Design of A Coordination Process for Cost Sharing

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Abstract

This paper proposes use of a coordination process for cost sharing. The underlying premise is that complexity may be as serious as free-riding for cooperative group outcomes. A coordination process is a group algorithmic search method to find an optimal solution to the group decision problem. The design is based on market-like principles: decentralization of computation tasks, privacy of preference information, individual rationality, group feasibility and optimality, and use of price as an equilibrating tool. Three types of coordination process described in this paper differ in terms of the nature of the message space, associated allocation rules, and procedures.

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This paper addresses the situation of a group that is trying to determine provision of a shared good. The shared good may be a public good, a club good, or even a good shared by divisions within a firm. In such cases, a cooperative agreement may be sought regarding the quantity of the good and how to share its costs among group members.

Much discussion of cooperation in economic literature has focused on the Prisoner's Dilemma as a rationale for why a cooperative solution may not occur. The dilemma has been used to emphasize the non-congruence between group-rational and individual game behavior. As explained by Binmore and Dasgupta (1986), it is not surprising that players in such games forego the fruits of cooperation, because "non-cooperative games like the Prisoner's Dilemma are constructed in such a way that players are not able to act jointly" (p.25).

To facilitate location of an agreement, this paper proposes use of a coordination process. A coordination process can facilitate cooperation among members of a group by providing an institutional structure to reduce complexity of the group decision problem. Complexity is due to the nature of the group decision situation: the shared good or goods may have multiple possible levels, the cost function for provision of the good may be nonlinear, and there may be heterogeneous preferences and heterogeneous endowments.

A coordination process is a group algorithmic search method for an optimal solution to the group decision problem (Reiter, 1995). The design of a coordination process is based on market-like principles: decentralization of choice, privacy with regard to preference, individual rationality, and the use of price as an equilibrating tool. Such a process is a special case of an adjustment process as defined by Hurwicz (1994); the term "coordination" is used to emphasize the goal of reaching a cooperative group outcome.

Similar to an auctioneer, there is a coordinator who has the role of collecting preference data from participants and executing the rules. However, this role need not be carried out by an actual person. A "smart market" can automate the rules of a coordination process (McCabe, Rassenti, and Smith, 1993).

Specification of a coordination process includes five types of rules:

1) the form of a policy instrument;

2) the nature of the message space;

3) allocation rules;

4) procedural rules;

5) "genuine implementation" rules.

Although these type of rules are generally applicable, here these rules will be described specifically for a policy instrument based on cost shares and personalized prices. The design of these rules is discussed in more detail below.

The basis for the cost sharing policy instrument is the cost share equilibrium defined by Mas-Colell and Silvestre (1989, 1991). The advantage of this solution concept compared to the Lindahl equilibrium is that it can be applied for any type of cost function without requiring production by a profit maximizing firm or distribution of profits or losses (Tian, 1994).

The type of coordination process developed here is related to the experimental work by Smith for public goods (1977, 1978, 1979, 1980). The process defined by Smith was for the case of constant marginal cost, whereas here the nature of cost is unrestricted.

Three alternative coordination processes for cost sharing will be described here. They differ in terms of the nature of the message space and related allocation rules. Each of the three processes has the potential to be Pareto optimal in an abstract economy, but they differ in terms of opportunies for misrepresentation through the message space. The message space will consist of bids and/or quantities demanded for given shares and prices.

Allocation rules specify how such messages determine shared quantity and cost shares. Procedural rules determine the sequencing of messages and allocation rules. Termination rules concern how such a process stops; as an example of "genuine implementation" rules. Here, consensus is the basis for termination. In Calculus of Consent. Buchanan and Tullock (1962) argued that any collective decision rule other than unanimity will have coercive aspects.

Besides the work of Smith (1979, 1980) and Banks and Plott (1988), there has been little work comparing the performance of alternative public good mechanisms. .1989, 1994). Alternative rule designs are compared here using simulation. Simulation is a design tool that requires specificity. That is, all pertinent operational rules must be identified.

The processes developed here differ from the implementation of a cost share equilibrium proposed by Tian (1994) in several ways. First, game implementation does not deal with how to find an equilibrium or consensus. Rather, implementation concerns how to provide incentives for truthful preference revelation. Here, the concern is "genuine implementation;" or how to develop an understandable process that has the potential for good performance. The discussion below provides background for this different emphasis.

Evolution of Public Goods Mechanisms

Discussion of public good issues can be found in general equilibrium theory, mechanism design, and experimental economics. The purpose of each is different.

General equilibrium theory considers equilibria in an abstract economy. A major focus has been a comparison of equilibrium to Pareto optimality by type of environment. The emphasis on Pareto optimality is based on its importance in much of welfare economics since the 1930's (Hurwicz, 1994).

Mechanism design literature concerns equilibrium in a game setting. The focus of mechanism design literature has been on the design of game rules that influence players' choice of messages toward Pareto optimality. Decentralization has been a major emphasis (Hurwicz, 1994): each player only needs to solve an individual optimization subproblem, and the set of individual subproblems is structured so as to be a decomposition of the social optimality problem. Privacy preservation and information cost minimization are benefits of decentralization.

Theoretical literature has resulted in impossibility theorems regarding whether a mechanism could be simultaneously Pareto optimal, incentive compatible, and privacy preserving. Because of impossibility results, less restrictive performance criteria have been formulated (Ferejohn, Forsythe, and Noll, 1979; Mas-Collel and Silvestre, 1989): at a minimum, an equilibrium outcome should be individually rational (superior to the status quo) and financially feasible both for individuals and for the group.

Hurwicz (1972) defined desirable characteristics of a mechanism by generalizing properties of a market. A nonwasteful mechanism is one that achieves a Pareto optimum outcome. In a game setting, Calsamiglia (1977) showed that because of the nonconvexities often associated with public goods, it is not possible to define a nonwasteful, incentive compatible, privacy-preserving mechanism with a finite message space.

Perhaps as important as impossibility theorems, Hurwicz (1979) formalized the description of an economic mechanism. A mechanism is a system of rules that determines a mapping from a vector (m) of messages (m_i) for each player to resource allocation h(m). The environment (e) is a combination of characteristics (e_i) of all players. Behavioral rules (b_i) are a mapping from the environment to a message space, given a set of strategies for each player:

$$m_i = b_i(e).$$

Performance of a mechanism is determined by the combination of behavior and allocation rules for the given environment: $P(e; h,b) = h(b(e))$.

Extending the concepts of institutional design to an experimental setting, Smith (1980, 1989) expanded Hurwicz's description of institutional rules. An economic system (S) consists of institutions or rules (I) and an environment (E): $S = (E,I)$. Messages from players are conditional on institutional rules: $m_i = b_i(e/I)$. Traditionally, institutional rules include property rights and market rules, but Smith also included termination rules. That such institutional rules influence resource allocation has been amply demonstrated for auction markets (Ketcham, Smith, and Williams, 1984; McCabe, Rassenti, and Smith, 1992).

Nash equilibrium has been the noncooperative solution concept "par excellence" (Maskin, 1985, p. 173). A Nash equilibrium is a situation, based on unilateral messages or actions chosen by each player, in which it is not in a player's interest to make a change from the equilibrium message, given that other players hold to the equilibrium. Implementation concerns whether a game solution is in the set of socially desirable outcomes. (See Maskin, 1985.)

