10-3-93

INCENTIVE STRUCTURE OF A COMMON-POOL RESOURCE SITUATION :A DYNAMIC GAME-THEORETIC MODEL OF IRRIGATION SYSTEM

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Preliminary, Partial Draft

This paper was prepared for presentation at the Fall Semester Colloquium, Workshop in Political Theory and Policy Analysis, Indiana University, Bloomington, November 30, 1992

Introduction

The incentive structure of common-pool resources (CPRs) situations has often been modeled as the prisoner's dilemma (PD) game in which the dominant strategy is of mutual defection (Wade 1988; E. Ostrom 1990; Tang 1992). Even though the PD game can give us useful insights with which we can understand the basic social dilemma problems in CPR situations, this line of logic is misleading since it ignores several important facts.

First, the incentive structures of CPR situations are not equal so that a single game model cannot explain all incentive structures of CPRs (Bloomquist, Schlager and Tang 1991). Even the appropriators of the two same kind of CPRs can face completely different incentive structures. If no one person's contribution is sufficient to gain a collective benefit but both person's contribution will produce the joint benefit, the incentive structure of this CPR dilemma can be best described by a Assurance game rather than by a PD game. And if (i) there is a minimum amount of work which must be done and (ii) either individual alone can do it all but (iii) each person prefers to the other to do all the work, then the incentive structure of this CPR dilemma can be best portrayed by a Chicken game rather than a PD game (for more detail, see Runge 1984; Taylor 1987; Isaac, Schmidtz, and Walker 1989; Gardner, Ostrom, and Walker 1990).

Second, people using CPRs face two kinds of collective action problems - "appropriation problem" and "provision problem". Appropriation problem refers to how to use CPRs efficiently, whereas provision problem refers to how to maintain CPRs well (E. Ostrom 1986b; Gardner, Ostrom, and Walker 1990; E. Ostrom 1990; Ostrom, Gardner and Walker 1992). It is because almost all CPRs have two aspects -- flow and stock aspects. In appropriation problems, we focus attention on the flow aspect of the CPR, whereas in provision problems we concentrate on the stock aspect of the CPR. In other words, solving appropriation problems focuses on the allocation of the flow of a resource, and solving provision problems focuses on the creation or maintenance of the stock of a resources (Gardner, Ostrom, and Walker 1990; E. Ostrom 1990). Provision problems are more important in the cases of man-made resources than in the cases of natural resources. This is why provision problems have been ignored and appropriation problems have been treated as the only problems in CPRs problems. Provision problem, however, can also be of great importance even in the cases of natural resources if CPRs are renewable so that maintenance works are required. Notice that these two collective action problems are highly inter-dependent. The incentive structure of the action situation of appropriation problem is to be affected by the outcomes of the action situation of provision problems, and vice versa. Thus, without considering the interaction between the two, it is difficult to understand the incentive structure of CPR situations.

Third, people will consider the effects of their decisions on future payoffs, as well as on present payoffs, at least to some extent. More important, appropriators' present behaviors can affect future payoff structure itself. This implies that the situation where appropriators interact with each other is time-dependent. Repeated game approach represents the former aspect by assuming that (i) game will be repeated over time; and (ii) players will maximixe the sum of the payoffs over time (Axelrod 1981; Taylor 1987). This approach is however still time-independent since the payoff structure of the game does not change over time (Friedman 1986). In repeated game models, past strategies do matter not because they affect the present and future payoff functions themselves but because they influence the current and future strategies of other players (Fundenburg and Tirole 1991). That is, the changes in the "physical environment" or the "payoff function" are not considered at all in time-independent game models. Instead, they are simply assumed to remain the same, no matter whether the game is repeatedly played or not.

A dynamic game-theoretic model of one particular CPR - the irrigation system - is developed here to depict these facts. In this model, (i) appropriators are assumed to make decisions about both appropriation and provision problem; and (ii) "physical environment" and "payoff function" themselves are allowed to change over time, unlike in repeated game-theoretic model.

The Model: A Hypothetical Irrigation System

Let us assume that there exist a canal irrigation system with no storage capacity and n (n>0) appropriators are entitled to get irrigation water from that system. The appropriators are composed of two types of players -- head-enders (j=1,...,m) and tail-enders (k=m+1,...,n), where n≥m. These two types of players differ from one another, that is there is an asymmetry between head-enders and tail-enders. For simplicity, however, it is assumed that there is no asymmetry within the two types of appropriators. That is, head-enders are assumed to be identical to every other head-enders in all aspects, and tail-enders are assumed to be identical to every other tail-enders, too. Appropriators will try to maximize the benefits from the irrigation system by making decisions on two variables -- the amount of irrigation water they appropriate and the amount of resources they invest in maintenance. Let the amount of irrigation water they appropriate and the amount of resources they devote to maintenance be "u_i (≥0)" and "m_i (≥0)", respectively[1]. They are the two control variables in our model. Within limits, appropriators decide the values of the two control variables once in each time period, from the time period 1 to the final time period T. Their decisions on the two control variables determine the payoff of appropriators[2].

I also assume that in this hypothetical irrigation system appropriators

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[1] There is, of cause, an upper limit for them. This will be explained later.

[2] They are not the only control variables in natural settings, of course. Other variables, such as choice of crops, the amount of fertilizer, also do have impact on the payoff of appropriators who are engaged in farming. In our model, however, all the variables other than our two control variables are simply assumed to be constant so that full attention can be paid to appropriation and provision problems.

first decide the amount of water they will appropriate during the time period t and after appropriation, they decide the amount of investment of their labor and resources on maintenance. This means that any single period t is composed of two time spans. The first one is appropriation period and the second is maintenance period. Time period t in this model, thus, can be understood as a season containing an appropriation period and a maintenance period. I define time period t this way because the product of irrigation activity is not obtained everyday but achieved only after a relatively longer time period, which can be called a crop season, is completed. For this reason, it is also assumed that (i) appropriation occurs over a appropriation period which is more than one day; and (ii) the sum of every day appropriation during the appropriation period determines the agricultural product of each time period, that is the total benefit of each time period.

Another important assumption of this model is that the marginal benefit of a unit of the irrigation water appropriated in any day is constant over the time period t. That is, there is no seasonality of the demand for irrigation water in this model -- in other words, appropriators can always get the same amount of total benefit from the same amount of water everyday over the entire time period t.

In real world settings, the above assumptions may not be fully met. First, there may exist some asymmetries between the head-enders and the tail-enders. By assuming more categories of the appropriators among the head-enders and the tail-enders, we may capture the asymmetries among the two types of players in this model. This, however, only will add complexity to the model. For this reason I choose not to do this. It is because we need as much simplicity as we can reasonably get in this model to get meaningful result, for this model is a dynamic game which requires complicated solution process.

Secondly, appropriators, of cause, do some maintenance works during the appropriation period, as well as during the maintenance period. But most of the maintenance works done during the appropriation period is likely to be on their individual field canals, which are not CPRs. They also do some emergency repairs during the appropriation period. But I ignore it in this model, assuming that emergency repair is so urgent that every body will participate -- That is, the incentive structure of emergency repair is far from that of social dilemmas.

