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11. Financial Transfers to Sustain Cooperative International Optimality in Stock Pollutant Abatement

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INTRODUCTION

It is well known that the transnational character of many environmental problems (for example, greenhouse gas emissions, acid rain, pollution of international waters) requires cooperation among the countries involved if a social optimum is to be achieved. The issues raised thereby have often been addressed, in the economic literature, using concepts borrowed from cooperative game theory (see, for example, Maler 1989).

However, most of these contributions have dealt only with static (one-shot) games, which are appropriate only for flow pollution models. When the environmental damage arises from the presence of stock pollutants that accumulate (and possibly decay), the problem acquires a dynamic and intertemporal dimension. In this case, differential game theory is a more appropriate tool for the analysis of cooperation, as is done in, for example, van der Ploeg and de Zeeuw (1992), Kaitala, Pohjola and Tahvonen (1992), Hoel (1992) and Tahvonen (1993). Such analysis proceeds by evaluating the gain to be obtained from cooperation, at an international optimum, in comparison with the non-cooperative state of affairs modelled as a Nash non-cooperative equilibrium (open-loop or closed-loop).

Most often, these contributions leave aside the issue of the voluntary implementation of the international optimum. This is an important drawback because in this context no supranational authority can be called upon to impose the optimum. In view of ensuring such implementation, it has often been suggested that financial transfers between the countries involved might reinforce incentives towards cooperation. This property, understood in the sense of the theory of the core of a cooperative game, has in effect been demonstrated by Chander and Tulkens (1995, 1996), who propose a particular transfer scheme based on parameters reflecting the relative intensities of the countries' environmental preferences. The result is obtained, however, for flow pollutants only, that is, in a static game model.

The present contribution establishes that the same property can be obtained for stock pollutants, within the wider context of differential games. It also shows that the transfer scheme implies a sharing rule of the aggregate emission abatement cost between the countries. Moreover, with this cost-sharing interpretation, the transfers appear to be a strategically stable form of joint implementation of the international optimum.

The paper is structured as follows. Section 11.1 presents the stock pollutant model; Section 11.2 characterizes emissions that correspond to an international optimum without transfers, and raises the sustainability issue addressed in the sequel. In Section 11.3, emissions corresponding to a non-cooperative (open-loop) equilibrium are similarly characterized and contrasted with the optimal ones. In Section 11.4, financial transfers are formulated that are shown to induce voluntary cooperation of each country individually, whereas in Section 11.5 these transfers are further specified to achieve cooperation in the core theoretic sense, that is, for all subsets of countries. Section 11.6 considers some issues that arise when the time profile of the transfers, and not only their aggregate present value, is examined. The concluding section emphasizes the political significance of our results.

11.1 THE MODEL

The model we use is a dynamic extension of the well-known one originally presented in Mäler (1989). It is formulated in discrete time. There are n regions or countries, indexed $i \in \mathrm{N} = \{1,...,n\}$. Human activity in each country entails pollutant emissions: let $E_t = (E_{1t},..., E_{nt})$ with $E_{it} \geq 0 \, \forall i, t$, be the vector of these emissions at date t. They spread across the various countries and contribute to the formation of a pollutant stock $S\ (\geq 0)$ which is determined by the equation

$$S_t = [1 - \delta]S_{t-1} + E_t \tag{11.1}$$

where by definition $E_t = \sum_{i=1}^{n} E_{it}$ is the sum of the emissions and δ is the rate of natural decay of the stock $(0 < \delta < 1)$. Equation (11.1) – often called a 'transfer function' – summarizes the ecological component of the model.

In this formulation, pollution is assumed to be global, in the sense that it spreads out uniformly across countries. Atmospheric CO_2 is the best example of this form of transfrontier pollution. Some pollutions, such as acid rain or river water pollution, do not obey that equation: they are called 'directional'. The results presented below could be extended to directional pollutions, by introducing other forms of transfer functions between emissions and depositions of pollutant (for example Germain, Toint and Tulkens 1995), where we use another function in the framework of a model without pollutant accumulation).

