# Workshop in Political Theory and Policy Analysis 

## Colloquium Presentation

October 5, 1992


Learning, sharing, and working
with the diverse languages in
communicating and using
knowledge.

> The Workshop in Political Theory and Policy Analysis combines teaching, research, and related activities where faculy, visiting scholars, and students have opportunities to participate in productive scholarship. The term "workshop" is used to emphasize a conviction that research skills are best acquired where students and faculty, working as apprentices and journeymen, participate in the organization and conduct of research.

Professor Edna Loehman, Departments of Agricultural Economics and Economics, Purdue University, will be the speaker for the Workshop Colloquium on Monday, October 5, 1992. Her presentation is entitled "Preference Under Risk: An Application of Nonexpected Utility Theories." An abstract of her paper is provided below.

Most decisions (political, economic, and social) are made in the context of risk so the subject of human behavior under risk is of interest to several disciplines. Social science researchers who apply experimental methods to study individual or group behavior in such contexts may also want to consider characterizing the risk preferences of experimental participants. The methods discussed in this paper could be used for such purposes.

This paper integrates concepts from economics and psychology regarding how to model preference over gambles. A method for measuring risk preference combining revealed preference and nonlinear programming is proposed. A small demonstration of the method was used to compare alternative theories of risk preference. The application reveals contradictions of economic and psychology theories in terms of the assumption of risk aversion and models of the utility function. One particular theory (expected utility with rank dependent probabilities) among those tested seems to provide an adequate model of behavior and thus is recommended for further experimental research.

A copy of her paper is available by calling the above telephone number. Colloquium sessions begin at 12 noon and adjourn promptly at $1: 30$ p.m. You are welcome to bring your lunch. Coffee is provided free of charge, and soft drinks are available. We hope you will be able to join us!

# Preference Under Risk: <br> An Application of Nonexpected Utility Theories 

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Thanks to Monte Vandeveer for his contribution to development of programming methods.

\section*{An Application of Nonexpected Utility Theories}

Preference under risk has received much attention from both psychologists and economists, but more integration of concepts from these two disciplines is needed for its explanation. Toward this end, this paper will: 1) review and test economic assumptions regarding risk aversion; 2) demonstrate a method to measure preference functions for nonexpected utility theories; 3) consider implications for the explanation of risk preference.

A heavy emphasis in recent risk preference literature, following the work by Allais (1984), is the inadequacy of expected utility theory to describe risk behavior. Several alternative nonexpected utility theories have been proposed by economists and psychologists (see Camerer, 1987 for a summary). In these theories, risk preference behavior is described in terms of both utility and probability weighting (subjective probability transformations) functions.

Concomitantly, the assumption of risk aversion commonly made in economic literature has been called into question. Kahneman and Tversky (1979) observed the "reflection effect", that the same person can be observed to be both risk averse and risk seeking depending on whether gambles are presented as gains or as losses. As a result, Kahneman and Tversky (KT) postulated in Prospect Theory that the utility function should be concave for gains and convex for losses. Since an intrinsic risk attitude should be invariant over choice regimes (Schoemaker, 1969), risk aversion would then not be an "intrinsic risk attitude".

As another part of Prospect Theory, Kahneman and Tversky postulated that people transform objective probabilities by reducing values for low
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probabilities and increasing values for high probabilities. They also did
not require subjective probabilities to sum to one, and they suggested that
certainty is evaluated differently from risk, i.e. there may be
discontinuities in the subjective probability transformation at zero and
one.
Here, using preference data for lotteries with combined gain and loss
outcomes, assumptions of expect utility and risk aversion are tested. A
nonlinear programming method is then applied to this data for the purpose of
measuring utility and subjective probability transformation functions for
several nonexpected utility theories including Prospect Theory. Utility
concavity opposite to Kahnemann and Tversky's postulated utility shape are
obtained. Below, theories of risk preference are summarized as background
for tests and measurement.

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\section*{Theories of Risk Preference}

Nonexpected utility theories have been proposed by both economists and psychologists. Below, economic and psychological theories are briefly reviewed with regard to their treatment of "riskiness" and risk aversion. Economic Theories of Risk Aversion

Decisionmakers are usually assumed to be risk averse in economic literature. The basic definition of risk aversion is in terms of the certainty equivalent (Pratt, 1969). Risk aversion is defined to occur when the certainty equivalent of a gamble is less than its expected value. More risk averse persons are said to have smaller certainty equivalents (larger risk premiums) for a given gamble.

Expected utility has been the predominant method of describing risk preference. Given that expected utility theory (equivalently, the von Neumann and Morgenstern axioms) represents preference, risk aversion has
been equated with concavity of the utility function. The ordering of risk preferences in terms of "more risk averse" has then been characterized by the degree of concavity of the utility function. The Pratt-Arrow risk aversion coefficient -- measuring relative curvature or concavity of the utility function -- has been extensively used as an index of risk aversion. The exponential utility function, with its constant Pratt-Arrow coefficient, has been applied in many economic studies.

The effect of wealth on risk preference has been the basis for characterizing types of risk aversion. Types of risk aversion -- constant absolute risk aversion, decreasing absolute risk aversion, and relative risk aversion -- are defined in terms of the wealth of effect (Pratt, 1964; Arrow, 1965). In defining these types, asset integration has been assumed; that is, the utility function has been defined over wealth, and no distinction has been made between risky gains or losses to wealth other than by addition or subtraction from an initial wealth level. However, asset integration has not been upheld in experimental studies (Thaler, 1987).

Yaari (1969) developed a different description of risk preference which, while consistent with expected utility, is more general. Risk aversion is defined in terms of an "acceptance set" -- the set of risky choices that a person will prefer to the status quo. A more risk averse person is defined to be a person with a smaller acceptance set than a less risk averse person. To link the two theories, Yaari showed that shape of the boundary of the acceptance set at the status quo is related to curvature of the utility function in expected utility theory.