Finally, the marginal benefit of the irrigation water of one day during the time period t could possibly be different from that of the other day of the same time period t, because of the seasonality of the demand for irrigation water. In my model, however, I assume that the marginal benefit of a unit of the irrigation water remains constant for the entire time period t. It is because what I want to highlight in this model is not the seasonality of the demand and supply for the irrigation water, but the effect of the appropriators' choice on the amount of irrigation water and investment in the maintenance on their payoff at the next time period. This does not means that the seasonality of the demand and supply for the irrigation water is of no importance in the incentive structure of the appropriators. It rather means that we have to sacrifice the seasonality to focus on what we want to do in this model since including both in one model will possibly make a model intractable, or at least extremely difficult to handle.

This hypothetical irrigation system will be helpful in understanding (i) the interaction between appropriation and maintenance and; (ii) the time-dependent characteristics of appropriators' incentive structure, even though it cannot capture all the details of the real world setting.

Game in Extensive Form

Next, let me explain the game appropriators in this hypothetical irrigation system will play.

(i) At period 1, all head-enders, j, choose simultaneously u_{j1}.

(ii) Knowing the choice of all head-enders, tail-enders, k, choose simultaneously u_{kt}.

(iii) All tail-enders, k, choose simultaneously m_{kt}.

(iv) Knowing the choice of all tail-enders, all head-enders choose simultaneously m_{jt}.

(v) At period t+1,...,T, stages from (i) to (iv) are repeated.

We can understand the way the game is played more easily by representing this game in extensive form. For simplicity let us assume that there are one head-ender and one tail-ender. This game is shown in figure 3.1. At the first node (node 1 hereafter), the nature will pick the values of parameters in this game, including the initial values of the state variables[3]. Given that, head-ender (player 1) will choose the amount of the irrigation water appropriated at time period 1 (u_{11}), then tail-ender (player 2) will choose the amount of the irrigation water appropriated at time period 1 (u_{21}). These two happen at the nodes 2 and 3. Notice that the upper limit on u_{i1} is determined by the amount of water at the source and the initial efficiency of water delivery. These two are external to this game. Also notice that the information condition at the node 3 implies that tail-ender knows what the head-ender chooses. This is the case at the nodes 4 and 5, where tail-ender and head-ender choose the amount of investment in the maintenance at time period 1, m_{21} and m_{11}, respectively. The information condition at node 3 is unnecessary since it is also the player 2 who chooses at the previous node. The information condition at node 5 implies that tail-ender decides first <u>and</u> head-ender knows what the tail-ender does at the node 4. I assume this because I think this will depict the strategic disadvantage of the head-ender. That is, it is possible that the head-ender even has to do the whole maintenance work without tail-ender's help when the tail-ender's has no incentive to invest in the maintenance since there is no enough water for

[3] They are the reliability of water supply and water delivery efficiency. They will be fully discussed later in this paper.

him/her. I will, however, see what happens when head-ender does not know what the tail-ender does. This is the end of the first time period. Players get the payoffs of the first time period Π_{11}.

This is, however, not the end of the game. And, from the second time period, the payoff structure is affected by the choices made at the previous time period. This happens at the nodes 6 and 7[4]. Here, the values of reliability and efficiency at time period 2 are determined. They are, notice that, not external to the game any more. They are determined indirectly by the players' decisions at the previous time period. And they will determine the payoff structure at time period 2. They will change the total benefit curve and the upper limit on the amount of water available to the players. We, therefore, need to represent this process in the extensive form game. Since they are not under the direct control of the players, however, there is no standard way of representing it in extensive form game. For this reason, I simply use the square symbol to represent this process at nodes 6 and 7. From the node 8, the process I explained so far will be repeated until the final time period. The payoffs for the players will be the sum of the discounted payoffs from period 1 to the final period.

What happens if we model this situation as a static game? The solution of this misspecified model will be the myopic one. This solution assumes that appropriators will appropriate and invest until the marginal benefit of each time period equals the marginal cost of each time period, without considering the existence of the effects of their choice on state variables. This solution is roughly parallel to the prediction of the PD game. That is, PD game assumes that appropriators have no foresight. But, if appropriators act with foresight, the assumption of myopic behavior will lead to an overstatement of the benefit loss resulting from management of irrigation system without central control. It is possible that the result of the long-term individual rationality is far from that of the short term individual

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4 It does not matter here which one is decided first.

rationality, and very close to that of social optimality. Then the static game model will be the "wrong way of simplification" (McGinnis 1991). This is why I employ a dynamic game approach.

Payoff functions

Benefit Component

Now, let us discuss the functional form of the payoff function of this model. For convenience, I will explain the benefit component and cost component separately. First, I assume that the total benefit of irrigation water is given by the area under a linear demand curve for irrigation water with negative slope[5]. This implies that the marginal benefit or "marginal value product" (Sparling 1990) is a diminishing function of the supply of irrigation water to the appropriator. The marginal benefit function of irrigation water use for individual appropriator i at time t can be expressed as:

(1) $$MB_{it} = q - r*u_{it} \qquad i=1,...,n, \quad t=1,...,T$$
$$q,r > 0$$

This is shown in Figure 3.2. Given this function, the total benefit of irrigation water is a quadratic function of the supply of the irrigation water to the appropriators. It can be easily obtained by simply integrating the marginal benefit function in equation (1). By integrating equation (1), we can have a total benefit function:

(2) $$\pi_{it}^{B} = q*u_{it} - .5*r*u^{2}_{it}.$$

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[5] This assumption is common in most previous models of irrigation water use. For examples, see Gotsch (1975), Kahn and Young (1979), Dixon (1989, 1991), Feinerman and Knapp (1983), Howe (1990), and Sparling (1990).

, where π_{it}^{B} = the benefit of water use of individual i at time t.

This is shown in Figure 3.3. Notice that the shadowed area under a linear marginal benefit curve in figure 3.2 is identical to TB_t in figure 3.3. They both represent the total benefit of u_t irrigation water.

Cost Components

Next thing to think about is the cost components. First, let us think about the appropriation cost. The appropriation cost will increase as appropriators obtain more water. For simplicity, let's assume that the appropriation cost is the amount of water appropriated times some constant, say e. This means that we can write the appropriation cost as '$e*u_{it}$'. This seems, however, unsatisfactory. It is because the appropriation cost for an individual i is also affected by the behaviors of other appropriators who can appropriate the irrigation water before he does or at least at the same time when he does. That means, the appropriation behavior of the tail-enders will not affect the head-enders' appropriation costs, whereas the appropriation behavior of the head-enders will affect the tail-enders' appropriation costs. It may become more and more difficult and costly to appropriate, as the amount of water appropriated by others increases. This can be summarized like this;

(3) $\pi_{jt}^{AC} = e*\Sigma u_{jt}*u_{jt}$ for head-enders

$\pi_{kt}^{AC} = e*(\Sigma u_{jt}+\Sigma u_{kt})*u_{kt} = e*\Sigma u_{it}*u_{kt}$ for tail-enders

, where π_{it}^{AC} = the appropriation cost for individual i at time t, e (>0) = costs coefficients.