As to the economic component of the model, it comprises two elements. On the one hand, the pollutant stock implies damages for each country i : at period t, and in monetary terms, these amount to $D_i(S_t)$, where D_i is supposed to be a positive, increasing and convex function $\left(D_i > 0 \text{ if } S > 0, D_i \geq 0 \text{ if } S = 0, D_i' > 0, D_i'' \geq 0\right)$. On the other hand, the only means the countries have to keep the stock of pollutant under control is at its source, that is, by reducing their emissions. More precisely, associated with country i is an abatement cost function $C_i(E_i)$, positive, decreasing and strictly convex $\left(C_i \geq 0, C_i' < 0, C_i'' > 0\right)$, that expresses the total costs incurred by the industries of country i due to the fact that total emissions in that country are restricted to E_{it}. The decreasing character of the function reflects the obvious phenomenon of increasing aggregate costs when emissions are abated.

These assumptions, identical to those of Mäler's, are justified in part by realism and in part by analytical convenience: realism in assuming that the damage cost functions $D_i(S_t)$ and the abatement cost functions $C_i(E_{it})$ be increasing and decreasing, respectively; convenience because without convexity the optimization approach we want to use below would require a much heavier mathematical apparatus without much gain in the understanding of the economics at stake.

11.2 INTERNATIONALY OPTIMAL POLLUTANT EMISSIONS AND STOCK

11.2.1 Determining the Optimum

For the economic–ecological system thus described, an international optimum is defined as the joint emission policy, and the ensuing stock, that minimize the sum, over all countries, of the total of both their damage and abatement costs. Formally, this corresponds to the solution of the problem:

$$\min_{\{E_t\}_{t\in T}} \sum_{t=1}^{T}\sum_{i=1}^{n} \beta^t \left[C_i(E_{it}) + D_i(S_t)\right]$$
$$\text{s.t.} \begin{cases} S_t = [1-\delta]S_{t-1} + E_t;\ S_0 \text{ given} \\ E_{it} \geq 0, \forall t \in \mathrm{T}, \forall i \in \mathrm{N} \end{cases} \tag{11.2}$$

where β is the discount factor $(0 < \beta \leq 1)$, $\mathrm{T} = \{1,\ldots,T\}$, T being the time horizon of the problem (T positive, integer and possibly infinite).

The necessary conditions for an interior minimum of problem (11.2) yield optimal trajectories for the emissions $\{E_t^*\}_{t\in \mathrm{T}}$ and for the pollutant stock $\{S_t^*\}_{t\in \mathrm{T}}$ which satisfy the following equations:

$$C_i'\left(E_{it}^*\right) + \sum_{\tau=t}^{T}\beta^{\tau-t}[1-\delta]^{\tau-t}\sum_{j=1}^{n} D_j'\left(S_\tau^*\right) = 0, \forall i \in \mathrm{N}, \forall t \in \mathrm{T}. \tag{11.3}$$

This expression means that at every moment of time, the marginal abatement cost of country i must be equal to the sum over all countries of their respective marginal damage costs over the entire horizon remaining. In this setting, the higher the damages (either intrinsically *via* the marginal damages D_i' and the stock S, or *via* the weight attributed to these in the future, that is, *via* β), the more important it is to abate and the lower are the optimal emissions; similarly, the slower the pollutant stock depreciates (the smaller δ is).

In the particular case where the damage functions are linear, that is, when

$$D_i(S_t) = \pi_i S_t, i \in \mathrm{N} \tag{11.4}$$

we can be more explicit on the time profile of the optimal emissions. In this case they do indeed obey the following equation:

$$C_i'\left(E_{it}^*\right) + \pi_N \frac{1-\beta^{T+1-t}[1-\delta]^{T+1-t}}{1-\beta[1-\delta]} = 0,\ i \in \mathrm{N},\ t \in \mathrm{T} \tag{11.5}$$

where by definition $\pi_N = \sum_{i=1}^{n} \pi_i$. Thus, we get the usual result that, with linear damages, optimal emissions do not depend upon the initial stock (Tahvonen 1993).

We also have that, because the functions $C_i(E_i)$ are decreasing and convex over their domain of definition, optimal emissions are increasing with time if T is finite, whereas they are constant when T is infinite.

