Preemption or Wait-and-See?* Endogenous Timing in Bargaining

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Abstract

Suppose two parties have to share a surplus of random size. Each of the two can either commit to a demand prior to the realization of the surplus, or wait until the surplus was publicly observed. Early commitments carry the risk that negotiations break down, because the surplus turns out too small. Still, when uncertainty is sufficiently small, commitment is a dominant choice. For more diffuse priors the equilibrium outcome depends on the distribution function and on risk aversion.

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1 Introduction

According to the "Nash program" in bargaining theory rational bargaining behavior should emerge as equilibrium behavior in a non-cooperative game - if necessary, because of multiplicities, by selection among the equilibria (Nash, 1953). Examples are Nash's model of simultaneous commitments (the "Nash demand" game), where among many equilibria the Nash-bargaining solution selects one, or the alternating-offers model (Stahl (1972), Krelle (1976), and Rubinstein (1982), among many others) with a unique subgame perfect equilibrium for the two-person case.

Interesting enough, most of these models lead to more or less the same outcome, at least in the bilateral case (with a sufficiently long horizon). When frictions, like discount rates, time costs, or break-down probabilities, are small, the equilibrium outcome is usually near the equal-split (see, however, Corchon and Ritzberger (1994) for a critique). This symmetry contrasts sharply with the highly asymmetric (subgame perfect) equilibrium of the simplest bargaining procedure. In "ultimatum bargaining" (see Guth (1976) and. for experimental analysis, Guth, Schmittberger, and Schwarze (1982)), where one party confronts the other with a take-it-or-leave-it offer, the proposer acquires essentially all the surplus. This is due to one party's commitment option.

In practise commitment opportunities are not far fetched at all. Delegates with rigid missions, one-way communication devices, or contracts with outside parties are examples. It is, therefore, of interest to study models where the timing decision of when to demand what (and for how long) is endogenous. This may imply that one party or the other ends up irreversibly committed to an excessive claim, creating a risk of breakdown. If such a breakdown is perfectly predictable, then efficiency considerations suggest that the parties will avoid it. Yet. when the effect of a commitment is uncertain, it is less clear that commitments will be avoided. In fact, Schelling (1966, chp. 3) suggests that if commitments are *not* certain to cause an impasse, they might constitute a viable bargaining strategy, even when disagreement is costly.

Indeed, that uncertainty together with parties striving for favorable commitments may imply the risk of impasse has been shown in the literature (Schelling (1956), Crawford (1982), Muthoo (1996)). In the latter two papers commitments are revokable at a known (Muthoo (1996)) or uncertain (Crawford (1982)) cost, while the size of the surplus is known. In this paper we show that impasses may arise even for irreversible commitments, if the size of the surplus is uncertain at the time when commitment devices are available.

in our model parties can try to preempt the other or wait. Preemption entails the risk of impasse, because parties commit before the surplus is observed and this decision is irreversible. Attempting an irreversible commitment seems to be suboptimal, because of its implied inefficiency. Yet, when the gains from being committed outweigh the losses from disagreement, equilibrium may still entail the possibility of impasse. In fact, it will be shown that this occurs in particular when the uncertainty is small. Hence, the phenomenon is robust against reducing uncertainty.

The model combines two well-known building blocks. One is an *uncertain size of the surplus* in bilateral bargaining, as introduced by Nash (1953) in the selection of the Nash-bargaining solution. The other is *endogenous timing of decisions*, as introduced by Hamilton and Slutsky (1990) (see also Spencer and Brander (1992), Mailath (1993), Sadanand and Sadanand (1996), Amir and Grilo (1999), and van Damme and Hurkens (1999)).

Endogenous timing models, ideally, would discriminate between other models for which the timing of decisions are fixed. In the present context, however, the results are conditional. It is more primitive data which determines (equilibrium) timing decisions. In particular, risk aversion and (the hazard rate of) the cumulative distribution of the surplus size will jointly determine whether parties attempt preemption or wait and see. Thus, equilibrium timing; decisions themselves depend on underlying environmental and preference parameters.

The remainder of the paper is organized as follows. Section 2 introduces the general model and basic assumptions. Section 3 states the general results. Section 4 contains an extended example which provides more precise insight **into what drives the equilibrium. Section 5 provides a brief discussion.**

2 The Model

There are two bargaining parties, players $i = 1$ and $i = 2$, who share a surplus which becomes available if they strike a deal. The size of the surplus is unknown initially, but becomes known at a later stage. Players have to decide on *when* to demand *how much.* Call a player, who commits to a demand prior to the observation of the surplus, a *0-type.* A 0-type attempts to confront the other party with an ultimatum. A *1-type,* on the other hand, postpones her demand until uncertainty is resolved. Accordingly, for each party *i — 1,2* there are two *behavioral dispositions* i, C {0, 1}

A 0-type will make her intention to commit known, like calling a press conference. A 1-type will not do so. Since a commitment represents an appeal to a third party, say, the public, calling upon this third party is observable to both players. Hence, the notion of endogenous timing to be invoked here is captured by the extension of a basic (bargaining) game, due to Hamilton and Slutsky, 1990 (see also Amir and Grilo, 1999). In this model the basic (bargaining) games show up as subgames after timing decisions became known.