Another cost component is the maintenance cost, 'm_{it}'. The amount of resources appropriators invest in maintenance is the maintenance cost. Maintenance work is done after appropriators appropriate the irrigation water. So, the maintenance work cannot add any positive utility to the benefit for an appropriator at time t. It can only add positive utility to the benefit for an appropriator at time (t+1). Under this assumption, the maintenance cost for an individual i at time t is;

(4) $$\pi_{it}^{MC} = m_{it}.$$

In sum, the payoff for an appropriator can be expressed by the above three equations (2), (3), and (4). That is, the payoff for an individual i at time t in the default situation is;

(5) $$\pi_{it}^{DF} = \pi_{it}^{B} - \pi_{it}^{AC} - \pi_{it}^{MC}.$$

Rule-Following Component

The payoff function in the equation (5) depicts the situation where there exists no rules concerning the appropriators' behaviors. That is, there is no sanctioning against over-appropriation and under-investment. Now, let's think about the situation where some rules exist. I assume that **when we consider only the rule-following payoff,** (i) the more cooperatively a player acts, the more benefit, in whatever form, he/she can get; and (ii) the less cooperatively he acts, the more cost, again in whatever form, he has to pay. Formally, the rule-following payoff is;

(6) $$\pi_{it}^{R} = -\{c(u_{it} - U) + d(M - m_{it})\}.$$

,where U = the amount of water assigned to a player,
M = the amount of investment in the maintenance assigned to a player,
c, d (>0) = penalty and monitoring coefficients[6].

This assumes that both U and M will be constant across a relevant time period. Ideally, they would be decided using dynamic optimization. That is,

[6] These coefficients can be thought of the probability of being caught and sanctioned by monitors times the amount of penalty against the rule-breaking behaviors.

somebody or a group of people, would calculate the best time path of U and M by dynamic optimization technique and decides the results as U_t and M_t, which vary over time. But, I doubt this actually happens in practice. For this reason, I treat both U and M as a constant instead of a variable.

If a player i appropriates more water than he/she is assigned <u>or</u> invests less than he/she is assigned, then the terms in the bracket of the equation (6) becomes positive and, consequently the equation comes to represent negative utility. It is because there exist sanctions against rule-breaking behaviors. And otherwise, that term represents positive utility[7].

Notice that, however, this is the case only in terms of <u>rule-following payoffs</u>. More cooperative behavior <u>can</u> pay less when we consider the total payoffs - both the sum of the default payoff (π_{it}^{DF}) and the rule-following payoff. If an appropriator gets more than his/her share and the increase in the default payoff is greater than the decrease in the rule-following payoff, then he/she will appropriate more that his/her share -- in other words <u>steals</u> water.

In addition, the enforcement of rule is not without costs. It is costly to enforce rules. Some organization can enforce the same rule less costly than the other organizations can do. We therefore introduce the cost of having rules and enforcing them into our payoff function. Let this cost be a constant ϵ, which can vary across the irrigation systems. We, then, can have a more general payoff function by adding rule-following payoff and enforcing cost to the default payoff. We can say that the payoff for a player i at time t is;

(7) $\Pi_{it} = \pi_{it}^{DF} + \pi_{it}^{R} - \epsilon$

Since head-enders and tail-enders have different appropriation costs,

[7] This is based on the assumption that appropriators get some types of payoff when they cooperate more than they are supposed to do. They could be in several forms such as respect from others, altruism, etc.

equation (7) should be re-written as:

$$(7') \quad \Pi_{jt} = \pi_{jt}^{DF} + \pi_{jt}^{R} - \epsilon$$

$$= q*u_{jt} - .5ru_{jt}^2 - e*\Sigma u_{jt}*u_{jt} - m_{jt}$$

$$- \{c(u_{jt} - U) + d(M - m_{jt})\} - \epsilon \qquad \text{for head-enders}$$

and,

$$\Pi_{kt} = \pi_{kt}^{DF} + \pi_{kt}^{R} - \epsilon$$

$$= q*u_{kt} - .5ru_{kt}^2 - e*\Sigma u_{it}*u_{kt} - m_{kt}$$

$$- \{c(u_{kt} - U) + d(M - m_{kt})\} - \epsilon \qquad \text{for tail-enders}$$

Players, then, will try to maximize the sum of the present value of Π_{it} where the future payoff are discounted by discount parameter ω. This present value of Π_{it} is formally;

$$(8) \quad \Psi_{it} = \Sigma_{t=1}^{T}\omega^{t-1}(\Pi_{it})$$

,where Ψ_{it} = the present value of Π_{it} at time t.

The choice on the control variable has been assumed to be free from any constraint so far. This is, however, not always the case. The choice on the control variables, especially on the amount of irrigation water, will be constrained by some exogenous factors as well as endogenous ones. This will be discussed in detail in the following section on the water delivery efficiency.

State Variables

The payoff for the appropriators is not determined entirely by the appropriators' choices on the two control variables. It is also affected by state variables, which change over time as they are affected by the choices on the two control variables. Payoffs are assumed to be influenced by two state variables - the reliability of water supply and water delivery efficiency of the irrigation system.

Reliability of Water Supply

First state variable in this model is the **reliability of the irrigation system.** As the water-depth in the example of ground water extraction model, reliability of the irrigation system can affect the payoffs. Ng (1988) defines reliability of the irrigation system as "the percentage occupancy of water level above CTL (Critical Tolerance Level)". According to Ng, appropriators will tolerate more water above some threshold, but not below it. This threshold level of water is "CTL". CTL can vary over time due to the seasonality of the demand for the irrigation water. The same amount of water at the field canal can produce different levels of reliability as CTL changes. In this model, however, CTL will be constant thanks to the assumption that the marginal benefit of the irrigation water is constant during the time period t.

It is previously assumed that u_{it}, the sum of the irrigation water appropriated for the time period t, determines the total benefit of time period t, Π_{it}. But, Π_{it} is also affected by the distribution of u_{it} during the appropriation period. Given that the CTL, as well as the marginal benefit of the irrigation water, is fixed over the time period t, the total benefit of the given amount of irrigation water (u_{it}) will be maximized when it is evenly distributed over the time period t. The following example helps to illustrate this point.