Indeed, when T is finite, (11.3) suggests that the incentive to abate (as measured by the discounted sum of future damages) decreases as one gets closer to the horizon; but this does not apply when T is infinite: the second term of (11.5) is then a constant.

11.2.2 The Sustainability Issue

In a Coasian spirit, one may interpret the international optimum as the outcome of a treaty voluntarily signed among the countries involved. However, defining an optimum does not imply that it is an enforceable agreement, not even, if it happened to be reached, that it can be sustained. One of the main reasons for non-sustainability lies indeed in the fact that some countries might consider that the total cost they incur with the optimal joint policy $\{E^*_{it}\}$ is higher than the one they would incur by acting on their own. Of course, this cannot be the case for *all* countries because if it were, the total costs represented by the value of the objective function (11.2) at the solution would not be a minimum.

To evaluate the strength of this argument, the cost of 'acting on her own' should be specified explicitly for each country, and compared with that of the optimal joint policy. This is examined in the next two sections.

11.3 THE OPEN-LOOP NON-COOPERATIVE NASH EQUILIBRIUM

Suppose that, at time 0, each country chooses once and for all its emissions for all future periods, given the pollutant stock S_0 existing at that time, and the emissions chosen by the other countries. Formally, each country i ($i \in \{1,\dots,n\}$) then solves the following problem:

$$\min_{\{E_{it}\}_{t\in T}} \sum_{t=1}^{T} \beta^t [C_i(E_{it}) + D_i(S_t)] \tag{11.6}$$
$$\text{s.t.} \begin{cases} S_t = [1-\delta]S_{t-1} + E_t;\ S_0 \text{ given} \\ E_{it} \geq 0, \forall t \in T;\ E_{jt}\ (j \neq i) \text{given} \end{cases}$$

The simultaneous solution of the n problems (11.6) consists of emissions and stock trajectories $\{E_t^N\}_{t\in T}$ and $\{S_t^N\}_{t\in T}$, respectively, that are also the solution of the first-order conditions:

$$C_i'\left(E_{it}^N\right) + \sum_{T=t}^{T} \beta^{\tau-t}[1-\delta]^{\tau-t} D_i'\left(S_T^N\right) = 0, \forall i \in N, \forall t \in T \tag{11.7}$$

Contrary to what is the case at an international optimum, country i here only takes account of the impact of its decisions *on its own* environment (hence the presence of $D_i'\left(S_t^N\right)$ in lieu of $\sum_{j=1}^n D_j'\left(S_t^*\right)$ in (11.7)).

The emissions and stock trajectories thus defined are of the nature of a Nash equilibrium in non-cooperative games. More specifically, they constitute an 'open-loop' Nash equilibrium, characterized by the fact that each country's strategy is expressed as a function of calendar time alone. See Fudenberg and Tirole 1993, pp. 130 ff. Exogenous changes in the transfer function, or possible deviations from the equilibrium strategies by other countries are not considered.

In the particular case of linear damage costs, the countries' emissions obey the following equations:

$$C_i'\left(E_{it}^N\right)+\pi_i\frac{1-\beta^{T+1-t}[1-\delta]^{T+1-t}}{1-\beta[1-\delta]}=0,\ i\in \mathrm{N},\ t\in \mathrm{T} \qquad (11.8)$$

As it was the case at the international optimum, and for reasons that are quite similar (see previous section), emissions are increasing if T is finite and constant if T is infinite.

In view of the fact that $\forall i, \pi_i < \pi_N$, we notice, by comparing (11.5) to (11.8), that when the D_is are linear, the Nash equilibrium emissions of country i are always higher than the internationally optimal ones. This is only natural since at a Nash equilibrium, the abatement decisions of each country are based on their impact on their own environment only.

