incomplete. Its ultimate worth depends on further theoretical and detailed empirical examinations. It does, however, have the virtue of turning attention away from descriptive detail at particular points in time and toward general institutional operating characteristics. In this fashion, I believe, we will secure more reliable knowledge about institutions and, for the normatively inclined, learn how to reform them and design better ones.

FOOTNOTES

1. Fenno (1962, 1966) focuses on appropriations politics and the House Appropriations Committee, while Manley (1970) analyzes the politics of finance with prominence given the House Ways and Means Committee. There have been other committee studies, following in the tradition begun by Fenno, but these are the best known and most often cited. Recently a major comparative committee study by Fenno (1973) has appeared.

2. Various sociological frameworks, most commonly of a structural-functional variety, have dominated studies of social institutions in general, and Congress in particular. The incorporation of economic thinking in studies of the Congress [see, e.g. Fiorina (forthcoming), Rohde and Shepsle(1973), Rothenberg (1965), Uslaner (1971), and Westefield (1973), reflects a more recent interest in economic modeling in political science. For a synthetic treatment in which both economic and sociological thinking is reflected, see Fenno (1973).

3. The Democratic CC is composed of the 15 (when the Democrats are the majority party) Democratic members of the Ways and Means committee. They are responsible for filling vacancies on all other committees while the entire party caucus (in effect, the party leadership) fills vacancies on Ways and Means. The Republican CC is composed of one member from each state with some Republican representation in Congress. Each member has as many votes as there are Republicans in his delegation. As a result of this majority rule--weighted voting arrangement, Republican appointments are dominated by a subcommittee of representatives from states with large Republican

delegations. The Republican leader chairs his party's CC. Until the 93rd Congress, Democratic party leaders did not officially intervene in its party's CC. Congressional Quarterly (1973) reports that "in 1973, for the first time, the Democratic assignment committee also included Speaker Carl Albert, Majority Leader Thomas P. O'Neill, and Caucus Chairman Olin E. Teague. It takes 10 votes from the 18-member expanded committee to win an assignment."

4. These statements are based on an examination of Democratic request lists for the 86th, 87th, 88th, and 90th Congresses. To what extent Republican requests differ is not known at this time, but they probably are not much different. The data set above was made available to me by Richard Fenno, John Manley and Robert Salisbury and has been analyzed in Rohde and Shepsle (1973).

5. If a nonfreshman holds two assignments in the previous Congress, he will sometimes prefer to hold one of those and trade the other for a more desirable second assignment. On occasion, usually at the request of the party leadership, a returning member will voluntarily yield his claim to one of his dual assignments.

6. Clapp (1964, p. 222) concludes that "normally an elaborate campaign is not essential to success. Sometimes delegation and regional leaders will, in order to obtain more effective representation for their area, undertake to intercede for the candidate and cover the important ground, making it unnecessary for him to do much. In other instances not even that is necessary; some freshmen do virtually nothing to press their claims for a particular assignment and yet, because of the post being sought, existing vacancies, and the background of the applicant, succeed in obtaining it." For additional details, see Clapp (1964, Chap. 5), Ronde and Shepsle (1973), Masters (1961), and Congressional Quarterly Service (1973).

7. The Committee on Standards of Official Conduct, created in the wake of the scandal surrounding Representative Adam Clayton Powell, is omitted from this analysis. Although it is the twenty-first standing committee, it appears to have a rather special status that distinguishes it from other standing committees.

8. A special provision for exclusive committees—Appropriations, Rules, Ways and Means—allows a disproportionate share of seats for the majority party.

9. Even if the Committee on Science and Astronautics, created in the 1950's, is eliminated from the calculation, the growth is still dramatic. In the 93rd Congress (1973-74) there are 674 committee slots. For some general theoretical considerations on negotiated committee structure, see Westefield (1973, esp. Chap. 5). Also see Gawthrop (1966).

10. For the remainder of this paper Democratic assignments are the focus. The theoretical formulation to be presented has been devised to account for the Democratic request data mentioned in note 4. To what extent it is appropriate for Republican committee assignments is unknown. Much of the model will obviously carry over to Republican assignments, though the rationale for some of the assumptions may have to be altered. There is, however, one unusual feature of procedure among Democrats that distinguishes it from the Republicans. The fifteen members of the Democratic CC are each assigned a geographically contiguous zone containing approximately the same number of Democrats. A zone representative is responsible for "processing" committee requests from his clientele in the sense that a request is not considered by the full CC unless it is nominated by the appropriate zone representative.

