Changing Commons

Diversity, Dynamics, and Design
in the Elementary Landscape of 2x2 Games

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Game theory concepts of cooperation and conflict in strategic interactions underlie much thinking about how people act in commons, often modeled in terms of a few fundamental social dilemmas. Payoff patterns from Prisoner's Dilemma, Chicken, and the rest of the symmetric 2x2 games combine to form an array of asymmetric games, revealing the diversity of possible incentive structures in even the simplest social situations. Changes in payoffs can switch the ranking of outcomes, transforming one game into another, pathways through which the potential for cooperation and conflict in commons may change.

The Robinson-Goforth topology of 2x2 games shows how games are linked by swaps in adjoining payoffs, mapping the diversity of social situations and visualizing how they might be transformed, moving dynamically within a design space. In this elementary landscape of cooperation and conflict, simpler games with ties lie between the strict ordinal games with four differently ranked payoffs, as do normalized versions of all 2x2 games with ratio or real value payoffs. This diversity of models enriches the tools for understanding institutional diversity and dynamics in collective action, including situations such as Jekyll-Hyde Type games, the High Dilemma game between Prisoner's Dilemma and Stag Hunt, and the archetypal Avatamsaka game of interdependence.

Deliberate crafting of rules to regulate boundaries, resource use, cost-sharing, monitoring, and sanctions, can shift expected payoffs, changing the incentive structures faced by commoners managing shared resources, such as farmers governing the mobile flows and physical infrastructure of irrigation systems. The diversity of 2x2 games and their elegant array of relationships offer insights into how incentive structures in commons may change and may be changed.

INTRODUCTION

Individual incentives may encourage cooperation, or discourage actions that could make things better for everyone. Simple models of two-person, two-move games particularly Prisoner's Dilemma, as well as its multi-person equivalents, the Tragedy of the Commons (Hardin 1968; Hardin 1998) and the Free Rider Problem (Olson 1971), underlie much thinking about collective action and how people may cooperate to govern shared resources (Ostrom 1990; Ostrom 2005).

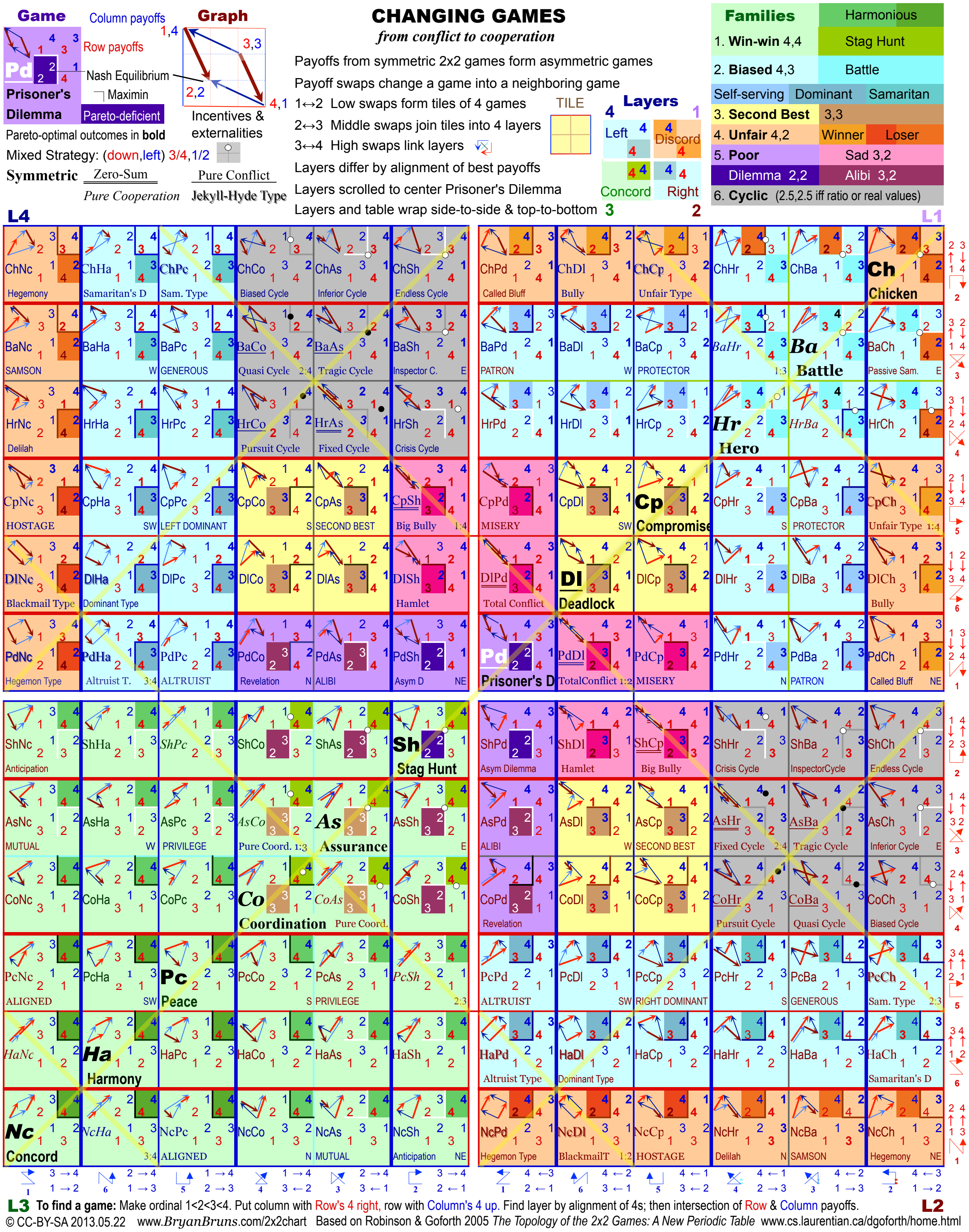
Research and discussion usually concentrate on a few well-known social dilemmas, ignoring the diversity of incentive structures possible even in elementary 2x2 strategic interactions. Preoccupation with the conflicts between individual incentives and cooperation may lead to neglecting the other challenges for collective action, and underestimating the prevalence of situations favorable for cooperation. Analysis is often restricted to a single static model, ignoring the potential for one incentive structure to transform into another, due to external factors or deliberate action. Rather than being trapped in a bad situation, it is often possible to change the game to get a better outcome.

Robinson and Goforth's (2005) topology of payoff swaps in 2x2 games maps the permutations of cooperation and conflict in strategic interactions, providing a tool for visualizing and understanding the possible structures of incentives where two people each have a choice of two moves and ranked preferences for the possible outcomes. The topology offers a framework for understanding the variety of game situations, showing how various incentive structures can be created as combinations of payoffs from a small number of symmetric games. The chart in Figure 1 shows how twelve symmetric games, in a diagonal from lower left to upper right, combine payoffs to form 144 strict ordinal games. Swapping payoffs turns one game into a neighboring game. The Technical Appendix further explains the topology of 2x2 games, including additional diagrams.

The topology provides a map of the various ways incentives for conflict and cooperation may combine. It offers an elementary "periodic chart" for students and researchers in game theory and related social sciences, as a foundation for learning and analysis. It provides an organized catalog of models of simple social situations, and shows the range of options available for designing experiments to examine strategic interaction. As a basis for comparative and cumulative research, a simple nomenclature helps identify equivalent and similar games, and their relationships. The chart shows how games are connected to each other, and so also provides a map for institutional design, including escaping Prisoner's Dilemma and crafting cooperation in commons.

The next section of the paper discusses how the topology helps to understand the diversity of games, their potential transformations, and options for institutional design. To the extent that payoffs occur randomly, then the chart shows the expected proportions of different games, the likely distribution of joint incentive structures. Given such a default expectation for the frequency of different patterns of cooperation and conflict, the subsequent section looks at how the topology may offer insights into basic questions about the prevalence of cooperation and inequality, robustness in social relationships, the role of norms, and ways of solving social dilemmas. The landscape of cooperation and conflict shapes the need and opportunities for different capabilities, such as anticipation, trust, and changing rules; and shapes how the application of institutional design principles and rules may change governance and outcomes.