Assume that there are two hypothetical appropriators and appropriation period is composed of two days. Both appropriators have the same amount of water at the source and the same amount of water at their field gate during that time period (say, 4 units of water, $4u_d$)[8], but they do not have the same

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[8] This means that they have the same level of efficiency, which will be explained in the following section. This example also will be helpful to see the difference between the reliability of water supply and the water delivery efficiency.

pattern of distribution of that amount over time for whatever reason. That is, appropriator A has one unit (u_d) for one day and 3 units ($3u_d$) for the other day. Appropriator B, on the other hand, has two units ($2u_d$) for both days. In this example, the total benefit for the two appropriators are different. It is because, to repeat, the total benefit of irrigation water is a quadratic function of the supply of irrigation water. See figure 3.4. An hypothetical total benefit curve for a day is shown in figure 3.4. Notice that the total benefit curve for one day should be identical to that of the other day, since the marginal benefit of irrigation water is assumed to constant over a time period t. Appropriator A, according to Figure 3.4, enjoys 4TB for the time period t (2TB for the one day plus 2TB for the other day), whereas appropriator B enjoys 3TB (TB for the one day plus 2TB for the other day) which is only a portion of 4TB. They get different total benefit even though they get the same amount of water at their field gates. Appropriator who enjoys higher level of reliability can gain more total benefits than those who enjoy lower level of reliability, from the same amount of water. More precisely, appropriators can get the maximum total benefit from the given amount of irrigation water when the irrigation water is distributed evenly over time. This is always the case when the appropriation period t is composed of two days[9]. This shows that appropriators can get the

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[9] This can be deduced from the characteristics of concavity. In my model, the total benefit function is concave. A function "g" is defined as concave if:

(a) $g(\lambda y+(1-\lambda)z) \geq \lambda g(y) + (1-\lambda)g(z), \quad \forall\ 0 \leq \lambda \leq 1.$

Let (i) x, y, and z be an amount of water such that $y+z=2x$ and $y,z \neq x$; and
(ii) $g(x)$ be the total benefit function which is concave.

If $\lambda=0.5$, then the equation (a) will be:

$$g(.5y+.5z) \geq .5g(y) + .5g(z).$$

This can be re-written as:

$$g(.5(y+z)) \geq .5g(y) + .5g(z).$$

Since $y+z=2x$, It also can be re-written as:

$$g(.5*2x) \geq .5g(y) + .5g(z).$$

maximum feasible total benefit from the given amount of irrigation water for the time period t when it is distributed evenly over the time period t. For this reason, appropriators will try to distribute the irrigation water appropriated as much evenly as they can over the time period t. And if the water supply of the irrigation system is completely reliable, then they can evenly distribute the irrigation water over the time period t. Therefore, under my assumption on non-seasonality of the demand for the irrigation water, an irrigation system is to be called "reliable" when the irrigation water is evenly distributed over time.

For this reason, I define the reliability of water supply like this:

(9) $R_t = 1 - \{(\Sigma_{d=1}^{N} | u_d - (u_t/N)|)/(u_t/N)\}$, $d=1,...,N$

,where u_d = amount of irrigation water at day d, $\Sigma_{d=1}^{N} u_d = u_t$.

Reliability lies between zero and one interval. It refers to how evenly the irrigation water is distributed over the time period t. If it is equal to one, it means that the water is evenly distributed over the time period t. If it is equal to zero, it means that one day gets the total amount of irrigation water available for the time period t (u_t) and other days get nothing.

Like this, the total benefit of the irrigation water depends on the reliability of the water supply of the irrigation system. To depict this, I

This is equivalent to:

$$g(x) \geq .5g(y) + .5g(z).$$

Multiply both sides with 2, then we have:

$$2*g(x) \geq g(y) + g(z) \quad \text{or}$$
$$g(x) + g(x) \geq g(y) + g(z).$$

As you see, by definition, the left hand side is always greater than or at least equal to the right hand side <u>if</u> the function g is concave. Notice that the left hand side of the equation (a) refers to the total benefit of the case where "2x" amount of water is evenly distributed over time (x for one day and x for the other day), whereas the right hand side refers to the total benefit of the case where it is unevenly distributed (y for one day and z for the other day). Based upon this, we can say that one can maximized his total benefit of the irrigation water when it is evenly distributed over time.

assume that the total benefit of irrigation water use is equal to the area under the linear marginal benefit curve shown in the equation (1) when the water supply is completely reliable (R_t=1). If the water supply is not completely reliable, then player i cannot only get a portion of that level of total benefit when R_t equals to one. This is possible if we change the marginal benefit function like this;

(10) $$MB_{it} = R_t q - ru_{it}.$$

If reliability is less than one, then the marginal benefit curve will shift downward. Consequently, the area under that curve, which is the total benefit, will decrease[10].

Using this marginal benefit function, we can say that the total benefit of water use is;

(11) $$\pi_{it}^{B} = R_t q^* u_{it} - .5ru_{it}^{2}$$

The change in the reliability of water supply over time is influenced by several factors. In ground water basin case, it can be affected by the amount of water extracted in the previous period. In this model, however, it is hardly affected by the amount of water extracted in the previous period. It is because the irrigation system in this model is assumed to have no storage capacity which can hold the water. Instead, the reliability of the water supply of the canal irrigation is affected by the maintenance work done in the previous period. If the canals were not maintained well in the previous period, then the canals cannot function well so that the reliability of the water supply will decrease. Whether or not the irrigation system is maintained well is determined by the minimum level of investment required to

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[10] See the dotted line in Figure 3.2 and 3.3. Also notice that ideally, the relationship would be non-linear. But here I use linear approximation for simplicity.

conserve the previous level of reliability. If the sum of investment exceeds that level, the reliability of a irrigation system will increase, but if it does not, then the reliability of that system will decrease.

This can be summarized by a state transformation equation which determines the transition of state variable - the reliability of the water supply. The transformation of the reliability of the water supply will be determined by the equation;

$$(12)\quad \begin{aligned} R_{t+1} &= 0 && \text{if } \{R_t - \alpha(K_{t+1} - \Sigma m_{it})\} < 0, \\ &= R_t - \alpha(K_{t+1} - \Sigma m_{it}) && \text{if } 0 \le \{R_t - \alpha(K_{t+1} - \Sigma m_{it})\} \le 1, \\ &= 1 && \text{if } \{R_t - \alpha(K_{t+1} - \Sigma m_{it})\} > 1 \end{aligned}$$

,where α = coefficient
K_{t+1} = the minimum investment required to maintain the previous level of reliability, R_t[11].

We may need more investment to maintain the previous level of reliability when the reliability is higher than it is lower. That is, K_{t+1} should also be a function of R_t. For simplicity, let K_{t+1} be a linear function of R_t:

$$(13)\quad K_{t+1} = \gamma * R_t.$$

Then equation (12) will be rewritten as:

$$(14)\quad \begin{aligned} R_{t+1} &= 0 && \text{if } \{R_t - \alpha(\gamma * R_t - \Sigma m_{it})\} < 0, \\ &= R_t - \alpha(\gamma * R_t - \Sigma m_{it}) && \text{if } 0 \le \{R_t - \alpha(\gamma * R_t - \Sigma m_{it})\} \le 1, \\ &= 1 && \text{if } \{R_t - \alpha(\gamma * R_t - \Sigma m_{it})\} > 1 \end{aligned}$$

,where α, γ = coefficients.

[11] In case of stationary CPRs such as ground water-basin, forest products, however, the reliability (or other indicators of the system) can be affected by the appropriation at the previous period. In this case, we should add the term $(-\beta\Sigma u_{it})$ into the equation (12) to depict this. It is also the case for the irrigation systems with storage capacity.

