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**CONFLICT AND STABILITY IN ANARCHIC INTERNATIONAL SYSTEMS**

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### Abstract

A considerable part of theory in international relations concerns the issue of whether cooperation and stability can emerge from the competition and self-interest of sovereign powers existing in a state of anarchy. Does anarchy, if ever, imply stability in the form of a balance-of-power, or does stability require restraints which arise from the complex nexus of interdependencies characterizing the contemporary world economy and its associated institutions? The analysis in this essay supposes that nation-states are each endowed with some infinitely divisible resource, which those states maximize and which also measures their ability to overcome adversaries in the event of conflict. In this context we reexamine and reformulate the realist view, by offering a noncooperative, extensive-form model of international conflict without exogenous mechanisms for the enforcement of agreements in order to uncover the conditions under which a balance-of-power as construed by our model ensures the sovereignty of all states in anarchic systems. Our primary conclusion is simple: there exists at least one world, albeit abstract and reminiscent of the frictionless planes with which we introduce the perspectives of physics, in which a balance-of-power ensures sovereignty.

# CONFLICT AND STABILITY IN ANARCHIC INTERNATIONAL SYSTEMS\*

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Institutional restraints on action in anarchic systems arise, by definition, endogenously, and the establishment, maintenance, and evolution of those restraints must be understood in terms of the individual incentives of relevant decision-makers. Correspondingly, if we accept the premise that "International relations continue to be a recurring struggle for wealth and power among independent actors in a state of anarchy (Gilpin, 1981:7)," then we can interpret a considerable part of contemporary theorizing about international relations as concerning the issue of whether cooperation and stability can emerge from the competition and self-interest of sovereign states. Can anarchy yield stability in the form of a balance-of-power, or does stability require restraints that arise from exogenously imposed institutions, from the complex nexus of interdependencies characterizing the contemporary world economy, or, in Riker's (1962) terms, from moral suasion? Burns (1968:249) states the realist position: "Classic balance-of-power theory can be interpreted as a hypothesis that in a more-than-two-Power world there are no non-autonomous causes of systemic change ... the purely political aspect of the system's power-political process always tends to produce a stable equilibrium that can be upset, if at all, only by autonomous changes." Such assertions, though, do not prove that nation-states can coalesce effectively to offset the ambitions of other states, that agreements can be maintained so as to ensure each nation's sovereignty, or that international institutions, as the byproducts of competition, merely facilitate the stability inherent in anarchy. If we equate the idea of regimes with balance, then Keohane (1984:99) states the problem succinctly: "The puzzle of compliance is why governments, seeking to promote their own interests, ever comply with the rules of international regimes when they view those rules as in conflict with ... their 'myopic' self-interest."

Despite the importance of such issues to international politics, they have a broader imperative, because they are fundamental to theories based on the rational choice paradigm and to game theory in particular. Von Neumann and Morgenstern's (1947) seminal work divided game theory into cooperative and noncooperative sub-parts. Nash (1951), however, suggested that cooperative action should be analyzed using noncooperative theory -- that

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coalitions and the like should be viewed as noncooperative equilibria in which the enforcement of agreements has as its basis the fact that no individual has a unilateral incentive to defect from an agreement. Until recently, game theorists largely ignored Nash's suggestion. Avoiding modeling the mechanisms of enforcement, they developed instead a variety of "solution hypotheses" such as the V-set, the bargaining set, and the competitive solution, which sought to identify the agreements that would be reached if enforcement were not an issue. Much of contemporary game theory, on the other hand, seeks to integrate the analysis of cooperative and noncooperative games along the lines suggested by Nash. The results to date are incomplete, but we now know, for example, how cooperation is sustained in the repeated Prisoners' Dilemma (Taylor, 1976, and Axelrod, 1984), how repetition and uncertainty sustain vote-trading in legislatures (Calvert, 1989), and how, in some circumstances, to rationalize cooperative solution hypotheses as noncooperative equilibria (Selten 1981, Sutton 1986).

Rationalizing realism's argument about stability, though, poses special difficulties. Cooperation arises in the repeated Prisoners' Dilemma, for example, because the punishments that repetition allows provide a mechanism whereby all players can avoid mutually distasteful outcomes and Pareto efficient outcomes can be enforced and sustained as equilibria. In the essential components of the realist view, on the other hand, all outcomes are efficient since power, a relational concept, is in constant supply. Hence, cooperation can only be directed at implementing or blocking outcomes that are disadvantages for some and advantageous for others. So anarchic systems, reduced to their basic character, seem susceptible to those instabilities we associate with majority rule, and the question remains as to whether there is a theoretical basis for realist thinking -- whether we can sustain stability in n-country systems if the primary rule is that countries or alliances of countries with more "power" can defeat those with less.

For special cases, of course, stability seems unexceptional, such as when one country is a hegemon or when there are only two equally powerful adversaries. However, we want to explore the possibility of stability in systems when there is no hegemon, when there are any number of countries, and when every country is the potential victim of some winning coalition. We proceed by building on a model in which nation-states, represented as unitary actors, are each endowed with some infinitely divisible and transferable resource that those states maximize and that also measures their ability to overcome adversaries in the event of conflict (Niou and Ordeshook, 1986, Niou, Ordeshook, and Rose, 1989). To reformulate the realist view, we differentiate between two forms of stability: system-and resource-stability. System-stability implies that all countries can ensure their sovereignty — that no country will have its resources reduced to zero. Resource-stability implies the prediction that no reallocation of resources will occur. We make this distinction because systems are rarely, if

ever, resource-stable and because the issue of sovereignty seems qualitatively different from the issue of how nations contend with the ebb and flow of economic and military capabilities: "disagreements about how benefits should be distributed permeate the relations among actors and persist because bargains are never permanently valid ... Apparent victory can be illusory or defeat ephemeral, for political bargaining and maneuver result not in definitive choices conferring power on some people rather than others, but in agreements that may in the future be reversed or in discord that signals a continuation of bargaining and maneuver" (Keohane,1984:18). In accordance with realism's fundamental concern, we want to focus on the conditions under which anarchic systems can be system-stable -- under which it is legitimate to assert that a balance-of-power, somehow construed, ensures the sovereignty of all states.

To establish these conditions we build on a second feature of international politics -- namely, that it does not correspond to a "single-play" game, but instead it is a process in which today's actions determine tomorrow's strategic possibilities. This fact necessitates modifying the assumption that nations maximize "power" in some myopic way. Specifically, if national leaders are concerned with a view of the future, and, hence, with what their actions imply about the ultimate, overall distribution of resources, then this interpretation of rationality is not a basic assumption but, at best, it is a deduction from some model that posits more fundamental goals. If we assume instead that the ultimate goal is the survival of their nations as sovereign entities (Waltz 1979, 1988), then those leaders must evaluate a decision to attack another nation, to form an alliance, or to cede resources, not in terms of immediate benefits, but in terms of what an action implies ultimately about the likelihood that its survival will be endangered. Thus we echo Kaplan's (1979:70) assertion that we must take account of the possibility that "...the weakest player, by joining a nearly predominant strong player, only creates a condition in which he will be the next victim," as well as Wagner's (1986:551) more technically stated implication: "the basic question that concerns us is whether states will act so as to eliminate other states. If one state is eliminated from a four-actor game, for example, the result is to precipitate a three-actor subgame. If a value can be assigned to such a subgame for each player, it is possible to determine whether any players have an incentive to eliminate other players."

Our previous analyses build on these observations, but they use cooperative solution theory to render predictions, and thus they circumvent the issue of enforcement and the precise logic whereby we can rationalize alliance formation. Here, we look more closely into the nature of collusive action in international systems, and, proceeding in much the same way as Wagner proposes, with a noncooperative model, we rationalize our previous conclusions about stability.<sup>1</sup> System-stability can prevail even if enforcement is endogenous, and the conditions under which this is true requires no special number of countries or distributions of resources.

In Section 1 we offer an extensive-form model of anarchic international systems. In Section 2 we explore equilibrium strategies in that model for 3-country systems, and in Section 3 we explore 4-country systems. In Section 4 we provide the general results for n-country systems that support our central conclusion -- that a system is system-stable if every member of S is in at least one minimal winning coalition. In Section 5 we survey the circumstances under which countries that are not members of any minimal winning coalition are eliminated, and we also suggest when a set of countries might voluntarily relinquish their sovereignty in order to form a new sovereign state. In Section 6 we offer some substantive conclusions. Before proceeding, we emphasize that our analysis takes no account of geography, resource growth, war costs, uncertainty, and ambiguities in the notion of sovereignty. Because our analysis rationalizes much of the same conclusions about system-stability that we derive using cooperative game theory, our earlier treatments of such matters stand. Our aim, though, is not to formulate a model of international political processes per se; rather, it is to establish a possibility result that provides a theoretical basis for the intuition guiding the realist view of international systems.

### 1. An Extensive-Form Model of Conflict

Beginning with some elementary notation, we denote a system by  $(S,r)$ , where  $S = \{1,2,\dots,n\}$  represents the set of countries, and where  $r = (r_1, r_2, \dots, r_n)$  corresponds to the distribution of resources across S. For convenience, we suppose that the countries are ordered  $r_1 > r_2 > \dots > r_n > 0$ , and we let R be the total resources in the system.<sup>2</sup> Assuming that r determines winning and losing coalitions, we denote the sum of resources controlled by the members of the coalition C by  $r(C)$ , so that C is winning (C is in the set W) if  $r(C) > R/2$ , it is losing if  $r(C) < R/2$ , and it is minimal winning if, for all i in C, C-{i} is losing. Thus, if  $r_i > R/2$ , i is predominant -- it is winning against all other countries and it can incorporate their resources at will -- so every country has an incentive to avoid the possibility that some other country becomes predominant. Countries that control precisely R/2 resources are near-predominant.

We next describe an extensive form game,  $\Gamma$ , that models conflict in which countries try to increase their resources by making and implementing threats against others and in which threatened countries try to maintain their sovereignty with counter offers or counter threats. Letting T be the set of all threats with respect to  $(r,S)$ , then

$$T = \{(r',C) : r(C) > r(S-C), C \subset S, \text{ where } i \in C \text{ iff } r'_i \geq r_i\},$$

so  $r'_i < r_i$  for all  $i \in S-C$ . We identify threats specifically by C thus:

$$T_C = \{(r',C) \in T : r'_i \geq r_i \text{ iff } i \in C\}.$$

The game  $\Gamma$ , now, is described as follows:

- (1) Nature randomly selects an  $i \in S$ ;
- (2)  $i$  offers a threat  $(r', C) \in T_C$ ,  $i \in C$ ; or  $i$  passes. If  $i$  passes, we return to step (1). If  $i$  threatens and if  $r'_j > r_j$ ,  $j$  is an active member of  $C$ , whereas  $r'_j = r_j$  for the passive members of  $C$ .<sup>3</sup>
- (3) The members of  $C - \{i\}$  choose between approving or rejecting  $i$ 's threat. If no member of  $C - \{i\}$  rejects, then  $r'$  becomes the current threat; otherwise, we return to step (1).
- (4) With  $(r', C)$  the current threat, nature randomly orders the members of  $S - C$ , and we denote this order by  $O = (m, l, \dots, k)$ .
- (5) With  $m$  in  $S - C$  offering the first counter, a counter takes one of two forms: a new threat,  $(r'', C'') \in T_{C''}$ ,  $m \in C''$ ; or a resource transfer from  $S - C$  to one or more members of  $C$ . We denote those party to the transfer by  $C''$ .
- (6) The members of  $C'' - \{m\}$  choose between approving or rejecting  $m$ 's counter. If a counter which is itself a threat is approved unanimously, it becomes the new current threat, and we return to step (4). Counter-threats that would be approved if proposed are viable. If one or more members of  $C'' - \{m\}$  reject the counter, we select the next country in  $O$  and return to step (5). For counters that are resource-transfers, unanimous acceptance renders the transfer the new status quo distribution and we return to step (1).
- (7) If the counter of the last country in  $O$  is rejected, the resource distribution of the current initial threat becomes the status quo, countries with no resources are eliminated, and we return to step (1).

