

# Asymmetric Payoff Mechanism and Information Effects in Water Sharing Interactions: A Game Theoretic Model of Collective Cooperation

R. Y. Wang<sup>1</sup>; C. N. Ng<sup>1</sup>; T. J. Zhao<sup>2</sup>

**Abstract:** General theoretic studies of collective action in common-pool resources (CPRs) dilemmas have been usually established on an assumption that actors all share symmetric access and position with regard to the commons. However, in real situations, most actors in a complex social-ecological system are heterogeneous in terms of their power, wealth, influence and so forth. The heterogeneity is also greatly attributed to diverse geographies, social hierarchies, skills, knowledge, and other features which are attached to those actors. Thus, analyses of internal and external asymmetric mechanisms are thus required to facilitate cooperation and achieve social optimum in CPR dilemmas. This paper uses surface water as an example to give a preliminary attempt to cope with the key issues mentioned above. We present an iterative N-Person Prisoner's Dilemma (PD) game theoretic model to quantitatively address the equilibrium conditions for collective cooperation in water sharing interactions. With a modified PD game payoff matrix and a simple evolutionary approach, asymmetric mechanism and information effects are incorporated into the model simultaneously. Numerical simulations are carried out in Matlab environment. The results demonstrate asymmetric mechanism leads to individual variations in terms of motivation to cooperate amongst different upstream and downstream actors. Tail-end actor is the one with least motivation to cooperate and up-midstream actors are more willing to cooperate. Moreover, higher level of asymmetries would increase upstream actors' incentives to cooperate yet downstream actors are barely affected by the asymmetric mechanism. "The shadow of the future" again is confirmed to increase the chances of universal cooperation, yet with the asymmetric mechanism, stronger motivation to cooperate is produced for downstream actors than upstream actors. The results also show greater level of information exchange relaxes overall equilibrium conditions, yet there is a negative effect on all actors' incentive to cooperate with the increase of total number of actors.

**Key words:** *game theory; asymmetries; modeling; collective cooperation; surface water*

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## Introduction

General theoretic studies of collective action in common-pool resources (CPRs) dilemmas have been usually established on an assumption that players all share symmetric access and position with regard to the commons (Janssen et al. 2010, Janssen and Rollins 2012, Janssen, Anderies, and Cardenas 2011, Lindahl 2012, Ostrom, Gardner, and Walker 1994, Ostrom and Gardner 1993). However, in real situations, most players in a complex social-ecological system are heterogeneous in terms of their power, wealth, influence and so forth (Du, Cao, and Hu 2009). The heterogeneities are also greatly attributed to diverse geographies, social hierarchies, skills, knowledge, and other features which are attached to those actors (Janssen and Rollins 2012). Beyond this, it has been noticed that such asymmetries, to a great extent, account for essential dynamics of the collective behavior in governing the CPRs. To facilitate cooperation and achieve better social outcome in CPR dilemmas, analysis of the elements that constitute the asymmetries are required to improve our understandings of the internal and external asymmetric mechanisms under which particular regularities might hold. In this paper, we will use surface water as an example to give a preliminary attempt to address the key issue mentioned above.

Surface water is a controversial CPR that flows across physical boundaries. In a river system, disparate users would be inevitably involved in conflicts of sharing water resources. Previous theoretical and experimental research suggests that asymmetries in terms of efficiency and equity would yield lower level of cooperation in CPR dilemmas (Tan 2008, Vandijk and Wilke 1995, Ahn et al. 2007). Yet various types of asymmetries exist among these actors in a complex water system. In particular, there has been lack of studies focusing on the asymmetric benefits and costs that are associated with the physical geographies of a river system. To put it more specifically, the relative upstream and downstream positions of actors have determined that their individual behaviors of conserving or appropriating water resources are by no means symmetrical to each other concerning their gains and losses. Knowledge of those spatial asymmetries would shed some light on how to design institutions that fit the characteristics of geographical contexts and further assist collective cooperation in coping with the CPR dilemma.

On the other hand, as much as the underlying structure of CPR dilemma might be globally analogous, more studies have suggested that various social-ecological variables might be able to substantially change the results and predictions of theoretic models (Janssen et al. 2010, Ostrom, Walker, and Gardner 1992, Sally 1995). There are diverse perspectives to study those variables. In this paper, we will focus on social reciprocity which refers to the repeated interactions between stakeholders involved in a social dilemma and their network relations such as reputation (Krippner and Alvarez 2007, Granovetter 1985, Raub and Weesie 1990, Raub, Buskens, and van Assen 2011). The rationale of studying social reciprocity lies in the fact that complex CPRs systems become increasingly diverse in terms of social interdependence and connectivity; and currently, more scholars and decision makers

are aware that the key approaches of coping with the CPR dilemma are, to a great extent, embedded in different scales of social networks and relations (Ostrom 2010a, b, c, Brondizio, Ostrom, and Young 2009, Young 2002). Therefore, we argue the effects of social reciprocity on collective action in CPR dilemmas warrants further study despite the volumes of previous literature which focus on behavior in economic exchange concerning the public good problem (Coleman 1988, Scott and Meyer 1994, Lindenberg 2001, 2006a, b, Coleman 1990, Raub, Buskens, and van Assen 2011, Axelrod and Hamilton 1981, Olson 1971, Granovetter 1985).

Combining the effects of asymmetric payoffs and social reciprocity simultaneously, this paper presents an iterative N-person Prisoner's Dilemma (PD) game theoretic model to quantitatively address the Nash Equilibrium conditions for collective cooperation in water sharing interactions. With a simple evolutionary approach, we allow every player in the game to respond to other players' behavior based on the information one has received. We incorporate the asymmetries into the model by modifying the standard PD game payoff matrix. The results demonstrate that the degree of asymmetries between upstream and downstream players would not only affect overall equilibrium conditions, but also alter different individual players' particularly the head-ender and the tail-ender's motivation in establishing cooperation; furthermore, in general a higher level of information exchange within the river system and a higher probability of future interactions would have positive effects on universal cooperation, whereas a larger group size might have negative effects on it. Various players would also be affected by these parameters in different ways, which will be discussed in later sections of the paper.

### The Model

The setup of the model is as follows. We adopt an iterative N-person PD game theoretic model of information effects to simulate the interactions among water users in a linear river system. A finite number of players ( $A_1, A_2 \dots A_i \dots A_n$ ) are located along the river in a fixed sequence and are assumed to make binary actions between Cooperate (C) and Defect (D) simultaneously. The actions are made on a discrete time scale ( $t = 1, 2, 3 \dots$ ) with a continuing probability  $0 < \beta < 1$ .

We first integrate the features of asymmetries into the model by modifying the standard PD game payoff matrix. We introduce an asymmetric parameter  $\alpha$  as indicated in Figure 1,

		$j$	
		C	D
$i$	C	$2R\alpha_{i,j}^i, 2R\alpha_{i,j}^j$	$2S\alpha_{i,j}^i, 2T\alpha_{i,j}^j$
	D	$2T\alpha_{i,j}^i, 2S\alpha_{i,j}^j$	$2P\alpha_{i,j}^i, 2P\alpha_{i,j}^j$

$T > R > 0 > P > S; 2R > T + S$

Figure 1. Asymmetric Payoff Matrix of Prisoner's Dilemma Game

where  $\alpha_{i,j}^i = \frac{i^\varphi}{i^\varphi + j^\varphi}$ ,  $\alpha_{i,j}^j = \frac{j^\varphi}{i^\varphi + j^\varphi}$ ,  $\alpha_{i,j}^i + \alpha_{i,j}^j = 1$ ,  $1 \leq i < j \leq n$ ,  $i, j \in \mathbb{Z}^+$ . Note that the asymmetric parameter  $\alpha$  is adjusted by the relative geographical distribution of any two players  $i$  and  $j$ , as well as the exploitation parameter  $\varphi$ . The parameter  $\varphi$  represents the degree to which one player differentiates from another player in terms of utilities. More importantly, the essential feature of PD game ( $T > R > P > S$ ) is still strictly satisfied for all  $\alpha$  that have been introduced in the model.