In sum, the equation (7') can be re-written as:

(15) $$\Pi_{jt} = \pi_{jt}^{DF} + \pi_{jt}^{R} - \epsilon$$
$$= R_t q^* u_{jt} - .5ru_{jt}^2 - e^*\Sigma u_{jt}^* u_{jt} - m_{jt}$$
$$- \{c(u_{jt} - U_t) + d(M_t - m_{jt})\} - \epsilon \qquad \text{for head-enders}$$

and,

$$\Pi_{kt} = \pi_{kt}^{DF} + \pi_{kt}^{R} - \epsilon$$
$$= R_t q^* u_{kt} - .5ru_{kt}^2 - e^*\Sigma u_{it}^* u_{kt} - m_{kt}$$
$$- \{c(u_{kt} - U_t) + d(M_t - m_{kt})\} - \epsilon \qquad \text{for tail-enders.}$$

And the equation (8) should be re-written as:

(16) $$\Psi_{jt} = \Sigma_{t=1}^{T} \omega^{t-1}(\Pi_{jt}) \qquad \text{for head-enders, and}$$
$$\Psi_{kt} = \Sigma_{t=1}^{T} \omega^{t-1}(\Pi_{kt}) \qquad \text{for tail-enders.}$$

Water Delivery Efficiency

Water delivery efficiency is another state variable which affects appropriators' payoff. It affects the payoff, however, in somewhat different way than the reliability of water supply does. It affects the payoff through its impacts on the quantity of the irrigation water available to individual appropriators. The quantity of irrigation water available to a irrigation system (Q) has been treated as a constant so far for simplicity. The amount of the water at the source (say, Q^s) is, of course, is pretty much given and beyond appropriators' control.[12] Therefore, unlike the reliability of water supply, it cannot be affected by appropriators' behavior at the previous round. But, we need to distinguish the amount of water at the source (Q^s)

[12] It also can be changed when, for example, a big dam is built. This sort of change is, however, considered as an exogenous change in my model.

from the amount of water available to the irrigation system at the field gate (say, Q^{FG}). They can be, of cause, identical if the irrigation system is attached to the source so that there is no canal between the source and the field. They can be different, however, if there exists a canal between the source and the field and there exists water leakage in the canal. This means that the amount of water at the field gate at time t, Q^{FG}_t, is a function of both the amount of water at the source (Q^s) and the water loses in the canal. Let the water loses be Q^l, then it can be written like this:

$$(17) \qquad Q^{FG}_t = Q^s - Q^l_t.$$

Q^l can be understood more easily with the second state variable in this model - the water delivery efficiency. With proper maintenance, appropriators can increase the water delivery efficiency and reduce the water losses in the stretch of the canal. Let the water delivery efficiency (E_t) be a measure of water losses in a stretch of canal. There may be several ways to define the water delivery efficiency. Here, I simply define it as an index of the efficiency of the canal which varies from zero to one. It can be "1" if the canal is so efficient that the water loss in the canal is reduced to its minimum. It can be "zero" if the canal is so inefficient that the water loss in the canal reaches its maximum[13]. The water loss is also affected by the length of the canal. If the field gate is directly attached to the source of the irrigation water, then there will be no water loss at all. But, Q^{FG} of a system will become smaller and smaller as the length of the canal from the source to the field gate becomes longer and longer, even though both Q^s and E_t are fixed. Let the length from the source to the field gate be l, then the water loses at time period t, Q^l_t can be written as:

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[13] Conceptually, it can be thought of as "the proportion of water entering the reach that is delivered to the other end" (Sparling 1990, 199) per a unit of length. If we use this concept, Q^{FG} will be expressed as a exponential function of E_t and we will have some problems with solving our game. So, I decided not to use this concept.

(18) $Q^l_t = a*(1-E_t)*l$,

,where a $\{0<a<(Q^s/l)\}$= coefficient.

With equations (17) and (18), the amount of water at the field gate at time t can be written as:

(19) $Q^{FG}_t = Q^s - a*(1-E_t)*l$.

This is the amount of water at the head-enders' field gate[14]. Since the length from the source to the field gate of tail-enders is always greater than that of head-enders, the tail-enders will always have less amount of water at their field gate than the head-enders can get even when the tail-enders appropriate nothing, due to the losses in the stretch of the canal[15]. And when the head-enders appropriate some amount of irrigation water, the amount of water appropriated by the head-enders will also be reduced from the amount of water at the field gate of the tail-enders. Let the amount of water at the tail-enders' field gate and the length of canal from the head-enders field gate to the tail-enders field gate be $Q^{FG}_t{}'$ and l', respectively, then:

(20) $Q^{FG}_t{}' = Q^s - a*(1-E_t)*(l+l') - \Sigma u^*_{jt}$.

Like the reliability of water supply, the water delivery efficiency is also influenced only by the maintenance work done by the appropriators in the previous time period. Again, whether or not the canal is maintained or not is

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[14] Since I assume that the head-enders are identical in all respects, I ignore the possible difference in Q^{FG}_t among the head-enders. This will be also the case for the tail-enders.

[15] Also notice that the water delivery efficiency at a particular part of canal can be different from the water delivery efficiency at another part of canal. That is, the water delivery efficiency of the tail-enters itself can be smaller than that of the head-enders. But, for simplicity, here I assume that there is a unitary water delivery efficiency for the entire irrigation at time t.

determined by the minimum level of investment required to preserve the previous level of efficiency, and it will be also a increasing function of the level of efficiency. Thus, the state transition equation for the water delivery efficiency is:

(21) $E_{t+1} = 0$ if $\{E_t - \theta(\gamma' * E_t - \Sigma m_{it})\} < 0$,

$= E_t - \theta(\gamma' * E_t - \Sigma m_{it})$ if $0 \leq \{E_t - \theta(\gamma' * E_t - \Sigma m_{it})\} \leq 1$,

$= 1$ if $\{E_t - \theta(\gamma' * E_t - \Sigma m_{it})\} > 1$

,where θ, γ' = coefficients.

Affected by the investment on the maintenance at the previous time period ($\Sigma m_{i(t-1)}$), the water delivery efficiency at time t influences the amount of water available to the individual appropriators, and this amount of water, then, affects their payoffs. Because appropriators physically cannot appropriate what is available to them, there must be an upper limit on the appropriators' choice on u_{it}. And, we also can easily think of lower limit, which is zero, since appropriators cannot get negative amount of water. If u^*_{it} falls between these two limits, then u^*_{it} will be the choice, but if it does not, then either one of the two limits will be the choice. This upper limit will be a function of Q^{FG}_t. This function, then, will be determined by both physical and institutional factors. If there exist 'm' head-enders and 'n-m' tail-enders with no institutional constraint, then the maximum amount of water available to the head-enders will be "Q^{FG}_t/m", which is the amount distributed evenly among them[16]. If (Q^{FG}_t/m) is big enough, then Σu^*_{jt} will be smaller than (Q^{FG}_t/m) and the rest will be available to the tail-enders. In this case, the upper limit for the tail-enders will be $\{Q^{FG}_t{}'/(n-m)\}$. If $\{Q^{FG}_t{}'/(n-m)\}$ is also greater than u^*_{kt}, then the tail-enders will also be able to get as much water as they want. But if $\{Q^{FG}_t{}'/(n-m)\}$ is small, it is possible that head-enders

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[16] It is because we assume that there is no asymmetry among head-enders so that there is no difference among them in all aspects.

can get as much water as they want <u>but</u> tail-enders cannot. And it is also possible that Q^{FG}_t is so small that even head-enders cannot get as much water as they want, which is u^*_{jt}[17]. So, appropriators' choice on the amount of irrigation water is subject to another set of constraints:

(22) $0 \le u_{jt} \le (Q^{FG}_t/m)$ for head-enders

$0 \le u_{kt} \le \{Q^{FG}_t'/(n-m)\}$ for tail-enders.