If both parties attempt to preempt the other, *{t1,t2}* = (0,0), the demands *X1* are chosen simultaneously *before* the observation of the surplus. Once the surplus has realized, each party receives x_i, if $x_1 + x_2$ does not exceed the surplus, and nothing otherwise.

If $(t_1, t_2) = (0, 1)$ or $(1, 0)$ the 0-type confronts the other party with a take-it-or-leave-it offer. The 1-type can, after the observation of the surplus, either collect the residual or call the deal off. In the latter case no party receives anything. Since the 0-type commits to a demand before the surplus becomes known, there is a risk of impasse, when the 1-type refuses the deal. Only in this case of asymmetric timing dispositions parties move sequentially, with the 0-type preempting the 1-type, but risking conflict.

If both parties are 1-types, $(t_1, t_2) = (1, 1)$, they wait until *after* they observed the size of the surplus and then determine simultaneously their demands $x_i \geq 0$. If the combined demands $x_1 + x_2$ do not exceed the surplus, each party i receives x_i. In the case of conflict, i.e., if the combined demands exceed the surplus, both obtain zero. Hence, two 1-types play the "Nash demand game".

Whenever the timing dispositions are the same, parties commit independently. In case of 0-types this is done before the observation of the surplus which entails a potential inefficiency, if the surplus turns out too small or larger than the sum of demands. If both parties are 1-types, conflict is avoidable, because the surplus is commonly known when parties choose their demands.

In all cases players make their demands in terms of absolute magnitudes rather than shares. If the size of the surplus is known, this is equivalent to demands in terms of shares. If it is yet unknown, commitment devices are available only in terms of absolute magnitude, because the size of the surplus is assumed to remain unverifiable to outside contractors even ex-post. Hence, this amounts to bilateral contracts with third parties as the only available commitment devices.

Players first decide simultaneously on their behavioral dispositions $t_i \in \{0, 1\}$. Then they publicly observe the outcome of these choices and move into the subgames described above. This allows an application of *subgame perfection* (Selten, 1965) in solving the game.

Preferences are represented by twice continuously differentiable, strictly increasing, and (weakly) concave *utility functions* u_i, i.e., $u_i'(x) > 0$ and $u_i''(x) \leq 0$, for all $x \geq 0$, for $i = 1, 2$. Normalize utility such that $u_i(0) = 0$, for $i = 1, 2$. Strict concavity ($u_i'' < 0$) models risk aversion, linearity ($u_i'' = 0$) models risk neutrality.

The environment is represented by the (cumulative) *distribution function* F of the surplus size. Assume that $F(z) = 0$, for all $z < 0$, and that F is twice continuously differentiable for all $z > 0$ in the interior of the support of F. Since $F(z)$ is a distribution function, it is nondecreasing, right-continuous even at the boundary of its support, and approaches 1 as z goes to infinity. F has a *density* $f = F'$ if it is continuous everywhere. More assumptions on F will be stated where they are needed. If F has density f, the *hazard rate* is $h(z) = f(z)/[1 - F(z)]$.

3 Results

Since the solution concept is subgame perfect Nash equilibrium, we start by investigating equilibria of proper subgames.

3.1 Subgames

First observe that a player i, who has decided for preemption, $t_i = 0$, will either be confronted with an opponent who also attempts preemption and demands $y > 0$, or with an opponent who waits until after the resolution of uncertainty and, therefore, currently demands nothing, $y = 0$. Her expected payoff from demanding $x \geq 0$ herself, accordingly, is

$$v_i(x, y) = u_i(x) \left[1 - F(x + y)\right] \tag{1}$$

Clearly, $v_i(0, y) = 0$ and $v_i(x, y) = 0$ whenever $F(x + y) = 1$. Moreover, $v_i(x, y) > 0$ for all $x > 0$ such that $F(x + y) < 1$. Therefore, if $F(y) < 1$, then the maximum of v_i with respect to x is interior. Hence, the first-order conditions

$$u_i'(x) \left[1 - F(x + y)\right] - u_i(x) f(x + y) = 0 \tag{2}$$

must obtain at a payoff maximum. Under the standard assumption of a *monotone hazard rate* the following result can be shown (for a proof see Appendix).[1]

Lemma 1 *If F is continuous and the hazard rate h is nondecreasing, then the reaction function*

$$\xi_i(y) = \operatorname{argmax}_{x \geq 0} v_i(x, y)$$

is a bounded continuously differentiable function of y with $-1 < \xi_i'(y) \leq 0$. Moreover, the maximum $V_i(y) = \max_{x \geq 0} v_i(x, y)$ is a continuous and strictly decreasing function of y.