11.4 TRANSFERS INDUCING AN INDIVIDUALLY RATIONAL INTERNATIONAL OPTIMUM

Let us recall that $\mathrm{E}_1^*,\ldots,\mathrm{E}_T^*$ and $S_0, S_1,\ldots, S_t^*$ are internationally optimal emissions and stock trajectories. Then:

$$W(S_0)=\sum_{t=1}^T \beta^t \sum_{i=1}^n\left[C_i\left(E_{it}^*\right)+D_i\left(S_t^*\right)\right] \qquad (11.9)$$

is the optimal aggregate total cost of all countries, and

$$W_i(S_0)=\sum_{t=1}^T \beta^t\left[C_i\left(E_{it}^*\right)+D_i\left(S_t^*\right)\right] \qquad (11.10)$$

is the share of $W(S_0)$ borne by country i $(i \in N)$. Of course

$$W(S_0) = \sum_{i=1}^{n} W_i(S_0) \tag{11.11}$$

Let us keep in mind also that $E_1^N, ..., E_T^N$ and $S_0, S_1^N, ..., S_T^N$ are the emissions and stock trajectories at the Nash non-cooperative equilibrium. Then :

$$V_i(S_0) = \sum_{t=1}^{T} \beta^t \left[C_i\left(E_{it}^N\right) + D_i\left(S_t^N\right)\right] \tag{11.12}$$

is the discounted total cost of country i at the Nash equilibrium and

$$V(S_0) = \sum_{i=1}^{n} V_i(S_0) \tag{11.13}$$

is the sum of these total costs over the set of all countries. Clearly, by definition of the optimum, $V(S_0) \geq W(S_0)$.

The above reasoning on sustainability of the optimum can now be formalized as follows. If $\forall i \in \{1, ..., T\}$ $W_i(S_0) \leq V_i(S_0)$, then the optimal trajectories are such that all countries save costs by adopting them; cooperation to achieve the optimum is sustainable in the sense of what is called 'individual rationality' in game theory.

By contrast, a country i, for which $W_i(S_0) > V_i(S_0)$, has no such interest in cooperating. Cooperation, although globally favourable for all countries taken as a whole, is not sustainable in the above sense. In order to gain the cooperation of such a country, some form of compensation must be designed, for example, in the form of financial transfers.

Borrowing from Chander and Tulkens (1992), who dealt with this issue in a static context (as opposed to the intertemporal framework considered here), we propose financial transfers between the countries that are of the following form:

$$\tau_i(S_0) = -[W_i(S_0) - V_i(S_0)] + \mu_i[W(S_0) - V(S_0)],\ i \in N \tag{11.14}$$

where the parameters μ_i are arbitrary fixed values chosen between 0 and 1 and satisfy

$$\sum_{i=1}^{n} \mu_i = 1 \tag{11.15}$$

This last condition guarantees that the financial transfers defined by (11.13) be balanced, in the sense that

$$\sum_{i=1}^{n} \tau_i (S_0) = 0. \tag{11.16}$$

If, with international cooperation, country i receives a financial transfer equal to $\tau_i(S_0)$, then its discounted total cost along the optimal trajectory becomes

$$\tilde{W}_i (S_0) = W_i (S_0) + \tau_i (S_0). \tag{11.17}$$

It is easy to verify that this optimal cost *with transfers* borne by i is smaller than or equal to the cost it would bear at the Nash equilibrium. Indeed, it follows from (11.13) and (11.16) that

$$\tilde{W}_i(S_0) = V_i(S_0) + \mu_i[W(S_0) - V(S_0)] \leq V_i(S_0), \forall i \in \{1,\dots,n\} \tag{11.18}$$

because μ_i is non-negative and $W(S_0) - V(S_0) \leq 0$ by definition of an optimum. Thus with the transfers defined by (11.13), international cooperation becomes individually rational.

11.5 TRANSFERS INDUCING RATIONALITY IN THE SENSE OF COALITIONS

Another dimension of the sustainability of the international optimum is suggested by the notion of 'rationality in the sense of coalitions' offered by the 'core' concept in cooperative game theory. Here, costs savings are considered not only at the level of each individual country but also for groups – called 'coalitions' – of countries. An international optimum is not sustainable for a coalition if its members can achieve lower costs for themselves than those they incur at the optimum; it is sustainable if there is no coalition for which this is possible. Our claim in this section is that the international optimum can be made sustainable in the sense of coalitions, thanks to further specification of the financial transfers. To do so we will adapt to our present dynamic framework the concepts and methodology also developed in Chander and Tulkens (1995, 1996) in a static framework. We first recall the methodology and then state our present result.