Figure 1. The Topology of 2x2 Games



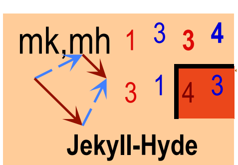
DIVERSITY, DYNAMICS, AND DESIGN IN 2X2 GAMES

Diversity

Prisoner's Dilemma, in which individual incentives conflict with a cooperative solution, and lead to a social trap where both get their second-worst result, is just one of many possible incentive structures. For the simplest strategic situations, where the results of one person's choice depends on another's choices, two people have a choice of two moves and each has ranked preferences for the four possible outcomes, there are still 144 possible incentive structures.[1] This includes not only asymmetric cousins of Prisoner's Dilemma, Chicken, battles, and coordination games, but also win-win games, biased and unfair games with unequal equilibrium payoffs, cyclic games without equilibria, and second-best games.

Elementary games are "toy" models that leave out much of the complexity of real situations, but reveal essentials of incentive structures that shape strategic interactions. The topology shows the large library of potential models, which may help to better understand social interaction, where conditions are often diverse and dynamic, incentive structures asymmetric, outcomes unequal, and motives mix cooperation and conflict.

Analysis of collective action often assumes that the underlying problem resembles a tragedy of the commons, a multi-person version of Prisoner's Dilemma, but other options are possible (M. Taylor and Ward 1982).[2] It may be sufficient for one to provide a public or common-pool good, but if neither does, both suffer, as in Chicken. Perhaps both must contribute, but then one or the other will end up doing somewhat better, as in a battle game. Cooperation might offer the best outcome, but risk aversion lead to getting stuck at second-best, a stag hunt (Skyrms 2004), also called a coordination or assurance game (Sen 1967; Runge 1986).[3] Incentive structures could be asymmetric, as in Samaritan's Dilemma (Buchanan 1977), where aid can help, but also encourage dependence (Buchanan 1977; Bruns 2010b). Irrigation water use is a major example of a common-pool resource where positions and associated payoffs are asymmetric, location along a canal affects the ability to appropriate water and associate benefits (Ostrom, Gardner, and Walker 1994; Janssen, Anderies, and Cardenas 2011; Janssen, Anderies, and Joshi 2011; Janssen and Rollins 2012). Changes in understanding, technology, markets, availability of government subsidies, expectations, sympathy, social capital, or other factors may change production functions and expected payoffs, for irrigation or other activities, transforming the situation into a different game.



An interesting example of how the topology enlarges the range of models available, can be seen in the mismatch of incentives and externalities in the Jekyll-Hyde games. Robinson and Goforth (2005) refined Schelling's (1960) categorization of games of conflict and cooperation. In games of pure conflict, such as Prisoner's Dilemma, one player's moves always reduce payoffs for the other, negative externalities. In games of pure cooperation, such as Harmony, one player's moves always improve payoffs for the other, what Greenberg (1990) called positive inducement correspondences. Robinson and Goforth identified what they called Type Games that mix these patterns; one player's incentives always lead to actions that improve payoffs for the other, while the other's incentives always lead to moves that make things worse for the first player. The incentive structure leads one person to be kind, and the other cruel. Four Type games lie in the lower left corners of Layers Two and Four on the chart. Since ties form simpler games located between the strict ordinal games shown on the chart (Robinson, Goforth, and Cargill 2007), the Jekyll-Hyde game, located between the Hegemon Type (*Dilemma-Concord*) and Biased Type (*Lock-Harmony*) games forms an idealized version of the Type games (Bruns 2012). Contrary to the happy endings usual in stories, in the Jekyll-Hyde Type games, the villain, whose actions always hurt the other, comes out best, while the nice person gets second-worst. Tragedy for one, and victory for another, outcomes that are highly unequal, and apparently inequitable and unfair, result from the structure of incentives, another troublesome situation it might be worth trying to escape by devising different rules.

Dynamics

Payoffs may change, due to external (exogenous) changes in the environment, or internal (endogenous) changes such as revised rules, side payments between players, and changes in perception and sentiments. The topology helps to understand the ways in which incentive structures may change, encouraging or discouraging cooperation in commons, as a result of various changes that affect expected payoffs:

- New *information* may update payoff values
- Changes in *technology* may reshape production functions, resulting in different outputs
- *Negotiations* may lead to side payments that transfer benefits from one person to another
- New *rules* may reward or penalize outcomes
- Changes in *monitoring*, and in *enforcement*, may shift expected sanctions and net payoffs that are a basis for decisions
- *Emotions*, sympathy or antipathy for others, may change how outcomes are weighed, for example valuing fairness or equity, or pursuing superiority or spite. Concern for the welfare of others, and for complying with norms and sanctioning violations may substantially shift final preferences regarding outcomes, transforming rankings and incentives.
- *Communication* may change perceptions and expectations
- *External changes* in assets, alternatives, and other factors may make outcomes more or less important (salient)
- Even where incentives are well-aligned initially, later shifts may be disruptive, encouraging post-contractual opportunism (Williamson 1996).

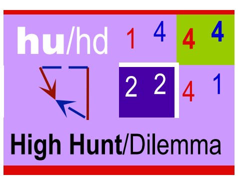
The most significant changes are those that shift to a new equilibrium, altering incentives and outcomes. The topology helps to map where such changes may occur, the boundaries between games that differ in properties including numbers of dominant strategies and equilibria, alignment of incentives and externalities, and equilibrium outcomes.

Assuming that swaps in the lowest ranked payoffs are least important, the topology also maps the distance between different games, how many swaps in adjoining payoffs it takes to convert one game into another. Thus, the chart reveals the distance to win-win, how many swap steps are needed to transform an initial game to one where both can get their best outcome. Just over half (56%) of the games on Layers One, Two, and Four can be changed to win-win by a single swap, while forty-two (39%) require two swaps. The hardest games to remedy are the zero-sum games, requiring three steps.[4] Overall, for 95% of games that are not win-win already, two swaps in the relative ranking of payoffs would be sufficient to reach win-win, compared to a maximum differences between games of six swaps. Empirically the feasibility of the changes needed to reach win-win may vary. However, rather than taking the structure of preferences and payoffs for granted, changes targeted at shifting key payoffs deserve consideration.

Win-win is a desirable goal, but may not always be the most feasible objective. Thus, if switching the high payoffs in Prisoner's Dilemma is not possible, then switching the middle payoffs can create a stable game of Deadlock where both can at least get second-best. The chart reveals not only the opportunities for reaching win-win or second-best, but also conversely which games are most vulnerable decaying into a worse situation.

Design

The relationships between games shown in the chart can help to understand how social institutions, such as communication, norms, monitoring, and sanctions may shift payoffs, helping to solve social dilemmas and promote cooperation. Even in Hardin's (belatedly revised) model of a tragedy of the commons (Hardin 1968; Hardin 1998), management rules, from government or from a community, could set limits for each person's share of livestock on the pasture, backed by monitoring and enforcement that offset the payoff from defection. Cooperative solutions are possible; dividing a commons into private property where individuals are only affected by their own behavior may be an option, but not the only one. Rules for sharing a commons, if well-designed, could swap the top two payoffs, the expected net benefits of cooperation or defection, making cooperation a win-win equilibrium preferable to the inferior outcome of overstocking and degraded pasture. Thus, Prisoner's Dilemma changes to a Stag Hunt, where communication and trust may be able to ensure cooperation on the Pareto-superior outcome. However, the situation may still be at risk of decaying back into a Prisoner's Dilemma, if rules are unclear, or application of penalties ineffective.



Another example of how the topology helps to locate interesting games and understand their context is the High Hunt game. Making ties for the top two payoffs in Stag Hunt or Prisoner's Dilemma creates a High Hunt in between, (in a sense in the middle of the chart) a borderline or transitional game containing an intriguing mix of the complexity of both.[5] Ties show indifference between outcomes. Solution concepts for this game with a narrow focus on dominant strategies would follow the presence of a weakly dominant strategy to a Pareto-inferior Nash Equilibrium (as in Prisoner's Dilemma), where both get second-worst and neither could unilaterally improve their payoff. Trusting cooperative strategies could achieve win-win, but face the risk (as in assurance games) that the other could defect and still hope to get their best payoff. If both acted that way, then lack of trust and avoidance of risk could mean that they again would end up stuck at second-best. An even simpler risk-minimizing maximin strategy to avoid the worst payoff would lead to the same Pareto-inferior payoffs as pursuing a weakly dominant strategy. High Hunt combines the Pareto-inferiority problem of Prisoner's Dilemma and the risk avoidance problem of Stag Hunts. From the perspective of collective action, this game poses interesting puzzles, but seems to have received little attention as an analytic model.