Hurwicz (1979) linked the Nash game equilibrium to an abstract economy equilibrium for Lindahl (public good) environments: assuming that equilibrium Lindahl prices are known (plus some other technical conditions), a Lindahl equilibrium is a Nash equilibrium. As Samuelson's (1954) seminal article on public goods pointed out, the problem is how to determine these prices in an incentive compatible way.

Following Samuelson's paper, a widely held belief was that free-riding would impede voluntary or cooperative solution of collective action problems. To alleviate free-riding, "demand revealing" mechanisms were proposed (Groves and Ledyard, 1977; Harstad and Marrese, 1981; Clarke, 1977;

Tideman and Tullock, 1977). The Groves-Ledyard mechanism does produce a Nash equilibrium that is Pareto optimal, but it is not ideal: it can (depending on the value of a certain parameter) lead to situations that do not satisfy individual budget constraints or individual rationality (Smith, 1980). The mechanism is also not free from process manipulation (Chen and Plott, 1993). The Clarke demand revealing process has been criticized for being inconsistent with individual budget constraints and group rationality (Groves and Ledyard, 1977).

Economic experiments have used the voluntary contribution mechanism to study regularities in behavior in a public goods game setting. It is well-known that such a mechanism cannot be Pareto optimal in all environments (Bagnoli and Lipman, 1989; Bergstrom, Bloom and Varian, 1986; Varian, 1994). Experiments focussed on free-riding and how efficiency is affected by the environment (Ledyard, 1993; Rappoport, 1994). The voluntary contribution mechanism was termed by Smith (1979) to be a free-rider mechanism because of its ability to be manipulated.

In an experimental context, environmental parameters shown to influence contribution include: heterogeneity in income distribution (Chan, Mestelman, Moir, Muller, 1995; Rapoport, 1988; Rapoport and Suleiman, 1993); group size (Isaac, 1993; 1994c); and reward level (Isaac and Walker, 1988). Institutional rules studied experimentally include the provision point rule (Isaac, Schmitz, and Walker, 1989) and communication rules (Ostrom and Walker, 1991). The effect of learning about other players has also been studied (Andreoni, 1988).

Generally, experimental evidence shows that people contribute at higher than noncooperative levels, though still below optimal (Isaac and Walker, 1988). Thus, the outlook for cooperative solution of social problems has brightened considerably. Some researchers have even suggested that the emphasis on demand revelation has been misplaced (Johansen, 1977).

Recently, it has been noted that incentive problems can be addressed in other ways than through direct economic incentives. Elster (1989) and Ostrom et al. (1991, 1992, 1994) discuss behavioral methods, such as sanctioning, that can regulate group behavior. Exit and voice are other mechanisms to influence behavior of other players (Hirschman, 1970).

Providing evidence that public good situations may be problematic because of complexity, multiple unit voluntary contribution experiments for public good provision were found to be less successful than the single unit case (Bagnoli, Ben-David, and McKee, 1992, p.87-88)):

"We also conclude that individuals have much more difficulty focusing on an equilibrium in the multiple units setting than they did in the single unit setting....However, we do find that the subjects are able to achieve the predicted equilibrium with some degree of regularity even in this complex setting."

Design Ingredients and Criteria

Environment

The term environment has been used to denote the nature of preferences for an economy (Hurwicz, 1979, 1994). Another important aspect of an environment is the types of goods (eg. private, public, externality) to be allocated.

Public goods have been described in a variety of ways (as having high exclusion costs, being nonrivalrous in consumption, being jointly supplied, exhibiting economies of scale, etc.). Here, we restrict attention to shared goods. A shared good is not divisible among group members; it may be jointly supplied to group members at any level, provided that payments from group members cover provision cost.

Public or shared goods may be associated with increasing returns to scale or other forms of nonlinear cost. Therefore, the notion of environment is extended to include the physical parameters that describe an abstract economy in terms of production and/or cost as well as preference parameters:

$$e = (e_u, e_\tau)$$

where e_u denotes preference-related parameters and e_τ denotes cost or technology-related parameters. A privacy-preserving process can have private preference information (e_u^i) common knowledge about production parameters (e_τ).

Policy Instrument

A policy instrument (P) is a rule that specifies, for an abstract economy, a mapping from the environment to an allocation:

$$P: e \rightarrow a$$

where the vector $\underline{a}$ describes total output and its distribution resulting as an abstract economy equilibrium determined by the policy P. For a given environment, each type of policy can have a corresponding equilibrium.

We distinguish between the theoretic form of a policy instrument (e.g. price as an instrument) and its operational determination (e.g. the numeric value of a price). Here, the form of the policy instrument for a cost share equilibrium involves shares and personalized prices (see below). Share and price values must be specified in order to operationalize the policy.

Theorems about the properties of a policy instrument can be based on only on its form. For example, the Pareto optimality of cost sharing equilibria was shown by Mas-Colell and Silvestre (1989, 1991), and the existence of equilibria was shown in Diamantaras (1992).

Allocation Rules and Behavior

Allocation rules serve to determine operational values of policy parameters given messages at each step (denoted by t) of a process. We extend the definition of an allocation rule given by Calsamiglia (1977, 1987) to include institutional rules as follows. An allocation rule (h) defines a mapping from messages by players (m_i^t) to resource allocation, given the environment (e), the policy instrument (P), and other institutional rules (I):

$$a^t = h(m_1^t, m_2^t, \ldots, m_n^t; e, I, P).$$

Institutional rules (I) include a rule that defines the nature of the message space. An averaging operator (A) is one example of an allocation rule: based on individual quantity proposal messages Q_i:

$$\overline{Q}^t = A(Q_1^t, Q_2^t, \ldots, Q_n^t).$$

Behavior can be related to preference, institutional rules, and environment as follows. A player selects messages based on preference over outcomes, given the strategies available to the player and information about messages of other players. Strategies are limited by the environment and the rules h and I. In a sequential process, given previous messages from other players, choice of a new message by a player I may be represented as:

$$\underset{m_i^{t+1}}{Max}\ [u^i(h(\overline{m_1^t}, \ldots m_i^{t+1}, \ldots \overline{m_n^t}; e, I, P)); m_i^{t+1} \in S^i_{e,h,I,P}],\ \overline{m_j^t} \in S^j_{e,h,I,P}, j \neq i.$$

If improvement in utility is possible, each player will select a new message from the strategy set, where $S^i_{e,h,I,P}$ is the strategy set for player I and $\overline{m}_j$ are other players' messages. The behavior response function b_i summarizes the result of preference interacting with institutional rules for the given environment:

$$m_i^{t+1} = b_i(m_1^t, m_2^t, \ldots, m_n^t; e, h, I, P).$$

A privacy-preserving process is one for which the response function b_i depends on only the preference characteristics e_u^i for person i (Hurwicz, 1994).

Hurwicz (1994) described an adjustment process in terms of the message space, response functions, and the allocation rule **h**. The sequential form of the response function highlights the possibility of a stationary point. A Nash equilibrium N(e; h,I,P) is a stationary point for an adjustment process, when no player has an incentive to change his/her message from the equilibrium message given the messages of other players.