By Lemma 1 a monotone hazard rate guarantees a *unique* equilibrium in the subgame where both parties attempt preemption, $(t_1, t_2) = (0, 0)$.

Proposition 1 *If F is continuous and the hazard rate h is nondecreasing, the subgame where both parties commit to early demands, $(t_1, t_2) = (0, 0)$, has a unique equilibrium in pure strategies.*

[1] Note that the assumption of a monotone hazard rate implies that the support of F must be connected (either a compact interval or the nonnegative reals).

Proof. By Lemma 1 reaction functions are bounded and nonincreasing. Hence, there is a rectangle $X = [0, \xi_1(0)] \times [0, \xi_2(0)]$ such that the product mapping $\xi = \xi_1 \times \xi_2$ is a continuous function from X to itself. By Brouwer's fixed point theorem, there exists $x^* \in X$ such that $\xi(x^*) = x^*$. Since x^* must be interior and corresponds to an intersection of the graphs of the reaction functions, the fact that the slope of reaction functions is between zero and -1 implies that x^* is unique. ■

Hence, with a monotone hazard rate, in the subgame $(t_1, t_2) = (0, 0)$ players will obtain payoffs

$$U_i(0, 0) = v_i(x^*), \text{ for } i = 1, 2. \tag{3}$$

where $x^* = (x_1^*, x_2^*)$ denotes the unique equilibrium in this subgame, for $i = 1, 2$.

In the subgames with asymmetric timing dispositions, $(t_1, t_2) \in \{(0, 1), (1, 0)\}$, the 0-type commits to a demand before the realization of the surplus. Learning this demand the 1-type effectively responds to an ultimatum, after having observed the surplus size. By subgame perfection, the 1-type accepts, if the surplus exceeds the 0-type's demand, and rejects otherwise. Therefore, at the first stage of the subgame the 0-type will choose her demand so as to maximize $v_i(x, 0)$ at $x = \xi_i(0)$, yielding

$$U_i(0, 1) = V_i(0) \tag{4}$$

where $t_i = 0$, for $i = 1, 2$. The 1-type will collect the residual, if it is nonnegative, and otherwise call the deal off. Her expected payoff, therefore is

$$U_i(1, 0) = \int_{\xi_{3-i}(0)}^{\infty} u_i\left(z - \xi_{3-i}(0)\right) dF(z). \tag{5}$$

where $t_i = 1$, for $i = 1, 2$.

When *both* parties wait until *after* the resolution of uncertainty, $(t_1, t_2) = (1, 1)$, there is a substantial multiplicity of Nash equilibria. Any efficient (viz. $x_1 + x_2 = z$) distribution of the surplus constitutes an equilibrium outcome and so does $x_1 = x_2 = z$, where z denotes the size of the surplus. Here we assume that in the subgame after $(t_1, t_2) = (1, 1)$ the parties share equally, i.e., $x_1 = x_2 = z/2$. This models that under certainty uncommitted parties can achieve something much more balanced than in the (subgame perfect) equilibrium of ultimatum bargaining.

There are several arguments in favor of this. One may appeal to Rubinstein's (1982) infinite alternating-offers model with random continuation. Its unique subgame perfect equilibrium will converge to the equal split as the continuation probability goes to 1. A random-proposer model with symmetric proposer-probabilities also yields the equal split. Alternatively, this subgame could be modeled as a Divide-and-Choose game, which again yields the desired outcome. Finally, for the symmetric case $u_1 = u_2$ one could stick to the Nash demand game and invoke *efficiency* and *symmetry invariance* to select the equal split, in the spirit of Nash (1950 and 1953). In short, if the equal split is a viable solution for two-person bargaining under certainty, then our model meets this criterion.

But the strongest argument in favor of the assumption is the present model itself. Suppose, for the moment, that uncommitted parties do not precisely learn the surplus size, but rather receive a public signal about it. We will show below that if the distribution F has most of its mass concentrated around the mean, then the present game has a (subgame perfect) equilibrium where both parties attempt preemption and receive approximately half the surplus each. Therefore, if the public signal is very precise, then repeating the game after $(t_1, t_2) = (1, 1)$, with F replaced by the conditional distribution given the public signal, yields (approximately) the equal split. Hence, the assumption that in the subgame $(t_1, t_2) = (1, 1)$ parties share equally may also be viewed as a shortcut modelling of repeating the game with superior information.