Redesigning Prisoner's Dilemma can turn it into a Stag Hunt, where mistrust may still trap participants in a Pareto-inferior equilibrium. By contrast, Chicken can be remedied more effectively. Raising the payoffs for the cooperative outcome for one player in Chicken to be greater than the outcome from "winning" would swap the ranks for the top two payoffs, changing Chicken into Hegemony (*Chicken-Concord*), with a single equilibrium that favors the other player. Raising the cooperative payoff for both players makes a win-win game of No Conflict with a single, stable equilibrium. Thus, for example, success in achieving equitable bipartisan agreements and passing legislation could become more rewarding, for both sides, than one-sided "victories" in political blame games. In that case, the structure would shift from an unstable one with two equilibria, with highly unequal payoffs, to a single equilibrium where both win. Thus, the topology maps a pathway for converting Chicken into No Conflict. A more nuanced model might measure benefits more precisely, analyzing differences in how gains are distributed, but the incentive structure still depends crucially on the ranking of the top two outcomes.

Table 1. Distribution of Games

Payoff Families	**Payoffs**	**# of games**	**%**
Win-win	4,4	36	25%
Biased	3,4 or 4,3	44	31%
Second-Best	3,3	12	8%
Unfair	2,4 or 4,2	19	13%
Sad	2,3 or 3,2	8	5.6%
Alibi	2,3 or 3,2	4	2.8%
Dilemma	2,2	3	2.1%
Cyclic	(2.5,2.5)	18	12.5%
Total		144	
Interest Alignments			
Pure conflict		14	9.7%
Zero–sum (fixed sum)		6	4.2%
Pure cooperation		14	9.7%
Jekyll-Hyde type		12	8.3%
Mixed interests		98	68%
Other Categories			
Asymmetric games		132	92%
Unequal payoffs at equilibrium		75	52%
Pareto-inferior equilibrium, or two equilibria: Dilemma, Alibi, Chicken, Battle, and Stag Hunt *Social Dilemmas*		25	17%
Both get best or second-best: Harmonious, Biased, and Second Best *Highlands*	≥3, ≥3	83	58%
Transitional: Best or second-best possible, but unsure. Upper right of layers *Borderlands*	≥3 or ≤2.5	19	13%
One or both get second-worst, or worst: Dilemma, Alibi, Sad, Unfair; Cyclic Tiles *Badlands*	≤2.5	42	29%
One winner, with a dominant strategy, or does best in unequal equilibria: Patron, Protector, Battle, Chicken, Unfair (Winner) *Feudal*	4,3 or 4,2	27	19%
One step from win-win; dominant strategy gets second-best or second-worst, or cyclic: Chicken payoffs on L2 & L4; E&S Generous (*Ha-Pc, Hr-Ba*) *Remediable*	≤3	20	14%

UNCOMMON GAMES: IMPLICATIONS OF A DEFAULT DISTRIBUTION

To the extent that payoffs occur randomly, the frequencies of games on the chart show not just the range of possibilities, but also the distribution, the expected proportions of different joint incentive structures. If payoffs occur randomly, then any of the 144 strict ordinal games is as likely as any other (Simpson 2010).[6] Thus, the chart illustrates the most likely, default, frequency of games, properties, and equilibrium outcomes, as summarized in Table 1.[7] Empirically, many things could affect the actual distribution of ordinal payoff structures, including characteristics of resources and capabilities of social actors. For any particular action situation, the incentive structure is something that needs to be assessed case by case. However, from a more general point of view for social theory, having an idea of the expected distribution of social situations if payoffs occur randomly provides a useful default, and may challenge preconceptions about the prevalence of social dilemmas and other situations.

Why so much cooperation?

The amount of research on Prisoner's Dilemma and other problematic games make it easy to think such games must be very common. However, to the extent that payoffs occur randomly, games where one or both sides have dominant strategies leading to successful cooperation would be much more common. One quarter of the games are win-win.

Games where both get at least second-best make up a majority of all games. The win-win games on Layer Three are linked by high swaps to the bands of biased games on the other layers, as well as the second best games, forming a contiguous region in the topology, shown in green, blue, and yellow (including the 1:3 Coordination-Battle hotspot, and southwest, south, and west pipes). This region could be called the *highlands*. Adjoining the highlands are *borderlands* in the upper right corner of each layer. In outer stag hunts and battles, one equilibrium includes a second worst payoff. The outer cyclic games have Pareto-optimal outcomes where both could get at least second best. The abundance of highland and borderland games shows how the elementary landscape of social situations is favorable for cooperation where both can get at least second-best.

In contrast to the relatively pleasant conditions for both players in the highlands, and slightly riskier situations in the borderlands, the remaining area might be characterized as *badlands*, filled with a variety of troublesome games where at least one definitely gets second-worst, including not only the dilemmas with their Pareto-inferior equilibria but also unfair and sad games, as well as the pair of cyclic tiles. Since these make up a bit over a third of games, capabilities for coping with or escaping such games would be worth developing. Since most games in this region are also more sensitive to even minor changes in payoffs, the need would be to deal not only with specific games, but also to cope with frequent changes between games with different equilibria and different kinds of problems.

The biased games on Layer One have relative good equilibrium payoffs and so may be considered part of the highlands, but are more problematic than those on Layers Two to Four. In the patron and protector games, the player with a dominant strategy already gets their best outcome, and so would presumably be less interested in changes, even though a single swap for the other player could reach win-win. In the battles, if the players have ended up at one equilibrium, the winner may also be similarly unmotivated to move, though the one who got second-best would have more incentive to seek change. The prevalence of inequality and lack of concern for improving conditions on the part of those who do best could be used to characterize these as *feudal* games. A priori, one could predict that these games would be less likely to be transformed into win-win.[8]

Conversely, on layers Two and Four, there are games where a dominant strategy leads to second-best or second-worst, or with the uncertainty of a cyclic game, which are only one swap away from win-win. Thus, these games are the most *remediable*. One could predict that they are more likely to be transformed into win-win than their neighbors with two dominant strategies (where there is a winner with a dominant strategy), or games that require two or three swaps to reach win-win.

The win-win games have often been treated as trivial or uninteresting by game theory researchers.[9] However, from the point of view of understanding collective action, win-win games represent well-aligned incentives, a desirable goal for institutional design. The prevalence of cooperation makes it worth understanding how incentive structures, capabilities, and other institutional factors contribute to its achievement.[10]

Why so much inequality?

Symmetric games, where both face the same payoff structure have been the principal focus of research (presumably because they seem easier to analyze). However, over ninety percent of games are asymmetric. To the extent that payoffs occur randomly, symmetric games would be expected to be uncommon. Asymmetry is the most likely state of affairs, and modeling and theory should go beyond the convenience of assuming symmetry.

A slight majority of games have unequal payoffs at equilibrium. A reasonable default expectation (Bayesian prior, absent evidence to the contrary) is that incentive structures will be asymmetric, and that equilibrium payoffs may well be unequal. Symmetry in incentive structures and equity in equilibrium outcomes cannot be taken for granted, and should not be assumed to be the most likely condition. Given the prevalence of asymmetric incentive structures and unequal outcomes, the question of what changes would be needed to yield more equal results is important, and supports a strong concern with equity in analyzing the governance of commons.

How robust are social relationships?

As discussed above, a variety of different factors could lead to changes in payoffs that switch ranks of different outcomes. These might occur randomly over the course of time, along with changes driven by particular trends and deliberate actions. Such variation could nevertheless change equilibria and lead to transitions. The proportions of games in the chart show that even for swaps in the highest payoffs, there is a large region of stability in the south and west of each layer where most low and middle swaps have no impact on outcomes, and high swaps still leave both at least getting second-best.[11]

Instability is concentrated around the dilemma family of games, where even games within the same tile, which differ only by swaps in the lowest payoffs, may have different equilibria.[12] For example, Prisoner's Dilemma, with its Pareto-inferior equilibrium lies on the same tile as Chicken with its two unfair equilibria, as well as the pair of Called Bluff games (*Dilemma-Chicken* and *Chicken-Dilemma*), with a single equilibrium with an unfair payoff where one gets their best payoff. Swaps in lowest payoffs turn the neighboring Alibi game (*Assurance-Dilemma*) in the Prisoner's Dilemma family into a cyclic game (*Assurance-Chicken)*.

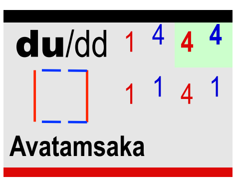
As mentioned earlier, Stag Hunt is a particularly interesting game since not only is the safe (maximin) strategy Pareto-inferior, but also the game is on the edge of decaying into a Prisoner's

Dilemma. While the games in the neighborhood around Prisoner's Dilemma are challenging for collective action, and interesting for social science, most situations have incentive structures that are much less vulnerable to shifts in equilibria and outcomes.