Coordination Process

A coordination process is a particular type of adjustment process. According to Reiter (1995), a coordination process is the set of institutional rules having to do with procedures to solve a joint optimization problem. It consists of: "(i) an algorithm for computing a function, the decision rule, and (ii) an assignment to individual agents of the steps required to execute the algorithm." Here, the process is based on an algorithm to determine an optimal shared good level and its finance. Marginal cost and sequential preference information define the gradient for the optimization algorithm.

The process is guided by a coordinator whose role (which could be automated) is to execute the process. Clearly, the role of a coordinator in such a process is more complex than that of an auctioneer who simply matches bids and offers (Jackson and Moulin, 1992).

Design Criteria

Achieving Pareto optimality as a decentralized equilibrium of an abstract economy is the benchmark for policy instrument design. Two criteria are implied: nonwastefulness, and privacy preservation. Necessary conditions are that a policy instrument be feasible (any outcome should satisfy the group budget constraint) and individually rational (individual budget constraints should be met, and individuals should be made better off compared to a status quo) (Hurwicz, 1994).

A distinction has been made between the properties of a policy instrument in an abstract economy and its performance in a game or "naturally occurring " setting with potential incentives problems. A policy instrument that produces a Pareto optimum outcome for an abstract economy can be called "potentially nonwasteful". The policy instrument associated with the cost share equilibrium is potentially nonwasteful.

Grafting incentive compatibility to cost share and ratio equilibria is the subject of Tian's work (1994, 1995). Such game implementation procedures can make genuine implementation much more cumbersome, and Clarke mechanisms, can have real social costs.

In judging "real world" mechanisms, Shorter (1995) has proposed the following design criteria: understandability (a mechanism should be understood by the agents who will use it); fairness (a mechanism should be perceived as fair to be acceptable for use); robustness (the success of the mechanism is not destroyed by error, and it can work for a wide range of environment characteristics); and profitability (participants should gain from using the mechanism).

Fairness applies both to the form of a policy instrument and the resulting outcomes. For example, equal cost shares may seem fair in some contexts but may not seem fair if benefits are unequally distributed.

Cost allocation methods in cooperative game theory such as the Shapley value (Loehman and Whinston, 1971, 1974, 1976; Loehman et al., 1979) express fairness through axioms that determine a particular form for cost sharing (Young, 1985). Young (1994) discusses several types of fairness rules. Equal cost shares has been the predominant outcome in games of fair division (see Davis and Holt, 1993, for a summary) and in contracting games (Hackett, 1993).

Stability of group agreement is another design criterion. Stability means that participants will not want to back out of an agreement once it is made. This criterion is similar to the Nash equilibrium in that, once a consensus is found, it is not to anyone's advantage to deviate. Without individual rationality, a process cannot be stable.

For stability reasons, simultaneous choice of shared quantity and cost shares is proposed here. Instead, there could be a two stage process in which shared good quantity is first determined and finance subsequently determined (as in Leyard and Palfrey, 1990). Cost allocation literature in cooperative game theory is consistent with a two-stage approach because public good determination is exogenous to the cost allocation procedure (Young, 1994). A two-stage process may not be stable: a participant may initially agree to a proposed quantity level but may later back out of agreement when his/her cost share is revealed.

A Policy Instrument for Cost Sharing

The cost share equilibrium is a decentralized solution concept based on cost shares and personalized transfer prices developed in theoretical form by Mas-Colell and Silvestre (1989, 1991), Weber and Wiesmeth (1991), and Diamantaras (1992). The cost share equilibrium is related to a ratio equilibrium defined by Kaneko, 1977, which uses endogenous shares but not personalized prices. Personalized transfer prices serve to make possible the existence of an equilibrium for given cost shares; see below. These prices must sum to zero for optimality reasons.

The Lindahl equilibrium also uses personalized prices; prices sum to marginal cost at the equilibrium (see Laffont, 1988). However, the use of Lindahl prices may not cover total costs if marginal cost is decreasing. The advantage of cost shares as a method of finance is feasibility: the cost for a shared good is covered by payments regardless of the nature of the cost function (Mas-Colell and Silvestre, 1989, 1991; Tian, 1994).

The conditions of production are not specified by the cost share equilibrium policy form. Zero profit, minimum cost production must be added as an assumption for the cost share equilibrium. Lump sum transfers may also be required to ensure that no one is worse off through group participation.

Pareto Optimality of Cost Share Equilibrium

Below, we use a simplified public goods economy to describe the policy instrument associated with a cost share equilibrium. The environment consists of a set of preferences and incomes for players and a cost function for shared good(s).

A Pareto optimum in such an economy is the solution of a vector optimization problem (Takayama, 1974):

$$\max_{Q,x_i} \Sigma\, \beta_i\, u_i(x_i, Q)$$

$$s.t. \quad \Sigma\, x_i + C(Q) \leq M$$

for private consumption for each player x_i and shared good Q, where C(Q) is the cost of producing the shared good relative to the private good, individual incomes are M_i which sum to M, and β_i are weights on players. Potentially, there is a Pareto optimum solution for each set of weights. Substituting for a player's weight in terms of his/her marginal utility of income, a Pareto optimum satisfies the well-known condition:

$$\sum_i \frac{u_Q^i}{u_x^i} = C'(Q)\ .$$

A cost share system is a family of personalized charge functions T_i (Q) such that $T_i(0)$= 0 and

$$\Sigma_i T_i(Q) = C(Q).$$

A cost share equilibrium system is a decomposition of the Pareto optimality problem as follows. Each person's determination of Q is represented by:

$$\max_{x_i, Q} \quad u^i(x_i, Q)$$

$$s.t.\ x_i + T_i(Q) \leq M_i.$$

This behavior generalizes price-taking behavior: rather than price, the charge function is taken as given. The quantity demanded of the shared good is determined by optimization given the charge function. A cost share equilibrium is a feasible state $\{x_i^*, Q^*\}$ such that no player would prefer a different shared good level for the given charge functions.

$$u_i(x_i^*, Q^*) \geq u_i(M_i - T_i(Q), Q) \text{ for all feasible } Q.$$

That is, a cost share equilibrium implies a unanimous choice Q^* for the shared good for the given charge function.

A linear cost share equilibrium is a special case that uses a set of personalized prices p_i and cost shares s_i (Mas-Colell and Silvestre, 1991). Here, we extend the definition to include lump sum transfers l_i that sum to zero. The form of the policy instrument is a nonlinear charge function:

$$T_i(Q) = s_i\, C(Q) + p_i\, Q + l_i.$$

For a vector of shared goods, the personalized price is a vector; there is a personalized price for each good for each player.

For the linear cost share instrument, first order conditions at an equilibrium are:

$$u_Q^i / u_x^i = s_i C'(Q) + p_i.$$

Balancing conditions for feasibility are that p_i and l_i should sum to zero and s_i should sum to one. Pareto optimality is implied by the requirement that p_i sum to zero and s_i sum to one.

If policy parameters (price, share, and transfers) corresponding to an equilibrium were known, a cost share equilibrium would be a Nash equilibrium, since the equilibrium quantity would be the preferred outcome for each player for the given parameters. However, as in the case of personalized prices for the Lindahl equilibrium, incentive problems arise because of the need to determine equilibrium parameter values.

Adding Incentive Rules to Cost Sharing

Incentives for truthful quantity revelation are consistent with a cost share equilibrium.