This sequence of moves is, of course, a gross simplification of possibilities. For example,  $i$ 's partners in  $C$  can merely accept or reject  $(r', C)$ , but they cannot propose modifications, and, thus,  $\Gamma$  does not model the negotiations that might precede an actual conflict. Nevertheless,  $\Gamma$  does model a system in which "power" is the sole determinant of winning and losing, in which threats and counters are the mechanisms whereby countries secure resources and ensure their sovereignty, in which countries join coalitions because it is in their individual interest to do so, and in which no exogenous constraints or complex set of economic interdependencies ameliorate conflict. Thus, if system-stability prevails here, we say that a balance-of-power exists.

Before proceeding, though, we must refine our assumptions about preferences. Specifically, if we think of resources as power and suppose that countries maximize power, we must nevertheless add an assumption about how countries evaluate elimination as an outcome distinct from the rest. Elimination, after all, is qualitatively different from merely possessing a small amount of resources. With a small resource base a country remains a potentially essential actor whereas with zero resources, it is forever eliminated from the game. We accommodate this fact by supposing that countries are risk averse with respect to the

possibility of elimination (equivalently, a country attributes a value of  $-\infty$  to any outcome in which it is eliminated):

If one act,  $a_1$ , ensures that  $i$  cannot be eliminated, whereas another act,  $a_2$ , leaves open the possibility that  $i$ 's resources are set to zero,  $i$  prefers  $a_1$  to  $a_2$ .

The principle analytic feature of the game  $\Gamma$ , now, is that threats and counters can continue in sequence forever. However, there are terminal points. First, we initially interpret nodes in which a threat is implemented as terminal, although we do this with the understanding that such nodes are merely the starting point for a new round of conflict with a new status quo resource distribution. Any evaluation that the countries might place on such reallocations depends, then, on what follows in the new round. Owing to the following assumption, though, there is a second type of terminal point, which arises whenever one country secures  $R/2$  resources:

if  $r_i = R/2$ , and if the threat  $(r', C)$  is implemented,  $i \in C$ , then even if  $r'_i = r_i$ ,  $i$  can take advantage of the conflict to become predominant.

Thus, systems with a near-predominant country are "frozen" -- no additional threats are worthwhile since the remaining countries will block any threat's implementation. Notice that we can rationalize this assumption by supposing that  $r_i$  measures resource proportion and that the conflict implicit in a threat's implementation destroys some part of the resources of the active antagonists. For example, if  $r = (50, 33, 17)$ , if the threat  $(50, 50, 0)$  is implemented, and if the conflict destroys some of 2 and 3's resources, then 1's relative share increases and it becomes near-predominant.

Terminal nodes in which one country becomes near-predominant can be reached, of course, if an appropriate threat is implemented, but they can also be reached if some set of countries transfer some share of their resources to another country. We have not specified the circumstances under which transfers are proposed, but the incentives for accepting them also derives from the idea that conflict is costly. Without modeling war costs directly, we can make allowances for such costs with this assumption (for an analysis of war costs see Niou and Ordeshook 1989):

If  $i$  can become near-predominant either by implementing a threat or by having some set of target countries transfer enough resources to  $i$  so as to render  $i$  near-predominant,  $i$  prefers the transfer.



In confronting the threat  $(r',C)$ , suppose the members of S-C find it in their interest to "freeze" the system. Letting  $\max[C]$  be the largest member of C, if  $r(S-C) + r_{\max[C]} > R/2$ , then S-C can freeze the system by offering  $\max[C]$  enough resources to render it near-predominant. Clearly, if S-C prefers freezing the system, it should transfer to  $\max[C]$ , since this choice minimizes the resources that S-C must surrender. And  $\max[C]$  accepts the offer: Because attempts to secure more than  $R/2$  will be blocked, securing  $R/2$  by transfer is  $\max[C]$ 's most preferred feasible outcome.

When the members of S-C prefer such transfers is an issue we cannot yet address, but we must consider a second type of transfer -- by S-C to members of  $C-(\max[C])$  such that  $r''_j \geq r'_j$  for all  $j \in C$ , where  $r''_j$  is the proposed post-transfer resources of  $j$ . For example, let  $r = (110,80,60,50)$  and let  $\{1,2,3\}$  threaten 4 with  $(150,85,65,0)$ . Barring a counter-threat, country 4, in lieu of rendering 1 near-predominant with a transfer of 40 resource units, might try instead to disrupt matters by offering 2 and 3 each 10 units of resources, taking the chance that it will fare better in a game with the distribution  $(110,90,70,30)$ . Whether 4 prefers such a proposal and whether 2 and 3 should reject this offer so as to eliminate 4 and freeze the system at  $(150,85,65,0)$  depends on expected value calculations, as well as attitudes towards risk. Notice, though, that if such a proposal is offered and accepted, it merely results in a new 4-country system, and if all such offers are accepted, we can cycle endlessly without eliminating countries, in which case  $(S,r)$  is system- but not resource-stable. However, we are not interested in resource-stability, and elimination, if it occurs at all, occurs only if countries are unwilling to propose or accept such non-terminal transfers. Throughout this essay, then, we suppose that such transfers will not be made or accepted, and we analyze the part of the extensive form in which transfers freeze systems or countries are eliminated.<sup>4</sup>

## 2. A Three-Country Example

Even after we eliminate non-terminal transfers,  $\Gamma$  remains a recursive game, because the sequence of threats and counters can proceed indefinitely, with each succeeding counter becoming a new threat that is itself subsequently countered.<sup>5</sup> However, we can proceed in the analysis by pretending that  $\Gamma$  is finite. First, notice that if a threat is approved, its characteristics and the status quo resource distribution are the sole relevant components of the situation. We suppose, then, that countries pursue the same strategy whenever they encounter the same threat (that is, strategies are stationary).<sup>6</sup> Second, letting  $\Gamma_{r'}$  denote the sub-game that follows acceptance of the threat  $(r',C)$ , suppose country  $i$  associates the value  $v_i(\Gamma_{r'})$  with playing that sub-game. Letting  $v(\Gamma_{r'}) = (v_1(\Gamma_{r'}), \dots, v_n(\Gamma_{r'}))$ , this vector -- the continuation value of  $\Gamma_{r'}$  -- specifies what the countries believe follows from the approval of  $(r',C)$ . Thus,  $v_i(\Gamma_{r'})$ , when compared against whatever follows if  $(r',C)$  is rejected, determines  $i$ 's preference for acceptance or rejection of  $(r',C)$ . Once values for all threats are specified we

can assume that the acceptance of a threat or counter is a terminal node with its continuation value as the "final outcome." We then analyze  $\Gamma$  as though it were a finite extensive-form game of complete information and we deduce sub-game perfect equilibrium strategies by working backwards from the terminal nodes in the same way we treat finite agendas in majority voting games -- we deduce what each country ought to do any time it must choose a threat, a counter, or accepting or rejecting a threat or counter.

The problem here, of course, is that we have merely pretended to know continuation values. Nevertheless, we can define a stationary equilibrium as a set of continuation values -- one for each threat (subgame) -- and a set of strategies for each country such that these values and strategies are consistent. Thus, in a stationary equilibrium, the choices that the continuation values imply -- the strategies that are a subgame perfect equilibrium given the continuation values, must, in turn, imply those continuation values.

To illustrate these ideas, consider a three-country system with  $(S,r) = ((1,2,3),(120,100,80))$ , and consider an initial threat by country 1 to eliminate country 2 and to share the system's 300 units of resources evenly with 3. Limiting our discussion for the moment to threats and counters of this type and to counters from one country to another that freeze the game, consider Figure 1's representation of the situation (a \* denotes a terminal node). After 1 proposes the threat, 3 must decide whether to accept or reject it. If 3 accepts, country 2 must then offer a counter that, given our limitations, is either a coalition with 1 to divide R, a coalition with 3 to divide R, or a transfer to 1 (which, if offered, 1 is certain to accept since this is the best possible payoff for 1 given that 2 and 3 will never allow an outcome that gives 1 more). Notice that 2 need not consider a transfer to 3 since, being larger, country 1 entails transferring fewer resources. Depending, then, on which counter-threat 2 chooses, either country 1 or country 3 must decide whether to accept or reject. In the event of a rejection, the threat is implemented and 2 is eliminated. In the event of an acceptance, the counter becomes the new current threat, and the subgame that follows is denoted by  $\Gamma$  with an appropriate subscript. Figure 1 portrays the next step in this process with either 1 or 3 offering a counter.

Consider, now, the following continuation values for the three threats that eliminate a country (we need not consider threats that give a country more than 150 since no country will assist in such a threat or allow it to go unchallenged):

$$v(\Gamma_{(150,0,150)}) = (150,70,80)$$

$$v(\Gamma_{(0,150,150)}) = (70,150,80)$$

$$v(\Gamma_{(150,150,0)}) = v(\Gamma_{(150,0,150)})/2 + v(\Gamma_{(0,150,150)})/2 = (110,110,80)$$

where we suppose that 3 counters with (150,0,150) and (0,150,150) with equal probability whenever it is indifferent between these two choices (or that 3 accepts each counter with equal probability if it is indifferent). With these values we can now deduce subgame-perfect equilibrium choices for the extensive form in Figure 1. For example, beginning at the top-right of this figure, we see that 3 prefers (0,150,150) over  $\Gamma_{(150,0,150)}$  since the continuation value to 3 of (150,0,150) is 80. We indicate this preference by an arrow. Similarly, 2 rejects  $\Gamma_{(150,150,0)}$  since  $v_2(\Gamma_{(150,150,0)})$  corresponds to a lottery between 150 and 70. Thus, if 2 counters the initial threat of (150,0,150) with (0,150,150) and if 3 accepts, then 1 prefers to transfer resources to 2, because to choose otherwise leads to 1's elimination. (We need not consider any other type of transfer: Transfers giving less than 150 are rejected since rejection implements the threat, and a transfer that renders 3 near-predominant is more costly than a transfer to 2.) Notice, now, that 1's choice of a transfer implies that  $v(\Gamma_{(0,150,150)}) = (70,150,80)$ , which corresponds to our initial supposition. Turning to the lower-right of Figure 1, country 3 is indifferent between proposing (0,150,150) and (150,0,150) in the event that 2 counters with (150,150,0) and 1 accepts, so let 3 choose with equal probability. Once again, then, we deduce a value  $v(\Gamma_{(150,150,0)})$  that is consistent with our assumption, so 1 rejects 2's counter of (150,150,0). Hence, since 3 rejects a counter of (0,150,150) and since 1 rejects a counter of (150,150,0), 2's sole course of action when confronting the threat of (150,0,150) is to transfer resources to 1 so as to render 1 near-predominant. Thus, as initially conjectured,  $v(\Gamma_{(150,0,150)}) = (150,70,80)$ .

Figures 2 and 3 repeat this analysis for the remaining allowed threats, and they establish the consistency of the conjectured continuation values. Thus, what remains is the specification of  $v(\Gamma)$  and an equilibrium identifying the initial choices and responses of all countries. Suppose first that  $v(\Gamma) = (120,100,80)$  -- that both system- and resource-stability prevail, which we presume arises because no country makes or accepts an initial threat. This supposition is sustainable, in fact, if we characterize equilibrium strategies thus: a country does not initiate or agree to a threat unless it gains resources. Since the continuation values for the game's threats imply that countries 1 and 2 can each gain from an initial threat, whereas 3 can neither gain nor lose, 3 has no positive incentive to participate in a threat. Further, since only 1 and 2's resources can be transferred, they are in a zero-sum game, and depending on the probability that 3 chooses one action or another when it is indifferent, 1 or 2 has no incentive to threaten (150,150,0). But if 1 threatens 2 with, say, (150,70,80), 3 rejects 1's proposal that it acquiesce to the attack on 2. Finally, neither 1 nor 2 has an incentive to threaten 3 alone, since 3 can counter with a threat that requires the originally threatening country to transfer resources to 3's partner in the counter. Hence, three-country systems can be both system- and resource-stable.

There is, however, another stationary equilibrium, characterized by "accept all initial threats if they promise no loss; otherwise reject" that sustains the continuation value  $v(\Gamma) = \alpha(150,70,80) + (1-\alpha)(70,150,80) = (70+80\alpha, 150-80\alpha, 80)$ , where  $\alpha$  is the probability that 3 coalesces with 1 if it is indifferent between coalescing with 1 and 2. Although country 3 cannot gain resources if it abides by such a strategy (the continuation values for all threats remain as before), 3 has no positive incentive to defect unilaterally to another strategy. Hence, there exists a stationary equilibrium in which 3-country systems are resource-unstable. However, regardless of which stationary equilibrium prevails, this 3-country system is system-stable. And since all 3-country systems without a predominant or near-predominant player are equivalent to our example, this fact establishes the possibility of stability in anarchic systems in the way that stability was envisioned by realist arguments.