The advantage of $(t_1, t_2) = (1, 1)$ is that all inefficiencies are avoided. The expected payoffs, according to the assumption, are

$$U_i(1,1) = \int_0^\infty u_i\left(\frac{z}{2}\right)\, dF(z) \tag{6}$$

for $i = 1, 2$. This completes the derivation of equilibrium payoffs from subgames.

3.2 The Truncation

Replacing subgames by equilibrium payoffs, the *truncation* with payoffs U_i and strategies $t_i \in \{0, 1\}$, for $i = 1, 2$, is a 2×2-bimatrix game. Table 1 represents the payoffs for the two players for all (t_1, t_2)-constellations.

t_1 \ t_2	0	1
0	$U_1(0,0)$; $U_2(0,0)$	$U_1(0,1)$; $U_2(1,0)$
1	$U_1(1,0)$; $U_2(0,1)$	$U_1(1,1)$; $U_2(1,1)$

Table 1 The truncation in matrix form

Consider first the case where players already have very precise information at the beginning of the game. Formally, suppose there is some small $\varepsilon > 0$ such that, with $z_o = \int_0^\infty z\, dF(z) > 0$ denoting the mean,

$$\varepsilon \geq \max\{F(z_o - \varepsilon), 1 - F(z_o + \varepsilon)\} \tag{7}$$

i.e., most of the mass is concentrated around the mean. Call a distribution which satisfies (7) for some small ε *concentrated.* An example of a distribution which satisfies this and has a monotone hazard rate is $F(z) = \left[1 + e^{(z_o - z)/\delta}\right]^{-1}$, for some sufficiently small $\delta > 0$.

The next result establishes that with sufficiently precise information both players commit and obtain approximately half the surplus each.

Proposition 2 *If F is sufficiently concentrated (i.e., (7) holds for some $\varepsilon > 0$ sufficiently small), then the truncation has a unique equilibrium in dominant strategies, $(t_1, t_2) = (0, 0)$. Moreover, equilibrium payoffs from this equilibrium are approximately $u_i\,(z_o/2)$, for $i = 1, 2$.*

Proof. If the distribution satisfies (7) for small $\varepsilon > 0$, then $v_i(x,y) \approx (1-\varepsilon)u_i(x)$, for all $x < z_o - y - \varepsilon$, and $v_i(x,y) \approx \varepsilon u_i(x)$, for all $x > z_o - y + \varepsilon$. Hence, there is some $\eta(\varepsilon) > 0$, which goes to zero as ε approaches zero, such that $z_o - \eta(\varepsilon) < \xi_i(y) + y < z_o + \eta(\varepsilon)$, for all $\varepsilon > 0$ sufficiently small. So, the reaction function will almost exhaust the surplus, i.e., $\xi_i(y) + y \approx z_o$. Therefore, the equilibrium $x^*(\varepsilon) = (x_1^*(\varepsilon), x_2^*(\varepsilon))$ of the subgame $(t_1, t_2) = (0,0)$ will satisfy $z_o - \eta(\varepsilon) < x_1^*(\varepsilon) + x_2^*(\varepsilon) < z_o + \eta(\varepsilon)$. Since $\xi_i'(y) > -1$, this implies that $x_i^*(\varepsilon) \approx z_o/2$, for $\varepsilon > 0$ sufficiently small and for $i = 1, 2$, and parties end up with approximately half the surplus each.

As ε approaches zero the payoff $U_i(0,0)$ approaches $u_i(z_o/2)$, for $i = 1, 2$. The payoff $U_i(1,0)$ approaches zero, because $\xi_i(0) \to_{\varepsilon \to 0} z_o$ and all the mass becomes concentrated at $z = z_o$. Finally, $U_i(0,1) = V_i(0)$ approaches $u_i(z_o)$ as ε goes to zero, and $U_i(1,1)$ approaches $u_i(z_o/2) < u_i(z_o)$. Therefore, by continuity, for all ε sufficiently small $t_i = 0$ constitutes a dominant strategy in the truncation, for $i = 1, 2$. ∎

That F is sufficiently concentrated is a sufficient, but not a necessary condition for bilateral preemption. Still it provides a novel foundation for simultaneous-move bargaining models under or close to certainty, like the Nash demand game or "Divide-and-Choose". Nash's (1953) approach was to smooth the discontinuous payoff functions of the Nash demand game and, thereby, select among the equilibria. Uncertainty about the surplus is but one way to smooth. But, under this interpretation of smoothing, why should the parties not wait until they know the surplus? We argue that they will *choose not to wait*, because sufficiently small uncertainty makes preemption a dominant strategy.