When do norms, rules, and trust matter?

If each person has a dominant strategy, then they may find the equilibrium solution without considering the other's incentives, without strategic interaction. However, only one fourth of games have two dominant strategies. In these games, Adam Smith's "invisible hand" leads to a stable equilibrium, without having to worry about the other's incentives and actions, except for assuming that they are able and inclined to pursue their own self-interest. In half the games, one player has a dominant strategy, which the other needs to anticipate. Thus, it becomes useful to be able to consider the other's incentives and actions, to have a theory that can be used to infer their expected behavior. Communication and social norms may simplify interaction in such situations by making it easier to predict how others will act. This offers a minimal level of cooperation, of going along with the decision made by someone else.

As has been analyzed in game theory research, norms, trust, and other cultural concepts become even more important in the quarter of games that have either two Pareto equilibria, (stag hunts and battles) (Skyrms 2004), or none, cyclic games. Norms, in the sense of shared expectations of what should be done, may be crucial in the stag hunts. In many stag hunts/assurance games, acting cooperatively when the other does not would lead to getting the worst outcome, instead of the best.[13] Risk avoidance, a maximin strategy, would lead to getting only the Pareto-inferior outcome. Trust, that others will act appropriately or will honor their agreements is particularly important to resolving such coordination problems.



The simpler Avatamsaka (*Double Hunt/Double Dilemma*) game of interdependence offers an interesting illustration of how norms, such as the Golden Rule, could contribute to a solution. In this *degenerate* game (Rapoport, Guyer, and Gordon 1976), neither player can directly affect their own payoff, which instead is dictated by the other's choice of move. The name Avatamsaka (Aruka 2001) comes from a Buddhist scripture telling the story of two people chained in place, each with a spoon too long to reach their own mouth, but able to feed the other. If each acts altruistically, "doing unto others as you would have them do unto you," then both get fed. Games such as this offer opportunities to explore the kind of reasoning used to choose strategies, whether norms about proper behavior, a pragmatic focus on an obvious solution, or other heuristics.

Norms may provide a focal point solution (T. Schelling 1960) for resolving a cyclic game. Over half of the cyclic games have a biased or second-best Pareto optima which may offer an attractive focal point. This leaves the central tile of cyclic games as the most intractable situations (where, as discussed later, a mixed strategy may be relevant, if payoffs are valued in a way that allows it to be calculated).[14]

The pure logical structure of game theory may offer no resolution to the choice of two competing equilibria (Gintis 2009), for example in the simplest coordination game, Double Coordination, or in a battle game. However, the usual recommendation is to find some kind of shared knowledge that would break the symmetry of the two options, making one more prominent, and so with a higher expected payoff. For Double Coordination, this would break the tie between the two different payoffs, turning it into a more complex middle ties game, Middle Coordination. In terms of expected payoffs in a strict battle game, such as Hero or Battle, this could be seen as changing the expected payoffs in a way that swaps the highest payoffs from one of the two equilibria, making it the best choice for both. In terms of expected payoffs, this swap would convert the battle into a coordination game.

For Prisoner's Dilemma, solution concepts tend to focus on repeated play, where with a suitable strategy such as tit-for-tat, playing cooperatively becomes the best choice in a supergame (Axelrod 1984). However, this does not solve the problem if play is not repeated (Binmore 2007), for example among strangers who never expect to interact again. In cases like these, rules (or norms) that include sanctions if others do not comply may become particularly important for changing games, for example escaping Prisoner's Dilemmas by imposing penalties or adding rewards that transform the incentive structure into a different game, such as Stag Hunt or Deadlock. For the classic Prisoner's Dilemma, a rule of silence, *omerta*, may be accompanied by punishments for violators, and rewards for those who adhere to the rule (Bruns 2012). On the other hand, a prosecutor's offer of a plea bargain represents a way to strengthen the Prisoner's Dilemma incentive structure, making defection more attractive, for one or both prisoners.

In summary, norms, social conventions that encourage a particular option, are most relevant for the quarter of games that either have two equilibria, or none. Rules are useful for changing games, to escape a Prisoner's Dilemma, and could also help craft better results in biased, unfair, or cyclic games.

Why change games?

The simple answer is that changing games is desirable when outcomes are unsatisfactory. The risk of bad outcomes is most conspicuous with the Pareto-inferior outcomes in the Dilemma and Alibi games. For stag hunts, norms and trust may achieve a win-win outcome, without changing the game. However, for nonrepeated play of Prisoner's Dilemma, changing the game, adjusting rules and payoffs and the resulting incentive structure offers a potential escape. The unfair games, with highly unequal payoffs also create situations where a strategic move between games may be preferable to staying trapped in the game, as may games with biased equilibrium payoffs, for the player who does worse. The poor payoffs in sad games may also motivate attempts to transform, although these games are harder to remedy, particularly the zero-sum games of Total Conflict (*Dilemma-Lock*) and Big Bully (*Hunt-Compromise*).

Why aren't social dilemmas more of a problem?

The preoccupation with social dilemmas, particularly the conflicts between individual and collective incentives in Prisoner's Dilemmas and Chicken-type[15] games, could lead to a grim outlook on social life.[16] The simplest answer to why such games are not more of a problem is that they are a minority. The most unfortunate games, where two dominant strategies lead to a Pareto-inferior outcome (Prisoner's Dilemmas and Alibi games), would be likely to occur less than 5% of the time. Even if a repeated play supergame solution is not available, it may often be possible to either avoid playing a Prisoner's Dilemma, or else to change the game, establishing norms and rules that shift payoffs and escape the social trap posed by the game. In sad and unfair games, the potential for a better solution may be less obviously apparent.

Prisoner's Dilemma is a game of pure conflict. For each strategy of the other player, the incentives of the first player encourages behavior that reduces the others payoff. However, such games of pure conflict are a small proportion, less than ten percent. The subset of pure conflict games that are zero (rank) sum games comprises less than half of that. Most games have mixed interests, combining positive and negative externalities. Even including the battles, and stag hunts, with equilibria where both could get at least second best, as social dilemmas, leaves social dilemmas as less than one fifth of all games.

Why aren't mixed strategies intuitive?

One of the most important and powerful results from the early days of game theory was to prove that mixed strategies, calculated to randomly play each of the available moves a fraction of the time, could guarantee a minimum payoff, regardless of the actions of the other player. However, mixed strategies are not necessarily an intuitive concept. The empirical frequency of mixed strategies has also been the subject of question. If payoffs are only ranks, then payoffs cannot be added or subtracted and the mathematical operations needed to calculate a mixed strategy are not even meaningful. Mixed strategies are only relevant if payoffs are valued on a ratio (interval) scale, or a real value scale (such as money), that makes payoffs comparable, where tradeoffs are possible, and mixed strategies can be calculated.

Furthermore, situations where mixed strategies would be the best option are relatively rare. For stag hunts and battles, other solution concepts may be able to reach a Pareto-optimal result, especially where communication, even "cheap talk," is possible. As discussed above, for the majority of cyclic games, focal points could also provide superior results. Thus, even where payoffs are measurable, mixed strategies would primarily be relevant for the tiles of cyclic games, eight games. Even then, a mixed strategy would only guarantee a middling payoff of 2.5, whereas changing the game might enable both to do even better.

PLAYING GAMES ON A FITNESS LANDSCAPE

The topology maps the payoff space of 2x2 games. It can be considered as a fitness landscape, where different areas may benefit from different capabilities. In the southeast corner, being able to pursue self-interest is sufficient to get to win-win, the invisible hand works. In the neighboring quadrants, being able to anticipate the others interests and actions leads to success. These resemble Mancur Olson's privileged games, where one person, such as a larger landowner, may have sufficient interest to provide the initial quantum of a public or common-pool good, after which other have an incentive to contribute. In the stag hunts, capabilities to communicate, to establish norms and build trust, help to solve assurance problems, moving from a risk-avoiding Pareto-inferior outcome to one that is better for both. As in small groups, mutual monitoring may make people believe that if they do not pursue the cooperative strategy, then others will also shift away, quickly collapsing to something worse for all. For the concord layer (L3) there is little need or advantage to being able to consider or carry through changes to the game.