### 3. Four-Country Examples

Three-country systems differ from larger ones in that, if no country is predominant, all countries are essential -- all are members of a minimal winning coalition. For a system with an inessential country -- one that is not a member of a minimal winning coalition -- we must consider systems with four or more countries. To see that this distinction is important, let  $E$  be the set of essential countries and  $E_0$  the set of inessential ones. Suppose  $r = (100,95,75,30)$ , in which case  $\{4\} = E_0$ , and let the presumed equilibrium be characterized by "countries make or accept threats if they do not lose resources from doing so." Now suppose that 3 threatens  $(100,95,105,0)$ . If 3 cannot gain resources from 1 or 2, 4 must coalesce with 1 or 2 -- 3 will reject any offer, thereby implementing  $(100,95,105,0)$ . But if 1 or 2 accepts 4's offer, each is vulnerable to counters by 3 in the form  $(150,0,150,0)$  or  $(0,150,150,0)$ .<sup>7</sup> Thus, 1 and 2 reject 4's overtures, and 4 succumbs to the initial threat and is eliminated.<sup>8</sup> Thus, the system  $(\{1,2,3,4\}, (100,95,75,30))$  is not system-stable.

On the other hand, if 4 is essential, as it is with the distribution  $r = (110,80,60,50)$ , every country can buy stability and, as with three countries, no one is eliminated. However, let us pursue this example to illustrate the way which we analyze our model and the role of certain key assumptions. Limiting discussion once again to specific types of threats and counters, consider the following continuation values:

$$\begin{aligned} v(\Gamma_{(150,150,0,0)}) &= (a,b,60,50) \\ v(\Gamma_{(150,0,150,0)}) &= (a,b,c,50) \\ v(\Gamma_{(150,0,0,150)}) &= (a,b,60,d) \\ v(\Gamma_{(150,85,65,0)}) &= (150,80,60,10) \\ v(\Gamma_{(150,85,0,65)}) &= (150,80,20,50) \\ v(\Gamma_{(150,0,75,75)}) &= (150,40,60,50) \\ v(\Gamma_{(0,150,75,75)}) &= (40,150,60,50) \end{aligned}$$

where  $a < 150$ ,  $b < 150$ ,  $c < 60$ , and  $d < 50$ , and where the values of  $a$ ,  $b$ ,  $c$ , and  $d$  depend on the threat in question.

We begin with the possibility that, in confronting any of these threats, instead of countering with a counter-threat from this list, a country in S-C proposes a resource transfer to  $\max[C]$ . Notice that if two countries are simultaneously threatened, then if the first country to counter proposes a transfer to freeze the system, it cannot require that its partner in S-C bear the greatest burden. If it proposes such a transfer, its partner can reject and propose instead that the first country bear nearly all of the burden -- and at this stage that country has little choice but to comply since choosing otherwise implements the original threat. Further, if any other counter by the first country is rejected, the second can again propose, if it is in its interest to do so, that the first country bear the greatest burden. We can imagine more complex bargaining schemes among threatened countries, but the details of who transfers to whom has little bearing on our general conclusions, and our representation implies that whatever country responds first must bear the greatest burden in the event of a transfer by S-C to  $\max[C]$ .

Figure 4 shows the extensive form when  $(150,85,0,65)$  is the current threat. Country 3 has five alternative actions (in accordance with our previous discussion, we let  $r$  correspond to a stage in the game in which no non-terminal transfers are proposed or accepted). If it proposes  $(150,0,150,0)$  as a counter, then 1 rejects since, by assumption,  $v_1(\Gamma_{(150,0,150,0)}) = a < 150$  and rejection implements a threat which gives 150 to 1. If 3 proposes  $(150,0,75,75)$ , then country 4 rejects since  $v_4(\Gamma_{(150,0,75,75)}) = 50 < 65$ , and if 3 proposes  $(0,150,75,75)$ , then 4 rejects for the same reason. And if 3 proposes  $(150,85,65,0)$ , then 2 rejects since  $v_2(\Gamma_{(150,85,65,0)}) = 80 < 85$ . Hence, the only alternative available to 3 is to transfer resources to 1, in which case, as conjectured,  $v(\Gamma_{(150,85,65,0)}) = (150,80,20,50)$ . An equivalent analysis holds for the threats corresponding to the distributions  $(150,85,65,0)$ ,  $(150,0,75,75)$ , and  $(0,150,75,75)$ .

The situation is more complicated if two countries are threatened, as when  $\{1,2\}$  threatens  $\{3,4\}$  with  $(150,150,0,0)$ . Figure 5 shows the part of the extensive form that pertains after such a threat is accepted, and after nature selects 3 to offer the first counter (the situation is symmetric if 4 counters first). As before, 3 has four counter-threats, but their rejection, rather than leading to the implementation of the threat, gives 4 an opportunity to offer a counter. Since 4's options are independent of 3's choice, Figure 5 portrays only one instance of 4's decision. Working backwards on the extensive form so as to identify subgame-perfect equilibrium strategies, and looking at 4's decision, we see that if 4 offers  $(150,0,0,150)$ , 1 is certain to reject since  $v_1(\Gamma_{(150,0,0,150)}) = a < 150$ . Similarly, if 4 offers  $(150,85,0,65)$ , then 2 rejects since  $v_2(\Gamma_{(150,85,0,65)}) = 80 < 150$ . However, if 4 offers  $(150,0,75,75)$  or  $(0,150,75,75)$ ,

then (1,3) and (2,3), respectively, accept. Thus, there is no reason for 4 to consider a transfer to 1, and, since it is indifferent between these two counters, we suppose that 4 chooses one or the other with equal probability.

Looking now at 3's choice in Figure 5, because rejection of any of 3's counters yields a lottery between  $v(\Gamma_{(150,0,75,75)}) = (150,40,60,50)$  and  $v(\Gamma_{(0,150,75,75)}) = (40,150,60,50)$ , if 1 prefers  $v_1(150,0,150,0)$  to a lottery between 150 and 40, 3 does not counter with  $(150,0,150,0)$  --  $v_3(\Gamma_{(150,0,150,0)}) = c < r_3$  and, as we see shortly, 3 has better choices. If, on the other hand, 1 prefers the lottery, then then the counter  $(150,0,150,0)$  is merely equivalent to the counter  $(150,85,65,0)$ , since 2 rejects this counter in favor of the lottery. Finally, the counters  $(150,0,75,75)$  and  $(0,150,75,75)$  -- both of which yield 3 a payoff of 60 -- are accepted by (1,4) and (2,4), respectively. Now, though, we can introduce an assumption that does not affect our conclusions here, but which simplifies proofs later; namely, suppose  $i \in S-C$ , to counter  $(r',C)$ , chooses  $(r'',C'')$  such that  $S-C \subseteq C''$  whenever it is otherwise indifferent. Later, we state this assumption formally, but its rationale is that the threat against S-C makes the formation of S-C less costly (also, if  $i$  is indifferent, such a choice can characterize an equilibrium strategy since  $i$  has no positive incentive to choose differently). Presently, this assumption implies that 3 counters with  $(150,0,75,75)$  or  $(0,150,75,75)$ . And, since an identical argument holds if 4 counters first,  $v(\Gamma_{(150,150,0,0)})$  is a lottery between  $(150,40,60,50)$  and  $(40,150,60,50)$  as originally asserted.

Finally, let  $(150,0,150,0)$  be the initial threat (see Figure 6). If 2 makes the first counter, and if that counter is rejected, then 4 should counter with  $(150,85,0,65)$ , which yields  $(150,80,20,50)$ . Thus, looking at each of 2's four possible counter threats, if 2's counters with  $(150,150,0,0)$ , 1 rejects so as to secure  $(150,80,20,50)$ . If 2 counters with  $(150,85,65,0)$ , 1 is indifferent (3 accepts), but regardless of how 1 chooses, 2 gets 80. If 2 counters with  $(150,85,0,65)$ , 1 is again indifferent and 2 gets 80. Finally, if 2 counters with  $(0,150,75,75)$ , 4 is indifferent (3 accepts), yielding 2 a lottery between 150 and 80. Thus,  $(0,150,75,75)$  is 2's best response. And since nature can choose either 2 or 4, this argument establishes that  $v_1(\Gamma_{(150,0,150,0)}) < 150$  and  $v_3(\Gamma_{(150,0,150,0)}) < 60$ . If nature selects 4 to make the first counter to  $(150,0,150,0)$ , Figure 6 shows that if 4's counter is rejected, then 2 must counter with  $(150,85,0,65)$ , because  $(150,85,65,0)$  and  $(0,150,75,75)$  is rejected by 3 whereas  $(150,150,0,0)$  is rejected by 1. This fact implies that 1 rejects  $(150,0,0,150)$ , 1 is indifferent between accepting and rejecting  $(150,0,75,75)$  (3 accepts), countries 2 and 3 accept  $(0,150,75,75)$ , and 1 and 2 are indifferent between accepting and rejecting  $(150,85,0,65)$ . Applying the same assumption as before, namely that 4 chooses a counter which includes S-C, 4 is indifferent between countering with  $(0,150,75,75)$  and  $(150,85,0,65)$ . However, as before, we see that  $v_1(\Gamma_{(150,0,150,0)}) < 150$  and  $v_3(\Gamma_{(150,0,150,0)}) < 60$ . A parallel analysis holds for  $(150,0,0,150)$ .

Having established that the posited continuation values are consistent with subgame perfect equilibrium strategies, we next identify a symmetric stationary equilibrium for the full game (by symmetric we mean that all countries abide by the same type of strategy). Consider the following two partial characterizations of equilibria: (1) no player makes or accepts an initial threat unless such action promises a gain; and (2) players make or accept initial threats if doing so promises no loss. For case (1), countries 1 and 2 prefer coalescing with 3 and 4, but, under the assumption of the presumed equilibrium, 3 and 4 do not accept any initial offer, because there does not exist an offer that yields them a resource gain. Moreover, neither 3 nor 4 gains by a unilateral defection from its presumed equilibrium strategy. Hence, a situation in which no threats are made is an equilibrium; however, it cannot be a perfect equilibrium.<sup>9</sup> If there is a chance that will 4 accept a threat in which it does not lose, then 3 should not forego participating in threats that freeze the system. This argument bears on case (2). If everyone accepts threats in which they do not lose, then no one has an incentive to switch to a strategy of accepting or making threats only if it gains. Thus, given the limitations on threats we impose, our 4-country example is necessarily system-stable, but only resource-instability corresponds to a perfect equilibrium.

There are, of course, other possibilities that we must consider before we can uttering definitive conclusions. In addition to the threats and counters in this 4-country game that we do not allow, larger systems introduce new possibilities. For instance, if  $r = (110,80,60,30,20)$ , everyone is essential, but 4 and 5 cannot individually freeze the system. How, then, might 5 respond to a threat of  $(115,85,65,35,0)$ ? For another example, let  $r = (70,65,60,55,50)$ . In this instance no country can buy stability, so might not countries 3, 4, and 5 look favorably upon a threat by 1 against 2, since implementation of the threat allows a subsequent defense by, if necessary, a resource transfer? And can country 3 respond effectively to a threat by, say, 4 and 5? We cannot answer such questions, though, using the methods we have applied to systems with three and four countries -- we simply do not have sheets of paper large enough on which to portray the situation's extensive form.

#### 4. A General Theorem about System-Stability

To formulate a general  $n$ -country analysis, we first partition  $S$  into two subsets,  $L$  and  $L_0$ , where each country in  $L$ , but no country in  $L_0$ , can be the largest member of a minimal winning coalition. Thus,  $L_0$  is a losing coalition and consists of the smallest countries in  $S$ , but  $L_0$  plus any member of  $L$  is winning. For example, if  $r = (120,100,80)$ , then  $L = \{1,2\}$  and  $L_0 = \{3\}$ . More generally, except for uniform resource distributions, neither  $L$  nor  $L_0$  is empty, and the significance of these sets stems from the fact that only the members of  $L$  will be recipients of a transfer to freeze a system and thus only the members of  $L$  can become near-predominant through a transfer -- in attempting to freeze a system with a transfer,

countries ought to buy as cheaply as possible, and it is never optimal to freeze the system with a transfer to a country other than the largest member of the threatening coalition.