A sufficient condition for simultaneous waiting, $(t_1, t_2) = (1, 1)$, as the unique equilibrium is not as straightforward. But it can be shown that the following is sufficient for $t_i = 1$ to be a best reply to $t_{3-i} = 1$, for $i = 1, 2$ (a proof is in the Appendix).

Lemma 2 *If F is continuous, the hazard rate h is nondecreasing, and*

$$F(\xi_i(0)) \geq \frac{2}{3} \tag{8}$$

holds, then $t_i = 1$ is a best reply against $t_{3-i} = 1$ in the truncation, for $i = 1, 2$.

Consequently, if (8) holds for both players $i = 1, 2$, then both parties waiting until after the resolution of uncertainty, $(t_1, t_2) = (1, 1)$, is a subgame perfect Nash equilibrium - but not necessarily the only one. Condition (8) is a joint condition on preferences (u_i) and the environment (F) and, therefore, not of the same status as (7). For more precise predictions we turn to a specific example.

4 An Example

Consider distributions F in the class $F(z) = c^a z^a$, for all $0 < z < 1/c$, for some $a > 0$ and $0 < c \leq 1$, with $F(z') = 0 = 1 - F(z'')$ for all $z' \leq 0$ and $z'' \geq 1/c$. Moreover, both parties are assumed to have *constant relative risk aversion*, i.e., their preferences are represented by $u_i(x) = x^{b_i}$ for some $0 < b_i \leq 1$, for $i = 1, 2$. The coefficient of relative risk aversion is $1 - b_i$. If $b_i = 1$, player i is risk neutral.

Distributions from the above exponential family are naturally ordered by *stochastic dominance* (Rothschild and Stiglitz (1970)). Independently of the parameter c, a distribution F with parameter a stochastically dominates another such distribution with parameter a' if and only if $a \geq a'$. This statement holds for first-order and, therefore, also for second-order stochastic dominance. Hence, a higher parameter a corresponds to an unambiguously more favorable distribution or, more intuitively, to *higher stakes*.

The parameter c can be used to vary the moments of F. If $c = \frac{a}{1+a}$, the mean is fixed at 1 and the variance is a decreasing function of a, with infinite variance as a goes to zero and zero variance as a goes to infinity. If $c = (1+a)^{-1}\sqrt{a/(2+a)}$, the variance is fixed at 1 and the mean is an increasing function of a, with zero mean as a goes to zero and infinite mean as a goes to infinity.

The main advantage of constant relative risk aversion and exponential distributions is tractability. Most payoffs can be calculated explicitly. For the subgame $(t_1, t_2) = (1, 1)$ it is

$$U_i(1,1) = ac^a \int_0^{1/c} \left(\frac{z}{2}\right)^{b_i} z^{a-1} dz = \frac{a}{a+b_i} 2^{-b_i} c^{-b_i} \text{ for } i = 1,2. \quad (9)$$

The distribution F has a monotone (nondecreasing) hazard rate if and only if $a \geq 1$. Hence, by Proposition 1, if $a \geq 1$, then the equilibrium of the subgame $(t_1, t_2) = (0, 0)$ is unique. If $0 < a < 1$, reaction functions may not be monotone, but direct computation shows that even in this case the subgame $(t_1, t_2) = (0, 0)$ has a unique equilibrium. For all $a > 0$ it is given by demands

$$x_i^* = \frac{b_i}{c(b_1+b_2)} \left(\frac{b_1+b_2}{a+b_1+b_2}\right)^{\frac{1}{a}} \text{ for } i = 1,2. \quad (10)$$

Substituting into v_i yields expected payoffs

$$U_i(0,0) = \frac{a}{a+b_1+b_2} \left(\frac{b_i}{b_1+b_2}\right)^{b_i} \left(\frac{b_1+b_2}{a+b_1+b_2}\right)^{\frac{b_i}{a}} c^{-b_i} \text{ for } i = 1,2. \quad (11)$$

It remains to solve the case of asymmetric timing dispositions, with one 0- and one 1-type. When choosing her demand, the 0-type will maximize $v_i(x, 0)$ at

$$\xi_i(0) = \frac{1}{c} \left(\frac{b_i}{a+b_i}\right)^{\frac{1}{a}} \text{ for } i = 1,2, \quad (12)$$

which yields her an expected payoff

$$U_i(0,1) = \frac{a}{a+b_i} \left(\frac{b_i}{a+b_i}\right)^{\frac{b_i}{a}} c^{-b_i} \text{ for } i = 1,2. \quad (13)$$

The 1-type, on the other hand, obtains the utility from the expected residual given by (5). The latter cannot be evaluated in terms of elementary functions when $b_i < 1$.