Things become more difficult moving onto the left and right layers, where best payoffs are aligned in the same row or column. Inequality in outcomes emerges. That may be locked into situations where both have dominant strategies. Cyclical games lack the attractors of even a Pareto-inferior equilibrium, opening up the need and potential for additional capabilities, other heuristics and solution concepts. The logic of dominant strategies becomes progressively more questionable in games such as Samson (*Concord-Battle*) where communication and a degree in trust could enable a credible threat and commitment, that might force an equitable second-best solution as an alternative to the unfair Nash Equilibrium.[17]

Diagonally opposed best payoffs on Layer One demand, or at least offer, opportunities for more sophisticated capabilities. Settling for second-best, here and on the adjoining layers requires at least the ability to stop pursuing the highest payoff at any cost, the sophistication to understand or at least act in a way that accepts that "the best is the enemy of the good." Beyond Compromise, in the battles, repeated play offers the opportunity to both gain by taking turns, if there is an ability to synchronize.[18] Chicken and Prisoner's Dilemma are even more challenging, since the temptation to defect is stronger; in battles, defection risks getting a low payoff, whereas in Chicken and Prisoner's Dilemma defection leads to a better payoff.

All of the games outside of the win-win layer (L3) may make it worthwhile to consider changing the game, rather than just trying to do as well as possible within the constraints of the game. However, in terms of a larger supergame, these raise questions of institutional design and choice, and trust in keeping commitments, as well as monitoring and enforcement, which may require solving second-level collective action problems of leadership and collective choice, and even third-level problems of constituting governance, making rules for making rules. The benefits of different capabilities may depend on the prevalence of different games in the landscape, and how likely it is that random shocks or other forces may shift the structure of incentives. These questions of capabilities matter in terms of different kinds of commons, the management challenges they offer, and how increasing scarcity and other stresses may realign interests to be more directly opposed, shifting from easier assurance problems to the more difficult issues of unequal outcomes, instability in cyclic games, opposing first preferences in battles, and Pareto-inferior social traps in dilemmas.[19]

Viewing the payoff space as a fitness landscape makes sense not only in thinking about humans trying to manage commons, but also in terms of biological evolution of various capabilities, from the most basic ability to move towards something attractive, to more sophisticated behavior like communication, anticipation, and taking turns, on towards building trust and detecting cheating. It also is something that could be explored with software agents, possessing or able to develop/express various capabilities.

INSTITUTIONAL DESIGN PRINCIPLES AND RULES

Elinor Ostrom's institutional design principles characterize successful ways of organizing to govern commons (Ostrom 1990; Cox, Arnold, and Villamayor-Tomas 2010), abstracting from the enormous diversity of practices in particular circumstances. Principles may be embodied in specific rules regarding who may do what with shared resources, how decisions are made, and rules enforced, forming an "institutional grammar" that shapes how people act in specific situations (Crawford and Ostrom 1995; Ostrom 2005).

1. Boundaries locate actors and resources, including some and excluding others, and thereby shape the potential payoffs. Boundaries may locate a social situation in a particular site or region in the landscape of cooperation and conflict, including by influencing the chances of interacting with those sharing ideas, interests, and inclinations to cooperate.

2. Proportional sharing of benefits and costs aligns incentives, making them congruent with local conditions, although outcomes nevertheless may often be asymmetric, with some benefiting more than others.

3. Inclusion of stakeholders in decision-making enables them to use their knowledge, and interests, to collectively make and modify rules, including finding ways to avoid or escape social traps that could discourage cooperation. Such involvement may also strengthen perceptions regarding the legitimacy and value of norms and rules, giving extra weight to compliance and against violation.
4. Monitoring by users, or those accountable to them, improves knowledge of resources and the flow of benefits they produce, as well as the likelihood that violations or rules and other problems will be detected.
5. Graduated sanctions can make rules more credible, reducing uncertainty in expectations. They allow violators to be educated about rules, and help sort out occasional acts of carelessness or desperation from persistent and deliberate violations. Sanctions are the "or else" consequences that adjust expected payoffs.
6. Institutions for efficiently resolving conflicts also help to clarify expected costs and benefits. Conversely, lack of such institutions increases uncertainty, making it harder to predict how others may act, and so also harder for people to anticipate the likely results from their own actions.
7. Government policies may support, tolerate, or disrupt local collective action, affecting the transaction costs of organizing, and the expected benefits. Markets shift the payoffs from resource units, and more broadly may affect the extent to which local actors are embedded in social relationships in which trust, norms, reputation, and other institutions influence their behavior.
8. Nested organizations may facilitate cooperation within smaller groups, where social dilemmas are easier to understand and overcome, for example by linking access to benefits with sharing costs, including detecting and responding to those who defect from cooperation, while providing a way to federate and organize cooperation at larger scales.

Institutional design occurs not only through deliberate crafting of rules, but also through trial and error evolution, trying out arrangements and making adjustments based on experience. As expected payoffs vary, the structure of incentives shifts, and so may lead to new choices and outcomes. Changes may destabilize existing patterns of cooperation, creating new threats or opportunities. Commons change, and the resulting changes in expected payoffs can transform incentives for cooperation and conflict, and so challenge commoners to creatively craft new solutions for continuing cooperation.

CONCLUSIONS

Commons are dynamic. Governance of the commons requires solving problems of collective action, which come in many forms. Even where institutions have aligned incentives to encourage collective action, changing conditions may shift outcomes, undermining and disrupting incentive structures or opening new opportunities. The topology of 2x2 games offers a library of elementary incentive structures. This range of models can help understand institutional diversity, threats to institutional stability, and opportunities for institutional design.

This paper has presented an enhanced visualization of the topology of 2x2 games, mapping the relationships between games, which can help to understand and use the diversity of elementary incentive structures. A simple naming scheme, based on how symmetric game payoffs combine to form asymmetric games, provides a binomial nomenclature, like scientific ("Latin") names, efficiently identifying not only the 144 strict ordinal games, but the complete set of 1,413 ordinal games with and without ties. The visualization also maps normalized versions of games with payoffs measured on real or ratio scales.

Game theory, and related social science research has concentrated on Prisoner's Dilemma and a small number of other particularly interesting and challenging symmetric games. To the extent that payoffs occur randomly, the chart maps the likely distribution of incentive structures in the full landscape of cooperation and conflict, offering a more realistic set of default expectations for how frequently different patterns of payoffs may occur.

- ***The landscape of 2x2 games favors cooperation***, with Prisoner's Dilemmas and other problematic games being less common.
- ***Inequality prevails***. Most games are asymmetric, so participants face different payoff structures. In a slight majority of games, equilibrium payoffs are unequal. Equality in situations and outcomes may be valued, and pursued, but should not be assumed.
- ***Equilibria are robust***. In many cases, even when changes in payoffs shift the relative ranking of different outcomes, equilibrium results are unchanged. There is a large zone of stability, *Harmonia,* on the landscape of cooperation and conflict. Instability is concentrated in the neighborhood of Prisoner's Dilemma.
- ***The invisible hand is insufficient***. Ignoring the other person's incentives only works in the quarter of the possible games where both have dominant strategies. The invisible hand only achieves win-win outcomes in nine games out of 144 (6%).
- ***Intelligence is useful.*** The half of games with a single dominant strategy demand strategic reasoning and interaction, since only one has a dominant strategy, while the other troublesome games, in the *Badlands*, require even more sophisticated capabilities.
- ***Culture matters.*** The quarter of games without dominant strategies require different solutions. Norms and trust can help achieve Pareto-optimal results in stag hunts (coordination/assurance games). Other norms can help break the symmetry of dual equilibria in battles or identify focal points in cyclic games.[20]
- ***Mixed strategies are rarely relevant.*** They are most useful in a few cyclic games, but only if payoffs can be measured on real or ratio scales.
- ***Change may be easy, or hard.*** Some games require more swaps to reach win-win, with zero-sum games the hardest to remedy. In *Feudal* games, the winner already gets their best result, and so lacks motivation to change the game. *Remediable* games are one step away from win-win, and the player with a dominant strategy gets second-best or second-worst, or cyclic outcomes are uncertain, making transformation to win-win a more feasible and desirable option.
- ***Changing games may be a better option.*** Dominant strategies and Nash Equilibria provide solutions for most games, but these may be unsatisfactory because they yield unequal or poor payoffs. Switching payoffs may transform incentive structures, yielding better results, so a map showing pathways for transformation may be helpful.

Understanding the topology of 2x2 games can help understand not only the diversity of incentive structures and institutions and likely frequency of different strategic situations, but also the opportunities and threats involved in coping with changes in commons and cooperating to govern commons.