With this partition of  $S$ , our analysis proceeds thus: we define two types of continuation values, and, after isolating a particular set of threats,  $T^P$ , we assign one type of value to members of  $T^P$  and the other type to all other threats. These values and the definition of  $T^P$  give  $T^P$  "core-like" stability in this sense: no member of  $T^P$  is an effective counter to any other member of  $T^P$ , and threats not in  $T^P$  are ineffective counters to the members of  $T$ . Consistency is secured by defining  $T^P$  so that two or more of the threats in it are subgame perfect counters to any threat outside of it, in which case the continuation value of a threat not in  $T^P$  equals a lottery over the continuation values of the threats in  $T^P$  that are effective counters to it. For example, in 3-country systems  $\{(150,0,150), (0,150,150)\}$ , when combined with the appropriate coalition structures and continuation values, illustrates  $T$ . The continuation value we assign to  $(150,150,0)$  determines that  $(150,0,150)$  and  $(0,150,150)$  are effective counters to it, thereby rendering that value consistent with our other assumptions and subgame perfection. We begin, then, with two types of continuation values:

- C1:  $(r',C)$  satisfies continuation condition C1 if  $v_i(\Gamma_{r'}) \leq r_i$  for all  $i \in C \cap L_o$ ,  $v_i(\Gamma_{r'}) < R/2$  for all  $i \in C \cap L$ , and  $v_i(\Gamma_{r'}) \leq r_i$  for  $i \in C \cap L$  if  $r_i < r_{\max[S-C]}$ .
- C2:  $(r',C)$  satisfies continuation condition C2 if  $v_i(\Gamma_{r'}) = R/2$  for  $\max[C]$ , and  $v_i(\Gamma_{r'}) = r_i$  for all  $i \in C - \{\max[C]\}$ .

We are especially interested, now, in a specific kind of threat, namely,

**Type 1 Threat:**  $(r',C)$  is a Type 1 threat --  $(r',C) \in T^1$  -- if

- i  $r_{\max[C]} + r(S-C) \geq R/2$ ,
- ii  $r'_j = 0$  for all  $j \in S-C$ ,
- iii  $r'_{\max[C]} = R/2$ ,
- iv  $\nexists C'' \in W$  such that  $C'' \cap C = \{k\} = \{\max[C'']\} \neq \{\max[C]\}$ .

For any  $C$  there is an infinity of threats in  $T_C$ , including an infinity of Type 1 threats (which differ only in the distribution of  $r(S-C) - [R/2 - r_{\max[C]}]$  among  $C - \{\max[C]\}$ ). We show shortly, however, that most such threats are strategically equivalent, in that they have the same continuation values, so, for the moment, if we can associate a Type 1 threat with  $C$ , we focus on one such threat and ignore the other threats that  $C$  might make. Formally, let  $C$  denote all coalitions that have Type 1 threats, let  $(r',C)$  be a particular Type 1 threat by  $C$ , and redefine the set of all threats  $T$  as  $T - \cup_{C \in C} [T_C - \{(r',C)\}]$ . Later, we reintroduce the



excluded threats to show that they leave our analysis unaffected, but now, with these threats removed from consideration, let  $T_0$  be the power set of  $T^1$ , and let  $T^P$  be any element of  $T_0$  that satisfies,

- i for no  $(r',C)$  and  $(r'',C'') \in T^P$  is  $C \cap C'' = \{\max[C]\} = \{\max[C'']\}$ ;
- ii  $\nexists C \in W$  such that  $\exists (r',C) \in T^1$  such that  $(r',C)$  can be included in  $T^P$  without violating condition i (i.e.,  $T^P$  is maximal).

We call a set  $T^P$  that satisfies conditions i - ii a set of **Primary Threats**. Before we illustrate these definitions, we first state a preliminary lemma:

**Lemma 0:** For each  $i \in L$ , there is at least one  $(r',C) \in T^P$  such that  $i = \max[C]$  (in particular,  $\{i\} \cup L_0$ ,  $i \in L$ , has a threat in  $T^P$ ); and for each  $j \in L_0$ , there is at least one  $(r',C) \in T^P$  such that  $j \in C$ .<sup>10</sup>

To illustrate,  $\{(150,0,150),(0,150,150)\}$ , with the appropriate coalitions, is the unique set of primary threats in a 3-country system. In our 4-country example, both  $\{(150,0,75,75), (150,85,65,0), (150,85,0,65), (0,150,75,75)\}$  and  $\{(150,0,150,0), (150,0,0,150), (150,0,75,75), (0,150,75,75)\}$  are sets of Type 1 threats, but only the first is in  $T_0$ , because  $(150,0,150,0)$  and  $(150,0,0,150)$  cannot coexist in any  $T^P \in T_0$ .<sup>11</sup> Finally, let  $r = (70,65,60,55,50)$ , so from condition i in the definition of  $T^1$ , threats in  $T^P$  must originate with coalitions having three members. Consider now the following distributions that, with appropriate coalitions, satisfy the definition of a Type 1 threat:

1.  $(150,0,75,75,0)$
2.  $(150,0,75,0,75)$
3.  $(150,0,0,75,75)$
4.  $(0,150,75,75,0)$
5.  $(0,150,75,0,75)$
6.  $(0,150,0,75,75)$
7.  $(0,0,150,75,75)$

There are three other threats that we might consider, namely  $(150,75,75,0,0)$  by  $\{1,2,3\}$ ,  $(150,75,0,75,0)$  by  $\{1,2,4\}$ , and  $(150,75,0,0,75)$  by  $\{1,2,5\}$ . However, each of these threats violates condition iv in the definition of  $T^1$  and, thus, they are not candidates for inclusion in a set of primary threats. Indeed, the seven threats listed constitute the unique set satisfying the definition of  $T^P$ .

Our next lemma establishes why we are interested in isolating primary threats.

**Lemma 1:** If all  $(r'',C'') \in T^P$  satisfy C1, and if all threats in  $T^P - \{(r',C)\}$  satisfy C2, then for any stationary equilibrium,  $(r',C) \in T^P$  satisfies C2.<sup>12</sup>

The difficulty with this lemma, of course, is that we have not yet established any result validating the assumption that C1 holds for non-primary threats. In fact, the proof that C2, the consequences of Lemma 1, and subgame-perfection are consistent requires an assumption bearing specifically on the nature of the stationary equilibrium that we can sustain. We begin with the observation that if a country is confronted with a threat it cannot counter so that its loss of sovereignty is inevitable, then it is indifferent among the various actions that it takes at that point. Thus, the actions countries choose in such circumstances are free parameters. It might seem, of course, that such free parameters are irrelevant to final conclusions, but it is important to understand that other countries may not be indifferent as to which action a country chooses when confronting elimination, and what they believe about the eventual choice of the threatened country can determine their prior actions. Indeed, these beliefs can even determine whether the country in question will confront the possibility of elimination.

This discussion leads to the notion of a credible threat. Briefly, a threat is credible if: (1) it is part of a subgame perfect equilibrium, which is to say that at the time of its implementation, no other choice yields a greater payoff to the country in question; and (2) the belief by others that the threat will be chosen improves the utility of the country that might choose it. The assumption, then, about the actions of countries threatened with inevitable elimination that we use to characterize a stationary equilibrium is:

**CC1:** If S-C has no counter to its threatened elimination, and if it cannot buy stability directly, then S-C transfers all of its resources to max[C].

This assumption is not without historical challenge. Although Nazi leaders at the final stages of World War II preferred capitulation to the U.S. and Britain as against Russia, the allied powers largely controlled the final allocation of the German resources. On the other hand, in addition to the imperatives of the Atomic bomb, Japan chose a timely surrender because of its desire to capitulate to the U.S. rather than contend fully with Russian territorial ambitions.

As second assumption about equilibrium strategies, which we introduce in our discussion of 4-country systems, facilitates the proof of our central results. Suppose  $(r', C)$  is the current threat and let  $i \in S-C$  offer a counter-threat. Any counter, of course, must either entail a transfer of resources or, since C is necessarily winning, it must coopt one or more members of C into a new coalition,  $C''$ . Our assumption is that if  $i \in S-C$  can form a counter that coopts only one member of C so that all of i's other coalition partners in the counter are in S-C, then, ceteris paribus, i chooses that counter. This is not to say that i will forego other alternatives if they are more valuable; but, whenever it is indifferent, i will take advantage of the fact that S-C is a coalition that, because of the threat by C, is "already nearly formed." Hence,

**CC2:** If  $(r', C)$  is the current threat, then, ceteris paribus,  $i \in S-C$  chooses a counter,  $(r'', C'')$ , such that  $C'' \cap C = \{j\}$ , and  $C'' - \{j\} \subseteq S-C$ .

We can now complete the specification of consistent continuation values:

**Lemma 2:** If all elements of  $T^P$  satisfy C2, if  $(r', C) \notin T^P$ , and if all other threats satisfy C1, then CC1 and CC2 characterize equilibrium strategies for  $\Gamma$  such that  $(r', C)$  satisfies C1.<sup>13</sup>

Lemma 2 exhausts all possibilities not encompassed by Lemma 1, so together they establish consistent continuation values for all subgames of  $\Gamma$ .<sup>14</sup>

We can proceed now to our central result -- the characterization of the stationary equilibria of  $\Gamma$ . What remains at issue is a specification of a country's choice whenever it is selected to make the initial threat, and the responses of its partners in a proposed initial threat. Postponing the question of the fate of inessential countries, assume that all  $S$  are essential. Limiting the discussion now to symmetric strategies -- strategies in which all countries in  $L$ , and all countries in  $L_0$  abide by the same strategy, consider this statement as a potential characterization of equilibrium strategies:

**CC3:** If  $i \in L$  is chosen to make the initial threat,  $i$  randomly chooses  $(r', C) \in T^P$ ,  $i = \max[C]$ ; and all  $j \in C - \{i\}$  accept. If  $i \in L_0$  is chosen to make the initial threat,  $i$  randomly chooses  $(r', C) \in T^P$ ,  $i \in C$ ; and all  $j \in C - \{i\}$  accept.

Lemmas 1 and 2 establish that if all other countries abide by the this characterization, and if  $i \in L$  can make the initial threat,  $i$ 's dominant choice entails selecting a threat  $(r', C) \in T^P$  such that  $i = \max[C]$ , because this choice renders  $i$  near-predominant and achieving near-predominance is the best feasible outcome for any country. Moreover, both lemmas imply that no country in  $C \cap L_0$  has an incentive to defect from its strategy of acceptance (they can never gain, and acceptance ensures that they cannot lose). Finally, suppose one of  $i$ 's partners in  $C$ , say  $j$ , is in  $L$ . The question is whether  $j$ , rather than abiding by CC3, has an incentive to unilaterally defect to a strategy whereby it rejects  $i$ 's offer. Keeping in mind that  $j$  neither gains nor loses resources if it accepts, the benefit of defecting is that  $j$  might be selected subsequently to make an initial threat, or some country in  $L_0$ , might be selected to make a new initial threat and this threat might be one in which  $j$  is the largest member of the threatening coalition. On the other hand, the hazards of defecting are that  $j$  will otherwise be the target of any initial threat. Complex expected resource calculations, however, are

unnecessary. First, since we assume that all aspects of the game, including people's attitudes towards risk, are common knowledge,  $i$  knows beforehand whether or not  $j$  will accept its offer to participate in a threat. So if  $j$  would accept its offer in accordance with CC3, either because  $j$  is sufficiently risk averse or because the probabilities of the various outcomes imply that  $j$ 's gain from defection is negative, then the following result holds:

**Stability Theorem:** If all  $i \in S$  are essential, CC1 - CC3, in conjunction with C1 and C2, describe a stationary equilibrium for  $\Gamma$  such that the system  $(S,r)$  is system-stable; but if  $r_i < R/2$  for all  $i \in S$ , then  $(S,r)$  is resource-unstable.

On the other hand, if  $j \in C \cap L$  is risk acceptant or if the gain from defecting is sufficiently great, all countries, including  $i$ , know this fact beforehand and  $i$  will not propose an initial threat with  $j \in C$ . Moreover, Lemma 0 establishes that for any  $i \in L$ , there exists a threat in  $T^P$  such that  $C$  contains only  $i$  and members of  $L_0$ . Thus, if  $j$  would reject its proposal,  $i$  can find a threat that is accepted, so a slight revision of CC3 that precludes the selection of threats with risk-acceptant partners maintains the conclusion that system-stability prevails.