In consistent with the ecological characteristics of a river system, we assume that  $\varphi > 0$  based on two main reasons. Firstly, it is natural to consider the relative downstream player  $A_j$  could obtain more benefits than the upstream player  $A_i$  due to either player's contribution to conserve the river system. Secondly, it is, however, also reasonable to consider the downstream player  $A_j$  could receive more damages than the upstream player  $A_i$  due to either player's defection to pollute the river system. As  $i$  and  $j$  represent the relative position of any two players in the game, we can notice that the more distant the two players are, the larger difference it caused on their respective utilities. In addition, the larger the parameter  $\varphi$  is, the greater effects on utilities are produced given the spatial asymmetries among players. Accordingly, we can address the utility function for any player  $A_i$  as follows,

$$U_{it} = f(V_t, \alpha_{i,j}^i) = \sum_j^{V_t} 2R\alpha_{i,j}^i + \sum_j^{N-V_t-A_i} 2S\alpha_{i,j}^i \quad \text{if } s_{it} = C \quad \text{Eq. 1}$$

$$U_{it} = g(V_t, \alpha_{i,j}^i) = \sum_j^{V_t} 2T\alpha_{i,j}^i + \sum_j^{N-V_t-A_i} 2P\alpha_{i,j}^i \quad \text{if } s_{it} = D \quad \text{Eq. 2}$$

where  $U_{it}$  denotes the utility of  $A_i$  at moment  $t$ ,  $s_{it}$  denotes the action of  $A_i$  at moment  $t$ .  $j \in V_t$ ,  $V_t$  represents a set of players who choose C apart from the focal player  $A_i$  at moment  $t$ .  $V_t$  is a subset of the whole set  $N = \{A_1, A_2, \dots, A_n\}$  which is a profile of all players in the game. Since we assume the event continues with a probability  $0 < \beta < 1$  in the indefinitely iterated game, therefore, the total utility  $\bar{U}_i$  that any player  $A_i$  could receive during the entire game is,

$$\bar{U}_i = \sum_{t=1}^{\infty} \beta^{t-1} U_{it} \quad \text{Eq. 3}$$

where  $\bar{U}_i$  is the exponentially discounted utility sum of  $A_i$  from  $t = 1$  till the indefinite end of the game.

After establishing the model, our next objective is to derive the conditions for Nash Equilibrium in which collective cooperation is selected. To achieve the objective, it involves two more procedures. One is to assign supergame strategy to all the players. Based on non-cooperative game theory, collective cooperation could be the outcome of individual rationality if cooperation for any player in all their interactions is the best strategy against each other (Nash 1951, Axelrod and Hamilton 1981). Yet there exist countless conditional cooperative supergame strategies as the game is indefinitely repeated. Also, the condition for universal cooperation varies with the supergame

strategies that are adopted by each player in the game. In our model we only focus on a classic conditional supergame strategy “Tit for Tat” (TFT). It should be noted that TFT has different definitions in an N-person game based on how a player interprets “defection”. To simplify our analysis, we adopt a very strict definition, that is, any player  $A_i$  enters the game with C and cooperate against all the other players at each round if he has no information that any other player defected in previous rounds; but if  $A_i$  receives information that at least one player has defected in previous rounds, then  $A_i$  chooses D as his action in the next round. The TFT strategy allows every player to response to other players’ behavior in every event moment. We assume all players use strict TFT in the model.

The other procedure to derive the necessary and sufficient conditions for cooperative Nash Equilibrium is to analyze the model in three different information scenarios. Information is one of the key elements of social reciprocity in the sense of generating reputation which might indirectly affect players’ long-term utilities. The three scenarios are defined based on assumptions regarding the information that is available to all the players, namely atomized interaction, perfectly embedded interactions and imperfectly embedded interactions. The effects of information on collective cooperation will be respectively examined in these three scenarios.

### Scenario One: Atomized Interactions

We begin with the simplest scenario. Assuming that the information any actor  $A_i$  can possibly receive is only from the actors who are located adjacent to him ( $A_{i+1}$  and  $A_{i-1}$ ). This information is assumed to be received right after an action is committed by  $A_{i+1}$  and  $A_{i-1}$  at moment  $t$ . Under this specific assumption, interactions are atomized in the sense that an actor only gets information from nobody else but his contiguous actors.

According to the definition of Nash Equilibrium, it is shown that either ALL-C (always play cooperation or any other strategy that never initiates a defection) or ALL-D (always play defection) is a best-response strategy should all other actors use strict TFT (Raub and Weesie 1990, Friedman 1977). Thus the conditions under which collective cooperation is established must satisfy,

$$E(\bar{U}_i | ALL - D) \leq E(\bar{U}_i | ALL - C) \quad \text{Eq. 4}$$

that is, for any player  $A_i$  his expected payoffs of using ALL-C is always no smaller than his expected payoffs of using ALL-D. Solving the inequality, we then have the necessary and sufficient condition for universal cooperative Nash Equilibrium in atomized interactions. We refer to the Appendix for outlines of the proof.

$$\gamma \leq \frac{\beta(\alpha_{i,i-1}^i + \alpha_{i,i+1}^i) + \beta^2(\alpha_{i,i-2}^i + \alpha_{i,i+2}^i) + \dots + \beta^{n-i-1}(\alpha_{i,2i-n+1}^i + \alpha_{i,n-1}^i) + \beta^{n-i}(\alpha_{i,2i-n}^i + \alpha_{i,n}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i} \quad \text{if } i \leq \lfloor \frac{n}{2} \rfloor$$

$$\gamma \leq \frac{\beta(\alpha_{i,i-1}^i + \alpha_{i,i+1}^i) + \beta^2(\alpha_{i,i-2}^i + \alpha_{i,i+2}^i) + \dots + \beta^{i-2}(\alpha_{i,2}^i + \alpha_{i,2i-2}^i) + \beta^{i-1}(\alpha_{i,1}^i + \alpha_{i,2i-1}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i} \quad \text{if } i > \lfloor \frac{n}{2} \rfloor$$

It should be noted that in all the expressions  $\alpha_{i,j}^i = 0$  if  $j > n$  or  $j < 1$ . We define  $\gamma$  as

the costs of cooperation (T-R) divided by the costs of defection (T-P),  $\gamma = (T - R)/(T - P)$ , where  $0 < \gamma < 1$  and  $\gamma$  is an invariant under the given payoff matrix. Basically,  $\gamma$  is an indicator of any player  $A_i$ 's short-term incentive for defection.

### Scenario Two: Perfectly Embedded Interactions

We then relax the first assumption by introducing a system which allows a player to obtain information from all the other actors rather than just his neighbors. It is also assumed that the information is perfectly embedded in the system. It implies that every actor receives full information about all the other actors' behaviors immediately after an action is made.