Risk aversion has been used to explain behavior in economic markets. For example, Ross (1981) described portfolio choice and insurance purchase. Using a measure of risk aversion stronger than the Pratt-Arrow measure, he showed that a more risk averse person will hold less of a more risky asset
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in a portfolio; also, a more risk averse person will pay a higher premium
for the same insurance contract than a less risk averse person. Similarly,
a risk averse person should always prefer a "fair" insurance contract
(premium equal to expected indemnity) to no insurance (Raviv, 1979).
Stochastic dominance has been widely used to compare gambles, although
it is well-recognized that stochastic dominance provides only a partial
ordering for pairs of gambles. The link between ordering gambles by second
order stochastic dominance and ordering by expected utility with risk
aversion was shown by Rothschild and Stiglitz (1970) and Bawa (1975).
In this context, "an increase in risk" i s a case when one gamble is a
"mean-preserving spread" of another (Rothschild and Stiglitz, 1970).
Psychology Perspectives on Riskiness and Risk Preferences
Clear distinctions have been made between "riskiness" and risk
preference in psychology literature. "Riskiness" refers to comparing
gambles in terms of objective characteristics such as moments, and "risk
preference" refers to how choices are made among gambles. In this context,
stochastic dominance can be viewed as a method of comparing the relative
riskiness of gambles, whereas utility theories describe preference.
Although variance has often been used in economic and financial studies
to measure riskiness, Coombs (1981) showed that perceived riskiness is not
well-modelled by moments of distributions. As an alternative, Weber (1988,
1986) developed a measure of the perceived riskiness of a gamble in which
probabilities of no gain, gain, and loss were each weighted separately in
additive terms. Experimental tests of this measure showed that it accounted
for subjects' ratings of riskiness significantly better than mean and
variance.
Using complex lotteries with more than two outcomes, Lopes (1984)
showed that the Gini coefficient (equivalent to stochastic dominance) was

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highly correlated with both ratings of perceived riskiness and preference ranking of gambles. Lopes used a traditional definition of risk aversion to classify subjects (those choosing a certain outcome in preference to a gamble with the same expected value), and she observed that preference orders for risk averse subjects were consistent with the Gini coefficient when the lotteries were in terms of gains. However, not all subjects were risk averse. From her observations, Lopes (1987) proposed other types of behavior besides risk aversion; "loss avoidance" and "gain seeking" are other types. Preference reversal for gain and loss lotteries as in Kahneman and Tversky (1979) was also observed by Schneider and Lopes (1985). Nonexpected Utility Theories

Theories such as "subjectively weightedutility" (Karmarkar, 1978) and Prospect Theory (Kahneman and Tversky, 1979) were developed to provide a systematic explanation of problematic behavior such as the Allais paradox. More recent nonexpected utility theories and their implications for risk preference are summarized below.

In Generalized Expected Utility, Machina (1982) defined the "local utility function" \(u(x, F)\) having the wealth level (x) and a base c.d.f. (F) as arguments. The expectation of this local utility function can be used to compare two gambles \(F\) and \(G\) when these gambles are "close." Machina showed that concavity of this local utility function in terms of \(x\) i s equivalent to the condition that mean-preserving increases in risk are not preferred. He also generalized the Pratt-Arrow risk aversion coefficient to be in terms of the concavity of the local utility function. He showed that the property that more risk averse persons will have smaller certainty equivalents is maintained by this formulation.

Another theory which allows the certainty equivalent is the Quasilinear Mean (Chew, 1983) or Weighted Expected utility (Hess and Holthausen). These
theories are equivalent to Fishburn's \(\operatorname{SSB}\) utility theory (Fishburn, 1986). In this theory, subjective probability weights depend on the outcome level. Hess and Holthausen (1990) further defined properties of this type of theory i \(n\) terms of the shapes of utility and weighting functions, expanding the typology of risk behavior to include "eccentricity" as well as risk aversion.

Another type of nonexpected utility theory is Anticipated utility (Quiggin, 1982) or Expected Utility with Rank Dependent Probability (EURDP) (Chew, Kami, and Safra, 1987). In both of these theories, the cumulative probability distribution is transformed. The subjective probability then depends on the rank ordering of outcomes. In anticipatedutility, the assumption is made that the subjective probability of . 5 is also. 5 , whereas no such requirement is made for EURDP. Chew, Kami, and Safra showed that this theory is not a special case of Machina's. For EURDP, Chew et al. also showed that concavity of both the utility function and the transformation function implies that mean-preserving increases in risk will not be preferred.

Loehman (1991) showed that the certain equivalent is also well-defined i \(n\) EURDP, and that second order stochastic dominance between gambles still relates to preference order. But, properties of the functions underlying EURDP allow more types of risk behavior then risk aversion. The utility function (u) and a subjective transformation (S) of the cumulative distribution are both used to describe preferences, and the combination of \(u\) and \(S\) determines the nature of preferences. Several combinations (concave or linear \(u\) with concave \(S\); or, concave \(u\) and symmetric \(S\) ) can imply risk aversion. Risk seeking occurs with \(u\) and \(S\) both convex. Other types of behavior are also included in the set of possible preference types.

Shapes for \(S\) can be interpreted in terms of optimism and pessimism. (These two alternative "attitudes toward fate" were first identified by Hey (1984) in a subjective expected utility context.) For strong subjective optimism, the subjective cumulative distribution dominates the objective distribution with first order dominance, implying a subjective mean greater than the objective mean. Strong pessimism is the reverse. For weak optimism, the subjective cumulative distribution dominates the objective distribution with second order dominance, implying a smaller subjective variance i f the subjective and objective means are the same. Weak pessimism is the reverse.

Yaari (1987) proposed theory of risk preference based on the inverse utility function. His theory is sometimes mistakenly interpreted as requiring that the utility function be linear (Camerer, 1987). Yaari's theory actually ranks lotteries equivalently to EURDP (Loehman, 1991). Roell (1987) showed that risk aversion in Yaari's theory can also be described in terms of the certainty equivalent. Preference Measurement and Problems with the Use of Certainty Equivalents

Following the methods of Becker, DeGroot, and Marschak (1964), the use of the certainty equivalent has been the predominant tool for utility measurement. As indicated above, the concept of the certainty equivalent remains valid even in a nonexpected utility context. Therefore, this elicitation method could also be applied with nonexpected utility theories for measurement purposes. However, this section discusses some difficulties in applying the certainty equivalent.