With respect to the possibility that there is a stationary equilibrium characterized strategies by "no country makes or accepts threats unless doing so promises an immediate gain," let  $r = (70,65,60,55,50)$ . Since no threat in  $T^P$  is accepted, let 1 threaten  $(120,65,60,55,0)$ . Ignoring the incentives that countries 2, 3, and 4 might have for avoiding a 4-country game, 5 is eliminated, because 2, 3, and 4 are unwilling to join in a counter in  $T^P$  unless such action promises a gain to each of and because all threats here have three members in  $C$ . On the other hand, 5 cannot individually threaten anyone, whereas if it can make the initial threat and if there is some chance that others will accept a threat in  $T^P$ , then 5's dominant strategy is to make a threat in  $T^P$  in which  $\max[C] = 2$ . Clearly, 2 has an incentive to accept, and 5's other partners should accept as well since doing so ensures that they cannot lose resources -- rejection merely allows a larger country in the next round to threaten them with elimination. Hence, the presumed strategy cannot be a perfect equilibrium.<sup>15</sup>

### 5. The Possibility of Instability

Because our stability result supposes that all countries are essential, we cannot preclude the survival of inessential countries. Of course, our analysis in Section 3 of 4-country systems illustrates the elimination of such a country, but 4-country systems are special and our model must be modified before we can eliminate inessential countries in general. To see why such systems are special, suppose  $r = (100,90,80,30)$ , and suppose 3, making the initial counter, proposes  $(110,100,90,0)$ . If 1 and 2 accept, 4 has no useful counter: Any counter in  $T^P$  must include 3, but 3 rejects such counters since it cannot gain resources with them and since 3

cannot lose resources in a 3-country system if (110,100,90,0) prevails. Alternatively, if 4 proposes a counter with 1 and/or 2 against 3, this leaves 1 and 2 vulnerable to a counter in T by 3 that requires one or the other to transfer resources; and since the game between them is zero-sum, one or the other will refuse to accept 4's proposal. With these consequences in mind, 1 and 2 accept 3's initial offer since, even if one or the other must transfer in the 3-country game that follows, each must transfer less with 4 eliminated than otherwise.

This reasoning, though, cannot be extended to a 5-country system such as  $\mathbf{r} = (100,70,60,55,15)$ . If 4 initially threatens, say, (105,75,63,57,0), then, in accordance with CC1, 5 transfers to 1 and is eliminated. But now 4 confronts the possibility of having to transfer resources in the 4-country game that ensues. Thus, since 4 cannot gain by threatening 5, and since it can lose resources if it fails to freeze the system, it (as well as 1,2, and 3) strictly prefers an initial threat in T. What makes a 4-country system with an inessential country different from larger systems, then, is that the unique essential country in  $L_0$  cannot lose resources in the 3-country game that follows the elimination of the inessential player, whereas all countries can lose resources in larger systems. Thus, there is an irresistible incentive for countries in large systems, when initially selected by nature, to choose Type 1 threats that freeze the system. Countries in L will not forego the opportunity to become near-predominant, whereas countries in  $L_0$  can be certain that they will not lose resources only if someone is rendered near-predominant. As a consequence of this "rush to stability," countries in L and  $L_0$  sacrifice the possibility of wholly absorbing inessential countries.

To see, however, how a modification of our analysis leads to the elimination of small countries, suppose 4, in our 5-country example, can propose a "sequential" initial threat — "3 and 4 eliminate 5 and distribute its resources between themselves, then require that 2 transfer so as to render 1 near-predominant; but if 2 rejects the transfer, implement (150,0,75,75,0)." If this threat "works," then 1 ought to accept 4's offer, since by "working" we mean that 1 is rendered near-predominant. And although 3 might prefer a different share of 5's resources than the one 4 proposes, it should accept participation in the threat for the reasons we specified in the discussion of our Stability Theorem.<sup>16</sup>

What is at issue, then, is 2 and 5's responses to 4's sequential threat. Because we want to show that inessential countries can be eliminated, it is sufficient to find a circumstance under which elimination occurs. So, supposing that 2 is first to counter 4's threat, 2 must transfer since it has no viable counter in T. But if 2 tries to save resources by proposing a transfer in which some of 5's resources are ceded to 1, 1 is indifferent between accepting or rejecting, whereas 5 can reject, secure the last move in the counter-threat sequence, and propose that 2 alone render 1 near-predominant (at which point 2 accepts since rejecting implements (150,0,75,75,0)). Thus, 2 has nothing to gain by not acceding to the transfer, and we can assume that it accedes in equilibrium.

If we represent 2's choice by the decision to "pass," 5 must counter next. Like 2, 5 has no counter in  $T^P$ , so it must consider transfers. A proposal by 5 to cede resources with 2 to 1 offers 1 no improvement over what it gets from 2 alone, so we can assume that, in equilibrium, 1 rejects 5 (and a preemptive transfer by 5 to 1 meets the same fate). Suppose, then, that 5 tries to disrupt matters with a preemptive transfer of something less than  $r_5$  to 3 or 4. Since 3 and 4 are dividing 5's resources in the current threat, both have some incentive to accept 5's offer. However, if both 3 and 4 are risk averse with respect to the possibility of replaying the game with all  $n$  players (that is, neither is willing to accept the chance of being the target of a threat in the next round), 5 is eliminated. If one or both is risk acceptant, 5 survives but remains inessential with diminished resources (notice that 5's preemptive transfer, unlike that of an essential country as embodied by CC1, does not remove the incentives of any member of  $C$  from proposing or accepting the initial threat). In the next round, though, there is a nonzero probability that some member of  $L_0$  makes the initial threat, and that this threat once again includes 5 as its target. Now, however, 5 has fewer resources with which to tempt a risk acceptant country, and, eventually, 5 is eliminated.

One final consideration remains -- whether the revision of the form of initial threats allows the elimination of essential countries. Let  $r = (70,65,60,55,50)$ , and let 4 propose, as before, to eliminate 5 and threaten 2 so as to secure an outcome such as  $(150,10,75,65,0)$ . Suppose 2 counters first in the same way -- acceding to the threat. But now 5's resources are essential to 2's ability to freeze the system, and 5, after 2, can propose a transfer to 1 in which 2 shares most of the burden. Clearly, 1 accepts 5's offer since it does not require implementation of a threat, and the final outcome -- 2 and 5 transfer to 1 -- is the same as when we allowed only simpler initial threats.

However, not all essential countries survive sequential threats. Let  $r = (140,122,24,8,6)$  and let  $\{1,3,4\}$  threaten  $\{2,5\}$ . Country 5's problem, now, is that 5's resources are inessential to 2's ability to render 1 near-predominant, so, as with inessential countries, a sequential threat eliminates 5. It is important to keep in mind, though, that the game  $\Gamma$  is not intended to model the entire process of international politics. Hence, we can imagine a "pre-game" in which countries 4 and 5, anticipating their mutual vulnerability in  $\Gamma$  (just as the threat by  $\{1,3,4\}$  eliminates 5, a threat by  $\{1,3,5\}$  eliminates 4), coalesce to form a confederation or a new country. The incentives to form such a confederation, of course, lie in the fact that, if  $r = (140,122,24,14)$ , no country is subsequently eliminated because all countries can individually buy stability. Thus, reminiscent of Riker's (1964) analysis of federalism, we detect in our analysis forces promoting the formation of new states. Notice, moreover, that confederations cannot ensure the sovereignty of inessential countries: a confederation of such countries alone is inessential, and a confederation of an essential country with inessential countries cannot preclude the essential country's vulnerability to sequential threats. In this way, then, we can

reassert the conclusion of our previous analyses that only inessential countries are eliminated. Of course, this discussion is not substitute for a formal analysis of the formation of states, but we now see how our analysis can be modified to yield insights into processes that otherwise seem outside the realm of realist theory.

## 6. Conclusions

In drawing lessons about war and peace from our analysis, we must keep in mind that system-stability does not imply resource-stability. Resource stability requires equilibria in which no initial threats are made or accepted, so, barring the possibility of a corresponding equilibria for  $\Gamma$  (which we cannot do), only systems with a near-predominant country are necessarily resource-stable. And because we do not model the substantive form of a threat's implementation, we cannot be certain that system-stability implies the absence of wars or that resource-instability is somehow less dangerous than system-instability. Substantive conclusions must be treated tenuously, moreover, because our analysis fails to accommodate important features of international processes, including the uncertainty inherent in such processes, the strategic complexity that uncertainty allows, the contemporary disjuncture between military and economic capabilities, the possibility that national leaders pursue goals that do not translate readily into some notion of power maximization, and the ambiguities inherent in the notion of sovereignty. Our analysis also implies that, barring non-terminal transfer cycles, systems are quickly frozen with one country becoming near-predominant. However, this consequence merely underscores the fact that our model does not yet accommodate the unequal resource growth of countries that can render systems unstable (for an extension using cooperative theory see Niou and Ordeshook 1987, and Niou, Ordeshook, and Rose 1989).

Our stability theorem is nevertheless important. Whether labeled Ho Tzung in the Warring States Period of China, identified as the policy of a Great Power in the 19th century, or named Realism in this century, there is the continuing attraction and salience of the idea that some notion of "balance-of-power" is an essential source of stability. However, there has long existed one problem for adherents to this view -- the absence of any formal, deductive proof that there exists an interpretation of balance-of-power ensuring stability. Our conceptualization of  $\Gamma$  and our stability theorem offers such a proof. One can argue, of course, that our theorem is merely a possibility result and that its descriptive relevance is questionable; but with that theorem we can address secondary issues that arise in the context of realist thinking. For example, we see that stability does not require any specific number of countries or great powers, nor does stability require either a uniform or a highly asymmetric resource distribution. We have also seen that the allowable form of initial threats can alter our assertions about the stability, and, thus, our conclusions are sensitive to a variable that had not

previously been appreciated. Finally, looking at the threats and counters in  $T^P$ , which are the primary instruments of resource reallocations, the target countries must be large enough that they can render  $\max[C]$  near-predominant. Thus, although we do not predict initial threats that correspond identically to Riker's (1962) size-principle hypothesis, we anticipate alliances that correspond approximately to this hypothesis.

Despite our emphasis on anarchic systems, our model also reveals the profound role of institutions as agents for facilitating stability. Common knowledge is an essential assumption in our analysis -- our conclusions follow only if all decision makers know  $\Gamma$ 's structure (the resources of other countries, and the ultimate consequences of alternative choices), if they each know that all others know, if they know that all others know that all others know ... and so on. Moreover, if a set of primary threats is not unique, then system-stability implicitly supposes that the set satisfying condition C2 is common knowledge. However, despite Blainey's (1973) compelling arguments about the consequences of violations of this assumption (a necessary and perhaps sufficient condition for war), we know little about the mechanisms that facilitate common knowledge. In vague terms, though, we know that strategic interaction and communication are "useful." We may not understand in any rigorous sense the way in which institutions facilitate common knowledge, but they are almost certainly essential towards that end. Minimally, our analysis identifies the variables and concepts that ought to be common knowledge if the stabilizing processes of balance-of-power are to operate, and, thus, we can identify the purposes that some institutions and processes might be designed to serve.

Our model also highlights the role of attitudes towards risk, and the corresponding relevance of domestic politics. We assume throughout that countries (or those who lead them) are risk averse with respect to elimination. Nothing we have said about system-stability applies, however, if decision makers are risk acceptant -- indeed, risk acceptance almost certainly destabilizes systems. Thus, to the extent that attitudes towards risk are determined by domestic politics -- to the extent, for example, that decision makers equate their personal survival with their country's sovereignty so that the pursuit of dangerous foreign adventures is seen as essential to maintaining domestic power -- otherwise stable systems are destabilized. We sympathize, then, with those analyses that interpret Hitler's personality and the political economy of Weimar Germany as critical to the outbreak of World War II rather than some breakdown in traditional balance-of-power forces (c.f. Muller 1989).