To solve the truncation consider first the case $t_{3-i} = 1$, i.e., the opponent waits. Using (9) and (13) one obtains

$$U_i(0,1) > (<)\ U_i(1,1) \Leftrightarrow b_i > (<)\ \frac{a}{2^a - 1} \equiv g(a)$$

independently of the parameter c, for both $i = 1, 2$. Since $\lim_{a \to 0} g(a) = 1/\ln(2) = 1.4427 > 1 \geq b_i$, $\lim_{a \to \infty} g(a) = 0 < b_i$ and $g'(a) < 0$, for all $a > 0$, there is a unique $a_i = a_i(b_i) > 0$ such that $2^{a_i} b_i = a_i + b_i$ with $a_i'(b_i) < 0$. It follows that

$$U_i(0,1) > (<)\ U_i(1,1) \Leftrightarrow a > (<)\ a_i(b_i) \text{ for } i = 1, 2. \tag{14}$$

For the special case $b_i = 1$ it is easily verified that $a_i(1) = 1$ for $i = 1, 2$.

The case where $t_{3-i} = 0$ is not as simple, because of the integral in (5). Yet the optimal behavorial disposition in this case is still independent of the parameter c. Since from (5) and (12)

$$ac^a \int_{\frac{1}{c}\left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}}^{1/c} \left[z - \frac{1}{c}\left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}\right]^{b_i} z^{a-1} dz =$$

$$ac^{-b_i} \int_{\left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}}^{1} \left[z - \left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}\right]^{b_i} z^{a-1} dz\ ,$$

by variable substitution, it follows that $U_i(0,0) > (<)\ U_i(1,0)$ if and only if

$$\Delta_i \equiv \frac{1}{a + b_1 + b_2}\left(\frac{b_i}{b_1 + b_2}\right)^{b_i}\left(\frac{b_1 + b_2}{a + b_1 + b_2}\right)^{\frac{b_i}{a}} - \tag{15}$$

$$\int_{\left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}}^{1} \left[z - \left(\frac{b_{3-i}}{a+b_{3-i}}\right)^{1/a}\right]^{b_i} z^{a-1} dz > (<)\ 0\ .$$

The left hand side of the inequality in (15) defines functions $\Delta_i(a, b_1, b_2)$, for $i = 1, 2$, the sign of which determines the best reply to $t_{3-i} = 0$.

For the special case of risk neutrality, $b_1 = b_2 = 1$, the integral in (5) evaluates to

$$U_i(1,0) = \frac{a}{c\,(1+a)}\left[1 - \left(\frac{1}{1+a}\right)^{1+\frac{1}{a}} - \left(\frac{1}{1+a}\right)^{\frac{1}{a}}\right]$$

and $\Delta_i(a, 1, 1)$ is positive if and only if $a > 1.4057$ (by numerical computation).

This allows for the following summary of the present example.

Proposition 3 *(a) If a is sufficiently large, $t_i = 0$ is a dominant strategy in the truncation and, therefore, in the unique equilibrium of the overall game both parties attempt preemption, $(t_1, t_2) = (0, 0)$.*

(b) If a is small enough, so that $b_i > 2a$, for $i = 1, 2$, then both parties waiting, $(t_1, t_2) = (1, 1)$, constitutes a subgame perfect equilibrium of the game. For small a and small risk aversion, $b_i \approx 1$, for $i = 1, 2$, this is the only equilibrium.

(c) If $1 < a < 1.4$ and risk aversion is small, $b_i \approx 1$, for $i = 1, 2$, then the truncation has two asymmetric equilibria in pure strategies, $(t_1, t_2) \in \{(0, 1), (1, 0)\}$ and one (completely) mixed equilibrium. The equilibrium probability of impasse in the two pure equilibria is given by $b_i/(a+b_i)$ when $t_i = 0$.

Proof. (a) Distributions in the present class satisfy (7) for $\varepsilon > 0$ if $\varepsilon c + \varepsilon^{1/a} > a/(1+a) > (1-\varepsilon)^{1/a} - \varepsilon c$. This is satisfied if a is large enough (as $a \to \infty$, the inequalities become $1 + \varepsilon c > 1 > 1 - \varepsilon c$). Hence, the first statement follows from Proposition 2.

(b) From (12) $F(\xi_i(0)) = b_i/(a + b_i) > 2/3$ if and only if $b_i > 2a$. Hence, the first claim of (b) follows from Lemma 2. Moreover, if $b_i \approx 1$ and a is small, then by continuity $\Delta_i < 0$, for $i = 1, 2$, and $t_i = 1$ is a dominant strategy in the truncation. This verifies the second claim.