Similar procedure applies when we solve the inequality 4. Then the necessary and sufficient condition for cooperative Nash Equilibrium in perfectly embedded interactions is as follows.

$$\gamma \leq \beta$$

### Scenario Three: Imperfectly Embedded Interactions

Either atomized or perfectly embedded interactions represent a relatively extreme situation of information exchange. Lastly, we introduce a more realistic assumption under which information is partly or imperfectly informed to all the actors. In particular, for any player  $A_i$ , it is still assumed that information can be immediately received from his contiguous actors. Meanwhile, he can also obtain information about the behavior of any other actor  $A_j$  ( $i \neq j$ ), but only after a certain time lag  $\pi_{ij}$ , which increases with the distance between actors  $A_i$  and  $A_j$ . We have to apply more complex procedures to solve the inequality 4 as a new parameter  $\pi_{ij}$  is introduced. On the other hand, it is worth noting that scenario one and two could also be interpreted as two extreme cases of scenario three. That is, when  $\pi_{ij} \rightarrow \infty$ , the information among distant actors travels so slow that the case is the same as atomized interaction; when  $\pi_{ij} \rightarrow 0$ , the information travels so fast that everyone will immediately be aware of the history of all the other actors as in perfectly embedded interactions. The necessary and sufficient conditions for cooperative Nash Equilibrium in imperfectly embedded interactions are as follows.

$$\text{if } i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n - i = 2m, m \in \mathbb{Z}^+$$

$$\gamma \leq$$

$$\frac{\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{\frac{n-i}{2}-1}(\alpha_{i,2i-n+2}^i + \alpha_{i,2i-n+3}^i + \alpha_{i,n-3}^i + \alpha_{i,n-2}^i) + \beta^{\frac{n-i}{2}}(\alpha_{i,2i-n}^i + \alpha_{i,2i-n+1}^i + \alpha_{i,n-1}^i + \alpha_{i,n}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

$$\text{if } i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n - i = 2m - 1, m \in \mathbb{Z}^+$$

$$\gamma \leq$$

$$\frac{\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{\frac{n-i}{2}-1}(\alpha_{i,2i-n+1}^i + \alpha_{i,2i-n+2}^i + \alpha_{i,n-2}^i + \alpha_{i,n-1}^i) + \beta^{\frac{n-i}{2}}(\alpha_{i,2i-n-1}^i + \alpha_{i,2i-n}^i + \alpha_{i,n}^i + \alpha_{i,n+1}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

$$\text{if } i > \left\lfloor \frac{n}{2} \right\rfloor \text{ and } i = 2m - 1, m \in \mathbb{Z}^+$$

$$\gamma \leq$$

$$\frac{\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{\frac{i-1}{2}-1}(\alpha_{i,3}^i + \alpha_{i,4}^i + \alpha_{i,2i-4}^i + \alpha_{i,2i-3}^i) + \beta^{\frac{i-1}{2}}(\alpha_{i,1}^i + \alpha_{i,2}^i + \alpha_{i,2i-2}^i + \alpha_{i,2i-1}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

if  $i > \left\lfloor \frac{n}{2} \right\rfloor$  and  $i = 2m, m \in \mathbb{Z}^+$

$$\gamma \leq \frac{\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{\frac{i}{2}-1}(\alpha_{i,2}^i + \alpha_{i,3}^i + \alpha_{i,2i-3}^i + \alpha_{i,2i-2}^i) + \beta^{\frac{i}{2}}(\alpha_{i,0}^i + \alpha_{i,1}^i + \alpha_{i,2i-1}^i + \alpha_{i,2i}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

## Results

The expressions of the equilibrium conditions are rather complex, yet there are still a few important regularities that should be noted. Firstly, all of the three equilibrium conditions are inequalities consisting of the “temptation to defect” on the left and a function on the right. For later reference, we define the right side function as

$f_a$  (atomized interactions),  $f_p$  (perfectly embedded interactions) and  $f_{im}$  (imperfectly

embedded interactions). The values of the three functions are denoted as  $V_a$ ,  $V_p$  and

$V_{im}$  respectively. Thus the equilibrium conditions could be translated into the following mathematical expressions.

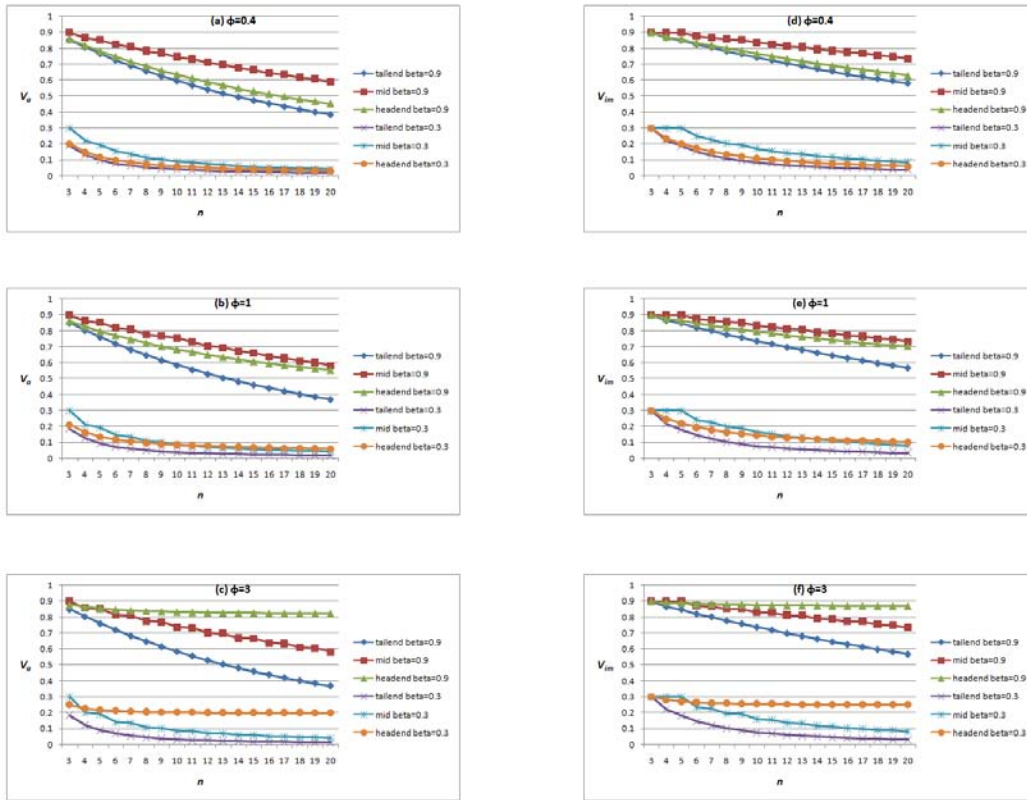
$$\gamma \leq V_a = f_a(n, \beta, \varphi, i) \quad \text{Eq. 5}$$

$$\gamma \leq V_p = f_p(\beta) \quad \text{Eq. 6}$$

$$\gamma \leq V_{im} = f_{im}(n, \beta, \varphi, i) \quad \text{Eq. 7}$$

Secondly, the equilibrium condition under the assumption of perfect information is only dependent on the discount parameter  $\beta$  regardless of the asymmetries among diverse players and the total number of players. Thirdly, the newly introduced parameter  $\alpha$ , along with the discount parameter  $\beta$  and the total number of players  $n$  are all critical elements in both atomized and imperfectly embedded interactions.