In one recent application, certainty equivalents were used to compare Prospect Theory and expected utility (Currim and Sarin, 1989). Similar to KT, their experiment used lotteries with either gains or losses (not combinations), and the Prospect Theory utility function shape was confirmed.
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Currim and Sarin assumed an exponential utility function with a risk
aversion coefficient which was estimated from certainty equivalents. Currim
and Sarin then measured subjective probabilities, allowing subjective
probability to be transformed differently for loss lotteries than for gain
lotteries (smaller subjective probabilities were obtained for losses than
for gains). That is, subjective probabilities and utility were not measured
simultaneously.
In the Becker, DeGroot, and Marschak (BDM) method, the certainty
equivalent for a lottery over gains is the same as minimum selling price or
willingness to accept for the sale of this lottery. BDM et al. also
developed a procedure to induce truthful behavior in the respondent in
revealing willingness to accept: a preliminary lottery is added to the
original lottery to determine whether or not a respondent will be paid
his/her stated minimum selling price. BDM show that a truthful response is
then the dominant strategy.
Based on Kahneman and Tversky's (1979) results, use of the BDM method
presents problems. Because a compound lottery results from adding the
incentive system to the original lottery, the new lottery will be perceived
as being riskier than the original lottery. Therefore, the revealed
certainty equivalent will not be the same as for the original gamble, but
the response is not necessarily the truthful selling price for the original
lottery.
The method proposed by BDM also requires use of a personal interview
and a nonstandard instrument. Kachelmeier (1989) proposed instead that this
type of measurement should be standardized and simple enough to be
administered as part of an experimental study, i.e. the elicitation of risk
preference should not be a major effort in itself. Kachelmeier's instrument
and method were based on a standardization proposed by Harrison (1986) to

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simplify the certainty equivalent elicitation process. Certainty
equivalents were elicited for prespecified probability levels, and then
econometric methods were used to estimate the certainty equivalent as a
function of probability; the utility function is then obtained as the
inverse function. However, this method assumes expected utility theory.
A fundamental criticism of use of certainty equivalents by McCord and
De Neufville (1986) is due to the "certaintybias" identified by Kahneman
and Tversky (1979) who suggested that the comparison of certainty to a
lottery may not lead to the same type of response as when one lottery is
compared to another.
An even more severe problem with the certainty equivalent is the problem of preference reversal. This effect has been widely studied following the work by Grether and Plott (1979). Their work showed an irreconcilable difference between the preference ordering and the certainty equivalent.
Laskey and Fischer (1987) tried to explain the Grether and plott results with a different criticism of certainty equivalent elicitation. The determination of the certainty equivalent is a matching task; that is, a dollar amount must be matched with a lottery. Laskey and fischer found greater consistency in responses for ranking tasks (the direct comparison of lotteries) compared to matching tasks, implying that subjects have more difficulty with matching than ranking tasks.
Clearly, there are several psychological and measurement problems when the certainty equivalent is the basis for nonexpected utility measurement.

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\section*{A New Method of Measurement for Nonexpected Utility Theories}

This section proposes a new method of measurement for nonexpected utility theories based on psychological measurement theory (Kranz, Luce, Suppes and Tversky, 1971). The method proposed here uses revealed preference instead of the certainty equivalent as the basis for measurement. It assesses subjective probabilities and utility simultaneously (rather then separately as in Currim and Sarin, 1989). Nonlinear programming is used to compute subjective probabilities and utility values from revealed preferences for an assumed nonlinear expected utility theory. Rather than estimating parameters for an assumed functional form (such as an exponential utility function), scale values for utilities and subjective probabilities are obtained to satisfy scaling principles.

Alternative nonexpected utility theories can be measured with the method. The theories applied here are described in more detail below. The preference elicitation instrument is given in the Appendix. Lotteries compared here are mixtures of gain and loss outcomes because such a mixture is realistic.

\section*{Elicitation Instrument}

Respondents were asked to give preference rankings for two sets of lotteries (indicated by \(I\) and I below). Lotteries in each set were compared pairwise, and the resulting preference ranking was implied by the pairwise comparisons. In each set, all lotteries have the same expected value in order to test for risk neutrality and consistency with second order stochastic dominance. The five lotteries in the first set (I) of the lotteries all have probabilities of \(.50 / .50\) for mixtures of gains and losses with an expected value of \(\$ 50\). One certain outcome of \(\$ 50\) is included in this set. In the second set (II) of five lotteries to be ranked, stated
probabilities vary from 0.1 to 0.9, including the value of 0.5 , and each lottery has an expected value of \(\$ 12.50\).

One certainty equivalent was also elicited from the first set. This certainty equivalent corresponds to the lottery having a 50-50 chance at the combination of the best (\$300) and worst (-\$200) outcomes in the choice set. Here, the degree of risk aversion will be classified by the size of this certainty equivalent. For respondents who like having a chance at this lottery, this certainty equivalent will be the minimum selling price as in BDM. Some respondents may actually dislike having a chance at this lottery; for such respondents this certainty equivalent will be the maximum willingness to pay to get rid of the lottery, as when insurance is purchased. No incentives for truthfulness were used.

As a test of risk aversion, questions about insurance behavior were included. Insurance gambles are similar in form to those used by Slovic, Fischoff, Lichtenstein, et al (1979), each of them having the same expected value with varying probabilities and loss levels. In each case, a gamble is compared to fair insurance. For fair insurance, risk averse persons should purchase all contracts whereas pure risk seekers would purchase none.