This procedure was altered slightly in the 92nd Congress:

> Until 1971, no candidate for a committee position could have his name placed in nomination unless his zone representative nominated him... The Democratic caucus changed the rules to permit nominations from the candidate's home state delegation directly to the assignment committee (Congressional Quarterly Service, 1973, p. 279).

The only noticeable bias this procedure has encouraged is found among Southerners: A Southerner from the same state as his zone representative is much more likely to receive one of his requests than his Southern counterpart whose state delegation differs from his zone representative's. Among non-Southerners there is no apparent bias. See Rohde and Shepsle (1973, Table A).

11. There is some dispute over whether the election should be treated as an exogenous shock. Much evidence suggests that committee assignments have little effect on reelection probabilities, though congressmen, from time to time, insist on the positive (or negative) effects a particular assignment had on his reelection efforts. On this proposition, see Bullock (1972). Some congressional views of this thesis are found in Fenno (1973). It seems reasonable to suppose that steps 1 and 2 of the temporal sequence above are not closely related, especially given the high likelihood of reelection for any incumbent. On the effect of the election on subsequent assignments, there is strong evidence that party CCs take extra care of marginally elected representatives. See Masters (1961), Clapp (1964), and Rohde and Shepsle (1973). For a dissenting view see Bullock (1972) whose dissent, I believe, can be accounted for by the data source he employed.

12. Request data is not part of the public record and typically is unavailable to scholars. Through the good offices of others (see note 4), Professor David Rohde and I have access to Democratic request lists in four Congresses. It is important to note that this request data is crucial for an understanding of individual motives at the request stage. Surrogates for this data--for example, employing actual assignments as an indicator of committee preferences--are often quite inadequate. See, e.g. Bullock and Sprague (1969, p. 501) and Bullock (1972, p. 997).

13. This does not add to 100%, reflecting the multiple goals held by this group of freshmen. See Bullock (1973b, Table 3).

14. Armed Services and Appropriations were requested with high frequency by both re-election oriented and policy oriented applicants (though not as frequently as the other committees mentioned). This reflects an acknowledged dual role played by both committees. See Bullock (1973b, Table 4).

15. Fenno (1973, p. 19) reports, moreover, that party leaders play an active, but selective, role at the recruitment stage. "They are instrumental (according to our interview results) in the appointment of about 40 per cent of the Ways and Means members and 30 per cent of the Appropriations members, while they have comparable impact on no more than 15 per cent of the appointments for the other four groups [in his study]."

16. I have limited discussion of motives at the request stage to three aspects: applicant preferences, the preferences of other House actors, and the opportunity structure. Bullock (1971) presents a

much more detailed account of the effects of state delegations on requests. Rohde and Shepsle (1973) examine the effect of constituence characteristics on requests. And Fenno (1973, pp. 34, 41, 43) presents some material on the role of outside clientele groups on requests.

17. Often conflict over the structure of the new Congress is as much an intra-party fight as it is one pitting majority against minority. Such was the case in the 1961 contest to expand the size of the Rules Committee. The alleged reason for this move by the Democratic leadership was to neutralize the blocking power of conservative Southern Democrats (in collision with Republicans), thereby increasing the prospects of passage for liberal Kennedy Administration legislation. See Cummings and Peabody (1963).

18. See Cummings and Peabody (1963), Fenno (1966, 1973) and Manley (1970).

19. Moreover, as I mentioned earlier, they are more likely to be used for policy purposes by the leadership, rather than as bargaining chits.

20. Another interesting consequence of Westefield's argument is the periodic necessity to revalue the currency. Hence he provides an explanation for the occasional reorganization laws which tend to alter and/or consolidate the committee system.

21. The recent change in the composition of the Democratic CC (see note 3) is quite compatible with this view. The party leadership quite clearly wants to insure that it receives the credit for accommodating applicant demands, thereby tying the members more closely to leadership wishes.

22. Manley (1970, pp. 77-78) reports that "The Ways and Means Democrats feel that they have enough autonomy in allocating committee seats to

see this as one of the most important things they do. For some, however, it is probably their most important committee function. Big-city Democrats, in particular, are inclined to rank committee assignments ahead of the job of molding substantive, and sometimes boring, legislation."