Groves-Ledyard. The Groves-Ledyard (GL) mechanism can be viewed as a penalty function for deviation from the group average quantity. To add GL to a cost share equilibrium framework, define the charge function as follows:

$$T_i(Q) = s_i\, C(Q) + p_i\, Q + \gamma\, [(n-1)/n\, (Q - Q_{-i})^2 - B_i^2]$$

where B_i is defined so that the sum of the terms [ ] over I is zero. With unanimity, the sum over i of the derivatives of [ ] with regard to Q is also zero, so that Pareto optimality holds for the corresponding cost share equilibrium. At a Nash equilibrium, any deviation from the group average demand would result in paying a penalty.

The parameter γ is arbitrary. Low γ values may not produce unanimity in Q since players may elect to pay the penalty while having different quantity demands. Larger values of γ force greater similarity of Q proposals, but then budget constraints may not be satisfied. Including personalized transfer prices with the GL mechanism can help assure that budget constraints are met.

Smith. Another incentive formula of the cost-sharing type generalizes Smith's rule for the constant marginal cost case (1979, 1980). To give incentives for truthful bidding, Smith (1979, 1980) proposed a rule in which a player's bid cannot directly influence his/her own price. Instead, price is based on the bids of other players. Smith's concept can be extended to the nonlinear case as follows:

$$T_i(Q) = s_i\, C(Q) + [(1 - s_i)\, C'(Q_{-i}) - \Sigma_{j\text{-}i}\, b_j]\, Q,$$

where b_j denotes a marginal willingness to pay bid to increase Q by one unit, $\Sigma_{j\text{-}i}$ denotes the sum over j excluding player i, and Q_{-i} denotes the group proposal excluding player i. Personalized prices are given by the [ ] term At an equilibrium, $\Sigma_j\, b_j = C'(Q)$ and $Q_{-i} = Q$; thus, Pareto optimality is satisfied.

Message Space:Willingness to Pay Interpretation

For application purposes, cost share equilibrium conditions may be interpreted to be in terms of marginal willingness to pay. This interpretation has implications for the message space. The message space can include not only messages about quantities demanded for given charge functions but also marginal bids (marginal willingness to pay) for an extra unit, given the current level of shared good. In comparison, the voluntary contribution mechanism uses "total" bids, or bids toward a given level of good.

Define $MWTP^i(x_i, Q)$ to be the maximum that a player would be willing to pay to increase Q by one unit, for a given expenditure on other goods x_i:

$$u^i(x_i - MWTP^i,\ Q+1) = u^i(x_i,\ Q)$$

Then, for a given Q, the marginal willingness to pay function has properties of a demand function; for example, marginal willingness to pay is declining as a function of Q (Loehman, 1991):

$$MWTP^i(x_i, Q) = \frac{u_Q^i}{u_x^i};$$

the right hand side is evaluated at the given Q and x_i.

Using a cost share mechanism with a charge function Ti(Q), each person chooses Q to maximize utility subject to the budget constraint. Substituting for private goods from the budget constraint, marginal willingness to pay is then only a function of Q. Therefore below we use the notation $MWTP^i(Q)$ for the cost share process. At an equilibrium $\{x_i^*, Q^*\}$, first order conditions imply that each person's marginal willingness to pay is equal to his/her marginal charge function:

$$MWTP^i(Q^*) = s_i C'(Q^*) + p_i .$$

Also at an equilibrium $\{x_i^*, Q^*\}$, the sum of marginal willingness to pay is equal to the marginal cost:

$$\sum_i MWTP^i(Q^*) = C'(Q^*) .$$

Full Information Solution of the Equilibrium System

Theory proving existence of a cost share equilibrium was presented by Diamantaras, 1992. Here, we present a solution algorithm for the full information case to explain the role of personalized prices.

Taking shares s_i as given, the number of equations for a linear cost share equilibrium (first order conditions, balance conditions, and budget constraints) exactly equals the number of unknowns (private good consumption, prices, and shared good quantity). If the Jacobian of the system is not zero, a solution for prices and quantities exists for given incomes and cost shares. A mapping is thus obtained from shares s_i to prices and quantities:

$$\{s_i\} \rightarrow \{Q, x_i, p_i\}.$$

The following method operationalizes the solution of the system. Under full information about preference, income, and given shares, the equilibrium system can be converted to an equivalent nonlinear programming problem, constrained by individual budgets:

$$\min_{l_i, x_i, p_i, Q} \sum (l_i)^2$$

$$\begin{aligned} s.t. \quad MWTP^i(x_i, Q) &= s_i C'(Q) + p_i ; \\ x_i &= M_i - s_i C(Q) - p_i Q + l_i \geq 0 ; \\ \sum_i p_i &= 0 ; \\ \sum_i l_i &= 0 . \end{aligned}$$

Note that there are 3n+1 unknowns and 2n+1 constraints. Any feasible solution is a cost share equilibrium. Lump sum transfers l_i ensure that budget constraints are met. The system is determined by the minimization of lump sum transfers (which may or may not be all be zero). This method is used in appendix examples for full information solutions.

This formulation shows the role of personalized prices: they serve as slack variables for the first order conditions to make solution of the system possible for any shares s_i.

Coordination Process for Cost Sharing

Rather than full information, preference data could be revealed sequentially as in a market process. Below, allocation rules, procedural rules, and "genuine implementation" rules are described for three coordination processes which are privacy preserving. Price adjustment rules presented below by message type bring the process to an equilibrium.

The three processes differ in terms of message space and corresponding allocation rules. Alternative message spaces are: quantities only (as in a price taking mechanism), bids only (as in an auction for a single good and voluntary contribution), and both bids and quantities (as in auctions, and in the auction mechanisms for public goods defined by Smith 1979, 1980). Table 1 summarizes rule sets for each type of process.

Allocation Rules

Allocation rules for the cost share equilibrium concern specification of shares and personalized prices. Each of these is discussed below.

Share Rules.. The specification of a share rule depends on the information available. For example, if incomes are not known, then share rules cannot be based on income.

Shares s_i may be defined according to a rule that determines them exogenously; examples are shares proportional to income or equal shares. For given shares s_i, ex post equilibrium cost shares are defined by the ratios $T_i(Q)/\Sigma\, T_i(Q)$. Because personalized prices depend on preferences, ex post shares are endogenous, or determined by execution of the mechanism.

One type of endogenous rule is that shares be proportional to marginal willingness to pay, or marginal benefit. Olson (1965, p. 30) believed that shares must be proportional to marginal benefit in order to satisfy Pareto optimality. Olson's rule is a special case of a cost share system that is sufficient but not necessary for Pareto optimality: personalized prices are zero and implied shares are proportional to marginal willingness to pay at the equilibrium Q and private consumption

$$s_i = MWTP^i(x_i^*, Q^*)/C'(Q^*) .$$

An equilibrium with zero personalized prices may not exist.

Another type of endogenous share rule is a consumption tax, or shares proportional to private consumption:

$$s_i = x_i/\Sigma x_i .$$

The consumption tax rule provides a "demi-incentive" for demand revelation. Solving simultaneously the system of equations, such shares satisfy

$$s_i = \frac{M_i - Qp_i}{\Sigma M_i}.$$

For relatively low expenditure on public goods, such shares would approximately be equal to income shares.