Two personality trait questions were also included in order to compare self-report with measures of risk preference. Preference Ranking Results and Problems with the Assumption of Risk Aversion

The instrument was given to 21 faculty members at Purdue in a variety of departments (about a third were in the business school); responses were anonymous. Responses are shown in Table 1, ordered by the value of the response to the single certainty equivalent question. None of the respondents was risk neutral.
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    Ranking by variance (equivalent to second order stochastic dominance
    and expected utility with risk aversion since means are constant), the
preference order over the first set of five lotteries would be 21534. Only
six subjects exhibited this order.
Seventeen respondents had a certainty equivalent less than the expected
value of \$50, indicating risk aversion in the classical sense. However, a
risk averse person should also rank the certain outcome of \$50 above all
other lotteries; in contrast eleven of these seventeen respondents (and
fourteen out of the twenty-one respondents) preferred to take the chance of
\$100 versus \$0 rather than receive a certain \$50 (indicating convexity in
the utility function for gains of \$100 or more).
Seven people had negative-valued certainty equivalents (i.e., they
would like to pay to get rid of the lottery). Six people had a certainty
equivalent of zero. Eight subjects had positive certainty equivalents
(ranging from \$10 to \$150). Among the four subjects with positive certainty
equivalents of \$100 and \$150, three are risk seeking in the classical sense
since they preferred the \$100 chance to the \$50 certainty. Two of these
"risk seekers" also chose to purchase insurance more often than most other
respondents.
Ranking by variance (equivalent to second order stochastic dominance
and expected utility with risk aversion), the rank order for the second set
of five lotteries would be 23541. No one exhibited this ranking. Ranking
strictly by the amount of loss, the ranking would be 54231; only three
subjects gave this ranking. The ranking 54213 indicates a desire to avoid
loss, but a higher chance of loss is also taken to obtain a higher gain with
lottery (1) preferred to lottery (3); this ranking was obtained for five
subjects. The set of lotteries (5,4) was preferred to the set (3,1,2,) by

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ten subjects. More than two-thirds of the respondents (fifteen) ranked
first either lottery (4) or (5), the two lotteries with the highest gains.
Eight out of twenty-one respondents had either intransitive or
inconsistent responses. Five subjects had intransitive responses for the
second set of ranking questions. Four subjects had inconsistent responses
to the certainty equivalent question. Consistency for set I requires that
CE > \$50 < lottery (4) preferred to lottery (2)
CE < \$50 < lottery (2) preferred to lottery

Note that all respondents with CE $>\$ 50$ had problematic responses.

For the insurance questions, the predominant insurance pattern for most subjects was to choose insurance for the higher loss/lower probability cases but not to choose insurance for the lower loss/higher probability cases. Only three respondents were consistent with risk aversion by choosing insurance in al cases. On the average, 3.5 contracts among the higher loss/lower probability cases were purchased. Thus, our results about insurance are opposite to those obtained by slovic et al. who found--for college students--that most respondents chose insurance only in lower loss/higher probability cases. The average number of insurance contracts purchased by the group of respondents having a negative certainty equivalent was 3.28, while the group having a positive or zero certainty equivalent purchased an average of 3.64 contracts. Thus, insurance purchasing behavior does not seem to correspond to risk preference as measured by this certainty equivalent. Also, Yaari's "acceptance set" definition of risk aversion does not correspond.

For the self-report questions, only two people reported themselves to be risk avoiders and only one gave the description of being a risk seeker. The rest generally described themselves as being willing to "take small risks provided the potential loss is not too great." None believed

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themselves to be unlucky, and eleven described themselves as being lucky.
These personality descriptions have no apparent link to revealed
preferences.
    It is clear that revealed preferences do not correspond very well to
classical assumptions of risk aversion. It is also clear that there is a
wide range of types of risk preference since twenty-one different preference
patterns were exhibited among the twenty-one respondents!
Alternative Nonexpected Utility Models
Alternative nonlinear models are easily incorporated in the nonlinear programming procedures demonstrated here. Four different models are compared here using the same preference data. Following the KT model, the utility function in each model is defined in terms of gamble outcomes rather than being in terms of wealth as in economic models such as Friedman and Savage (1948) and Pratt (1964).
In all cases, lotteries have two outcomes \(x\) and y with probabilities p and 1 -p. Subjective probabilities in each model are required to be monotone. The certainty of \(\$ 50\) is represented by \(u(50)\) in each model. (Other assumptions could be tested with the method.) Prospect Model. The Kahneman-Tversky model is of the form \(U(F)-p(p) u(x)+p(1-p) u(y)\),
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where the sum $p(p)+p(1-p)$ is not required to equal one. Following KT, the requirement $u(0)-0$ is imposed. The normalization requirement $u(300)-u(-200)-=500$ is also imposed in order to obtain a unique solution to the nonlinear programming problem.

Expected Utility with Rank Dependent Probability (EURDP). For the two
outcome case, this model is similar to Prospect Theory except that subjective probabilities for the two outcomes are required to sum to one:

$$
U(F)=\rho(p) u(x)+(1-\rho(p)) u(y)
$$

The normalization requirements $u(-200)=0$ and $u(300)=1$ are imposed in order to determine a unique ratio scale for utility. This ratio scale can be transformed to have $u(0)=0$ and $u(300)-u(-200)=500$ as in the Prospect model above.

Subjective Distribution Model (SDM). Following Currim and Sarin (1989) and Chew (1983), subjective probabilities for gains and losses may be different. In a model similar to Prospect Theory, for lotteries with mixed negative and positive outcomes, the form of this model is

$$
U(F)=\rho_{\ell}(p) u(x)+\rho_{g}(1-p) u(y)
$$

where $\rho_{\ell}(p)+\rho_{g}(p)$ may sum to less than one. As for the Prospect Theory model, $u(0)=0$ and normalization to a scale of length 500 are also imposed. Chew Model

Following Chew (1983), subjective probabilities could vary with the outcome level. The form of this model for two outcomes is

$$
U(F)=\frac{\alpha(x) p}{\alpha(x) p+\alpha(y)(1-p)} u(x)+\frac{\alpha(y)(1-p)}{\alpha(x) p+\alpha(y)(1-p)} u(y)
$$

Note that the subjective probabilities sum to one. Ratio scaling requirements for utility as in EURDP are also imposed.