Technical Appendix: A Brief Overview of the Topology of 2x2 Games

Swapping adjoining payoffs changes one game into another. Robinson and Goforth (2005) follow this logic to show how payoff swaps link all 2x2 games in a network. Their topology maps the elementary landscape of cooperation and conflict, the payoff space when two players each have a choice of two strategies, and ranked preferences for the outcomes.

In the chart, twelve symmetric games, where both players face the same payoffs, form a diagonal from lower left to upper right. Payoffs are shown in normal form, and put Row's highest payoff in the right column, and Column's highest payoff in the upper row.[21] Payoffs from the symmetric games combine to form asymmetric games. This provides binomial names to identify games, like scientific names for species.[22] Pairs of games on either side of the diagonal are mirror reflections, switching positions for row and column.[23]

The topology can be generated starting with any game. Swapping the lowest payoffs creates a *tile* of four games, such as the second-best tiles. Additional swaps of middle and low payoffs create a *layer* of nine tiles and thirty-six games. After that, the numbers repeat, so the layer forms a torus, which maps onto a rectangle that wraps around from left to right and top to bottom. Swapping the top two payoffs, for example changing Stag Hunt into Prisoner's Dilemma, starts a new layer, completed by additional low and middle swaps. Layers differ by the alignment of best outcomes, win-win concord in Layer Three, and diagonally-opposed discord on Layer One.[24]

Scrolling the torus to put Prisoner's Dilemma near the center elegantly displays the relationships between games.[25] In the lower-left quadrant of each layer, both have dominant strategies, the best move whichever choice the other makes, leading to a single Nash Equilibrium, where neither can unilaterally improve their payoff. In the adjoining quadrants, only one has a dominant strategy, still yielding a single equilibrium. In the upper-right quadrant, neither has a dominant strategy, resulting in either two equilibria, as in the stag hunt and battle games, or no equilibrium in pure (unmixed) strategies, in the cyclic games. Payoffs at Nash Equilibria categorize games into families.[26]

High swaps link each band of three tiles in a layer (two rows or two columns of games) with an equivalently located band of tiles on a different layer, weaving the layers together.[27] Six *hotspots* link a pair of tiles on different layers, such as the coordination-battle hotspot, and the cyclic hotspot. Six *pipes* form sets of four tiles, equivalently located on each layer, such as the southwest tiles. Scrolling Prisoner's Dilemma to the center splits open tiles. The entire chart is then a torus, showing high swaps linking layers, wrapping around left to right and top to bottom.

Half-swaps make (or break ties) (Robinson, Goforth, and Cargill 2007). Thus, Volunteer's Dilemma (*Middle Battle)*, with ties on middle payoffs, lies midway between Chicken and Battle, at the intersection of borderlines between games. Six types of ties produce a total of 38 symmetric ordinal games (see Figure 3), which combine to form the complete set of 2x2 ordinal games (Bruns 2012). Normalized payoffs can also be mapped, for symmetric games (Robinson and Goforth 2005; D. Goforth and Robinson 2010) and for normalized versions of all games (Bruns 2010a). The ordinal games in the topology form not only a "periodic table" of discrete elementary games, but also, like a number line or Cartesian coordinates, chart the landscape of 2x2 games and transformations that change one game into another.

Figure 2. Structures in the Topology of 2x2 Games

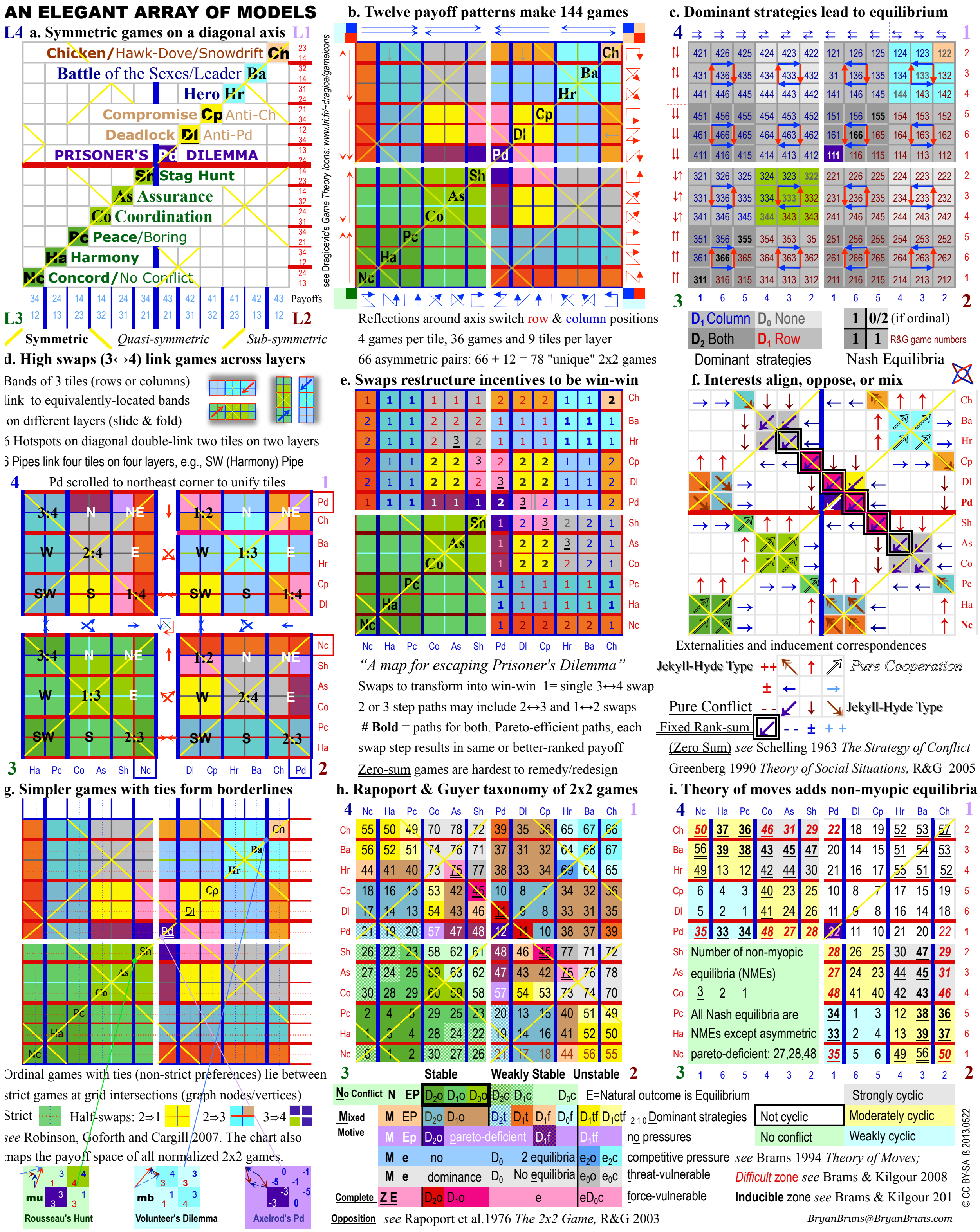
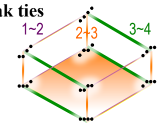


Figure 3. 2x2 Games with Ties

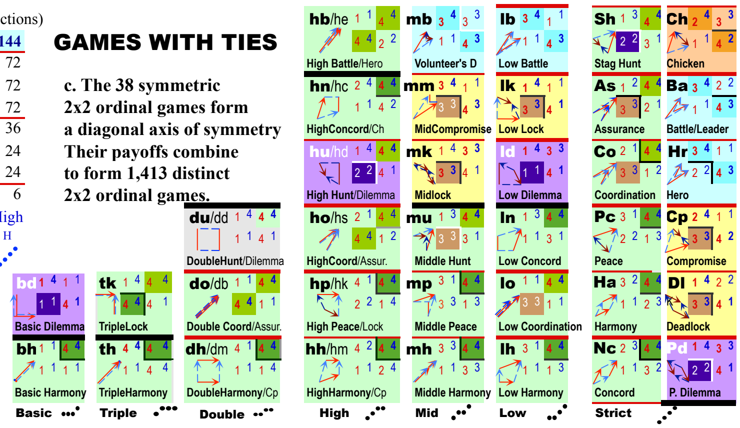
a. Preference Classes: Type of Ties Number of games (including reflections)