Reliability and Efficiency

The reliability of water supply and the water delivery efficiency can be thought of as two different measures of the performance of a irrigation system. The reliability of water supply refers to how reliable the water supply is or, in other word, how evenly the supply of water is distributed over time during the time period t. The water delivery efficiency, on the other hand, refers to how efficiently the water at the source is delivered to the field gate. Reliability is about *the distribution of a given amount of water* and efficiency is about *the amount of water delivered to the field gate*. If a certain crop needs less amount of water to grow than some other crop does, then the critical tolerance level for the former will be lower than that for the latter. In this case, the level of reliability of water supply for the appropriator who grows the former can be higher than that for the appropriator who grows the latter even though they all enjoy the same amount of water at the field gate.

As you see, there assumed to be no effects of the two state variables on each other's state transition equation. There should be such effect if Ng's definition of reliability is used. It is because perfect reliability is impossible if there is not enough water in the field gate and the amount of

[17] This situation may be called as a "default situation". If there exists some rules concerning the allocation process, the upper limit may be determined in totally different ways.

water at the field gate is a function of the water delivery efficiency as well as the amount of water at the source. But, when we employ the definition in the equation (9), just as our model does, we can have a perfect reliability even without enough water. We can have a perfect reliability whenever the water is distributed evenly over the time period t, even though u_t is smaller than the lower critical tolerance level for one day. This definition should be used carefully. It makes sense in our model since the marginal benefit of the irrigation water is assumed to be fixed over the time period t. Since the marginal benefit of the irrigation water is fixed over the time period t, "even distribution" always means "timely distribution" and consequently "reliable water supply". But if the marginal benefit of the irrigation water is not constant over the time period t, then we cannot say that even distribution means reliable water supply.

In sum, players in our game will maximize the present value of Π_{it} in the equation (16) subject to a set of constraints that characterize the reliability and the efficiency of the irrigation system (equations (14) and (21)) <u>and</u> the constraints on the amount of water available to the appropriators at each time period (equation(22)). If the upper limit on the amount of water available to an appropriator at time period t is greater than u_{it}^* maximizing Ψ_{it}, then the upper limit will be the choice at time t. And if not, u_{it}^* will be the choice at time t. The choice on the amount of resources invested in the maintenance at time t, then, will decide the reliability of water supply and water delivery efficiency at time (t+1), and this will affect the payoff functions at time (t+1). This process will be repeated until the final time period T.

Solution of the Game

Two solution concepts are used for the dynamic game model. The first one is the open-loop solution. In the open-loop solution, each appropriator maximizes net present discounted value given the strategy paths of the other appropriators. This solution is a Nash equilibrium in which each player has no incentive to deviate from his/her strategy path given the path of the other players. This solution can capture forward-looking behavior but this solution assumes that each appropriator does not take into account the effect of his/her behavior on the behavior of other appropriators. That is, this solution assumes that each appropriator will not think that other appropriators will respond to their actions and accordingly they have no reason to alter his/her own action during the course of play.

In most situations, a more realistic assumption about appropriators behavior is that each appropriator will adjust behavior in response to the action of other appropriators. The outcome of this assumption is called the closed-loop solution of a game. The term "closed-loop solution" means a subgame-perfect equilibrium of the game where players can observe and respond to their opponents' action at the end of each period (Fundenburg and Tirole 1991; Dixon 1991). That is to say, the closed-loop solution assumes that at any point in the game, each player will respond to an action by picking the strategy path that maximize personal payoff for the rest of the game.

I will find closed-loop solution of this difference game using computer simulation based on backward induction. The basic logic which will be used in the computer simulation is:

(1) solve the maximization problem of stage (v) at the final round (t=T) to get the best reaction function, m_{JT}[18];

(2) solve the maximization problem of stage (iv) to get m_{KT}^*;

(3) substitute the specific value of m_{KT}^* into the best reaction function m_{JT} to

[18] We can only calculate the best reaction function rather than a specific value here. It is because the tail-enders are assumed to act before the head-enders act. This is also applied to the calculation step (4).

Also notice that m_{JT}^* and m_{KT}^* will be zero. It is because there is no future round. This is also the case when this game is played as a single-shot game or played without a transition mechanism. This is close to the prediction of the single-shot PD game.

get m_{JT}^*. Notice that both m_{kT}^* and m_{JT}^* are all to be expressed in terms of R_T and E_T;

(4) solve the maximization problem of stage (iii) to get the best reaction function u_{kT};

(4) solve the maximization problem of stage (ii) to get u_{JT}^*;

(5) substitute the specific value of u_{JT}^* into the best reaction function u_{kT} to get u_{kT}^*. Notice that both u_{kT}^* and u_{JT}^* are also to be expressed in terms of R_T and E_T;

(6) rewrite Π_{iT} in terms of R_T and E_T.

This is the maximization problems at T. The maximization problem at (T-1) will be to maximize the sum of the payoff at time (T-1) <u>and</u> the discounted payoff at time T (formally, $\Pi_{i(T-1)} + \omega\Pi_{iT}$). Notice that R_T and E_T can be re-written in terms of R_{T-1} and E_{T-1}, respectively. That implies Π_{iT} can be re-written in terms of R_{T-1} and E_{T-1}. And, using the stages from (1) to (6), we can also calculate $u_{i(T-1)}$, $m_{i(T-1)}$, and $\Pi_{i(T-1)}$, which then can be expressed in terms of R_{T-1} and E_{T-1}. Repeating these stages until t=1, we can get the time path of R_t and E_t, and consequently u_{it}, m_{it}, and Π_{it}. They all can be expressed in terms of the initial values of R_t and E_t. This is the closed-loop solution. Notice that this solution is subgame-perfect equilibrium. This process is very simple in the light of its logic but very complicated in the light of its actual calculation. For this reason, I will analyze my game model by parameterizing and running computer simulation.

<u>Parameterization and Simulation</u>

<u>Conclusion</u>

These two sections are not finished yet.