## Nonlinear Programing Method and Preferences Measurement Results

Subjective probability and utility values were determined from revealed preferences by constrained nonlinear programming. The solution algorithm was written in GAMS. For EURDP, Prospect, and SDM models, two optimization problems are solved iteratively. The first problem fits utility values using preference ranking $I$ and the subjective probability tranformation of $p=.5$; in the initial iteration, $p=.5$ is not transformed. The second problem fits subjective probabilities for $p=.1, .25, .5, .75, .9$ using preference ranking II and utility scale values from the first problem.

In each succeeding iteration, the subjective probability transformation of $p=.5$ from the second problem is used in the first problem. These two nonlinear programs are solved sequentially and iteration continues until a non-changing subjective probability for $p=.5$ is found.

Constraints for the nonlinear problems are derived from the revealed preference orders. Additional constraints impose monotonicity for utility and subjective probability. The single certainty equivalent elicitation (CE) was also included as one constraint: eg. for the Prospect model,

$$
u(C E)=p(.5) u(300)+\rho(.5) u(-200) .
$$

The objective function for problem $I$ is to minimize the length of the piecewise linear curve through the utility scale values. The minimum length, provided it were feasible, would be obtained for a linear utility function. Similarly, for the subjective probability problem II, the length of the piecewise linear curve through the subjective probabilities is minimized, with a linear function (as in expected utility) having minimum length.

Convergence in the iterative procedure at the third decimal place required three to eighteen iterations depending on the model. The Prospect model was simplest, requiring the least iteration, whereas the SDM model with two sets of probabilities required the most; the EURDP model was intermediate in terms of iterations.

For the Chew model, the weights $\alpha(x)$ and utility values $u(x)$ were determined simultaneously, i.e only one optimization problem was solved over both preference rankings I and II. The objective function was to minimize the length of the utility function. For the Chew model, monotonicity was imposed for both the weights $\alpha(x)$ and the utility function. The initial value of $\alpha(x)$ was a vector of ones.

Tables $2-4$ show results for the Prospect, EURDP, and SDM models for representative respondents. Figures $1-4$ illustrate the shapes of utility and probability transformation functions for Prospect, EURDP, SDM, and Chew models for one respondent. Table 5 shows the Chew model for one respondent.

Nonlinear preference measurement for the Chew model was generally not successful since a feasible solution was not possible for many respondents. When there was a solution, it did not yield a smooth utility function. The SDM model also did not yield a smooth utility function, and the utility function gave little difference for $\$ 200$ and $\$ 300$. Therefore, discussion below focuses on results for the more successful EURDP and Prospect models.

Utility slopes -- rather than utility scale values -- are given in Tables $2-5$; models are then readily compared to the risk-neutral case with a utility slope of one. Concavity is indicated when the utility slope decreases, and convexity is indicated when this slope increases.

Respondents a-e were chosen to be representative of a range of preference types including negative, zero, and positive certainty equivalents. Respondent a with a certainty equivalent of $\$ 0$ is risk averse
 first set of lotteries but different certainty equivalents. Respondents a, b, d, and e have the same ordering over the second set of lotteries but different certainty equivalents. Respondents b and d have the same certainty equivalent of $-\$ 50$ and the same ranking over the second set but different rankings over the first set.

Zero Certainty Equivalent (Respondent a). Both EURDP and Prospect models have a risk-seeking portion for gains above $\$ 100$ and concavity below \$100 in contrast to Kahneman and Tversky's postulate of convex utility for loss and concave utility for gains (see Figure 1). In both cases, the slope of the piecewise linear utility function is steeper about zero for losses than for

Figure 1. Prospect Model for Respondent A.



Figure 2. EURDP Model for Respondent A.



Figure 3. SDM Model for Prospect A.



Figure 4. Chew Model for Respondent A.


gains, consistent with Prospect Theory. The fitted Prospect and EURDP models are similar in the utility function shape, but the EURDP model shows more deviation from a linear utility function.

Different subjective probabilities are obtained from the two models. For EURDP, subjective probabilities for probabilities greater than $p=.1$ are less than the objective probabilities; second order dominance does not hold since the subjective probability for $p=.1$ is also greater than objective probability. For the Prospect model, the opposite effect is obtained: for probabilities above $p=.1$ but less than .9 , subjective probabilities are greater than objective.

Negative Certainty Equivalent ( $C E=-\$ 50$ ) (Respondent b).
Again for both Prospect and EURDP models, utility is concave for losses and then changes to convex for gains. Compared to the person with $C E=\$ 0$, this person has utility slopes which are more steep for losses and less steep for gains for both EURDP and Prospect models. The EURDP model is also more concave than the Prospect model in the loss domain.

Probability transformations for both Prospect and EURDP models are of a similar pattern to those for the $C E=\$ 0$ person. For the Prospect case, probabilities less than $p=.5$ are overweighted. The pattern for subjective probabilities for EURDP is similar to the $C E=\$ 0$ case, but subjective probabilities for probabilities less than $p=.75$ are even more underweighted.

Positive Certainty Equivalent ( $C E=\$ 10$ ) (Respondent $c$ ).
The Prospect utility model for this person is actually concave. The EURDP model again has the shape of concave for losses and gains up to $\$ 100$, and then changes to convex but the change in concavity is very small. Corresponding to the positive certainty equivalent, in comparison to respondents $a$ and $b$, this person has the smallest utility slope for losses
and largest slope for gains. The Prospectutility model for this person is actually concave.

The subjective probability transformation is nearly linear for both Prospect and EURDP models. The subjective probability for p - . 1 is less than one i n both models; allother respondents show the reverse relation. Other Cases. Respondent $d$ has CE - $\$ 50$ but has a difference preference pattern than Respondent b for the first lottery set. Still, the utility and probability functions for the Prospect and EURDP models are very similar for Respondents $b$ and $d$.

Respondent e has a positive certainty equivalent of $\$ 25$ with a completely different preference order for the first preference set. The Prospect utility shape is irregular, while the EURDP model maintains the utility shape $o f$ concave changing to convex.