Strict	STRICT	1,2,3,4	H	6	24	24	36	72	72	72	**144**
Low Tie		1,1,3,4	D	3	12	12	18	36	36	**36**	72
Middle	EDGE	1,3,3,4	F	3	12	12	18	36	**36**	36	72
High Tie		1,2,4,4	G	3	12	12	18	**36**	36	36	72
Double		1,1,4,4	C	3	6	6	**12**	18	18	18	36
Triple	VERTEX	1,4,4,4	E	1	4	**4**	6	12	12	12	24
Basic		1,1,1,4	B	1	**4**	4	6	12	12	12	24
Zero	ORIGIN	0,0,0,0	A	**1**	1	1	3	3	3	3	6
Total 1,413				Zero A	Basic B	Triple E	Double C	High G	Middle F	Low D	High H

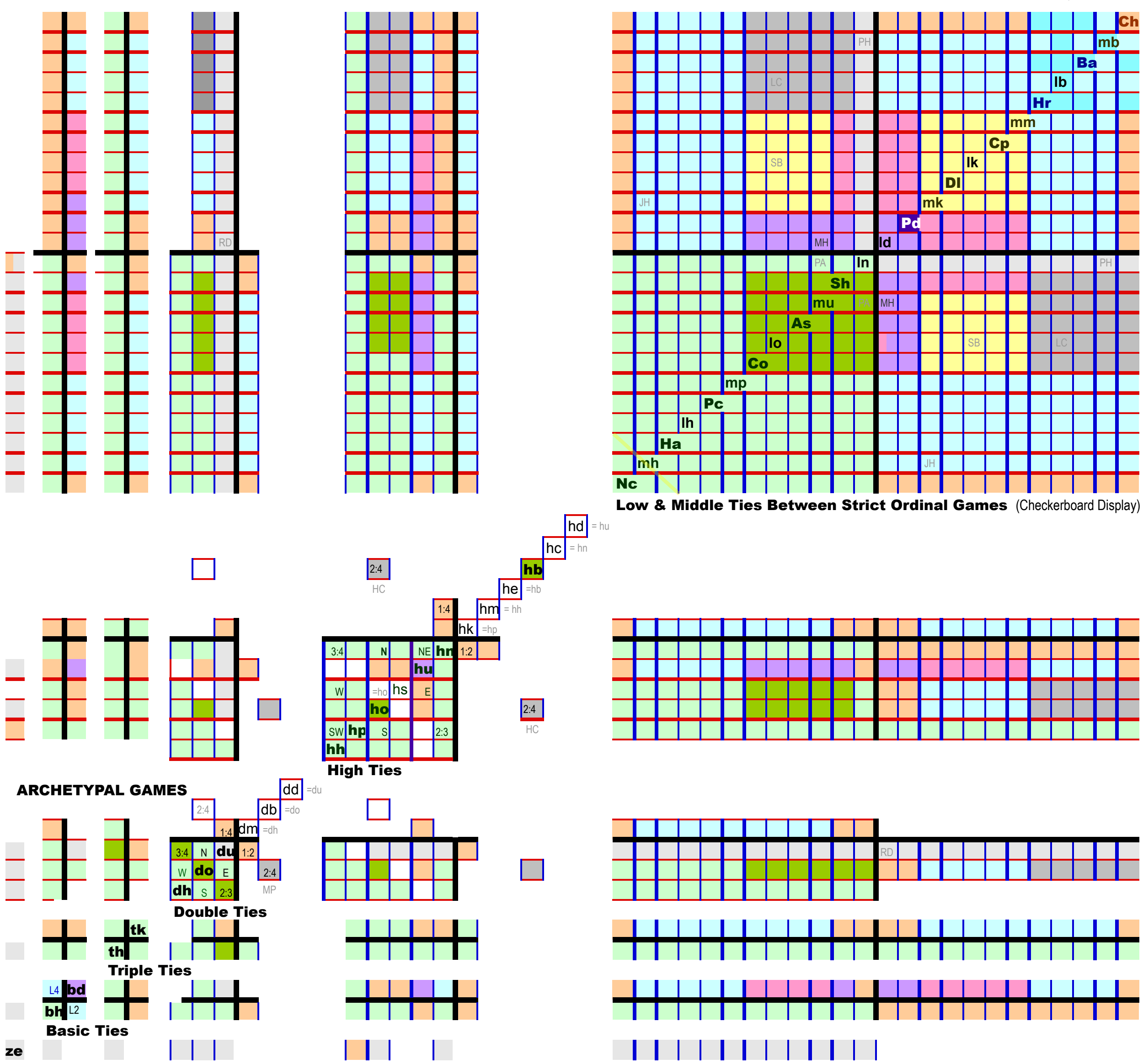
b. Half-swaps make or break ties



Figures a & b adapted from Robinson, Goforth and Cargill 2007, and see Fraser and Kilgour 1986, K&F 1988



d. Swaps & half-swaps link 2x2 ordinal games



e. To find a game: Make ordinal: Lowest = 1 Highest = 4 Middle ties = 3. Find class by type of ties, for each player. Put column with Row's 4 right, row with Column's 4 up. Find Layer by alignment of 4s, then intersection of Row and Column payoffs. For high, double, and triple ties, interchange rows and columns if necessary.

ENDNOTES

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[1] This assumes games formed by switching rows or columns are equivalent (Rapoport, Guyer, and Gordon 1976). The12 symmetric games and 66 reflected pairs match their total of 78 distinct strict ordinal 2x2 games. For the topology, reflections equivalent by switching row and column positions must be treated as different.

[2] See Taylor (2006) for his later critique and rejection of rational choice approaches to social analysis. The discussion in this paper assumes bounded rationality, and that elementary games are just some of the tools that may be useful for institutional analysis.

[3] In this paper, the terms stag hunt, assurance, and coordination games will be used synonymously for strict ordinal games with a second Pareto-inferior equilibrium. Robinson and Goforth used stag hunt as the name for the win-win symmetric game with a second, Pareto-inferior equilibrium where both get their second-worst result. This is analogous to Rousseau's story about the hunter who must choose between getting a stag, if others also cooperate, or the certainty of catching a rabbit on his own, a much lower payoff.

Robinson and Goforth used the name coordination for both the other two symmetric ordinal win-win games with a second equilibrium with second-best payoffs. One has the most extreme case of an assurance problem (Sen 1967) where acting cooperatively risks getting the worst result if the other does not cooperate. It would be safer to choose the strategy that avoids the worst outcome although that leads to getting second-best, and so is best named as Assurance. In the other, cooperating is not only the Pareto-optimal win-win outcome, but is also the safe, maximin strategy, that also guarantees not getting the worst outcome. It does not pose as severe a conflict between individual and collective benefits, but still requires coordinating on the same choice. In Coordination, acting cooperatively if the other does not still risks getting the second-worst outcome.

The simplest, *archetypal* game of Double Coordination with ties for highest and lowest payoffs, shown in Figure 3 in the Appendix, illustrates a coordination problem alone, since the two Nash Equilibria have equal win-win payoffs. There is no risk-avoiding maximin strategy or Pareto-inferior equilibrium.

[4] See Figure 2e in the Appendix for a visualization showing the distance to win-win. For ordinal games, the "zero-sum" or fixed-sum ordinal games are fixed rank-sum games, where increases in the payoff rank for one person are matched by equivalent decreases in rank for the other. These lie on a diagonal linking the cyclic games, from Crisis Cycle (*Hero-Hunt*) on the upper left to its mirror image (*Hunt-Hero*) on the lower right. For ratio or real value payoffs mapped on the surface of the topology, the zero-sum games lie on the line linking the centers of those cyclic games. The only symmetric ordinal zero-sum game, Midlock lies where that line intersects the axis of symmetry, between Prisoner's Dilemma and Deadlock (and, surprisingly, seems to have received little attention, and lacks an established common name).