23. Committee chairmen are alleged to have considerable influence in assignments to their committees. In recent years, however, their success has been mixed at best. Congressional Quarterly (1973, p. 281) reports that when a request that a Black be appointed to Armed Services was under consideration, the chairman, F. Edward Hebert (D La.), was consulted. He reportedly said that he would accept anyone but Ronald Dellums (D Calif.). Dellums was appointed. Also, in the last few years a number of liberals have been appointed to the powerful Appropriations Committee over the objections of its chairman, George Mahon (D Tex).

24. As is the case with any assumption, my assumption that members of the CC behave as managers is an oversimplification. There are, without a doubt, many other motives operative. For a discussion of the management goal, as well as several others, see Rohde and Shepsle(1973).

25. As I noted earlier, I focus in this paper on Democratic committee assignments. Moreover, it will be less complicated to deal initially with freshmen congressmen. Thus, I do not examine committee transfer phenomena.

26. For present purposes the notation is left in general form. One of the reasons for this is that the status of some of the Congressional committees has changed over time. During the 86th through 90th Congresses,

the period for which empirical examination of this model is conducted (Rohde and Shepsle (1973) and work in progress), the committees in each status category were: exclusive: Appropriations, Rules, Ways and Means; semiexclusive; Agriculture, Armed Services, Banking and Currency, Education and Labor, Foreign Affairs, Interstate and Foreign Commerce, Judiciary, Public Works, Science and Astronautics; nonexclusive: District of Columbia, Government Operations, House Administration, Interior and Insular Affairs, Merchant Marine and Fisheries, Un-American Activities, and Veterans Affairs. In addition, Post Office and Civil Service was changed from semi exclusive to nonexclusive status at the beginning of the 88th Congress. Since the time period of this study, Science and Astronautics has been changed to nonexclusive status and the name of Un-American Activities has been changed to Internal Security (its status remains the same).

27. As an empirical matter, it came as something of surprise to discover the frequency with which constraints in class [I] are satisfied as inequalities. The table below gives the number of unfilled vacancies, for the 86th through 93rd Congresses:

Unfilled Vacancies, 86th - 93rd Congresses
(number and committee by party)

Congress	Democratic	Republican
86th	none	1 - Educ. & Labor 1 - Vet. Affairs
87th	3 - Post Office 1 - Vet. Affairs	1 - DC 1 - Post Office 1 - Pub. Works
88th	none	1 - Vet. Affairs
89th	none	none
90th	1 - D C 1 - Educ. & Labor 2 - Interstate	none

(Table continued next page)

91^{st}	1 - Pub. Works 1 - Vet. Affairs	1 - Mer. Marine 1 - Un-Amer. Act.
92^{nd}	1 - Sci. & Ast.	1 - Mer. Marine
93^{rd}	1 - Educ. & Labor 1 - For. Affairs 1 - Gov. Oper. 1 - Pub. Works	1 - Interior 1 - Mer. Marine

It is apparently a rather common phenomenon, not **related** to party. The unfilled vacancy phenomenon merits further attention.

28. This class of constraints is violated, during the period under study, in several instances for one of two reasons. It was not unusual for the twenty-first standing committee, Standards of Official Conduct, to serve as a third appointment for some congressmen. It is evident, however, that this committee is a standing committee in name only and has been excluded from the analysis. Also, some congressmen who held a third committee assignment prior to the Legislative Reorganization Act of 1946 were not forced to give them up. These comments apply to constraint class [IV] as well.

29. A recent reform passed by the Democratic caucus supplements this constraint in a significant way. Not only is a congressman restricted to at most one semiexclusive committee; if he does not serve on an exclusive committee he must be assigned to <u>at least</u> one semiexclusive committee. This is patterned after the Johnson Rule in the U.S. Senate. See <u>Congressional Quarterly Service</u> (1973, p. 279). I do not incorporate this recent reform in the analysis.

30. In note 4 I allude to some relevant data. Those data consist of the preference orderings of freshman Democrats for four recent Congresses. If a freshman lists the j^{th} committee on his application, then $p_{ij} = 1$ for that

person; otherwise $P_{ij} = 0$. Notice that this operational definition does not employ all the information available. In particular it makes no use of the <u>order</u> in which an applicant lists his request, instead relying on a crude dichotomy (a committee is either on his list or it is not). I have explored alternative operationalizations which I plan to report on elsewhere. The particular operational definition used here does have the merit of being simple and direct.