11.5.1 The Methodology

A cooperative game in characteristic function form (with transferable utility) is defined by the pair $[\mathrm{N}, w(.; S_0)]$ where $\mathrm{N}=\{1,\ldots,n\}$ is the set of players (that is, the n countries) and w is the characteristic function. The space of the players' strategies on which this function is defined is specified as follows: for each country i, this space is the interval of all possible emission levels, that is, $[0, \infty]$. For any coalition $U \subseteq \mathrm{N}$, it is the set product of these intervals over the members of U.

The characteristic function of the game may then be defined by using the concept of *partial Nash equilibrium with respect to a coalition*, proposed by Chander and Tulkens (1995, 1996), subject to some adaptation to the present framework. Specifically, the function specifies as follows the vectors $\mathrm{E}_t (t \in \mathrm{T})$ of the strategies adopted by all players when a coalition $U \subseteq \mathrm{N}$ forms:

(i) For the members of U, the trajectories are described by $\{E_{it}^U : i \in U; t \in \mathrm{T}\}$ which is the solution of

$$\min_{\{E_{it}\}_{i\in U, t\in T}} \sum_{t=1}^{T} \sum_{i\in U} \beta^t [C_i(E_{it}) + D_i(S_t)] \text{ s.t. } (1), \qquad (11.19)$$

where $\forall j \in \mathrm{N}\backslash U, \forall t \in \mathrm{T}, E_{jt} = E_{jt}^U$ as defined by *(ii)*;

(ii) For the players out of the coalition U, the trajectories $\{E_{jt}^U : t \in \mathrm{T}\}\ j \in \mathrm{N}\backslash U$, are solutions of the simultaneous resolution of the problems

$$\min_{\{Ejt\}_{t\in T}} \sum_{t=1}^{T} \beta^t [C_j(E_{jt}) + D_j(S_t)] \text{ s.t.} (1), j \in \mathrm{N}\backslash U \qquad (11.20)$$

where $\forall i \in U, \forall t \in \mathrm{T}$, $E_{it} = E_{it}^U$, as defined by *(i)*.

Thus one supposes that if a coalition forms, its members minimize *together* the sum of their total discounted costs, while each country outside of it reacts by minimizing its own total discounted cost. The hypothesis about this last behaviour justifies the expression 'partial Nash equilibrium' used above.

On this basis, the characteristic function may be written

$$w(U; S_0) = \sum_{t=1}^{T} \sum_{i\in U} \beta^t \left[C_i\left(E_{it}^U \right) + D_i\left(S_t^U\right)\right], \ U \subseteq \mathrm{N} \qquad (11.21)$$

where $S_t^U = [1-\delta]S_{t-1}^U + E_t^U, \forall t \in T$ (with $S_0^U \equiv S_0$ given). Because of (11.18), one notes that $w(N; S_0)$ is equal to $W(S_0)$ defined by (11.9), in other words to the optimal total cost of all countries.

For the game $[N, w(.; S_0)]$, any n-dimensional and non-negative vector whose components sum up to $w([N; S_0])$ is called an *imputation* of the game. Thus an imputation may be seen as a way of sharing the optimal total cost between the different players.

The vector $(W_1, (S_0), \ldots, W_n(S_0))$, where $W_i(S_0)$ is defined by (11.10), constitutes such an imputation, where each country bears itself the abatement and damage costs induced by the optimal strategy $\{E_t^* : t = 1, \ldots, T\}$. But the possibility of financial transfers between countries implies that there exists (an infinity of) other imputations associated with the same strategy. Indeed, any vector $(\tilde{W}_i(S_0), \ldots, \tilde{W}_n(S_0))$, defined by (11.16) such that (11.15) is verified, is an imputation.

A *solution* of the game is an imputation that verifies certain properties. Among the imputations we have just defined thanks to the transfers $\tau_i(S_0)$, those who verify the condition

$$\sum_{i \in U} \tilde{W}_i(S_0) \leq w(U; S_0), \ \forall U \subseteq N, \forall S_0 > 0 \qquad (11.22)$$

are said *to belong to the core* of the game. The core is thus the set of imputations that attribute to any coalition a share of the aggregate cost $W(S_0)$ less than or equal to the mimimal cost $w(U; S_0)$ this coalition could achieve by itself.