Personalized Price Rules. Below, price rules are defined in terms of message space: quantities only, or bids and quantities. Price rules are described as part of an iterative process to determine the cost share equilibrium (see below).

Allowing both bid and quantity messages, the first order condition can be used directly to define the allocation rule for personalized prices. The personalized price is determined from the player's marginal bid minus the marginal cost share, evaluated at the average $\overline{Q}$ of players' demands for the current situation:

$$p_i = MWTP^i(\overline{Q}) - s_i C'(\overline{Q})$$

A new quantity proposal can then be determined from the new price. Let Q_i denote the message about quantity demanded by player i given the charge function $T_i(Q)$; Q_i satisfies:

$$MWTP_i(Q_i) = s_i C'(Q_i) + p_i.$$

The equilibrating property of this price rule can be seen as follows. Suppose for the given cost share function, that the quantity demanded by a player is greater than the group average. Marginal willingness to pay is a declining function:

$$MWTP^i(\overline{Q}) > MWTP^i(Q_i) \ .$$

Thus, if the marginal cost is increasing, there is an implied price increase for the next round:

$$\begin{aligned} p_i' - p_i &= [MWTP^i(\overline{Q}) - MWTP^i(Q_i)] + s_i(C'(Q_i) - C'(\overline{Q})) \\ &> s_i\,[C'(Q_i) - C'(\overline{Q})] > 0. \end{aligned}$$

So, on the next round, a smaller quantity will be demanded by this player.

In "real world" applications, the price rule must differ from the theoretical condition because marginal willingness to pay is not observable. The “marginal bid” rule is based on marginal bid messages rather than marginal willingness to pay. The following allocation rule derives from the first order condition assuming truthful messages ($b_i = MWTP^i$) and the equilibrium condition $\Sigma b_i = C'(Q)$:

$$p_i' = b_i - s_i \Sigma_i b_i.$$

Note this rule automatically satisfies the condition that the sum of personalized prices be zero. Appendix 1 discusses the optimality properties of this rule and shows that when outcomes are discrete, the bid need not reveal true willingness to pay in order to achieve a Pareto optimum.

The resulting charge formula is

$$T_i(Q) = s_i\, C(Q) + [b_i - s_i\, \Sigma_i\, b_i]\, Q.$$

$$= b_i\, Q - (\Sigma_i\, b_i - C(Q))\, s_{i.}.$$

This rule has the following: the bid determines a personalized price per unit, and there is a rebate of the excess of total bids over cost according to the specified shares.

With only quantity messages, the rule for price can only be based on cost and quantities. Given the current price $p_{i,}$ the "marginal cost" rule for price adjustment is :

$$p_i' = p_i + [C'(Q_i) - C'(\overline{Q})]\ s_i$$

where $\overline{Q}$ denotes the average of players' demands for the current charge functions. The new personalized price for a player is based on the difference in marginal cost between the player's demand Q_i and the group average demand. Out of equilibrium, prices must be normalized to sum to zero for feasibility reasons. To do this, each player's share is applied to the sum of prices; then this share of the sum is subtracted from the each person's price.

The origin of the marginal cost rule is the first order condition applied iteratively. For the given share s_i, defining price from the first order condition at $\overline{Q}$ and substituting the optimality condition for Q_i:

$$p_i' = MWTP_i - s_i\, C'(\overline{Q}) = p_i + s_i\, C'(Q_i) - s_i\, C'(\overline{Q})$$

This price rule has an equilibrating property: if a person proposes a quantity greater than the group average, then that person's price will increase. So, for increasing marginal cost on the next round, this player will reduce the quantity proposed. (A different rule could be formulated for the decreasing cost case to give incentives to increase quantity.)

<u>Procedural Rules for Cost Sharing: Alternatives</u>

Below, we describe three types of sequential search processes to locate a cost sharing equilibrium. Each process represents an algorithm for solving the equilibrium problem for the given message space. Each of the three processes could potentially result in a Pareto optimal outcome in an abstract economy. The sequential voluntary bid process is relevant only for discrete quantity.

<u>Quantity Process</u>. For this process, players act as price takers to choose quantity proposals in response to proposed charge schedules. Personalized price is computed from the marginal cost rule.

The following describes a round of the iterative process. The sequence of messages and computation is as follows:

(i) Each player is asked to propose a quantity Q_i given his/her charge function.

(ii) The coordinator determines the average group quantity based on players' proposed quantities.

(iii) Given the current charge for the current group quantity, each player determines if he/she wishes to continue the proposal process.

(iv) On continuance, the coordinator computes new prices.

The process returns to step (i). The process repeats until it converges to a common quantity..

Bid/Quantity Process. This iterative search process uses both bid and quantity messages. It is more complex than the quantity process, and bids add another dimension to the message space.

To provide an incentive for bids, a rule similar to the provision point for voluntary contribution can be added: the sum of marginal willingness to pay bids should exceed the incremental cost in order for quantity to increase to the next level. The incremental cost amount provides an anchor for bidding.

The process is as follows:

(i) Each player is asked to propose a quantity Qi given his/her charge function.

(ii) The coordinator determines the average quantity based on players' proposed quantities.

(iii) Given the current charge for the current group quantity, each player determines if he/she wishes to continue the proposal process.

(iv) If the process is continuing:

(a) For the current group quantity, each player states a marginal willingness to pay bid to obtain the next higher level.

(b) The provision point test is then applied: if the sum of the marginal bids to increase from the current quantity does not exceed the required marginal cost, quantity proposals for the next round are constrained to be the same or lower on the next round.

(v) The coordinator computes new prices based on bids.

The process returns to step (i). The process repeats until convergence to a common quantity is reached.

Sequential Voluntary Bid Process. Voluntary contribution can be put in an algorithmic framework similar to the method for multiple units suggested by Bagnoli and McKee (1991). This is similar to the process proposed by McKee et al., 1992, except that we use marginal rather than total bids, and the provision point test is also in terms of marginal bids and marginal cost, rather than total bids and total cost. The reason for this is optimality (see Appendix 1). As in the Bagnoli and McKee game, the excess of bids over total cost should not be returned for incentive reasons.

The following process is repeated for successive levels of the shared good:

(i) Given the current shared good level and each player's total charge to date, each player proposes a bid to increase the current quantity by one unit.

(ii) Marginal provision point test: the coordinator checks the sum of bids to see if they exceed the marginal cost to go to the next higher level. If not, the process stops at the current quantity level.

(iii) Given sufficient bids to increase to the next level, the current quantity is increased to the next level;

(iv) The coordinator computes the corresponding cumulative bids for each player.

The process repeats until the provision point test fails.

Genuine Implementation Rules

Voting. The processes tested by Smith (1977) and Banks and Plott (1988) had voting to determine whether or not to stop the process. However, Appendix 4 examples show that the process produces moves that are not Pareto improving; so players may not actually agree to an outcome with common quantity proposals. Therefore, voting over generated outcomes is needed to ensure consensus and verify that an equilibrium is actually the preferred choice of all participants.

Discreteness. One important difference between theoretical and experimental settings is that quantity in a real world setting may be discrete rather than continuous. Because of the set of choices is finite, the discrete situation may actually be more robust in terms of convergence than the continuous case. On the other hand, for discrete optimization, a coordination process may not reach an equilibrium if intermediate adjustment is not possible. For discrete choices, rather than marginal conditions, inequalities and incremental conditions are relevant for personalized prices (see Appendix 1 and Appendix 4).