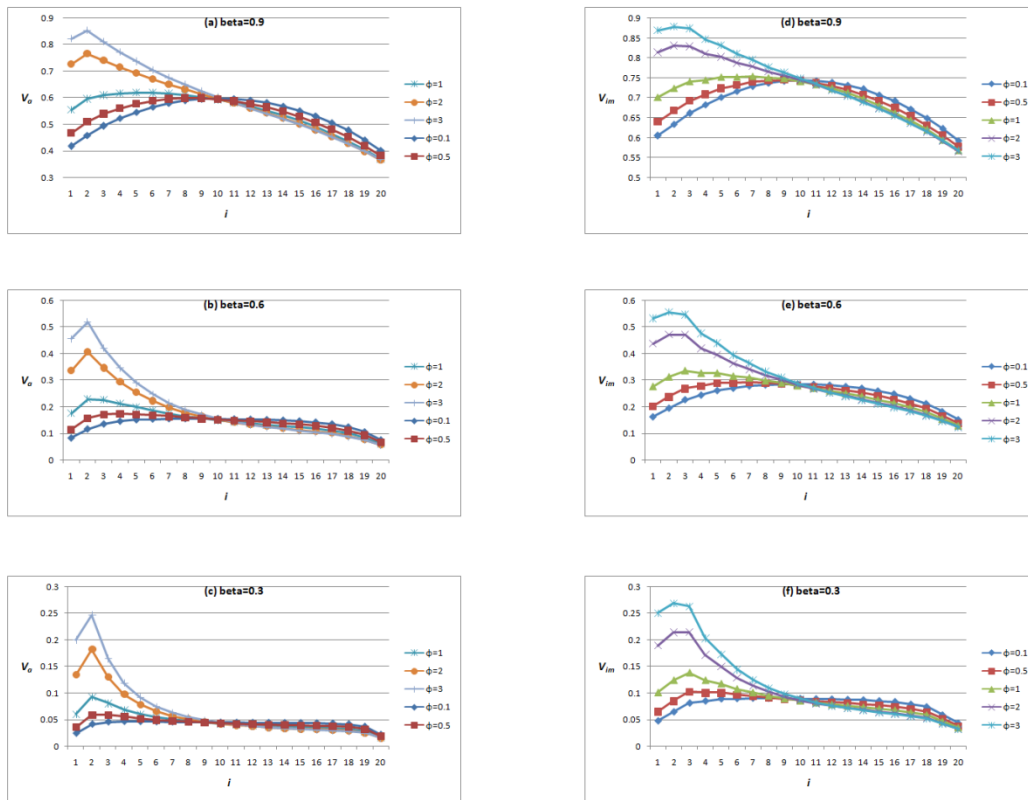
The goal of our analysis is to understand how asymmetric payoffs and information affect players' behavior in collective cooperation based on the Nash Equilibrium conditions derived in the last section. However, as the four parameters  $n, \beta, \varphi$  and  $i$  affect the value of the functions concurrently, it is impracticable to establish explicit understandings of the effects of these independent parameters. In other words, we cannot intuitively predict how  $V_a$  and  $V_{im}$  changes if either of the four parameters varies. To obtain a better insight into the functions, we performed numerical simulations in Matlab. The simulations were designed to examine  $V_a$  and  $V_{im}$  under circumstances when parameters  $n$  varies from 3 to 20 at a 1 interval,  $\beta$  varies from 0.01 to 0.99 at a 0.01 interval,  $\varphi$  varies from 0.1 to 3 at a 0.1 interval and  $i$  varies from 1 to  $n$  at a 1 interval.



**Figure 2.** The trend of  $V_a$  and  $V_{im}$  as the total number of players  $n$  increases.

We first examine the effects of parameter  $n$ , then for simplicity, our analysis in this paper will only focus on how the variations of  $\beta$ ,  $\varphi$  and  $i$  affect  $V_a$  and  $V_{im}$  when  $n$  equals to 20. It is worth noting that the larger  $V_a$  and  $V_{im}$  are, the more likely the focal player could voluntarily cooperate in the game. Figure 2 indicates the trend of  $V_a$  and  $V_{im}$  for the headend, tailend and midstream players as the total number of players  $n$  increases from 3 to 20 while  $\varphi$  equals to 0.4, 1 and 3. Each subplot in Figure 2 also compares the value of  $V_a$  and  $V_{im}$  for the above mentioned three players when  $\beta$  equals to 0.3 and 0.9. In general, the results demonstrate a descending trend of  $V_a$  and  $V_{im}$  in most scenarios, which confirmed Olson's influential argument about "the logic of collective action" which states that the larger a group is, the less likely they are to create social incentives that lead its members to provide collective goods (Olson 1971). Nonetheless, as shown in Figure 2 (c) and Figure 2(f), we also discovered that the headend player's incentive to cooperate is hardly affected by group size when a high level of asymmetries ( $\varphi = 3$ ) exists during the course of interactions.



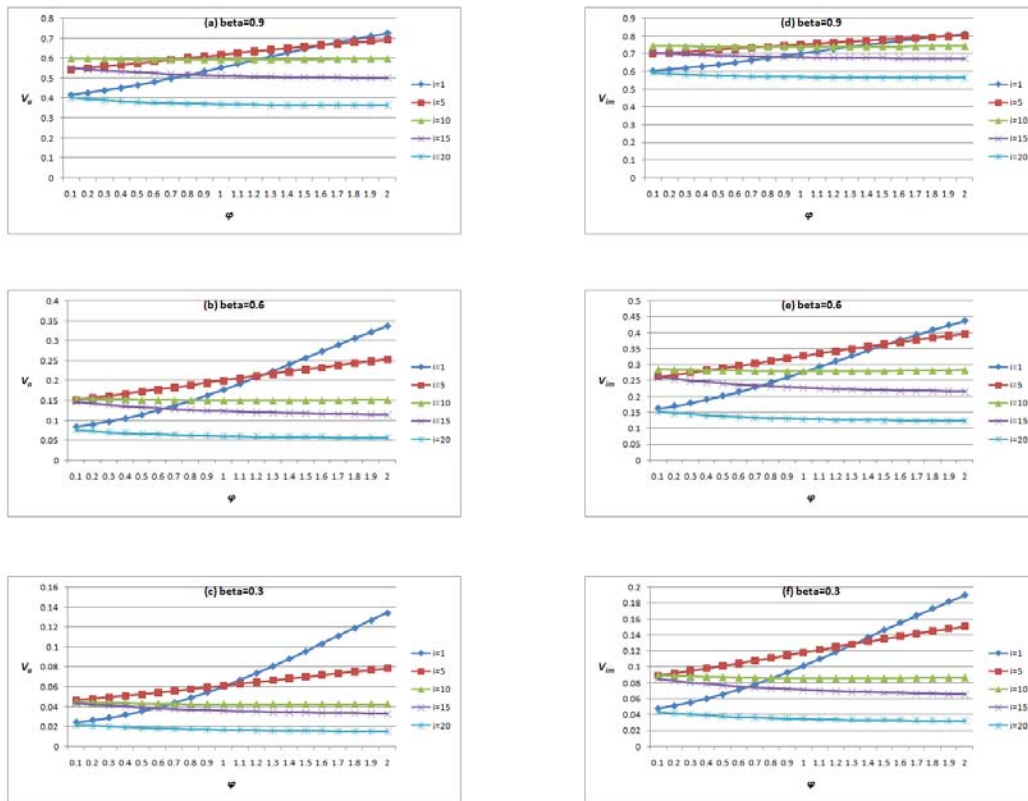


**Figure 3.**  $V_a$  and  $V_{im}$  for different upstream and downstream players as  $\beta$  is fixed

Figure 3 indicates the value of  $V_a$  and  $V_{im}$  for different upstream and downstream players while  $\beta$  equals to 0.3, 0.6 and 0.9 in atomized and imperfectly embedded interactions respectively. Each sub-image in Figure 3 also compares how upstream and downstream players react to variations of the levels of asymmetric payoff mechanism. A few remarks can be drawn from this figure. 1) The value of  $V_a$  and  $V_{im}$  increases with  $i$  at the beginning and then decreases after  $V_a$  reaches its apex. Although the position of apex varies with  $\phi$  and  $\beta$ , it implies relative up-midstream players are more likely to cooperate than the other players when other parameters are invariant; 2) The curves in each sub-image became more steep as  $\phi$  increases. It shows that greater individual differences exist in games with higher level of asymmetries. 3) It is shown from the figure that  $V_a$  and  $V_{im}$  reach their bottoms at the last point, which implies the tailend actor is meanwhile the one who has least motivation to cooperate.