We will call 'rational in the sense of coalitions' any imputation which belongs to the core of the game described above: with such a share, indeed, no coalition has an interest to form because its members, taken as a whole, would bear a total cost greater than what is proposed to them.

11.5.2 The Result

We now show that when *the damage functions are linear* (see 11.4)), such an imputation exists and can be exhibited in terms of specific values of the parameters μ_i appearing in (11.13).

Theorem: Let $\{E_t^* : t = 1, \ldots, T\}$ be the trajectory of the optimal emissions solution of problem (11.2), and $\{E_t^N : t = 1, \ldots, T\}$ be the trajectory of emissions at the Nash equilibrium solution of problems (11.6). Let $(W_1, (S_0), \ldots, W_n(S_0))$ and $(V_1, (S_0), \ldots, V_n(S_0))$ be the vectors of total discounted costs of each country along these two trajectories (as defined in

(11.10) and (11.11)). Then, under the assumptions of convexity of the functions C_i and of linearity of the functions $D_i\ (i \in \mathrm{N})$, the imputation $(\tilde{W}_i(S_0),\dots \tilde{W}_n(S_0))$ defined by

$$\tilde{W}_i(S_0) = W_i(S_0) + \tilde{\tau}_i(S_0),\ i = 1,\dots, n \tag{11.23}$$

where

$$\tilde{\tau}_i(S_0) = -[W_i(S_0) - V_i(S_0)] + \frac{\pi_i}{\pi_N}[W(S_0) - V(S_0)] \tag{11.34}$$

and $W(S_0)$ and $V(S_0)$ are respectively defined by (11.9) and (11.12), belongs to the core of the cooperative game $[\mathrm{N}, W(.; S_0)]$.

It is easy to see that the coefficients defined by $\delta_i = \pi_i / \pi_N$ (where the π_i are the coefficients of the damage cost functions (11.4)) have values between 0 and 1 and verify (11.14), and that the financial transfers $\tau_i(S_0), i \in \mathrm{N}$ are balanced in the sense of (11.15). By analogy with individual rationality, we call the optimal trajectory with transfers (11.22) 'strategically stable' or 'rational in the sense of coalitions'.

One may note also that this property is satisfied whatever are the initial stock S_0 and the temporal horizon T (which may be infinite).

11.6 THE TIME DIMENSION OF THE FINANCIAL TRANSFERS

The transfers introduced in Sections 11.4 and 11.5 have been defined on a global basis: they are given as total sums related to the complete period $\mathrm{T} = \{1,\dots,T\}$ where cooperation takes place. As a consequence, the question of their precise allocation in time during this period remains open, in particular when the horizon is infinite (see also Zaccour 1994). Even in the case where it is finite, the countries may benefit from parts of the cooperation dividends before the horizon is attained. The purpose of the present section is to briefly analyse some simple situations that may occur.

The first observation is that, if the international optimum globally dominates the Nash equilibrium (in other words $W(S_0) < V(S_0)$ where W and V are defined by (11.9) and (11.12)), this domination is not *monotonic*, in the sense that it does not imply that, for each elementary period (a year, for instance) $t \in \mathrm{T}$, the global cost for this period is inferior at the international optimum to that at the Nash equilibrium.

Let us define

$$J_t^* = \sum_{i=1}^{n} J_{it}^* = \sum_{i=1}^{n} \left[C_i\left(E_{it}^*\right) + D_i\left(S_t^*\right)\right]\beta^t, \forall t \in \mathrm{T} \quad (11.35)$$

$$J_t^N = \sum_{i=1}^{n} J_{it}^N = \sum_{i=1}^{n} \left[C_i\left(E_{it}^N\right) + D_i\left(S_t^N\right)\right]\beta^t, \forall t \in \mathrm{T} \quad (11.36)$$

In order to focus the exposition, we will consider here only the three following assumptions.