(c) If $1 < a < 1.4$ and $b_i \approx 1$, then by continuity $\Delta_i < 0$, for $i = 1, 2$. At the same time $a/(2^a - 1) < 1 \approx b_i$, for $i = 1, 2$. This verifies that asymmetric timing dispositions are equilibria of the truncation. The existence of a third (mixed) equilibrium follows. The probability of impasse follows from (12). ■

More cases can emerge. In particular, with asymmetric risk aversion there are borderline cases where the truncation has a unique asymmetric equilibrium in pure strategies. The outcome of this resembles an ultimatum bargaining game.

The message of Proposition 3 is clearly "anything goes". More precisely, it depends on parameters of preferences and the environment which bargaining behavior will prevail. When the stakes are high, both parties will attempt preemption. When the stakes are low, they will wait and see before they strike a deal. In between, timing may be asymmetric.

Before leaving the example, a formal similarity is worth pointing out. For the special case $a = b_1 = b_2 = c = 1$, where F is uniform on the unit interval, the results resemble duopoly models with a linear demand function, with slope -1 and intercept 1, and zero marginal costs. For $(t_1, t_2) = (1, 1)$ and $U_i(1, 1) = 1/4$ for $i = 1, 2$ the situation corresponds to one of perfect symmetric price discrimination. The case $(t_1, t_2) = (0, 0)$ is analogous to the Cournot (1838) solution with $x_i^* = 1/3$ and $U_i(0, 0) = 1/9$ for $i = 1, 2$. The results $\xi_i(0) = 1/2$, $U_i(0, 1) = 1/4$, and $U_i(1, 0) = 1/8$ for $(t_1, t_2) = (0, 1)$ or $(1, 0)$ similarly capture the equilibrium in the Stackelberg (1951) game. This suggests that our results may also be of relevance for duopoly models, where F corresponds to the shape of the (inverse) demand function.

5 Discussion

This paper has considered endogenous timing in bilateral bargaining when the surplus is uncertain. Whether parties will commit initially or wait until the surplus has become known depends on parameters of preferences, in particular, risk aversion, and the environment, i.e., the distribution of the surplus size. If uncertainty is small, parties will choose to commit immediately. This provides a novel foundation for the Nash bargaining solution. With more uncertainty also both parties waiting, until uncertainty resolves, and asymmetric ultimatum-like timing behavior may constitute equilibria, depending on parameters.

Two restrictive assumptions were used. One is that timing decisions are publicly observed before demands are chosen. This is but one possible model of endogenous timing (see van Damme (1997) for an alternative analysis). Yet, in our view, both models, with and without observable timing decisions, deserve attention.

The other model, where players choose demands simultaneously with timing decisions, yields generally multiple asymmetric equilibria (in pure strategies). Hence, a coordination problem arises. To highlight the dependence of timing decisions on more primitive parameters of the environment and preferences, therefore, the present setup seems more suitable, since it yields (depending on parameters) unique equilibria.

The other restrictive assumption is that commitments are in terms of absolute magnitudes. In other words: We portray a situation where commitments are bilateral contracts with outside parties, where the latter cannot verify the overall surplus size ex-post. Therefore, such contracts cannot be conditioned on the ex-post size of the surplus, but must be written in terms of absolute magnitudes which are the only verifiables among the outside contractor and the committed party.

This we adopt for two reasons. First, it keeps the model within the realm of received models of bilateral bargaining. Second, in our view, contracts written in terms of absolute magnitude are the more realistic scenario. This is because verification of the overall surplus size would have to involve a party which has not signed the contract. If "trilateral" contracts are feasible, more strategic options become available which we view as outside the scope of the present study. Still, of course, such enlarged models constitute a challenge for further research.

6 Appendix

Proof of Lemma 1: Differentiating (2) at a point where it equals zero with respect to x yields

$$u_i''(x)\,[1 - F(x+y)] - 2u_i'(x)f(x+y) - u_i(x)f'(x+y)$$

$$
\begin{aligned}
&= u_i''(x)\,[1 - F(x+y)] - \frac{2u_i(x)f(x+y)^2}{1-F(x+y)} - u_i(x)f'(x+y) \\
&\leq u_i''(x)\,[1 - F(x+y)] - \frac{2u_i(x)f(x+y)^2}{1-F(x+y)} + \frac{u_i(x)f(x+y)^2}{1-F(x+y)} \\
&= u_i''(x)\,[1 - F(x+y)] - \frac{u_i(x)f(x+y)^2}{1-F(x+y)} < 0,
\end{aligned}
$$

because the assumption of a monotone hazard rate,

$$
\frac{d}{dx}\left(\frac{f(x+y)}{1-F(x+y)}\right) = \frac{f'(x+y)}{1-F(x+y)} + \left(\frac{f(x+y)}{1-F(x+y)}\right)^2 \geq 0,
$$