A great many questions remain. Are there alternative classifications of threats and specifications of continuation values that yield different conclusions about stability? What outcomes can we sustain if strategies are more complicated (non-stationary), such as when decision makers punish coalition partners who defect from threats and counters? And since we have already seen the import of allowing sequential threats, how sensitive are our



conclusions to other changes in  $\Gamma$ 's extensive form? For example, how is the game changed if we allow a threatening country's coalition partners to propose modifications to the original proposal before a threat becomes the current threat? Answers to these questions are topics for future research, and the questions themselves simply emphasize the fact that the preceding analysis is but a tentative first step towards a theory of international processes. Nevertheless, we are now certain of one thing -- there exists at least one world, albeit abstract and reminiscent of the frictionless planes that introduce physics, in which a balance-of-power ensures the sovereignty of essential participants.

## Footnotes

1. Our analysis, although similar to Wagner's, differs from his in several ways. First, Wagner offers no general results about  $n$ -country systems and he proceeds instead on a case-by-case basis. However, we do not accomplish greater generality costlessly. Wagner pays greater attention to the mechanisms whereby nations war and secure resources from others, whereas we assume simply uncountered threats are implemented. Another difference is that we allow preemptive resource transfers. Wagner argues against such an assumption, but it is our contention that such transfers are not without historical precedent and are much a part of international processes (Niou, Ordeshook, and Rose, 1989).
2. In addition to assuming that  $r$  strictly orders  $S$ , we assume that there are no blocking coalitions, except when  $r_i = R/2$ . Neither assumption is of any consequence to our results, but their imposition greatly simplifies argument.
3. Assuming that  $j \in C$  if  $r'_j = r_j$  precludes some silly possibilities; e.g., after  $(1,2,3)$  threatens  $(4,5)$ ,  $(4,5)$  counters by threatening  $(3)$ , or  $(4)$  counters by threatening  $(5)$ .
4. Several assumptions preclude "non-terminal transfers" without recourse to complex expected value calculations. For example, countries are risk averse in the extreme -- that  $i$  prefers  $r_i$  to any lottery that promises both gains and losses with respect to  $r_i$ . Or, owing to uncertainty about risk attitudes, countries accept terminal transfers with certainty but non-terminal transfers with probability less than 1. Since a non-terminal transfer leaves open the possibility of elimination, risk attitudes with respect to elimination precludes such transfers. Referring to an idea that we introduce shortly, notice that both assumptions implicitly associate a continuation value with non-terminal transfers that render them unacceptable choices for  $S-C$  or  $C-\{\max[C]\}$  when compared to a system-freezing transfer.
5. This is not to say that  $\Gamma$  proceeds indefinitely. Indeed, barring non-terminal transfers, our analysis implies that  $\Gamma$  ends quickly, after the first

threat and counter. Nevertheless, we must accommodate the possibility of an infinite sequence of threats and counters.

6. Assuming stationarity removes from the scene a great many possibilities, including punishment strategies. For example, we do not consider that a country, instead of choosing threats randomly whenever it is indifferent, might limit its choices to those that punish specific countries because they had previously defected from an earlier agreement. We know that allowing non-stationary strategies expands the set of sustainable equilibria, and it is certainly worthwhile to explore various types of non-stationary strategies to learn more about the outcomes that can be supported as equilibria. At this stage, though, stationary strategies pose a sufficiently complex analytic hurdle, and since our purpose is to offer a possibility result about stability in anarchic systems, we limit discussion to the simplest form of strategy. For discussion of recursive games and the role of stationary strategies in a different political context, see Baron and Ferejohn (forthcoming and 1989).
7. We can extend the previous analysis to show that  $(150,0,150,0)$  yields  $(150,75-\epsilon,75,\epsilon)$  or  $(150,45+\epsilon,75,30-\epsilon)$ , whereas  $(0,150,150,0)$  yields  $(75-\epsilon,150,75,\epsilon)$  or  $(45+\epsilon,150,75,30-\epsilon)$ . For example, if 3 counters with  $(150,0,150,0)$ , the final outcome depends on whether 2 or 4 has the last move in offering a counter, which necessitates a transfer. If 2 moves last, then it can propose  $(150,75-\epsilon,75,\epsilon)$ , and 4 has little alternative but to accept, whereas if 4 moves last, it can propose  $(150,45+\epsilon,75,30-\epsilon)$ .
8. If 4 is threatened with elimination and cannot find a counter, it is indifferent among all moves, including a preemptive transfer to, say, 1. Later, we characterize equilibria with such an assumption, but here this transfer is of no consequence: It eliminates 4, and, since 3 makes or accepts threats if doing so yields no loss, 3 proceeds with the threat. In the resulting 3-country system,  $(130,95,75)$ , 3 cannot lose resources (also, 1 and 2 are better off with the elimination of 4 since, although one or the other must eventually transfer resources, both must transfer less with 4's elimination as a result of 3's initial threat).
9. An equilibrium is perfect if, for each  $i \in S$ , no arbitrarily small probability that others defect from their equilibrium strategies yields an incentive for  $i$  to defect.

10. First, with respect to  $T^1$ ,  $C = \{i\} \cup L_0$  has a threat satisfying condition i (otherwise,  $L_0 \in W$  and  $\max[L_0] \in L$ ), and it follows that conditions ii and iii can be satisfied. With respect to condition iv, if  $C'' \cap C = \{j\}$ ,  $j \neq \max[C]$ , then  $j \in L_0$  and  $C'' \notin W$ . Thus,  $\exists (r', C) \in T^1$ . Second, with respect to  $T^P$ , it is sufficient to show that, regardless of the other threats in  $T^P$ ,  $C$  has a threat in  $T^P$ . From condition i,  $C$  cannot have a threat in  $T^P$  only if  $C'' \subseteq L$  has a threat in  $T^P$ , but from condition iv in the definition of Type 1 threats,  $C''$  does not have such a threat. Hence, from condition ii in the definition of  $T^P$ , every  $T^P$  must contain a threat by  $C$  in  $T^1$ . It follows that every  $i \in S$  is in some  $C$  that has a threat in  $T^P$ . Notice that, by this argument, every  $\{i\} \cup C^* \in W$  has a threat in  $T^P$ , where  $i \in L$  and  $C^* \subseteq L_0$ .
11. This example, however, shows that  $T^P$  is not unique:  $\{(150, 0, 150, 0), (150, 85, 65, 0), (150, 0, 75, 75), (0, 150, 75, 75)\}$ , and  $\{(150, 0, 0, 150), (150, 85, 0, 65), (150, 0, 75, 75), (0, 150, 75, 75)\}$  are also sets in  $T_0$ . But notice that these sets are asymmetric in that each renders some  $i \in L_0$  invulnerable to threats in  $T^P$ . Only  $\{(150, 85, 65, 0), (150, 85, 0, 65), (150, 0, 75, 75), (0, 150, 75, 75)\}$  treats 4 and 5 symmetrically, and so if there is the common knowledge presumption of "equal treatment" by all members of  $S$ , then this set is the "primary" primary set of  $\Gamma$ .
12. Let  $(r', C) \in T^P$  and let nature order  $S-C$  as  $O$ . We want to show that  $S-C$  must transfer to  $\max[C]$ . Let  $j \in S-C$  be last in the order  $O$ , and consider the counter  $(r'', C'')$ ,  $j \in C''$ . Clearly,  $C \cap C'' \neq \emptyset$ , and so we have three cases. **First**, if  $i \in L_0 \cap C \cap C''$ , then  $i$  rejects  $(r'', C'')$  since  $r'_i > r_i$  and, from the assumption that  $(r'', C'')$  satisfies C1 or C2,  $v_i(\Gamma_{r''}) \leq r_i$ . **Second**, suppose that  $\max[C] \in C \cap C''$ . If  $\max[C''] = \max[C]$ , then, since  $(r', C) \in T^P$ ,  $(r'', C'') \notin T^P$ , in which case, from C1,  $r'_{\max[C]} (= R/2) > v_i(\Gamma_{r''}) (< R/2)$ , and  $\max[C]$  rejects  $(r'', C'')$ . Similarly, if  $\max[C''] \neq \max[C]$ , then  $\max[C]$  again rejects  $(r'', C'')$ . **Finally**, let  $L' = \{i \in L : r'_i < R/2\}$ , and  $C \cap C'' = L'$ . So, if  $(r'', C'') \in T^P$ , then, from C2,  $r'_i > v_i(\Gamma_{r''})$  for all  $i \in L'$ , and each such  $i$  rejects  $(r'', C'')$ . If  $(r'', C'') \notin T^P$ , it satisfies C1. Letting  $i^* = \min[L']$ , since  $C \cap C'' = L'$ , it must be the case that  $r_{i^*} < r_{\max[S-C'']}$ , so from C1,  $v_{i^*}(\Gamma_{r''}) < r_{i^*}$ , and  $i^*$  rejects  $(r'', C'')$ . Hence, the last country in  $O$  must propose a transfer, and this reasoning can be applied to the next-to-last country in  $O$ , and so forth. Thus, the only counter to  $(r', C)$  is a transfer to  $\max[C]$  by  $S-C$ , which establishes C2 with respect to  $(r', C)$ .

13. Let nature order S-C as O. We have three cases: **First**,  $r(S-C) + r_{\max[C]} \geq R/2$ . Let  $K = \{(r'', C'') \in T : C'' \in W, C'' \cap C = \{k\}, k = \max[C''] \neq \max[C] = \{i\}\}$ . Clearly,  $K \neq \emptyset$ . It is sufficient to show that  $\exists (r'', C''), (r_+, C_+) \in T^P \cap K$  such that  $C'' \cap S-C \neq \emptyset$  and  $C_+ \cap S-C \neq \emptyset$  and  $\max[C''] \neq \max[C_+]$ . To establish that at least one  $(r'', C'') \in K$  is in  $T^P$ , we show first that  $K \subseteq T^1$ , which requires that, for any  $(r'', C'') \in K$ ,  $\exists (r_o, C_o)$  such that  $C_o \in W$ ,  $C_o \cap C'' = \{j\}$  and  $j = \max[C_o] \neq \max[C''] = k$ . Suppose  $(r_o, C_o)$  exist, and let  $C_o = \{h \in S-C'' : r_h < r_j\} \cup \{j\}$ , where  $j \in C''$ . By assumption,  $C_o \in W$ , so, since  $r_i > r_k > r_j$ ,  $\{h \in S-C'' : r_h < r_j\} \cup \{i\} \in W$ . But  $i \in S-C''$ , which implies  $\{h \in S-C'' : r_h < r_j\} \cup \{i\} \subseteq S-C'' \notin W$ , so  $(r'', C'') \in T^1$ . Let  $C'' = S-C + \{k\}$  with  $k = \max[C'']$ . Since  $(r'', C'') \in T^1$ ,  $(r'', C'') \notin T^P$  only if  $\exists C_{oo} \in W$  such that  $C_{oo} \cap C'' = \{k\}$ ,  $\max[C_{oo}] = k$ . However, since  $r_i > r_k$  and since  $C_{oo} \in W$ , then  $C_{oo} - \{k\} + \{i\} \in W$ , but  $C_{oo} - \{k\} + \{i\} \subseteq S-C'' \notin W$ , so  $C_{oo} \notin W$ . Hence,  $(r'', C'') \in T$ . Now substitute  $i$  for  $k$  in  $C''$  to obtain  $C_+$ . To see that  $C_+$  has a threat  $(r_+, C_+) \in T^P$ , notice that  $(r_+, C_+) \in T^1$ , because the definition of  $T^1$  concerns only those  $j \in C_+$  such that  $r_j < r_i$ . So  $(r_+, C_+) \notin T^P$  only if  $S-C_+ + \{i\}$  has a threat in  $T^P$ . But  $S-C_+ + \{i\} = C$ , and  $C$  has no threat in  $T$ . So  $(r_+, C_+) \in T$ . Hence,  $\exists$  at least two counters in  $T^P$  to  $(r', C)$ , namely  $(r'', C'')$  and  $(r_+, C_+)$ , with  $\max[C''] \neq \max[C_+]$ , so  $v_i(\Gamma_{r'}) < R/2$  for all  $i \in L$ . And since all counters are in  $T^P$ ,  $v_i(\Gamma_{r'}) \leq r_i$  for all  $i \in L_o$ . To establish that  $v_i(\Gamma_{r'}) \leq r_i$  for all  $i \in L \cap C$  for which  $r_i < r_{\max[S-C]}$ , we must show that no counter in  $T^P$  by any  $j \in S-C$  has such an  $i$  as its maximal member. Using CC2, let  $G \subseteq S-C$  and  $C'' = G \cup \{i\}$  with  $\max[C''] = i$ . If  $C'' \in W$ , then  $C'' - \{i\} + \{\max[S-C]\} \in W$ , but  $C'' - \{i\} + \{\max[S-C]\} \subseteq S-C \notin W$ , which is a contradiction.
- Second**,  $r(S-C) + r_{\max[C]} \geq R/2$  and  $(r', C) \in T^1$  but  $(r', C) \notin T^P$ . Hence,  $\exists (r'', C'') \in T^P$  such that  $C'' = S-C + \{\max[C]\}$ ,  $S-C'' = C - \{\max[C]\}$ . If nature orders S-C as O, then if all counters up to the last player in O are rejected, this last player in O proposes  $(r'', C'')$ , which, from C2, is accepted by  $\max(C)$  and all other members of  $C''$ . Therefore, if the next-to-the-last player in O proposes a threat by  $\{i\} + L_o$ ,  $i \in L$ , by Lemma 0 this threat is in  $T^P$ , so it is accepted. This argument can be repeated to establish that the first player in O can propose a threat in  $T^P$  which has a positive probability of being accepted, and, thus, condition C1 is established.
- Third**,  $r(S-C) + r_{\max[C]} < R/2$ . Hence, the last country in O has no counter in  $T^P$  that is accepted: Any counter in  $T^P$  which includes  $i \in O$  must include two or more members of  $C$  since  $S-C + \{j\} \notin W$ ,  $j \in C$ , and, because C2 implies that all threats in  $T^P$  yield a gain to only one country, at least one member