Figure 3.1: Game in Extensive Form

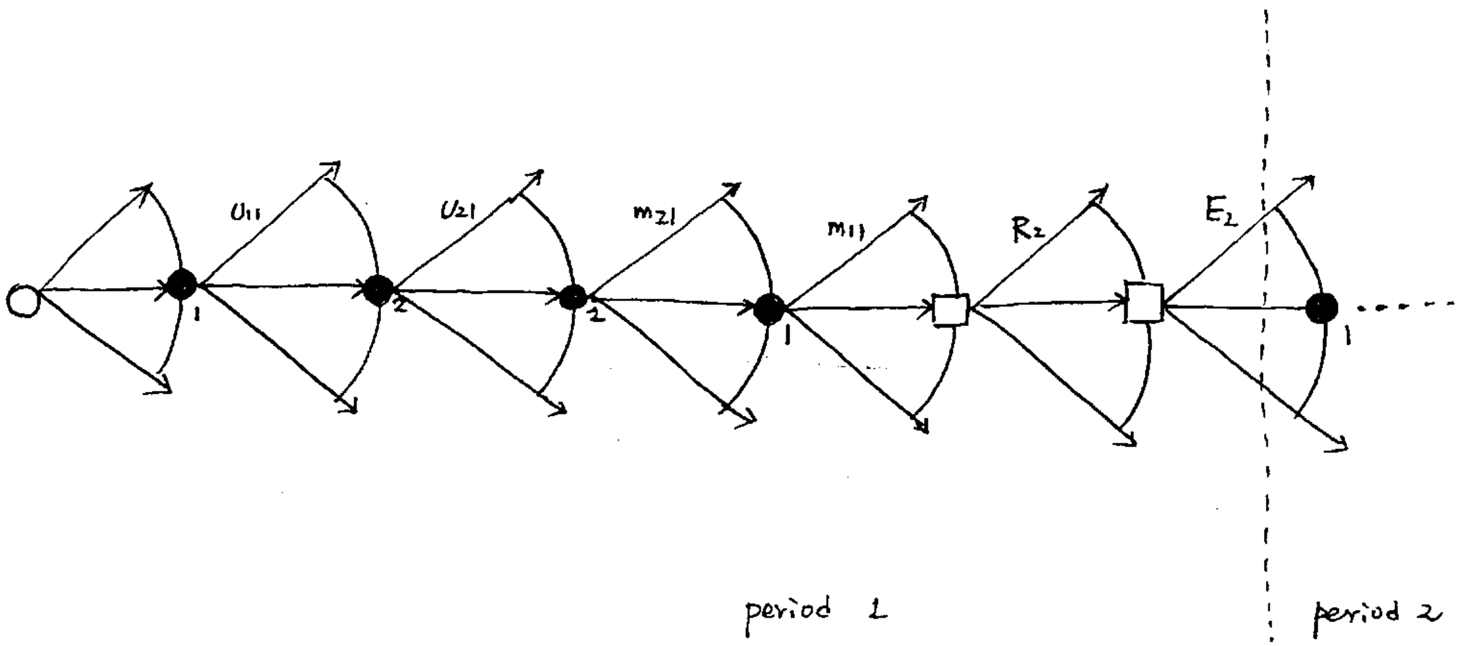


figure 3.2: Marginal Benefit Function

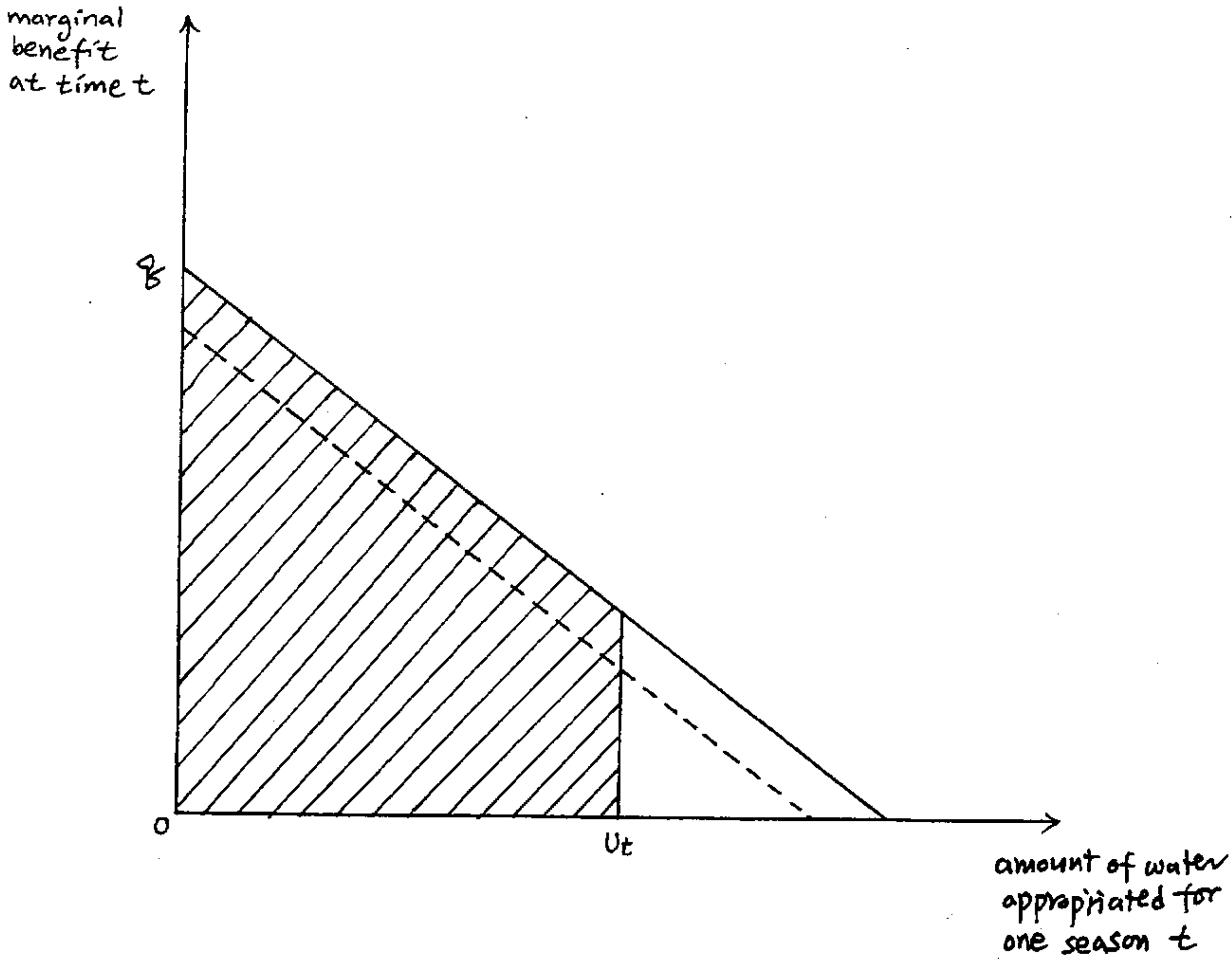


Figure 3.3: Total Benefit Function