Figure 4 indicates the trend of  $V_a$  and  $V_{im}$  for a particular player as  $\phi$  increases when  $\beta$  equals to 0.3, 0.6 and 0.9 in atomized and imperfectly embedded interactions respectively. Figure 4 also compares the value of  $V_a$  and  $V_{im}$  for five different upstream and downstream players when  $\beta$  is constant. A few important remarks can be also drawn from this figure. 1) In general, the value of  $V_a$  and  $V_{im}$  for downstream players is smaller than upstream players which implies the former is less likely to cooperate than the latter. The tailend player is meanwhile the one who

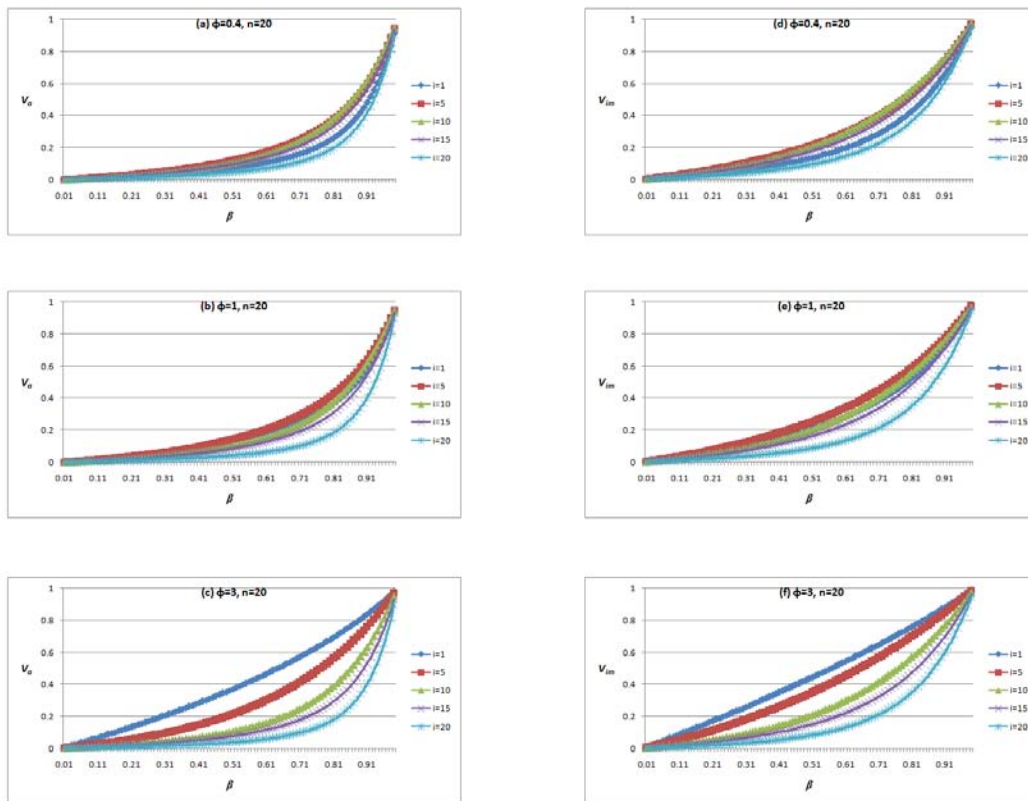
has least motivation to cooperate; 2) The values of  $V_a$  and  $V_{im}$  for upstream players increase with  $\varphi$ , yet for downstream players they decrease slightly with  $\varphi$  and tend to stabilize though  $\varphi$  increases. It implies upstream players are sensitive to the degree of asymmetries and more likely to cooperate when higher level of asymmetries became manifest. Nevertheless the asymmetries have little effect on downstream players in terms of their motivation for cooperation.



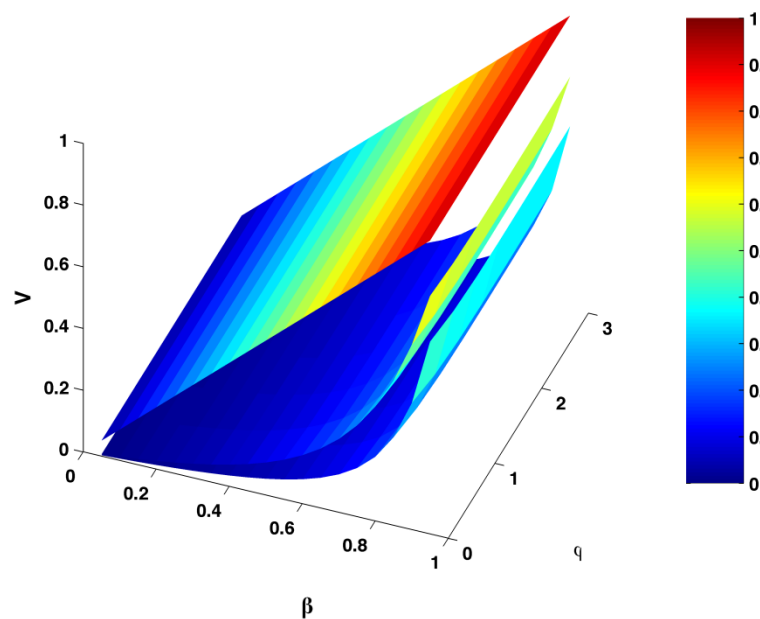
**Figure 4.** The trend of  $V_a$  and  $V_{im}$  for five particular players as  $\varphi$  increases

Figure 5 indicates the value of  $V_a$  and  $V_{im}$  for a particular player as  $\beta$  increases while  $\varphi$  equals to 0.4, 1 and 3. Figure 5 also compares the value of  $V_a$  and  $V_{im}$  for five different upstream and downstream players when  $\varphi$  is constant. The value of  $\beta$  is generally referred as “the shadow of the future” which indicates the possibility of future interactions between all involved players. Hence, the larger  $\beta$  is the more likely the game will continue. According to Figure 5 we can draw the following remarks. 1) For a particular player, the value of  $V_a$  increases with  $\beta$  when the degree of asymmetries is constant. It suggests that a player is more likely to cooperate when “the shadow of the future” is significant; 2) the slope of the curves for downstream players became more steep than upstream players when  $\beta$  is relatively large; the situation is reversed when  $\beta$  is relatively small. It implies that downstream players’ motivation for cooperation increase faster than upstream players when there is a greater possibility that future interactions will remain to happen. To the contrary,

upstream players are more motivated to cooperate than downstream players even when there is greater chance that the game could end soon.



**Figure 5.** The value of  $V_a$  and  $V_{im}$  for five particular players as  $\beta$  increases



**Figure 6.** A comparison of equilibrium conditions for universal cooperation under three scenarios

After analyzing the effects of each parameter individually, in Figure 6 we display the equilibrium conditions for all three scenarios in a single three dimensional graph. The X, Y and Z axis represent  $\beta$ ,  $\varphi$  and  $V$  respectively. This figure examines information effects on collective cooperation by comparing the equilibrium conditions. The equilibrium condition in perfectly embedded interactions is only dependent on  $\beta$  and thus is a flat surface on the graph regardless of spatial asymmetries. The flat surface is above the other two curved surfaces which the latter represent imperfectly embedded and atomized interactions. Meanwhile, the asymmetric payoff mechanism has slightly changed the shape of the curved surfaces, and yet importantly, it does not change the fact that the conditions for universal cooperation become less restrict when information becomes better exchanged in the model. The result once again reveals the significance of information on collective action in dilemma situations. To sum up, the results suggest significant individual variations in terms of motivation to cooperate amongst different upstream and downstream players. With the increase of total number of actors, there is a negative effect on all actors' willingness to cooperate. More specifically, with the asymmetric payoff mechanism in the linear system, we have a counter intuitive conclusion that the tailend player becomes the one with least motivation to cooperate and the up-midstream players are the ones who are most willingly to cooperate. Moreover, our numerical simulation shows that greater level of asymmetries would increase upstream players' incentives to cooperate yet downstream players are barely affected by the degree of asymmetries pertaining to the conditions for their cooperation. With regard to the effects of "the shadow of the future", it is suggested that greater probability of future interactions would increase all players' willingness to cooperate, yet with the asymmetric payoff mechanism, stronger motivation to cooperate is produced for downstream players than upstream players. Last but not least, it is shown that greater levels of information exchange relaxes the overall equilibrium conditions for universal cooperation among multiple heterogeneous actors, yet

## **Discussions**

Successful management of complex common-pool resources (CPRs) systems requires high levels of cooperation. As human society becomes increasingly interdependent and hierarchical, such successes, to a great extent, are attributed to mechanisms which enable various stakeholders to cooperate when they are heterogeneous with regard to their social-ecological statuses. In this paper we investigate the effects of asymmetric payoff mechanism and information effects simultaneously. We establish a quantitative model of iterative N-person Prisoner's Dilemma game and study the game from an evolutionary perspective. Under different information scenarios, our analysis produces mathematical equilibrium condition for universal cooperation in each scenario and shows the dynamics of interactions between multiple players who obtain asymmetric payoffs during the course of the game.