[5] Without being aware of the full set of connections that form the topology, Rapoport, Guyer, and Gordon (1976) presciently used the term "borderline" for games such as the one between Chicken and Prisoner's Dilemma, here called Low Dilemma. In the chart, games with ties lie on borderlines dividing ordinal games (and normalized versions of ordinally equivalent games). As with other high ties games, High Hunt can be formed by transformations from two starting

points, either Prisoner's Dilemma or Stag Hunt. To provide a compact display of the topology, win-win games on Layer Three, such as High Hunt, are treated as the primary or preferred versions where available.
[6] More precisely, these will generate 4 x 144 = 576 games, but if row or column swaps are considered to make equivalent games, then this leaves only 144 distinct games, in the same proportions.
[7] The chart also shows expected proportions of normalized versions of games with ratio or real values, mapped onto the square containing each strict ordinal game. (Note that a precise mapping on the chart would be based not on directly plotting the normalized payoff, but instead on the relative distance of the second lowest value between the lowest and second-best, for both players. This can be visualized in terms of moving the second-lowest payoff, so that it approaches 1 or 3, at which point a tie would be formed and further changes would switch to another game. High swaps moving the second-best outcome towards the best would move in additional dimensions for Row and Column respectively.
[8] In formal terms, this could be treated as adding a third move for the player whose high swap would reach win-win (a strategic move between games). In remediable games, this move would dominate the other strategies available to the loser, encouraging a shift, while in feudal games there would be no gain for the winner with the added move. In battles, a swap to win-win would offer a dominant strategy for the loser, the one who came out worse at equilibrium. In other words, remediable games represent a one-sided social dilemmas, someone may realize that their payoffs, their incentive structure, has trapped them into a situation that leads to an undesirable result, and one which may be relatively feasible for them to change.
[9] The concentration on conflicts, and neglect of win-win games applies to the study of non-cooperative games, where there is no third-party enforcement of agreements. Cooperative game theory *assumes* that binding agreements can be made, and then examines how gains may be distributed even when the incentive structure favors cooperation, depending on the available payoffs, options available to participants, and other factors.
[10] See *Supercooperators* (Nowak and Highfield 2011) for an interesting discussion of recent theory and research on cooperation.
[11] This zone of robustly pleasant outcomes extends from the Harmony tile, formed by the southwestern or harmony, pipe and its neighbors, the south and east pipes, and the aligned-samaritan hotspots, in the south and west of each layer, linking harmonious, biased, and second-best games.
[12] Robinson and Goforth (2005) discuss the prevalence in instability in the games near Prisoner's Dilemma (on the Prisoner's Dilemma Pipe and the Prisoner's Dilemma family that includes the asymmetric games with poor, Pareto-deficient equilibria). They note that Prisoner's Dilemma lies in a "bad neighborhood."
[13] Kollock (1998) conjectured that assurance games, also known as stag hunts or coordination games, were more common than Prisoner's Dilemma. The topology shows that if the distribution of payoffs is random, then stag hunts (9 games) are more common than Prisoner's Dilemmas (7 games), for strict ordinal 2x2 games, (and for games with ratio or real payoff values) verifying Kollock's conjecture.
[14] If payoffs can be measured on a scale with real or interval values, then a mixed strategy can be calculated that will guarantee a payoff regardless of the other's behavior. In a majority of cyclic games (5/9=55%) the payoff from a mixed strategy is Pareto-inferior, so solution concepts that

could coordinate behavior to achieve the Pareto-superior result would be preferable.
[15] This includes the four games that combine payoffs from Chicken with those from Battle and Hero, with two unequal and asymmetric payoffs. One of these, *Battle-Chicken,* is the game Buchanan (1977) analyzed as Passive Samaritan's Dilemma.
[16] Schmidtchen (2002) quotes Buchanan as saying his analysis of Samaritan's Dilemma came from his "pessimistic working period."
[17] Under a somewhat different set of rules, allowing a series of shifts in moves until stability is reached, this game, *Concord-Battle*, discussed in terms of the Biblical story of Samson and Delilah, is a centerpiece of Bram's (1994) Theory of Moves, with three "non-myopic" equilibria.
[18] In the symmetric games, this boundary where taking turns could achieve higher total returns is demarcated by what Goforth and Robinson (2010) call a Reconciliation Line.
[19] The display with Prisoner's Dilemma scrolled near the center is very convenient for showing properties of games related to number of dominant strategies and equilibria. However, the version with tiles reunited and Prisoner's Dilemma in the corner is arguably more "natural" given the way it aligns the symmetric and quasi-symmetric diagonals (Villarceau circles in the topology) to intersect at the center. On the flat display, this puts Harmony at one extreme on the southeast corner, and Prisoner's Dilemma on the other extreme, as opposing "poles" in the topology.
[20] See Gintis (2009) *Bounds of Reason* for an extensive critique of the limits of conventional game theory, and the need to utilize the strengths of game theory while incorporating it within a broader framework of analysis.
[21] Robinson and Goforth's convention for locating payoffs puts win-win outcomes in the upper left or northeastern corner. This is consistent with the usual Cartesian convention of plotting higher values to the right and upwards. (Their additional rules for displaying second-best games are not used here.) In the game theory literature, symmetric games are usually shown with the cooperative outcome in the upper left, northwestern cell. The four variations by switching rows and/or columns are usually considered equivalent versions of the same "game." Psychologically and empirically, they could be perceived differently, and "geographic" nomenclature, e.g. NW or NE, provides a way to specify the different variants if necessary.
[22] To make the binomial ("scientific") names easier to say, Prisoner's Dilemma is shortened to *Dilemma*, Deadlock to *Lock*, Stag Hunt to *Hunt*, and No Conflict replaced by *Concord*.
[23] Twelve symmetric games plus 66 pairs of reflected games make up the total of 78 "unique" 2x2 games identified by Rapoport and Guyer (1966). The topology structure requires all (12+2x66=) 144 games. The binomial nomenclature, with abbreviations, used here makes it easy to identify the mirror pairs, and is intended to be easier to remember and use than Robinson and Goforth's index numbers based on layers and row and column payoff patterns, and also extends much more easily to identify games with ties. The strict symmetric game abbreviations start with an initial capital, while symmetric games with ties start with a non-capitalized letter based on the type of ties, which distinguishes them and also indicates the transformation from a strict game that would create the game with ties. Thus, the game between Prisoner's Dilemma and Chicken formed by ties on the two lowest payoffs is called Low Dilemma.
[24] Robinson and Goforth's original "Periodic Table" display put Layer One, with Prisoner's Dilemma in the lower left, and their numbering scheme started with it as the first game, with index number 111 (roughly like starting the Periodic Table of the Elements with uranium, interesting, but also highly complex and dangerous. The enhanced visualization used here puts

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the simpler win-win games on Layer Three in the lower left, and the more complex games on Layer One in the upper left, consistent with the Cartesian convention of putting higher values to the right and upwards. Compared to their original periodic table, this enhanced visualization adds a range of features, including payoff values in normal form, colors for payoff families, fonts to distinguish hotspots and pipes, borderlines between games (showing ties, bands, and high swap destinations), and other details. For additional information, see Bruns (Forthcoming) *Changing Games: An Atlas of Conflict and Cooperation*.

[25] Scrolling the torus to put Prisoner's Dilemma near the center splits open tiles of games. Switching the lowest two payoffs for one player turns Prisoner's Dilemma into Called Bluff (*Dilemma-Chicken*), and then switching the lowest payoffs for the other then creates Chicken. Even though on the torus they are next to each other, these four games now lie in the corners of the layer, in the projection of a torus onto a flat surface. Similarly, low swaps turn Stag Hunt into Anticipation (*Concord-Hunt*), and then No Conflict (*Concord*).

[26] Robinson and Goforth identified the Prisoner's Dilemma family, cyclic games, and battles of the sexes, as well as coordination (stag hunt) and no conflict (win-win) games. Categorization according to equilibrium payoffs adds biased, second-best, and unfair families, and includes sad games in a family with poor payoffs (Bruns 2010a).

[27] High swaps can be though of as connections in hyperspace, beyond the two dimensions of the flat display. These are more easily visualized with Prisoner's Dilemma scrolled into the outer corner, reuniting tiles, as in Figure 2d in the Appendix. One can envision vertical bands linked by blue bridges (for column high swaps), and horizontal bands linked by red tunnels (for row high swaps). The combinations of high swaps form the *hotspots* double-linking pairs of tiles, and *pipes* linking sets of four tiles, creating a complex weave of links.

Robinson and Goforth (2005) proved that the topology of the 2x2 ordinal games could be shown in three dimensions, on a torus with 37 holes. However, attempting to visualize this for links between individual games creates a tangled spaghetti of links. These links can be seen and explored in the online version of the Robinson and Goforth's periodic table (Goforth and Robinson 2009).

The visualization of bands linked by bridges and tunnels, as described above, is equivalent to adding two more dimensions to the toroidal layers formed by low and middle swaps: a blue dimension for column high swaps (bridges) and a red dimension for row high swaps (tunnels). This suggests that the structure of the topology, including the normalized versions of all 2x2 games, has an orderly arrangement in five dimensions. The projection of layers onto two dimensions, wrapping side-to-side and top-to-bottom, plus bridges and tunnels for high swaps, provides a three-dimensional way of approximately visualizing the links created by swap transformations in the payoff space of 2x2 games.

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