Incentive Problems. Appendix 1 shows that truthful bids are not required in the discrete bid/quantity case since bids and prices need only satisfy a system of inequalities. However, the sequential voluntary process may stop at a quantity less than Pareto optimal. Incentive rules such as those described above may need to be added to the potential processes to reduce misrepresentation.

Simulation of Alternative Rules

Simulation results shown here compare alternative share rules, test convergence of algorithms, and study the effects of misrepresentation.

Comparison of Share Rules for Full Information

For a full information solution, equilibrium outcomes for alternative cost share rules are compared in Appendix 2. Alternative rules are: shares proportional to income, proportional to marginal benefit, consumption tax, and equal shares. (Zero personalized prices are implied by the rule of shares proportional to marginal benefits.)

Appendix 2A shows results for quasi-linear utility functions (a predominant example in public goods literature); quasi-linear utility implies a closed form solution for quantity and price, with quantity independent of the cost share rule (see Bergstrom and Cornes, 1983). Simulation results demonstrate that shares s_i need not be correlated with relative welfare at an equilibrium: a high share can be offset with a subsidized price. Or, a person with a relatively low share may receive a relatively high price! Surprisingly in the quasilinear example, cost share equilibria for a wide range of share allocations give fairly similar utility gains. That is, without income redistribution, cost share equilibria may provide access to only a small range along the Pareto frontier. If the range of possible equilibrium outcomes is small, fairness arguments regarding share rules could be less compelling.

Appendix B shows the implications of alternative share rules for logarithmic utility. The logarithmic utility case exhibited more variation in the distributions of utility gain than the quasi linear case. The consumption tax share rule produced the largest equilibrium quantity as well as the greatest sum of utility changes.

Convergence of Processes

Appendix 3 compares bid/quantity and quantity processes in terms of convergence for a nonlinear utility function and continuous quantity. Simulation is based on truthful responses. Convergence means agreement in the first two decimal places among players' quantity demands. (The sequential voluntary bid process was not simulated. It would terminate at a step number which is no greater than the number of discrete quantity levels.)

The simulated process using bid/quantity messages converged much faster than the quantity process; the solution was obtained in 5-6 iterations was nearly identical to the full information solution. The quantity process was subject to "cobwebs." Similar convergence results were obtained for both consumption tax and equal share cases.

For discrete outcomes (0, .5, 1.5, etc), simulations of quantity and bid/quantity processes are shown in Appendix 4. The quantity mechanism performs as well as the bid/quantity mechanism in the discrete example: both converge in two steps to the equilibrium.

The Smith pricing rule ~ that price be independent of a player's own bid for incentive reasons ~ was also simulated. It produced even worse convergence results than the quantity mechanism. Evidently, the equilibrating role of price is sacrificed in the name of demand revelation.

An equilibrating procedure based only on bid messages called the M.P. planning process was developed by Dreze and de la Vallee Poussin (1971) (see also Roberts, 1987). A similar process ~ that computed quantity to equalize total marginal willingness to pay and marginal cost ~ did not converge after many iterations.

Misrepresentation

Appendix 4 simulates quantity and bid/quantity processes with and without misrepresentation. With misrepresentation, the quantity mechanism converged less rapidly for the discrete example. After round-off to the nearest discrete level, the quantity process converged in five steps to a quantity lower than Pareto optimal. The bid/quantity process still converged, after two iterations, to the optimal quantity. However, the "cheater" is better off compared to the truthful case. Still, all parties are better off with cheating than if there were no group solution. Of course, if more than one person cheats, the process could fall apart!

Conclusions

This paper proposes use of a coordination process for group decision making. The underlying premise is that complexity may be as serious as free-riding for cooperative group outcomes, and that coordination may relieve complexity.

Through coordination, shared good quantity and cost shares are determined as the stationary point of a structured group decision process. A consensus rule ensures individual rationality since all participants must agree to the outcome. Because of potential incentive problems, this type of process may be most successful for small groups when it is more difficult to misrepresent preferences.

For public or shared goods, it is well-known that a market process is not possible. Voting has been the predominant method of group decision making; however, voting also may not lead to optimal decisions (Frohlich and Oppenheimer, 1978). The bureacratic approach to resource allocation for public goods has been based on benefit-cost analysis. An associated method of benefit estimation for public or nonmarket goods has been contingent valuation to elicit willingness to pay statements. Contingent valuation has been criticized because willingness to pay statements are hypothetical and possibly biased (U.S. Dept. of Commerce, 1993).

Compared to contingent valuation, the bid/quantity process described here obtains marginal willingness to pay in a binding agreement context. Marginal willingness to pay for a quantity change may be more readily comprehended than "total" willingness to pay for a given quantity. The provision point rule that bids must cover marginal cost provides a "demi-incentive", while the cost requirement provides an anchor for bidding.

Three types of coordination processes described in this paper differ in terms of the nature of the message space and associated allocation rules. The quantity and bid/quantity processes have many desirable properties of a market: decentralized decision making and privacy coupled with an equilibrating process that produces individual- and group- rational decisions. The sequential voluntary contribution process is simpler, because it does not include an equilibriating process, but it may be more subject to incentive problems.

The usefulness of simulation as a design tool under both full and partial information was demonstrated through the comparison of a variety of rules. Not surprisingly, alternative share rules could have different distributional consequences. However, the share need not become a bone of contention, because different shares could result in similar ex post cost distributions. Equal shares can be a satisfactory rule because personalized prices allow for differences in preferences to be expressed.

Comparison of quantity and bid/quantity processes showed that the bid/quantity process can be more robust in terms of convergence for a continuous quantity. However, both bid/quantity and quantity processes can perform well when quantity levels are discrete. Truthfulness about bids is not required for the bid/quantity process to find an equilibrium quantity that is Pareto optimal.

Future work should compare alternative processes in an experimental framework to test the interaction of rules and behavior.

Table 1. **Comparison of Alternative Allocation Rule Sets**

rule set:	Seq. Voluntary Bid	Quantity	Bid/Quantity
Message:	bid	quantity	bid & quantity
Share s_i:	none	given[a]	given[a]
Price:	zero	marginal cost	marginal bid
Incentive:	provision pt.		provision pt.
Quantity Aggregation:	sequential	weighted avg.	weighted avg.

[a]Equal shares can be used. Other rules are possible.

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Appendix 1: Cost Share Equilibrium for Discrete Quantity Outcomes and the Need for Truthful Bids

For the discrete version of a cost sharing equilibrium, the following discussion includes:

(1) necesssary and sufficient conditions for Pareto optimality;
(2) neccessary and sufficient conditions for a cost sharing equilibrium;
(3) properties of bidding for bid/quantity and sequential voluntary bid processes.

It is shown that bids need not reveal true willingness to pay in order to find an equilibrium.

Pareto Optimality

Below for simplicity, instead of using utility functions, $B^i(Q_j)$ denotes the willingness to pay function for person i and discrete levels of quantity Q_j. Also, Pareto optimality is expressed in terms of the sum of willingness to pay rather than as a weighted sum. The same type of argument would apply with a more complicated problem.