Summary of EURDP and Prospect Model Results. For both Prospect and EURDP preference models, a utility shape which is concave for losses and convex for gains above a certain value was generally obtained, opposite to the $K T$ utility model with concavity for gains and convexity for losses. (In one case -- Respondent $c$-- for the Prospect model, a purely concave utility was even obtained.) Also in contrast to Prospect Theory, zero was not the inflection point; instead it was around $\$ 100$. The EURDP model exhibited wider differences among preference types than did the prospect model.

Except for one case (Respondent c) and $p=.9$, overweighting of probabilities was obtained with the Prospect model. Overweighting for large probabilities is in accord with $K T$ postulates; however their postulate that probability values would be reduced for small probabilities is not supported. Here the subjective probability of $p-.1$ is overweighted except for Respondent c.

EURDP gives quite different subjective probability results than the Prospect model. With EURDP, underweighting was obtained for all probabilities greater than $p=.1$ for all respondents except for Respondent c at $p=.9$; for $p=.1$, overweighting was obtained in all cases except for Respondent $c$.

Differences from Kahnemann and Tversky's results for Prospect Theory can be attributed to the use here of lotteries over combined gains and losses, rather than KT's use of gains and losses separately.

Insurance Purchase Predictions. In all cases, insurance purchase was predicted by comparing $u(-\$ 10)$ with the utility if a loss occurred, where

$$
U(F)=u(-x) \rho(p)
$$

and $x p=\$ 10$ in each case. $(u(0)$ was transformed to be zero for the EURDP and CHEW models.) Insurance purchase was predicted to occur if $\mathrm{U}(\mathrm{F})<\mathrm{u}(-\$ 10)$.

Insurance purchase predictions were not very successful because linear extrapolation had to be used for ranges of outcomes and probabilities not included in the two sets of lotteries in the elicitation instrument. A more basic explanation is that different utility and probability transformation functions may apply for preferences over pure loss cases as compared to preferences over combined loss/gain cases. The KT conjecture that certainty (as with insurance) should be treated differently than risk also may apply.

Table 6 shows model predictions for Respondent a. (Other respondents had similar patterns.) The Prospect model incorrectly predicted that insurance would be bought in all cases because of the pessimistic transformation of loss probabilities and concave utility in the loss region. EURDP predicted that insurance would be purchased for high loss cases but not purchased for the lowest loss case. The reason for this switch with EURDP is that - even though utility is concave in the loss region .- the
subjective probability for $p=.25$ is underweighted, and therefore the subjective expected loss is less than the objective value of $\$ 10$, so that insurance is less attractive than in the objective case.

Integrating Economic and Psychology Concepts of the Utility Function. In spite of being $S$-shaped, utility functions measured above in terms of gamble outcomes are not of the Friedman-Savage form in terms of final wealth. As an alternative to combining wealth and risk by $u(W+x)$, the two types of utility concepts could be integrated as follows: define the utility function $v$ to be additively separable in terms of gamble outcomes $x$ and initial wealth $W_{o}$, where $V$ is a Friedman-Savage utility:

$$
v\left(x ; W_{0}\right)-u(x)+v\left(W_{0}\right)
$$

(see figure 5 for an illustration). Then, for a person with initial wealth $W_{0}$, the overall utility $U$ for a gamble $F$ is

$$
\begin{aligned}
U\left(F ; W_{0}\right) & =\Sigma v\left(x ; W_{0}\right) \rho(p) \\
& =\Sigma\left[u(x)+V\left(W_{0}\right)\right] \rho(p) \\
& =U(F)+V\left(W_{0}\right)
\end{aligned}
$$

for subjective probabilities summing to one. Overall utility is then sum of a Friedman-Savage utility $V$ over certain wealth $W_{0}$ and a utility $U(F)$ for gambles $F$. With $W_{0}$ held constant for a person, comparison of lotteries with $U$ is equivalent to comparison in terms of $U$. The case of "no risk" can be represented by a gamble with an outcome of zero received with certainty; with the normalization $u(0)=0$ as above,

$$
\mathrm{U}(\text { no risk }) \mathrm{u}(0)=0
$$

"Overall" utility then collapses to the certain part $V$ in the case of no risk.


Figure 5. Combining the Friedman-Savage utility model with utility over gambles.

## Conclusions

This paper has demonstrated a nonlinear programming method for simultaneously determining the shapes of utility and subjective probability functions for nonexpected utility theories. The method is based on preference order information rather than information about certainty equivalents. As demonstrated here, the nonlinear programming method is useful because alternative theories and assumptions can be tested within the same framework.

Responses used as preference data confirmed problems with risk aversion as a global rule. Correspondingly, measured utility shapes were generally not concave. However, the certainty equivalent for a 50-50 chance at the best and worst outcomes correlated well with the shape of the utility function: a more negative certainty equivalent corresponded to a utility function which was more steep for losses and less steep for gains. Therefore this single certainty equivalent could be useful to categorize the type of risk preference.

Results different from Kahneman and Tversky's hypothesized shape for the utility function were obtained. The predominant utility function shape obtained for both Prospect and EURDP models was concave for losses and gains less than $\$ 100$ and convex for gains higher than $\$ 100$. Differences from Kahneman and Tversky may in part be attributed to the use here of lotteries over combined gain and loss outcomes, whereas Kahneman and Tversky tested gain and loss lotteries separately. Therefore, the nature of lottery outcomes being compared can affect the nature and representation of preferences.

Estimation results for four different nonlinear models were compared. A feasible solution for the Chew model could not be found for most respondents. The $S D M$ model was feasible but gave an irregularly shaped

```
utility function. Only EURDP and Prospect Theory resulted i n a consistently
```

smooth and regular shape for the utility function for a range of preference types.

The EURDP model was more successful than the Prospect model in predicting insurance purchasing behavior since it showed a switch from insurance purchase to nonpurchase for low valued losses with high probability. The EURDP model also has better mathematical properties than the Prospect model; for example it easily extends to lotteries with more than two outcomes, whereas Prospect Theory is. limited to at most three outcomes i f one is the zero outcome. EURDP also has behavioral interpretations in terms of optimism and pessimism. Therefore the. use of EURDP is supported here both in terms of its behavioral and mathematical properties.