of a countering coalition will prefer  $(r',C)$ . If  $i \in O$  can offer a counter not in  $T^P$ , then by the assumption of the lemma,  $v(\Gamma_{r'})$  satisfies C1. If no counter is available to  $i \in O$ , then S-C is eliminated and  $i \in S-C$  is indifferent as to the final allocation of its resources. So let  $i$  propose a transfer of the resources of S-C in accordance with C1. This argument applies also to the next-to-the-last country in O, etc. so, in accordance with C1, the first country in O might as well propose the transfer the last country in O would propose or a counter that satisfies C1 if such a counter exists that is accepted, in which case  $v_{\max[C]}(\Gamma_{r'}) < R/2$  and  $v_i(\Gamma_{r'}) = r_i$  for all  $i \in C - \max[C]$ .

14. We can now reintroduce those previously excluded threats by showing that they are strategically equivalent to threats already considered, where  $(r',C)$  and  $(r'',C)$  are equivalent if  $v(\Gamma_{r'}) = v(\Gamma_{r''})$ . Recalling that C denotes the coalitions with a threat in  $T^1$ , let  $T'_C$  be all the Type 1 threats by C (threats that differ only by the distribution within  $C - \{\max[C]\}$  of  $r(S-C) - [R/2 - r_{\max[C]}]$ . From the proof of Lemma 1, all such threats are strategically equivalent (since the proof of that lemma requires only that  $r'_i \geq r_i$  for all  $i \in C - \{\max[C]\}$ ). Thus,  $\Gamma$ 's strategic structure is unaffected if we select one element of  $T'_C$  as representative of the set, and "discard" the rest. There remains the threats in  $T''_C = T_C - T'_C$ , such as, with 4-countries  $((130,0,90,80),\{1,3,4\})$ . In this example, the only coalition with a threat that might be viable against  $((130,0,90,80),\{1,3,4\})$  is  $\{1,2\}$ , such as  $((150,150,0,0),\{1,2\})$ , because C1 does not exclude the possibility that  $v_1(\Gamma_{(150,150,0,0)}) > 130$ . If this is true, then  $v(\Gamma_{(130,0,90,80)}) = v(\Gamma_{(150,150,0,0)})$ . However, no simultaneity confounds the determination of  $v(\Gamma_{(150,150,0,0)})$ , since this value is not a function of  $v(\Gamma_{(130,0,90,80)})$ : if  $i \in S$ , whenever it is otherwise indifferent, chooses a counter in  $T^P$ , then  $v(\Gamma_{(130,0,90,80)})$  has no bearing on  $v(\Gamma_{(150,150,0,0)})$ . More generally, for any  $(r',C)$ ,  $C \in C$ , no threat by another  $C'' \in C$  is viable against  $(r',C)$ . If no threat is viable against  $(r',C)$ , then  $(r',C)$  is strategically equivalent to the threat in  $T^P$  by C and, thus, we can "discard"  $(r',C)$ . If  $\exists (r'',C'')$ ,  $C'' \in C$ , that is viable against  $(r',C)$ , then  $v(\Gamma_{r'})$  satisfies C1. In either event,  $(r',C)$  satisfies C1 or C2.
15. This argument does not establish the uniqueness of the equilibrium in our stability theorem. Even stationary equilibria are not unique since we can manipulate the probability that a country chooses one action as against

another whenever it is indifferent. More fundamentally, though, we should ask whether there are equilibria in which essential countries are eliminated. To this point we are unable to find such equilibria (assuming equilibrium refinements such as perfection), but we do not to prove that such an equilibrium does not exist.

16. That 1 and 3's responses do not depend on the possibility that the reallocation of  $r_g$  alters  $\Gamma$ 's strategic character among  $\{1,2,3,4\}$  follows from two readily established facts. First, no reallocation of  $r_j$ ,  $j \in E_o$ , renders  $i \in E$  inessential or  $k \in E_o$  essential. Second, if  $i \in E$  but  $r_1 + r_i < R/2$ , no reallocation of  $r(E_o)$  renders  $\{1,i\} \in W$ . Hence, if  $i \in E$  cannot initially buy stability, it cannot do so after an inessential country is eliminated.