The optimal quantity level Q_j^* will satisfy:

$$\mathrm{Max}_j\ \Sigma_i\, B^i(Q_j) - C(Q_j).$$

Necessary and suffficient conditions that Q_j^* is a maximum for declining $\Sigma_i\, \Delta B^i - \Delta C$ are:

$$\Sigma_i\, \Delta B^i(Q_j^*) \geq \Delta C(Q_j^*)$$

$$\Sigma_i\, \Delta B^i(Q_{j+1}^*) < \Delta C(Q_{j+1}^*)$$

where the increments are defined as follows:

$$\Delta B^i(Q_j^*) = B^i(Q_j^*) - B^i(Q_{j-1}^*)$$
$$\Delta C(Q_j^*) = C(Q_j^*) - C(Q_{j-1}^*).$$

That is, level Q_j^* is the last level for which incremental benefits exceed incremental costs.

Cost Share Equilibrium

The charge system $T_i(Q)$ for the linear cost share case is defined as before:

$$T_i(Q_j) = s_i\, C(Q_j) + p_i\, Q_j$$

with cost shares s_i summing to one and personalized prices p_i summing to zero. For a cost share equilibrium, players take prices and shares as given (i.e. price-taking behavior) and determine quantity proposals by solving:

$$\underset{Q_j}{\mathrm{Max}_j}\ B^i(Q_j) - T_i(Q_j)$$

A cost share equilibrium is a set of personalized prices, cost shares, and Q_j^* satisfying price-taking behavior for each person i and the balancing conditions. Thus, necessary and sufficient conditions for equilibrium for given shares and declining ΔB^i and increasing ΔC are:

$$\Delta B^i(Q_j^*) \geq s_i \, \Delta C(Q_j^*) + p_i$$

$$\Delta B^i(Q_{j+1}^*) < s_i \, \Delta C(Q_{j+1}^*) + p_i$$

$$\Sigma p_i = 0.$$

$$\Sigma s_i = 1.$$

Summing the inequality conditions for each player over i, the restrictions on prices and shares imply that a cost share equilibrium satisfies the conditions for Pareto optimality.

The Quantity Equilibrium

Quantity messages are chosen to maximize net benefits for the given charge system. Becasue of price-taking behavior, an equilibrium of the quantity process directly satisfies the cost share equilibrium conditions.

The Bid/Quantity Equilibrium

Bids are the messages chosen in response to the question of willingness to pay to increase Q from the current aggregate group level to the next higher level. Quantity messages are chosen to maximize net benefits for the given charge system. b_{ij} denotes the bid by player i to increase from level Q_j to Q_{j+1}.

A bid/quantity equilibrium is a set of shares and prices, together with equilibrium bid messages and quantity Q_j^* messages, that satisfy the following rules:

(i) bids are no greater than marginal willingness to pay:

$$\Delta B^i(Q_j) \geq b_{ij};$$

(ii) the provision point rule: at the terminating quantity Q_j^*,

$$\Sigma_i \, b_{ij}^* \geq \Delta C(Q_j^*);$$

$$\Sigma_i \, b_{ij+1}^* \leq \Delta C(Q_{j+1}^*);$$

(iii) by price-taking behavior, for the given charge function, the equilibrium quantity satisfies:

$$\Delta B^i(Q_j^*) \geq s_i \, \Delta C(Q_j^*) + p_i$$

$$\Delta B^i(Q_{j+1}^*) < s_i \, \Delta C(Q_{j+1}^*) + p_i;$$

(iv) personalized prices are defined by:

$$p_i = b_{ij} - s_i \Sigma_i b_{ij}.$$

By (iii), a bid/quantity equilibrium is a cost share equilibrium and is therefore a Pareto optimum.

By (i), (ii), (iii) and (iv): equilibrium quantity and prices satisfy the following set of inequalities:

$$b_{ij}^* \leq \quad s_i \Delta C(Q_j^*) + p_i \leq \Delta B^i(Q_j^*)$$

$$\Delta B^i(Q_{j+1}^*) \geq b_{ij+1}^* \geq s_i \Delta C(Q_{j+1}^*) + p_i.$$

That is, rather than exact conditions, prices satisfy a set of inequalities. Many price vectors may satisfy the required inequalities. Satisfying the above conditions does not require that bids be equal to true marginal willingness to pay.

<u>The Sequential Voluntary Bid Equilibrium</u>

Bid b_{ij} denotes the amount player i proposes to increase from Q_j to Q_{j+1}. For a Sequential Voluntary Bid equilibrium:

(i) bids satisfy

$$\Delta B^i(Q_j) \geq b_{ij} \text{ for all } Q_j;$$

(ii) marginal provision point rule: at the terminating quantity Q_j^*,

$$\Sigma_i b_{ij}^* \geq \Delta C(Q_j^*);$$

$$\Sigma_i b_{ij+1}^* \leq \Delta C(Q_{j+1}^*).$$

Combining (i) and (ii), the sequential bid equilibrium satisfies one necessary condition for Pareto optimality, that the summed marginal benefits at the equilibrium Q exceed the marginal cost. However, unless bids are truthful, the second required inequality may not be satisfied. That is, the process could stop on a quantity less than Pareto optimal unless bids are truthful.

Appendix 2. Equilibrium Solutions for Alternative Cost Share Rules

2A. Quasilinear Utility

Each of three players has a utility function of the form: $u_i(x_i, Q) = x_i + \gamma_i \log(1 + Q)$. Preference parameters γ_i are respectively 115, 230, 115. Income levels are respectively 200, 300, and 100. The cost function is of the form: $C(Q) = 100 + 10Q - 5Q^2 + 5Q^3$.

The efficient Q will be independent of the distribution of income (Bergstrom and Cornes, 1983). The cost at the equilibrium is \$220, with marginal cost of \$115. Equilibrium prices satisfy:

$$p_i = \frac{\gamma_i}{1+Q} - s_i C'(Q).$$

Utility changes are in comparison to a status quo of Q=0 and $x_i=M_i$.

<u>Full information solutions for alternative share rules:</u>

s_i:	.17[e]	.33[a]	.25[b]	.45[c]	.33[d]
	.66	.50	.50	.49	.33
	.17	.16	.25	.06	.33
Q:	3.0	3.0	3.0	3.0	3.0
p_i:	9.59	-9.20	0	-23.00	-9.55
	-19.16	0	0	1.15	19.20
	9.58	9.20	0	21.85	-9.55
T_i:	65.40	45.	57.50	30.80	44.69
	89.18	110.	115.	110.	130.61
	65.40	65.	57.50	79.20	44.69
$\Delta u_i/\Sigma\, \Delta u$:	.22	.27	.24	.31	.27
	.56	.50	.49	.49	.45
	.22	.23	.24	.19	.27

a: Shares proportional to income
b: Shares proportional to marginal benefit at equilibrium
c: Shares proportional to private consumption (consumption tax):

$$s_i = \frac{M_i - \gamma_i \frac{Q}{1+Q}}{M - C'(Q)\,Q}.$$

d:Equal shares
e: Arbitrary

2B. Nonlinear Utility

Utility functions:
$u_i = \log(1+x_i) + \gamma_i \log(1+Q)$.
Utility Parameters: $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = 1$.
Incomes: $M_1 = 20$, $M_2 = 10$, $M_3 = 10$
Cost function: $C(Q) = 10 + 10\,Q - 5\,Q^2 + 5\,Q^3$.