The development of new methods and theories is an evolutionary process, and the interaction of economists, psychologists, and other decision theorists can help to develop improved methods and new theories to be tested. Further consideration shouldbe given to design of elicitation instruments (eg. how many comparisons and what range and types of outcomes are needed to describe preferences adequately) and estimation methods. The nonlinear programming method demonstrated here gives a powerful tool for these purposes.

Table 1.
Responses to Questions

## Certainty Equivalent $\quad$| Ordering |
| :---: |
| $(I)$ |

| Risk Preference |  |
| :---: | :---: |
| Ordering | Insurance |
| (II) Purchase |  |

Risk

| (II) | Purchase | Attitudes |  |
| :---: | :---: | :---: | :---: |
|  | $\underline{Y / N^{i}}$ | Risk Taking ${ }^{\text {t }}$ | Luck $^{1}$ |
| 31245 | $3 / 3^{n}$ | RT | N |
| 54213 | 3/3 | RT | L |
| 54231 | $2 / 4$ | RS | N |
| 54213 | 4/2 | RT | L |
| 45213 | 3/3 | RT | L |
| 54213 | 4/2 | RA | N |
| 53241 | 4/2 | RT | L |
| 54213 | 3/3 | RT | L |
| 54231 | 6/0 | RA | N |
| 52143? | 3/3 | RT | N |
| 45213 | 4/2 | RT | N |
| 32145? | 0/6 | RT | N |
| 53214 | 3/3 | RT | N |
| 45321? | 4/2 | RT | N |
| 34521 | 5/1 | RT | L |
| 15423 | 3/3 | RT | N |
| 54213 | 6/0 | RT | N |
| 52431? | 5/1 | RT | L |
| 41235? | 3/3 | RT | N |
| 23514 | 6/0 | RT | $L$ |
| 13542 | indiff. | RT | L |

```
a-e cases analyzed in Tables 2.6
*"." indicates WTP to remove lottery
    "+" indicates WTA to sell lottery
```

i"Yes" to insurance for larger losses, "no" for smaller losses.
n a nonmonotonic insurance choice pattern.
$t_{R T}=$ Risk Trader
$1_{L}=$ Lucky
RA = Risk Avoider $\quad U=$ Unlucky
RS = Risk Seeker $N=$ Not lucky or unlucky
? indicates an intransitive order
$x$ indicates inconsistency between the $C E$ and the ranking

| Prospect Model for Respondents a-e |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| objective probability | subjective probability |  |  |  |  |
|  | a | b | c | d | e |
| . 1 | . 25 | . 32 | . 08 | . 31 | . 16 |
| . 25 | . 36 | . 40 | . 27 | . 39 | . 30 |
| . 5 | . 52 | . 53 | . 51 | . 52 | . 52 |
| . 75 | . 76 | . 76 | . 745 | . 755 | . 759 |
| . 9 | . 898 | . 895 | . 899 | . 894 | . 899 |
| outcome |  |  | y slo |  |  |
|  | a | b | c | d | e |
| - 100 | 1.25 | 1.65 | 1.20 | 1.66 | 1.26 |
| -50 | 1.25 | 1.65 | 1.20 | 1.66 | . 97 |
| 0 | 1.25 | . 76 | 1.20 | . 77 | 1.06 |
| 50 | . 91 | . 76 | . 92 | . 72 | . .93 |
| 100 | . 75 | . 65 | . 85 | . 70 | . 87 |
| 150 | . 83 | . 71 | . 85 | . 69 | . 99 |
| 200 | . 83 | . 71 | . 85 | . 71 | 1.08 |
| 300 | . 83 | . 71 | . 85 | . 71 | . 78 |

## Table 3 <br> EURDP Model for Respondents a-e

objective probability
.1
.25
.5
.75
.9

| subjective probability |  |  |  |  |
| :--- | :---: | :---: | :--- | :--- |
| a | b | c | d | e |
| .13 | .13 | .09 | .13 |  |
| .13 | .16 | .249 | .16 | .12 |
| .21 | .20 | .48 | .20 | .36 |
| .58 | .60 | .72 | .60 | .62 |
| .83 | .84 | .901 | .84 | .84 |

outcome

| -100 | 1.75 | 2.65 | 1.24 | 2.64 | 1.44 |
| :---: | ---: | ---: | ---: | ---: | ---: |
| -50 | 1.75 | 2.65 | 1.24 | 2.64 | 1.44 |
| 0 | 1.75 | .42 | 1.24 | .42 | 1.44 |
| 50 | .64 | .42 | .91 | .40 | 1.06 |
| 100 | .24 | .08 | .82 | .13 | .59 |
| 150 | .52 | .08 | .83 | .06 | .64 |
| 200 | .52 | .35 | .83 | .35 | .64 |
| 300 | .52 | .35 | .83 | .35 | .64 |


| SDM Model for Respondents a-e |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| objective probability | subjective probability |  |  |  |  |
|  |  |  | 10ss |  |  |
|  | a | b | c | d | e |
| . 1 | . 13 | . 12 | . 09 | . 12 | . 07 |
| . 25 | . 245 | . 17 | . 28 | . 17 | . 17 |
| . 5 | . 38 | . 26 | . 49 | . 26 | . 42 |
| . 75 | . 67 | . 64 | . 70 | . 65 | . 67 |
| . 9 | . 84 | . 84 | . 83 | . 85 | . 81 |
| objective probability |  |  |  |  |  |
|  | gain |  |  |  |  |
|  | a | b | c | d | e |
| . 1 | . 16 | . 16 | . 17 | . 15 | . 18 |
| . 25 | . 30 | . 17 | . 28 | . 17 | . 26 |
| . 5 | . 58 | . 56 | . 49 | . 56 | . 42 |
| . 75 | . 75 | . 76 | . 70 | . 76 | . 75 |
| . 9 | . 87 | . 88 | . 89 | . 88 | . 90 |
| outcome | utility slope |  |  |  |  |
|  | a | b | $c$ | d | e |
| - 100 | 1.49 | 2.27 | 1.11 | 2.27 | . 98 |
| - 50 | 1.53 | 2.27 | 1.21 | 2.27 | . 98 |
| 0 | 1.53 | . 12 | 1.21 | . 19 | 1.19 |
| 50 | 1.19 | . 14 | 1.71 | 1.08 | 1.43 |
| 100 | . 82 | . 85 | 1.27 | . 87 | 1.98 |
| 150 | . 97 | . 02 | 1.17 | . 02 | 1.10 |
| 200 | . 97 | 1.01 | 1.17 | . 99 | 1.30 |
| 300 | . 01 | . 01 | . 01 | . 01 | . 01 |