is equivalent to $f(x+y)^2/[1-F(x+y)] \geq -f'(x+y)$ whenever $F(x+y) < 1$. Therefore, the maximum of $v_i(x,y)$ is unique and given by the solution ξ_i of (2) with respect to x. By the implicit function theorem ξ_i is a continuous function of the parameter y. Implicitly differentiating (2) yields

$$
\frac{d\xi_i}{dy} = \frac{u_i'(\xi_i)f(\xi_i+y) + u_i(\xi_i)f'(\xi_i+y)}{u_i''(\xi_i)\,[1-F(\xi_i+y)] - 2u_i'(\xi_i)f(\xi_i+y) - u_i(\xi_i)f'(\xi_i+y)} \leq 0,
$$

because at a point where $u_i' = u_i f/(1-F)$ holds, $u_i'f + u_i f' = u_i f^2/(1-F) + u_i f' \geq u_i f^2/(1-F) - u_i f^2/(1-F) = 0$, by the monotone hazard rate assumption, and $\partial^2 v_i/\partial x^2 < 0$ by the second-order condition. That $d\xi_i/dy > -1$ follows from $u_i'(\xi_i)f(\xi_i+y) > 0$.

The first order condition (2) holds if and only if $u_i'(x) - u_i(x)h(x+y) = 0$, where $h(z) = f(z)/[1-F(z)]$ is the hazard rate. Since $u_i'(x)/u_i(x) \leq u_i'(0)/u_i(0)$ and $h(x)$ is nondecreasing, $\xi_i(0)$ is bounded and, by $\xi_i'(y) \leq 0$, the whole function ξ_i is bounded.

Finally, by the envelope theorem, the derivative of the maximum is given by $\partial V_i(y)/\partial y = -u_i(\xi_i)f(\xi_i+y) < 0$, as required in the second claim.

Proof of Lemma 2: If F is continuous, it can be written in terms of its hazard rate by

$$
F(z) = 1 - e^{-\int_0^z h(x)\,dx}, \text{ for all } z \geq 0.
$$

By concavity and monotonicity of u_i, integrating by parts, the monotone hazard rate, and (2)

$$\begin{aligned}
&\int_0^\infty u_i\left(\frac{z}{2}\right)\, dF(z) = \int_0^\infty u_i\left(\frac{z}{2}\right) h(z) e^{-\int_0^z h(t)\,dt}\, dz \\
\geq\ & \frac{1}{2}\int_0^\infty u_i(z)\, h(z) e^{-\int_0^z h(t)\,dt}\, dz \geq \frac{1}{2}\int_0^{\xi_i(0)} u_i(z)\, h(z) e^{-\int_0^z h(t)\,dt}\, dz \\
+\ & \frac{1}{2} u_i(\xi_i(0)) e^{-\int_0^{\xi_i(0)} h(z)\,dz} = \frac{1}{2}\int_0^{\xi_i(0)} u_i'(z)\, e^{-\int_0^z h(t)\,dt}\, dz \\
\geq\ & \frac{1}{2} u_i'(\xi_i(0)) \int_0^{\xi_i(0)} e^{-\int_0^z h(t)\,dt}\, dz = \frac{1}{2} u_i(\xi_i(0)) h(\xi_i(0)) \int_0^{\xi_i(0)} e^{-\int_0^z h(t)\,dt}\, dz \\
\geq\ & \frac{1}{2} u_i(\xi_i(0)) \int_0^{\xi_i(0)} h(z)\, e^{-\int_0^z h(t)\,dt}\, dz = \frac{1}{2} u_i(\xi_i(0)) \left[1 - e^{-\int_0^{\xi_i(0)} h(z)\,dz}\right] .
\end{aligned}$$

Since $U_i(0,1) = V_i(0) = u_i(\xi_i(0)) \exp\left\{-\int_0^{\xi_i(0)} h(z)\,dz\right\}$, it follows that (8) implies

$$\begin{aligned}
\frac{1}{3} \ &\geq\ e^{-\int_0^{\xi_i(0)} h(z)\,dz} \Rightarrow \frac{u_i(\xi_i(0))}{2}\left[1 - e^{-\int_0^{\xi_i(0)} h(z)\,dz}\right] \geq u_i(\xi_i(0)) e^{-\int_0^{\xi_i(0)} h(z)\,dz} \\
&\Rightarrow \int_0^\infty u_i\left(\frac{z}{2}\right)\, dF(z) \geq u_i(\xi_i(0)) e^{-\int_0^{\xi_i(0)} h(z)\,dz} = V_i(0) .
\end{aligned}$$

This is precisely the condition required for the statement of the Lemma.

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