In general, our model reveals how individual heterogeneity affects cooperative behavior of stakeholders who are embedded in different social-economic categories. This paper is a preliminary theoretical attempt to connect asymmetric mechanisms with various stakeholders' collective action in sharing limited surface water resources. In this paper, we would not, and do not intend to conclude with a deterministic argument about the causal relationship between individual asymmetries and collective cooperation in water governance. A lot more theoretical models and empirical case studies remain to be conducted. However, we do expect to shed a light on the significance of economic asymmetries among various stakeholders. With the increase of global changes and diversification of individual interests, it is critical that players with heterogeneous attributes are coordinated by varied institutions which are suitable for the characteristics of these players. Likewise, successful reform of water resources management should be made base on better knowledge of the asymmetries that are embedded in complex water systems.

In addition, the implications of asymmetries and social reciprocity are not only restricted to water resources, for many complex CPR systems, it has been also occurred to researchers that mismatches between the essential features of a CPR system and its institutional context are accountable for negative externalities and inefficient governance of the CPRs. Therefore, successful CPR management requires a higher level of fit, not only between the physical geographies of the natural resources systems and the institutions with which they interact, but also between the associated institutions and heterogeneous social norms, economies, politics and culture that are attached to the system at various scales (Moss 2012, Johnson et al. 2012, Young 2002). Addressing the problem of fit itself is accompanied with both theoretical and empirical challenges. Yet regardless of the complexity and challenges, recent research has reached a consensus that more subtle understandings of the dynamics of human-nature interactions and substantial mechanism of fit rather than deterministic approaches are required to better address misfits in CPRs management (Moss 2012, Johnson et al. 2012, Folke et al. 2007, Folke et al. 2005). To design certain institutional arrangements that are better suitable for the physical and social-economic characteristics of a CPR system, scholars, practitioners and decision makers need to better understand the dynamics of interactions occur in the system and grasp the internal and external mechanisms under which particular social phenomena might take place. The asymmetries embedded in stakeholders at various levels are important elements that make up these critical mechanisms of collective action in governing limited CPRs, and thus merits further investigation in future studies.

There are a number of approaches that could enrich the study we initiated. Firstly, researchers could investigate variations of the presented model by introducing alternative games, supergame strategies and information conditions. As a result, more subtle nuances of the effects of asymmetries and social embeddedness could

be examined and further understood in theory. Secondly, there is still a wide gap between the predictions we generated from theoretical models and empirical cases in real situations. Arguably, we can offer some possible explanations for the results of the model. For instance, pertaining to motivation for cooperation, the results suggest upstream players are more sensitive to the asymmetries and downstream players are more sensitive to the probability of future interactions. An intuitive justification for the outcome might be that downstream players as the explorer need a more secure environment of long term interactions to cooperate since they bare relative larger risks once their cooperation is defected; whereas upstream players are more concern about the inequalities therefore would show up nice gestures to win cooperation from downstream players because they would not lose too much even if their trust is defected. Nevertheless, it would substantially complement our research if laboratory experiments and empirical cases studies could be carried out to test and adjust the theories produced by the models. Last but not least, there remains a significant research gap in asymmetries and social embeddedness that warrants further studies despite the volumes of previous literature. For example, many forms of asymmetries such as access to information, access to resources, eco-political influences still need further exploration. In addition, it is also of significant importance to extend the study on asymmetries from surface water system to other CPR systems in which distinctive social-ecological network structure would function.

## Appendix

The appendix summarizes the proofs and calculating procedures of the three theorems discussed above. Followed by the deduction in section 3, the critical step to derive the equilibrium conditions is to solve the following inequality for any player  $A_i$  under the three different scenarios (Wang, Ng, and Buskens 2012).

$$E(\bar{U}_i|ALL - D) \leq E(\bar{U}_i|ALL - C)$$

A. 1

One established formula will be used in following deductions.

$$S = a_1 + \beta a_1 + \beta^2 a_1 + \dots = \frac{a_1}{1-\beta}$$

A. 2

Where  $a_1$  is an invariant real number and  $0 < \beta < 1$

With the asymmetric payoff settings, obviously each player will receive different utility in each round. Focusing on the right side of the inequality, however, C will be chosen by all of the players throughout the game as no player will ever initiate a defection according to their supergame strategy (ALL-C or TFT). Hence for any player  $A_i$ , based on the utility function in section 2, his expected payoff of using ALL-C against all the other players who stick with TFT will be identical as presented in A.3 under all three assumptions regardless of their information situations.

$$E(\bar{U}_i|ALL - C) = \frac{2R}{1-\beta} (\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i)$$

Yet player  $A_i$  will receive different expected payoff if he uses ALL-D as his supergame strategy under different assumptions; because, other players who use TFT will response to defection in different ways based on the information they receive. Different responses will further affect the utility that  $A_i$  could receive in each event moment. Therefore, we will compute the expected payoff of  $A_i$  when he uses ALL-D under all the three different assumptions respectively. Then we will derive the equilibrium conditions by putting  $E(\bar{U}_i|ALL - C)$  and  $E(\bar{U}_i|ALL - D)$  back to the inequality A.1. Please note in all the following calculations that  $\alpha_{i,j}^i = 0$  if  $j > n$  or  $j <$

1. Some situations in which  $j > n$  or  $j < 1$  may occur in order to keep the expressions of theorems more consistent and in a more regular pattern.

### Assumption One: Atomized interactions

In this scenario, it is assumed that any player  $A_i$  could only receive information from his contiguous players  $A_{i-1}$  and  $A_{i+1}$ . The information is expected to be received right after an action is committed. It implies that if  $A_i$  initiates a defection in the game at moment 1, then  $A_{i-1}$  and  $A_{i+1}$  will be aware of the defection and start to defect at moment 2. Players  $A_{i-2}$  and  $A_{i+2}$  will realize the defection of  $A_{i-1}$  and  $A_{i+1}$ , and then start to defect at moment 3, and so forth. In general, once a defection emerges, it will diffuse through contiguity in a linear atomized network towards both upstream and downstream directions. Considering the relative geographical location of  $A_i$  will affect the equilibrium condition, the calculation will be carried out in two situations in which  $A_i$  is located either in relative upstream ( $i \leq \lfloor n/2 \rfloor$ ) or downstream ( $i > \lfloor n/2 \rfloor$ ) of a river.