31. For ease of reference a complete list of the constraints and the objective function is included at the end of this paper.

32. This section follows the very excellent paper by Koopmans and Beckman (1957).

33. Uslaner (1971) employs a variant of linear programming in his examination of the committee assignment process. He makes use of it, however as an algorithm, and is far less interested in the <u>operating characteristics</u> of the process, itself. Moreover, his is a normative inquiry; thus his specification of the problem is considerably different than the one here.

34. The psychologist Thorndike's concern with personnel assignment (Thorndike, 1950) appeared before much of the theoretical development in the general assignment problem had taken place. He reports that he took his problem to a mathematician colleague who dismissed him, in a patronizing manner, with the remark, "It's trivial"! The literature appearing since that time, some of which is cited in the text, suggests the contrary.

35. Notice that in the <u>simple assignment problem</u> there are exactly the same number of slots as there are members. Most of the technical literature, to my constant frustration, deals with this simple situation. There is,

of course, no such neat equality between vacancies and personnel in the committee assignment problem.

36. When I discuss the committee assignment problem the definition of feasibility is the same, except (C·1) through (C·3) is replaced by [I] through [VII].

37. (C·3) requires nonnegativity. To show that the a_{ij}'s are bounded from above, assume the contrary: $a_{i*j*} > 1$. Then from the $i*^{th}$ equation of (C·1) or the $j*^{th}$ equation of (C·2), it must be the case that some other a_{ij} is negative, contrary to (C·3). This completes the proof.

37a. Throughout I do not worry about the existence of optima. The structure of the problems with which I am concerned always have optimum points, so that existence never poses a problem. Rarely, however, are these optima unique. Thus Figure 1b, where multiple optimum points are illustrated, is more typically the case.

38. This is an especially prominent problem at the computing stage. I had the opportunity to confer with members of the Washington University Computing Center who had been commissioned by the university to devise a computing algorithm to process student requests for dormitory rooms--another illustration of an applied assignment problem. Their algorithm, as well as all others with which they were familiar, specifically prohibited multiple assignments. This, of course, makes sense in their problem--a student cannot occupy more than one room (at least in principal, the social climate notwithstanding!); it makes sense in most other assignment problems as well. There is, then, a rather fundamental analytical difference between the simple assignment problem and the committee assignment problem. The net effect for my own empirical work (not reported on here) is that I must rely on a much

less efficient (and much more expensive) standard linear programming algorithm.

39. Although I have earlier in the text provided an operational definition for the P matrix in the CC objective function (namely that P_{ij} is zero or unity for all i and j), I have alluded to other possibilities (see note 30). In order to retain some generality at this point I only assume that the objective function is linear in assignments, making no specific assumption about weights (the P_{ij}'s). Shortly I relax this by assuming that the weights are nonnegative.

40. That is, they subjectively appraise the revealed preferences and, according to some criterion (e.g. the productivity of an assignment in securing an applicant's reelection), infer utilities and intensities of preference.

41. While this example employs a special kind of weighting function, with the CC in this instance serving as a social welfare function, the theorem is much more general than the example would lead one to suspect. In fact, with the earlier operational definition of the P_{ij}'s, there are instances in which no complete assignment is optimal.

42. See Gale (1960, Chap. 5) and Balinski (1968). This is the approach, for a modified specification of the committee assignment problem, taken by Uslaner (1971). For a comparison of the two approaches, see Gomory (1558

43. See Fenno (1973, p. 19). In the 93rd Congress, a freshman was assigned to the Rules Committee. As Congressional Quarterly Service (1973) reports, however, it was clear that this slot (and the other two Rules slots) "went to Democrats loyal to the Albert - O'Neill leadership." The "freshman, by the way, is one of Albert's fellow Oklahomans. In the same Congress, the powerful chairman of Ways and Means, Wilbur Mills,

attempted to place one of his Arkansas freshmen on Appropriations and was defeated in the CC -- an extremely rare event.

44. Actually, the latter term is sometimes used in a slightly different fashion. I use unimodularity and Dantzig property interchangeably.