Utility changes are in comparison to a status quo of $Q=0$ and $x_i=M_i$.

<u>Full information solutions for alternative share rules:</u>

s_i:	.17[e]	.50[a]	.25[b]	.54[c]	.33[d]
	.66	.25	.50	.20	.33
	.17	.25	.25	.26	.34
Q:	.948	.968	.965	.97	.964
p_i:	4.51	-.99	0	-1.68	1.80
	- 5.57	1.32	0	2.25	-.03
	1.06	-.33	0	-.56	-1.77
T_i:	7.55	8.80	8.59	8.94	8.17
	7.41	6.16	6.38	6.00	6.39
	4.28	4.56	4.50	4.61	4.91
$\Delta u_i / \Sigma\,\Delta u$:	.37	.16	.19	.15	.25
	.35	.66	.61	.69	.64
	.28	.17	.19	.16	.11

a: Shares proportional to income
b: Shares proportional to marginal benefit at equilibrium
c: Shares proportional to private consumption (consumption tax)
d: Equal shares
e: Arbitrary

Appendix 3: Quantity and Bid/Quantity Coordination Processes for Continuous Quantity

Utility functions:
$u_i = \log(1+x_i) + \gamma_i \log(1+Q)$.
Parameters: $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = 1$
Incomes: $M_1 = 20$, $M_2 = 10$, $M_3 = 10$
Cost function: $C(Q) = 10 + 10\,Q - 5\,Q^2 + 5\,Q^3$.

Utility changes are in comparison to a status quo of Q=0 and x_i=M_i.

3A. Consumption Tax Share Rule

Full Information: Δu_1 =.123; Δu_2 = .567; Δu_3 = .134; sum=.824.

Q	.97
p_1	-1.68
p_2	2.25
p_3	-.56
s_1	.541
s_2	.195
s_3	.264

Quantity Process: no convergence after 15 iterations:
Δu_1 = .112; Δu_2 = .599; Δu_3 = .130

Iteration	1	4	7	10	13	15
Q	.93	.96	.96	.964	.963	.97
Q_1	1.17	.88	.92	.93	.94	.95
Q_2	.97	1.13	1.09	1.04	1.05	1.01
Q_3	.66	.96	.96	.96	.97	.96
p_1	.40	-.312	-.778	-1.08	-1.29-	-1.37
p_2	.05	.764	1.245	1.56	1.78	1.87
p_3	-.45	-.451	-.46	-.48	-.49	-.496
s_1	.49	.51	.52	.526	.53	.533
s_2	.25	.23	.22	.21	.21	.205
s_3	.26	.26	.26	.262	.26	.262

Bid/Quantity Process: converged in 6 iterations:
Δu_1 =.123; Δu_2 =.568; Δu_3 = .135.

Iteration	1	2	3	4	5	6
Q	.93	.96	.972	.967	.973	.97
Q_1	1.17	.68	.965	.97	.979	.970
Q_2	.96	1.18	.950	.99	.953	.976
Q_3	.66	1.18	1.003	.98	.969	.970
p_1	2.72	-1.61	-1.66	-1.85	-1.66	-1.70
p_2	.006	2.46	2.23	2.41	2.21	2.26
p_3	-2.726	-.85	-.55	-.56	-.55	-.56
s_1	.44	.54	.54	.545	.540	.541
s_2	.25	.19	.19	.192	.196	.195
s_3	.31	.27	.26	.264	.263	.264

3B. Equal Shares

Full Information: Δu_1 =.182; Δu_2 = .479; Δu_3 = .083; sum= .744.

Q	.964
P_1	1.80
P_2	-.03
P_3	-1.77

Quantity Process: converged after 23 iterations:
Δu_1 =.183; Δu_2 =.479; Δu_3=.083

Iteration	3	11	23
Q	.945	.963	.964
Q_1	1.09	.983	.965
Q_2	.95	.958	.964
Q_3	.80	.947	.964
p_1	.947	1.68	1.80
p_2	.08	.003	-.03
p_3	-1.027	-1.68	-1.77

Bid/Quantity Process: converged after 5 iterations:
Δu_1 = .183; Δu_2 = .480; Δu_3 = .083

Iteration	1	2	3	4	5
Q	.927	.96	.964	.965	.965
Q_1	1.17	1.00	.971	.966	.965
Q_2	.96	.96	.962	.964	.964
Q_3	.66	.92	.958	.964	.965
p_1	1.49	1.75	1.79	1.80	1.80
p_2	-.02	-.02	-.03	-.03	-.03
p_3	-1.47	-1.73	-1.76	-1.76	-1.77

Appendix 4: Processes with Discrete Quantity and Misrepresentation

Example same as in Appendix 2A, with equal shares.

4A. Discrete Quantity Process, no misrepresentation

The process converges in two steps.

Iteration	1	2
Q_g	2.84	3.00
Q_1	2.50	3.00
Q_2	3.50	3.00
Q_3	2.50	3.00
p_1	-8.97	-8.97
p_2	17.95	17.94
p_3	-8.97	-8.97
dU_1	113.36	113.74
dU_2	189.59	190.20
dU_3	113.36	113.74

dU indicates change from the status quo of no provision.

4B. Discrete Bid/Quantity Process, no misrepresentation

The process converges to the Pareto optimum in two steps.

Iteration	1	2
Q_g	2.84	3.00
Q_1	2.50	3.00
Q_2	3.50	3.00
Q_3	2.50	3.00
p_1	-5.70	-9.20
p_2	11.39	18.40
p_3	-5.70	-9.20
dU_1	104.04	114.42
dU_2	208.22	188.84
dU_3	104.04	114.42

dU indicates change from the status quo of no provision.

4C. Discrete Quantity Process with misrepresentation

Player three deflates quantity: $Q_3 = .75\, Q_3^*$. With round-off to the nearest discrete quantity, the last step represents convergence to a quantity lower than Pareto optimal.

Iteration	1	2	3	4	5
Q_g	2.63	2.75	2.71	2.70	2.54
Q_1	2.50	3.00	2.50	3.00	2.50
Q_2	3.50	3.00	3.00	2.50	2.50
Q_3	1.87	2.25	2.62	2.62	2.62
p_1	-5.19	.42	-4.55	2.55	1.64
p_2	21.85	27.63	34.83	29.82	28.88
p_3	-16.66	.42	-30.27	-32.37	-30.52
dU_1	101.66	86.92	50.35	81.19	83.43
dU_2	176.99	162.16	159.03	156.16	157.81
dU_3	131.88	165.33	173.23	175.71	165.16

dU indicates change from the status quo of no provision.

4D. Discrete Bid/Quantity Process with misrepresentation

Player Three reveals a bid that is .75 of true value. The equilibrium is still the Pareto optimum, but the distribution of benefits is different than in the truthful mechanism.

Iteration	1	2
Q_g	2.84	3.0
Q_1	2.50	3.0
Q_2	3.50	3.0
Q_3	2.50	3.0
p_1	- 2.98	- 6.83
p_2	14.18	20.84
p_3	-11.19	-14.02
dU_1	96.34	107.31
dU_2	200.29	181.52
dU_3	-119.67	128.87

dU indicates change from the status quo of no provision.