(1) If  $i \leq \lfloor n/2 \rfloor$ , then it takes longer time for the defection diffuses towards relative downstream players than upstream players. Apparently, player  $A_n$  will be the last one who realizes another player had defected in an earlier round.  $A_n$  will start to defect at moment  $n - i + 1$ , and since then all players will defect afterwards. Therefore, we can divide the total expected utility of  $A_i$  into two parts on the basis of whether cooperative behavior still exists in the interactions.

$$E(\bar{U}_i|ALL - D) = \sum_{t=1}^{\infty} \beta^{t-1} U_{it} = \sum_{t=1}^{n-i} \beta^{t-1} U_{it} + \sum_{t=n-i+1}^{\infty} \beta^{t-1} U_{it}$$

A. 4

For part one,

$$\begin{aligned} \sum_{t=1}^{n-i} \beta^{t-1} U_{it} = & 2T(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i) + \beta[2T(\sum_{j=1}^{i-2} \alpha_{i,j}^i + \sum_{j=i+2}^n \alpha_{i,j}^i) + 2P(\sum_{j=i-1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{i+1} \alpha_{i,j}^i)] + \\ & \beta^2[2T(\sum_{j=1}^{i-3} \alpha_{i,j}^i + \sum_{j=i+3}^n \alpha_{i,j}^i) + 2P(\sum_{j=i-2}^{i-2} \alpha_{i,j}^i + \sum_{j=i+2}^{i+2} \alpha_{i,j}^i)] + \dots + \beta^{n-i-2}[2T(\sum_{j=1}^{2i-n+1} \alpha_{i,j}^i + \sum_{j=n-1}^n \alpha_{i,j}^i) + \\ & 2P(\sum_{j=2i-n+2}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-2} \alpha_{i,j}^i)] + \beta^{n-i-1}[2T(\sum_{j=1}^{2i-n} \alpha_{i,j}^i + \sum_{j=n}^n \alpha_{i,j}^i) + 2P(\sum_{j=2i-n+1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-1} \alpha_{i,j}^i)] \end{aligned}$$

Times  $\beta$  on each side of the equation and with some simple algorithm we will have

$$(1 - \beta) \sum_{t=1}^{n-i} \beta^{t-1} U_{it} = 2T(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i) + 2(P - T)[\beta(\alpha_{i,i-1}^i + \alpha_{i,i+1}^i) + \beta^2(\alpha_{i,i-2}^i + \alpha_{i,i+2}^i) + \dots + \beta^{n-i-1}(\alpha_{i,2i-n+1}^i + \alpha_{i,n-1}^i)] - \beta^{n-i}[2T(\sum_{j=1}^{2i-n} \alpha_{i,j}^i + \sum_{j=n}^n \alpha_{i,j}^i) + 2P(\sum_{j=2i-n+1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-1} \alpha_{i,j}^i)]$$

A. 5

For part two,

$$\sum_{t=n-i+1}^{\infty} \beta^{t-1} U_{it} = \frac{\beta^{n-i} 2P(\sum_{j=2i-n}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i)}{1-\beta}$$

A. 6

Combining A.5, A6 and put them back to A.1 with A.3, we will have

$$\frac{T-R}{T-P} \leq \frac{\beta(\alpha_{i,i-1}^i + \alpha_{i,i+1}^i) + \beta^2(\alpha_{i,i-2}^i + \alpha_{i,i+2}^i) + \dots + \beta^{n-i-1}(\alpha_{i,2i-n+1}^i + \alpha_{i,n-1}^i) + \beta^{n-i}(\alpha_{i,2i-n}^i + \alpha_{i,n}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

A. 7

(2) If  $i > |n/2|$ , then it takes longer time for the defection to diffuse towards relative upstream players than downstream players. In this case, player  $A_1$  will be the last player who realizes another player had defected in an earlier round. Player  $A_1$  will start to defect at moment  $i$ , and since then all players will defect afterwards. Likewise, we divide the total expected utility of  $A_1$  into the following two parts.

$$E(\bar{U}_i | ALL - D) = \sum_{t=1}^{\infty} \beta^{t-1} U_{it} = \sum_{t=1}^{i-1} \beta^{t-1} U_{it} + \sum_{t=i}^{\infty} \beta^{t-1} U_{it}$$

A. 8

Without repeating a similar calculation as in A.5 and A.6, we will have the following equilibrium condition.

$$\frac{T-R}{T-P} \leq \frac{\beta(\alpha_{i,i-1}^i + \alpha_{i,i+1}^i) + \beta^2(\alpha_{i,i-2}^i + \alpha_{i,i+2}^i) + \dots + \beta^{i-2}(\alpha_{i,2}^i + \alpha_{i,2i-2}^i) + \beta^{i-1}(\alpha_{i,1}^i + \alpha_{i,2i-1}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

A. 9

### Assumption Two: Perfectly Embedded Interactions

In this scenario, it is assumed that any player  $A_i$  could receive information from all of the other players right after an action is committed. It implies that if  $A_i$  initiates a defection in the game at moment 1, then all the other players will be aware of the defection immediately and start to defect at moment 2. Hence, it is easy to calculate the expected utility of  $A_i$  when he uses ALL-D against all the other players who use TFT under a perfect information scenario.

$$E(\bar{U}_i | ALL - D) = \sum_{t=1}^{\infty} \beta^{t-1} U_{it} = 2T(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i) + \frac{2P\beta(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i)}{1-\beta}$$

A. 10

Put A.10 and A.3 back to A.1 we will have



$$\frac{T-R}{T-P} \leq \beta$$

### Assumption Three: Imperfectly Embedded Interactions

In this scenario, it is assumed that any player  $A_i$  could receive information from all of the other players, but only after a certain time lag  $\pi_{ij}$  which is dependent on the geographical distance between any of the two players  $A_i$  and  $A_j$ . It is still assumed that  $A_i$  could only receive information from his contiguous players  $A_{i-1}$  and  $A_{i+1}$  immediately. This assumption implies that if  $A_i$  initiates a defection in the game at moment 1, then all the other players will be aware of the defection sooner or later and start to defect upon their receipt of the defective information. Clearly, the longer time it takes for information to transfer, far fewer players could have been notified that the defection was made in earlier rounds, and vice versa. Moreover, Assumption One & Two could also be interpreted as two extreme cases of Assumption Three. That is, when  $\pi_{ij} \rightarrow \infty$ , the information among distant players travels so slow that the case will be the same as atomized interaction; when  $\pi_{ij} \rightarrow 0$ , the information travels so fast that everyone will immediately be aware of the history of all the other players as in perfectly embedded interactions.

Although information might travel at a different speed under Assumption Three, to find an equilibrium condition, we only need to concentrate on a scenario in which player  $A_i$  could gain maximum expected utility if he uses ALL-D against all of the other players who use TFT. Obviously, this scenario should allow information travel as slowly as possible given the constraints of Assumption Three; because, it will give  $A_i$  the most short-term benefits before the whole game turns into a situation with universal defection. More specifically, it implies that if  $A_i$  initiates a defection at moment 1, then  $A_{i-1}$  and  $A_{i+1}$  will be aware of the defection and start to defect at moment 2 as they are  $A_i$ 's neighbors; besides that, players  $A_{i-2}$  and  $A_{i+2}$  will also receive the information and start to defect at moment 2 as information is better embedded in this scenario than in atomized interactions and they are geographically closer to  $A_i$  than other players. Yet  $A_{i-2}$  and  $A_{i+2}$  are the only two more players who can receive the defective information so that  $A_i$  is ensure to gain maximum benefits by using ALL-D in Assumption Three.

The deduction of the equilibrium condition in Assumption Three is more complicated than the other two scenarios. Four different scenes will occur considering the relative geographical location of  $A_i$  and the unequal number of players who are located on both upstream and downstream directions of  $A_i$ . The results will have subtle differences. We will only present one deduction process in which  $A_i$  is located in relative upstream ( $i \leq \lfloor n/2 \rfloor$ ) and the total amount of downstream players is an even number ( $n - i = 2m, m \in \mathbb{Z}^+$ ).