FIGURE 1

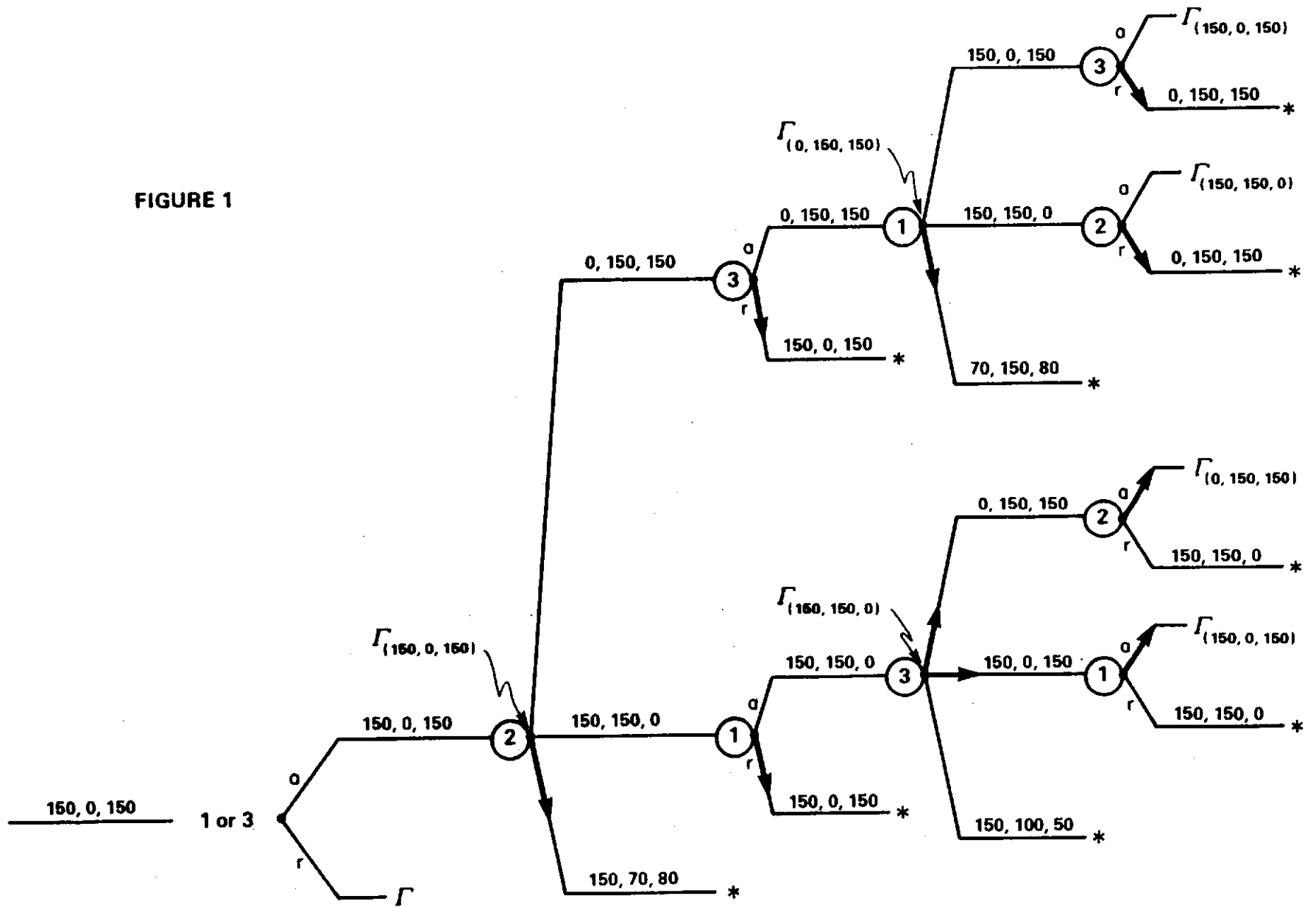






FIGURE 3

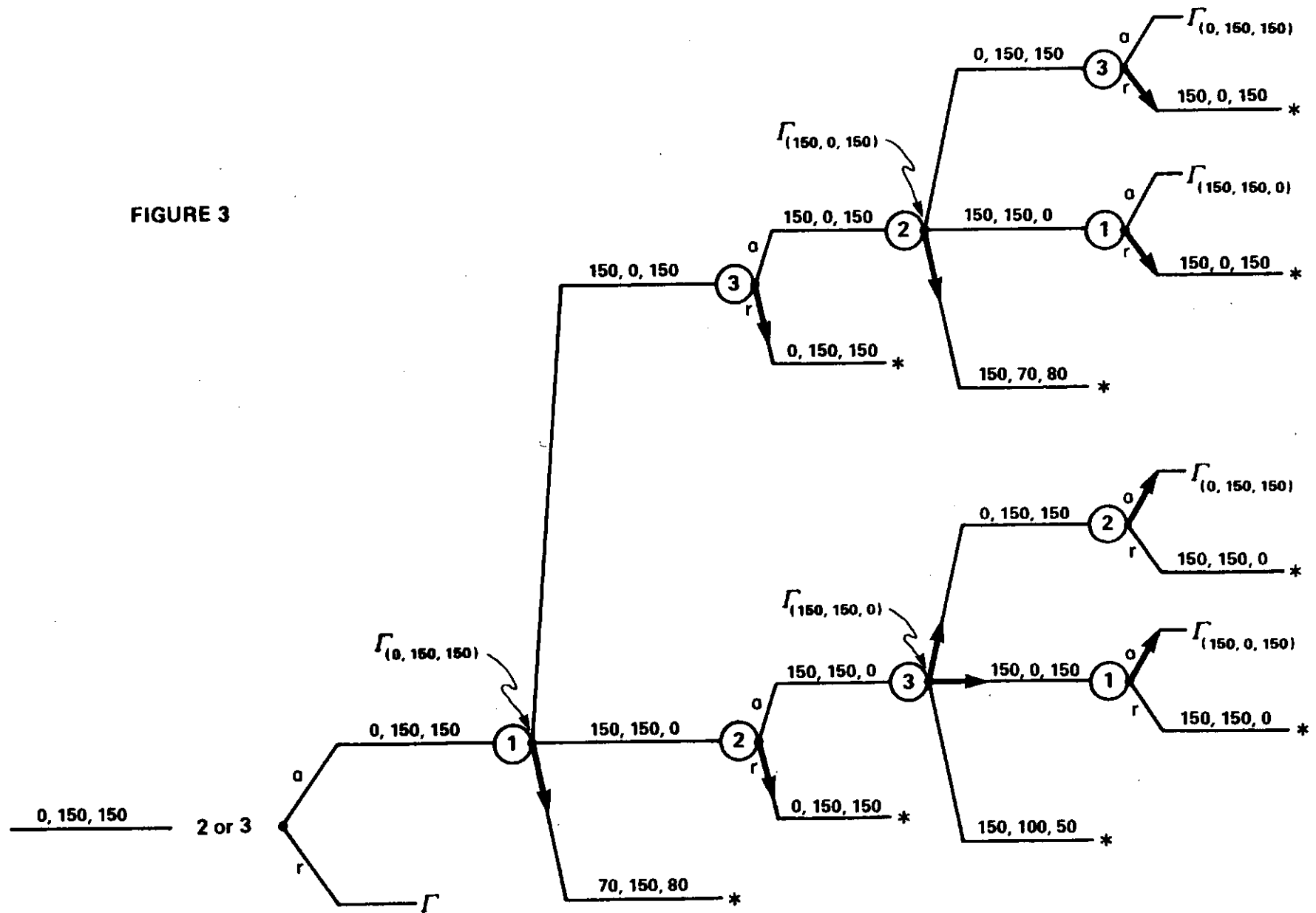
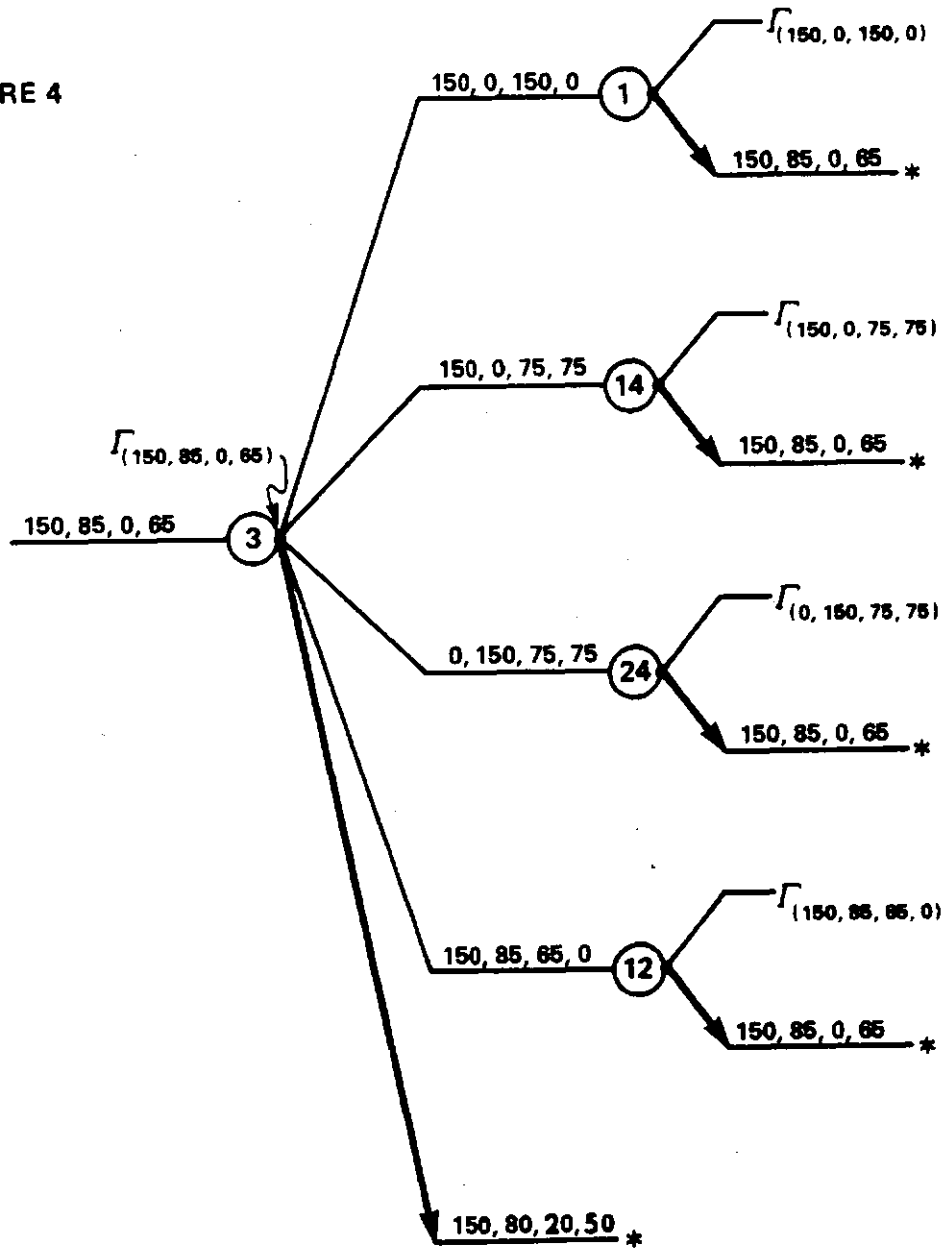


FIGURE 4



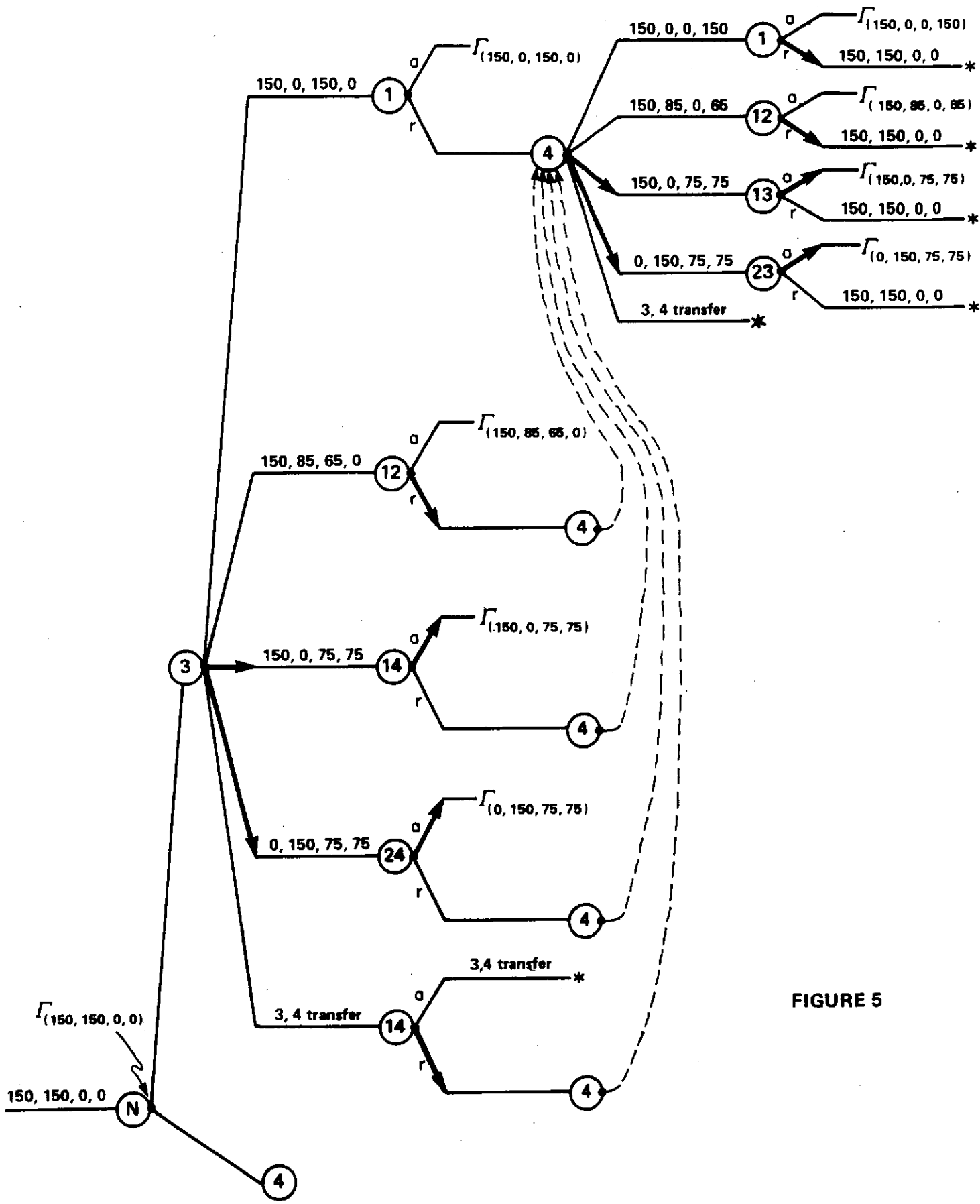


FIGURE 5

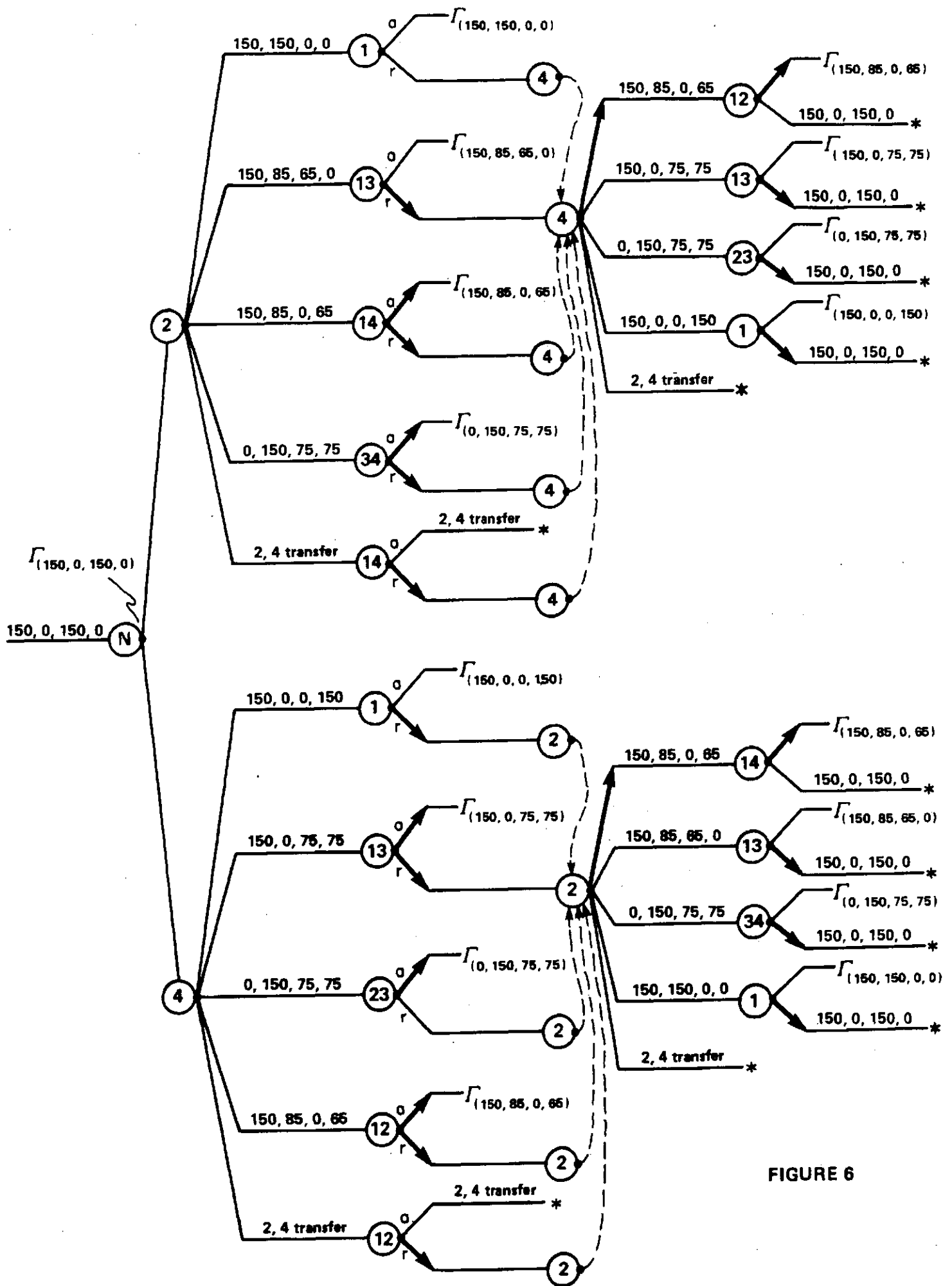


FIGURE 6

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