Assumption 1:

$$J_t^* \le J_t^N, \forall t \in \mathrm{T}. \quad (11.37)$$

The international optimum monotonically dominates the Nash equilibrium. In this case, the global surplus generated by cooperation increases every year and the global transfers defined by (11.13) may be decomposed in annual transfers of the form

$$\Theta_{it} = -\left[J_{it}^* - J_{it}^N\right] + \mu_i\left[J_t^* - J_t^N\right] i \in \mathrm{N}, t \in \mathrm{T} \quad (11.38)$$

One easily verifies from (11.9)–(11.13) and Assumption 1 that $\tau_i(S_0) = \sum_{t=1}^{\mathrm{T}} \Theta_{it}$.

Assumption 2:

$$\exists M : 1 \le M < T : \begin{cases} J_t^* < J_t^N \text{ if } 1 \le t \le M, \\ J_t^* > J_t^N \text{ if } M < t \le T. \end{cases} \quad (11.39)$$

The international optimum dominates the Nash equilibrium until a certain year M, starting from which the domination order is reversed. In this case, one verifies that

$$W(S_0) - V(S_0) = \sum_{t=1}^{M}\left[J_t^* - J_t^N\right] + \sum_{t=M+1}^{T}\left[J_t^* - J_t^N\right] > \sum_{t=1}^{M}\left[J_t^* - J_t^N\right] \quad (11.40)$$

The global surplus is inferior ***in absolute value*** to the surplus obtained during the subperiod $\{1, \ldots, M\}$ (both sides of (11.24) indeed represent negative cost differences). If one applies the annual transfer formula (11.23), country i will save $\mu_i \sum_{t=1}^{M}\left[J_t^* - J_t^N\right]$ at the optimum compared to the Nash equilibrium during the first M years, and will lose $\mu_i \sum_{t=M+1}^{T}\left[J_t^* - J_t^N\right]$ during the each of the years $\{M+1, \ldots, T\}$. Since the whole procedure is individually rational, the gain in the first period is necessarily superior (in absolute value) to the loss in the second. In this context, if this country wishes to restore, as far as it is concerned, the domination of the international optimum over the Nash equilibrium, it is enough that it levies the loss of the second period from its gains at the first, and then transfers the resulting sum to the second period. Compared to the Nash equilibrium, it will then obtain, at the international optimum with transfers, a gain of $\mu_i [w(s_0) - v(s_0)]$ during the period $\{1, \ldots, M\}$ and of zero during period $\{M+1, \ldots, T\}$.

Assumption 3:

$$\exists M : 1 \leq M < T : \begin{cases} J_t^* > J_t^N \text{ if } 1 \leq t \leq M, \\ J_t^* < J_t^N \text{ if } M < t \leq T. \end{cases} \tag{11.41}$$

The international optimum is dominated by the Nash equilibrium until year M, after which the order of domination is reversed. This is the most plausible situation, in that the optimum implies abatement costs higher than those of the Nash equilibrium, costs which must financed from the date where they are decided while the gains in damages that they induce are obtained only later. In this context, the countries obviously cannot transfer future gains to cover present losses, in order to ensure the domination of the international optimum over the Nash equilibrium during the early subperiod $\{1, \ldots, M\}$. However, if the countries globally lose until $t=M$, the impact of this loss may be very different from country to country. The most affected countries could then benefit from financial transfers from the less affected ones.

11.7 CONCLUSIONS

Using the theory of non-cooperative games in the framework of a model of intertemporal transboundary pollution, we have constructed by means of appropriate transfers a sharing scheme for the abatement costs between the countries involved, which makes their cooperation sustainable in the sense of 'coalitional rationality', that is, that no coalition of countries can guarantee to its members a lower total cost than that obtained at the international optimum with the proposed financial transfers.

That the result was obtained under two restrictive assumptions (linearity of the damage functions, and the open-loop character of the Nash equilibrium considered as reference) should not undermine its political significance. Indeed, the issue at stake is the one whether worldwide agreements on transboundary pollution problems such as CO_2 emissions are at all achievable, and it is a disputed one, as witnessed by the controversy reported in Tulkens (1997). Burden sharing, in particular, is an important economic component of the debate.

The present paper reinforces the optimistic perspective opened by the results of Chander and Tulkens (1995, 1996), by extending these results to stock pollutants. In addition, the constructive nature of this extension – contained in the burden sharing formula (11.34)- lows for computations that we intend to provide in forthcoming simulations.

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