45. Cell entries in C are zero unless otherwise indicated. For a similar diagram pertaining to the transportation problem, see Karlin (1959, p. 133).

46. In Shepsle (1972), an approximation formula is given for the number of extreme points as a function of the parameters m and n and the particular distribution of v_i's. To give an example, there were nineteen 87^{th} Congress Democratic freshmen (m=19) applying for 28 vacancies ($\sum_j v_j=28$) on 12 committees (n=12). With these parameters there are approximately <u>2×10^{22} extreme points</u>. These points are located in a space of dimensionality mn = 228.

While 228 - space is extremely large, the magnitude of the number of extreme points suggests to me that the problem of optimal assignments is very sensitive to the specification of the objective function (the P-matrix). That is, unlike Figure 1, where the slope of the objective function may vary considerably before a different extreme point is "picked up," in this case the large number of extreme points that dot the landscape may well upset this relative invariance. This is an unfortunate empirical fact. I have several theoretical results, however, which demonstrate a still broad class of objective functions for which optimality remains invariant. I do not report on these here.

A second disturbing empirical fact follows from the large number of extreme points. If the Democratic CC could evaluate one such extreme point (assignment configuration) per second, 6.35 x 10^{14} years are required for it to complete its task.' Thus, even if the CC is "intendedly rational" (to use Simon's term), in the absence of an efficient algorithm or a decentralized mechanism for information processing, outcomes may well be far from "optimal." The zone system may serve the Democrats as a decentralized allocation mechanism. This proposition is well worth investigating further, perhaps in the context of team theory (see Marschak and Radner, 1972). On the general problem of designing informationally decentralized mechanisms for resource allocation, see Hurwicz (1973).

47. I write the nonnegativity condition separately here. Thus, C is composed of constraints [I]-[II] and [V]-[VI].

48. For the relationship between shadow prices and quasi-market phenomena in centrally planned economies, see Koopmans and Beckmann (1957) and Hurwicz (1973). Also see Dorfman, Samuelson and Salow (1958).

49. These latter values provide insight about committee prestige (a topic of concern in the literature -- see Bullock (1973a), Bullock and Sprague (1969), and Gawthrop (1966)) as a function of supply and demand conditions.

50. The λ_{ij}'s (if they exist) should prove useful in examining hypotheses about what kinds of congressmen get what kinds of committees. In particular, we may determine whether regional biases exist, whether regional strategies seem to be played (see, e.g. Masters (1961, p. 52) on the "collective strategy" of Southerners; on the same point see Bullock (1973a, p. 96)), whether the CC gives preference (weight) to marginally elected freshmen, etc.

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CONSTRAINTS

[I] $$\sum_{i=1}^{m} a_{ij} \leq v_j \qquad j = 1,\ldots,n$$

The number of assignments to c_j may not exceed v_j.

[II] $$-\sum_{j=1}^{n} a_{ij} \leq -1 \qquad i = 1,\ldots,m$$

Every congressman must serve on at least one committee.

[III] $$\sum_{j=1}^{n} a_{ij} \leq 2 \qquad i = 1,\ldots,m$$

No congressman may serve on more than two committees; in particular no congressman may serve on more than 2 non-exclusive committees [multiple exclusive committee assignments and semi-exclusive assignments are ruled out by constraints IV and V, respectively.

[IV] $$3\sum_{j=1}^{e} a_{ij} + 2\sum_{j=e+1}^{s} a_{ij} + \sum_{j=e+1}^{n} a_{ij} \leq 3 \qquad i=1,\ldots,m$$

A congressman may serve on at most one exclusive committee; if he does he may serve on no other. A congressman serving on a semi-exclusive committee may serve on at most one non-exclusive committee.

[V] $\sum_{j=e+1}^{s} a_{ij} \leq 1$ $\quad i = 1,\ldots,m$

A congressman may serve on no more than one semi-exclusive committee

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[VI] $a_{ij} \leq 1$ $\quad i=1,\ldots,m \quad j=s+1,\ldots,n$

A congressman may not receive a multiple appointment to the same non-exclusive committee

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[VII] $-a_{ij} \leq 0$ $\quad i=1,\ldots,m \quad j=1,\ldots,n$

nonnegativity constraint

<u>Objective Function:</u> $\max_{A} \sum_{i} \sum_{j} p_{ij} a_{ij}$