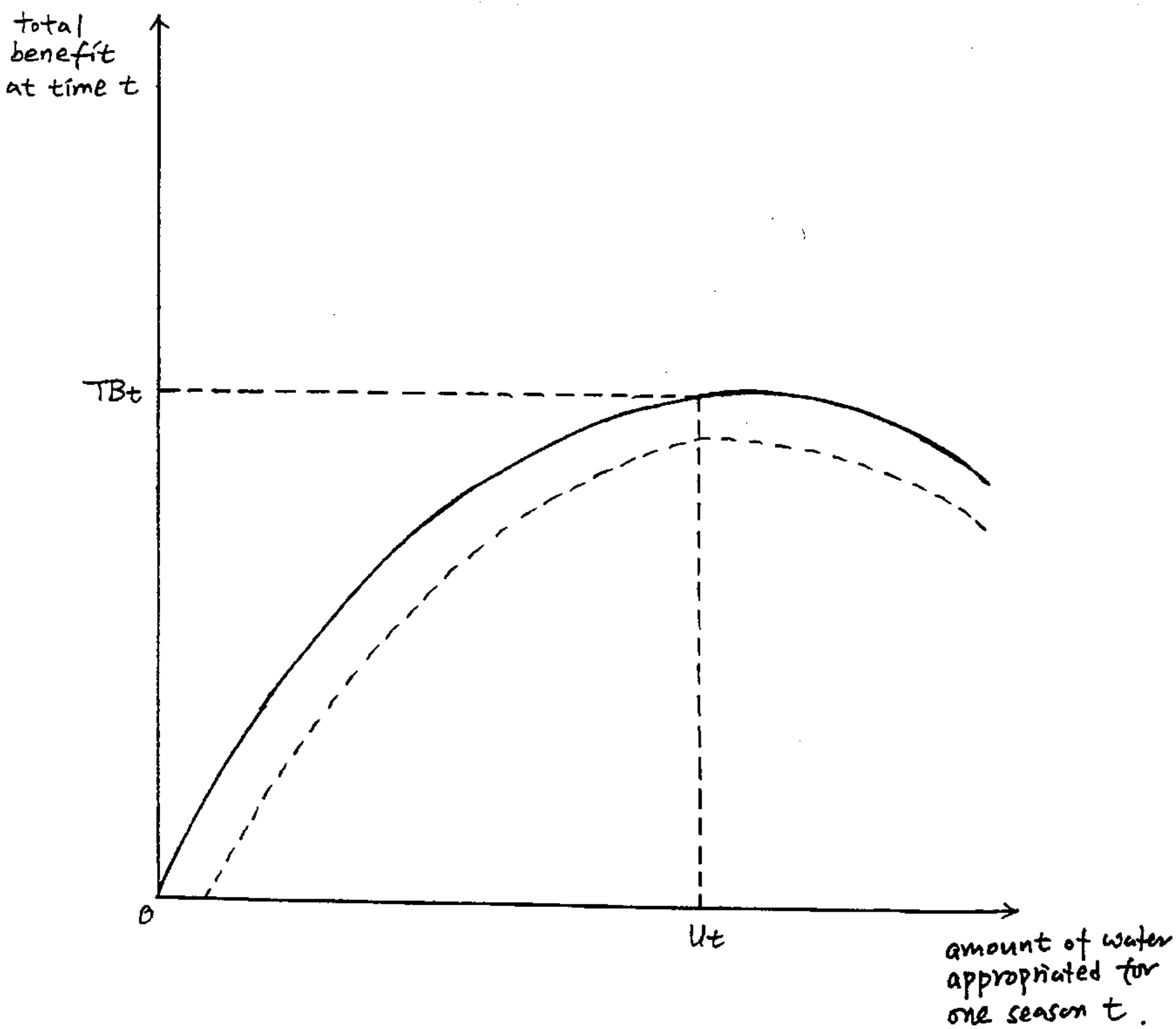
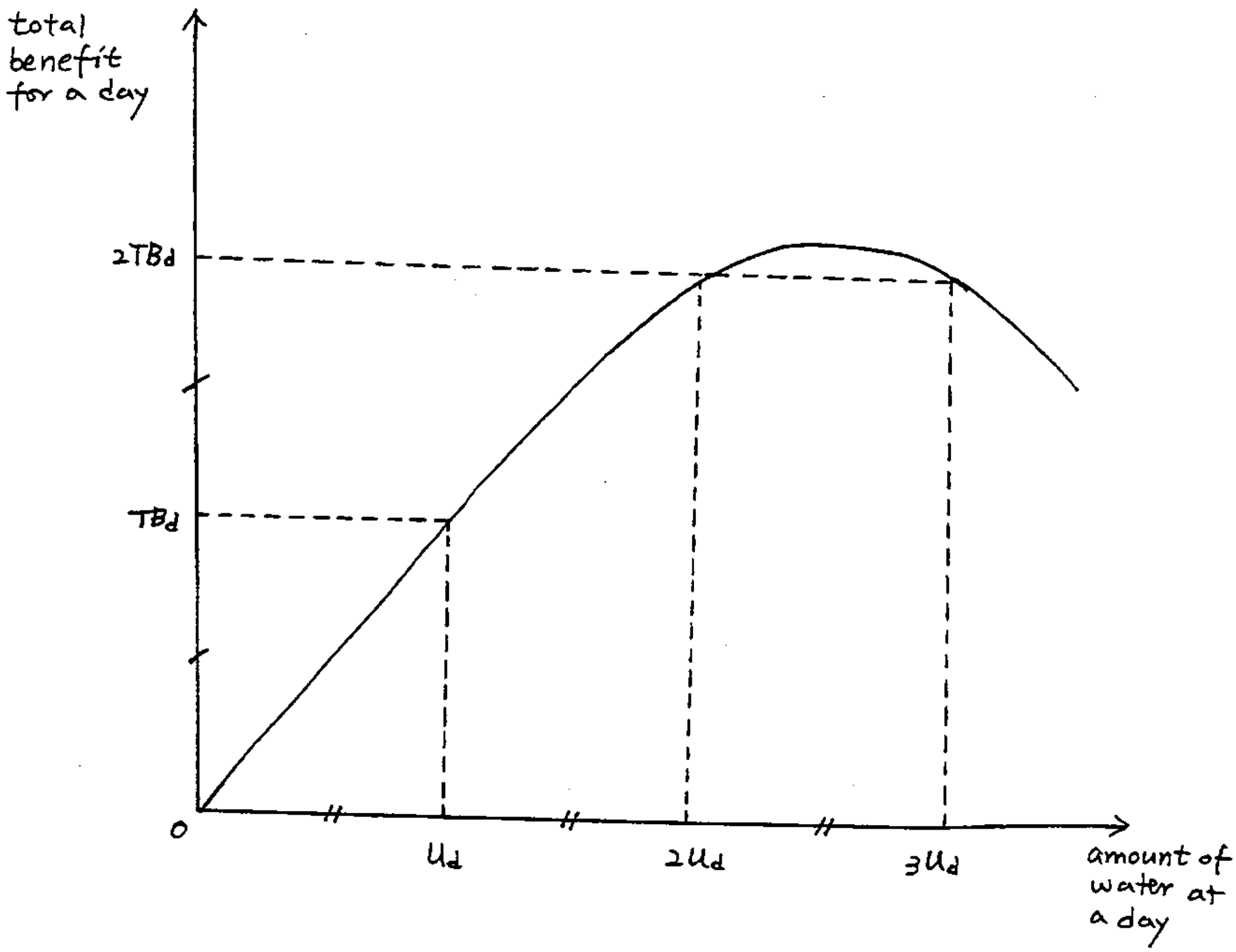


Figure 3.4: Total Benefit Function for One Day



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