Table 5
Chew Model, Respondent a

| Outcome ( $\$$ ) | Utility Slope | 人-weight |
| :---: | :---: | :---: |
| -200 | - |  |
| -100 | 2.19 | 1. |
| -50 | .81 | 1. |
| 0 | 2.45 | 1. |
| 50 | .89 | 1. |
| 100 | .46 | 1.81 |
| 150 | .75 | 2.62 |
| 200 | .02 | 3.15 |
| 300 | .01 | 3.23 |

Table 6
Model Predictions for Insurance Purchase, Respondent a

| Outcome | Prob | EURDP | Prospect |  | SDM | Chew | Actual |
| ---: | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| -4000 | .0025 | yes | yes | yes | yes | yes |  |
| -2000 | .005 | yes | yes | yes | yes | yes |  |
| -1000 | .001 | yes | yes | yes | yes | yes |  |
| -200 | .05 | yes | yes | yes | yes | no |  |
| -100 | .1 | yes | yes | yes | yes | no |  |
| -40 | .25 | no | yes | no | yes | no |  |

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Appendix<br>RISK PREFERENCE INSTRUMENT

Thank you for helping me with my research on risk preferences! I am developing a way to characterize people's risk preferences in terms of how they answer the questions below.

Please do not do very deep thinking or calculations to answer the questions on the next pages. Just answer the questions "off the top of your head". There are no right or wrong answers.

The figure below

indicates a lottery in which you could either gain $\$ 10$ with a $50-50$ chance (probability of .5) or else lose $\$ 5$ with a $50-50$ chance.

Other probability levels will also be used and these will be indicated by a decimal number on figures similar to the above.
I. You have inherited the right to a lottery ticket which gives you a fiftyfifty chance at winning or losing money.

You will be able to choose which lottery ticket you prefer from the list below. Go on to the next page to indicate your preferences.

Lottery
(1)

(2)

(3)

(4)

(5)


Please indicate your preferences among pairs of the lotteries shown on the preceding sheet.
In each case just indicate the preferred lottery:
I prefer:
(1) vs. (2):
vs. (3):
——_
vs. (4)
vs. (5):
(2) vs. (3):
vs. (4):
vs. (5):
(3) vs. (4):
vs. (5):
(4) vs. (5):
II. Suppose you have inherited the lottery ticket with a $50-50$ chance of gaining $\$ 300$ or losing $\$ 200$.

Which case (a, b, or c) most closely describes your attitude toward this lottery? (Indicate with a check below.)
(a) You would not like to have a chance at this lottery. In this case, you would be willing to pay someone to take the lottery away from you, thereby relieving you totally of its risk (both gain and loss).
(b) You would like to have a chance at this lottery. But you would be willing to give up this chance at the lottery for a price.
(c) You are neutral. You don't dislike the lottery but you would be willing to give it away to someone for free.

Please answer either (a) or (b) below if you checked (a) or (b) above.
(a) If you selected (a) above, and someone offers to take the lottery from you for a price, what is the most you would pay someone to accept the lottery (i.e., for any higher price, you would rather just keep the lottery).

The most $I$ would pay to have someone take away this lottery is $\qquad$ .
(b) If you selected (b) above and a buyer appears, what is the least amount you would accept as a selling price (i.e., for any lower price, you would just keep the lottery).

The least I would accept to give up this lottery is $\qquad$ .
III. Now, please consider the following lotteries with gains and losses where the probabilities are mostly not 50-50 chances. Again, you will be asked to indicate your preferences on the next page.

## Lottery

(1)

(2)

(3)

(4)

(5)


Please indicate your preferences among the following lottery.
In each case please indicate the preferred lottery:
I prefer:
(1) vs. (2):
vs. (3):
vs. (4)
vs. (5): $\qquad$
(2) vs. (3): $\qquad$
vs. (4):
vs. (5):
-
(3) vs. (4):
vs. (5):

(4) vs. (5):
IV. You are faced with a risky situation in which you may experience a loss. In each of the following six cases (A through $F$ ) you can buy insurance for a $\$ 10$ premium. Then if a loss occurs, it will be fully covered by insurance.

In each of the six cases below, indicate by checking "yes" if you would buy the insurance or "no" if you would rather take the chance of a loss.

|  | CHANCE |  |  |
| :--- | :--- | :--- | :--- |
| CASE LOSS | OF LOSS INSURANCE? |  |  |


| A | $\$-4,000$ | 1 IN 400 | $\$ 10$ | - | - |
| :--- | ---: | ---: | ---: | :--- | :--- |
| B | $-2,000$ | 1 IN 200 | 10 | - | - |
| C | $-1,000$ | 1 IN 100 | 10 | - |  |
| D | -200 | 5 IN 100 | 10 | - |  |
| E | -100 | 10 IN 100 | 10 | - |  |
| F | -40 | 25 IN 100 | 10 | - | - |

V. How would you characterize yourself in terms of risk behavior? Please check the closest descriptions below.

1. In my daily life,
__ I don't mind taking small risks provided the potential loss is not too great.
_ I try to avoid taking risks whenever I can.
_I enjoy taking risks because I like it when things work out in my favor.
$\qquad$ Other $\qquad$
2. In daily life, $I$ think $I$ am
__ often lucky in that things work out my way.
___ often unlucky.
$\qquad$ I don't consider myself lucky or unlucky.

ANY COMMENTS? $\qquad$

If you want to know how your risk preferences compare to others in this study, please indicate by writing your name below.

NAME