When ( $i \leq \lfloor n/2 \rfloor$ ) and ( $n - i = 2m, m \in \mathbb{Z}^+$ ), player  $A_n$  will be the last one who realizes another player had defected in an earlier round. Player  $A_n$  will start to defect at moment  $(n - i)/2 + 1$ , and since then all players will defect afterwards. Therefore,

we can divide the total expected utility of  $A_i$  into two parts on the basis of whether cooperative behavior still exists in the interactions.

$$E(\bar{U}_i | ALL - D) = \sum_{t=1}^{\infty} \beta^{t-1} U_{it} = \sum_{t=1}^{\frac{n-i}{2}} \beta^{t-1} U_{it} + \sum_{t=\frac{n-i}{2}+1}^{\infty} \beta^{t-1} U_{it}$$

A. 12

For part one,

$$\begin{aligned} \sum_{t=1}^{(n-i)/2} \beta^{t-1} U_{it} &= 2T(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i) + \beta[2T(\sum_{j=1}^{i-3} \alpha_{i,j}^i + \sum_{j=i+3}^n \alpha_{i,j}^i) + 2P(\sum_{j=i-2}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{i+2} \alpha_{i,j}^i)] + \\ &\beta^2[2T(\sum_{j=1}^{i-5} \alpha_{i,j}^i + \sum_{j=i+5}^n \alpha_{i,j}^i) + 2P(\sum_{j=i-4}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{i+4} \alpha_{i,j}^i)] + \dots + \beta^{(n-i)/2-2} [2T(\sum_{j=1}^{2i-n+3} \alpha_{i,j}^i + \sum_{j=n-3}^n \alpha_{i,j}^i) + \\ &2P(\sum_{j=2i-n+4}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-4} \alpha_{i,j}^i)] + \beta^{(n-i)/2-1} [2T(\sum_{j=1}^{2i-n+1} \alpha_{i,j}^i + \sum_{j=n-1}^n \alpha_{i,j}^i) + 2P(\sum_{j=2i-n+2}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-2} \alpha_{i,j}^i)] \end{aligned}$$

Times  $\beta$  on each side of the equation and with some simple algorithm we will have

$$\begin{aligned} (1 - \beta) \sum_{t=1}^{(n-i)/2} \beta^{t-1} U_{it} &= 2T(\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i) + 2(P - T)[\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \\ &\beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{(n-i)/2-1}(\alpha_{i,2i-n+2}^i + \alpha_{i,2i-n+3}^i + \alpha_{i,n-3}^i + \alpha_{i,n-2}^i)] - \\ &\beta^{(n-i)/2} [2T(\sum_{j=1}^{2i-n+1} \alpha_{i,j}^i + \sum_{j=n-1}^n \alpha_{i,j}^i) + 2P(\sum_{j=2i-n+2}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^{n-2} \alpha_{i,j}^i)] \end{aligned}$$

A. 13

For part two,

$$\sum_{t=\frac{n-i}{2}+1}^{\infty} \beta^{t-1} U_{it} = \frac{\beta^{(n-i)/2} 2P(\sum_{j=2i-n}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i)}{1 - \beta}$$

A. 14

Combining A.13 and A.14 and put them back to A.1 with A.3, we will have

$$\frac{T-R}{T-P} \leq$$

$$\frac{\beta(\alpha_{i,i-2}^i + \alpha_{i,i-1}^i + \alpha_{i,i+1}^i + \alpha_{i,i+2}^i) + \beta^2(\alpha_{i,i-4}^i + \alpha_{i,i-3}^i + \alpha_{i,i+3}^i + \alpha_{i,i+4}^i) + \dots + \beta^{\frac{n-i}{2}-1}(\alpha_{i,2i-n+2}^i + \alpha_{i,2i-n+3}^i + \alpha_{i,n-3}^i + \alpha_{i,n-2}^i) + \beta^{\frac{n-i}{2}}(\alpha_{i,2i-n}^i + \alpha_{i,2i-n+1}^i + \alpha_{i,n-1}^i + \alpha_{i,n}^i)}{\sum_{j=1}^{i-1} \alpha_{i,j}^i + \sum_{j=i+1}^n \alpha_{i,j}^i}$$

There are three other scenarios which in mathematical languages are

$i \leq |n/2|$  and  $n - i = 2m - 1, m \in \mathbb{Z}^+$ ,  $i > |n/2|$  and  $i = 2m - 1, m \in \mathbb{Z}^+$  and  $i > |n/2|$  and  $i = 2m, m \in \mathbb{Z}^+$ . The calculations of the other three scenarios are basically the similar. We will not repeat the calculations and please refer the results to section 3.

## Reference

- Ahn, T., Myungsuk Lee, Lore Ruttan, and James Walker. 2007. "Asymmetric payoffs in simultaneous and sequential prisoner's dilemma games." *Public Choice* no. 132 (3):353-366. doi: 10.1007/s11127-007-9158-9.
- Axelrod, R, and WD Hamilton. 1981. "The Evolution of Cooperation." *Science* no. 211 (4489):1390-1396. doi: 10.1126/science.7466396.
- Brondizio, E. S., E. Ostrom, and O. R. Young. 2009. "Connectivity and the Governance of Multilevel Social-Ecological Systems: The Role of Social Capital." *Annual Review of Environment and Resources* no. 34:253-278.
- Coleman, J. S. 1988. "Social Capital in the Creation of Human Capital." *American Journal of Sociology* no. 94 (Supplement: Organizations and Institutions: Sociological and Economic Approaches to the Analysis of

Social Structure):95-120.

- Coleman, J. S. 1990. *Foundations of Social Theory*. Cambridge, Massachusetts: Harvard University Press.
- Du, Wen-Bo, Xian-Bin Cao, and Mao-Bin Hu. 2009. "The effect of asymmetric payoff mechanism on evolutionary networked prisoner's dilemma game." *Physica A: Statistical Mechanics and its Applications* no. 388 (24):5005-5012. doi: 10.1016/j.physa.2009.08.026.
- Folke, C., T. Hahn, P. Olsson, and J. Norberg. 2005. "Adaptive governance of social-ecological systems." *Annual Review of Environment and Resources* no. 30:441-473.
- Folke, C., L. Pritchard, F. Berkes, J. Colding, and U. Svedin. 2007. "The problem of fit between ecosystems and institutions: Ten years later." *Ecology and Society* no. 12 (1):30.
- Friedman, James. 1977. *Oligopoly and the Theory of Games*. Amsterdam: North-Holland.
- Granovetter, M. 1985. "Economic Action and Social Structure: the Problem of Embeddedness." *American Journal of Sociology* no. 91 (3):481-510.
- Janssen, M. A., J. M. Anderies, and J. C. Cardenas. 2011. "Head-enders as stationary bandits in asymmetric commons: Comparing irrigation experiments in the laboratory and the field." *Ecological Economics* no. 70 (9):1590-1598.
- Janssen, M. A., R. Holahan, A. Lee, and E. Ostrom. 2010. "Lab Experiments for the Study of Social-Ecological Systems." *Science* no. 328 (5978):613-617.
- Janssen, M. A., and N. D. Rollins. 2012. "Evolution of cooperation in asymmetric commons dilemmas." *Journal of Economic Behavior & Organization* no. 81 (1):220-229.
- Johnson, T. R., J. A. Wilson, C. Cleaver, and R. L. Vadas. 2012. "Social-Ecological Scale Mismatches and the Collapse of the Sea Urchin Fishery in Maine, USA." *Ecology and Society* no. 17 (2):15. doi: <http://dx.doi.org/10.5751/ES-04767-170215>.
- Krippner, G. R., and A. S. Alvarez. 2007. "Embeddedness and the intellectual projects of economic sociology." *Annual Review of Sociology* no. 33:219-240.
- Lindahl, Therese. 2012. "Coordination problems and resource collapse in the commons — Exploring the role of knowledge heterogeneity." *Ecological Economics* no. 79 (0):52-59. doi: 10.1016/j.ecolecon.2012.04.016.
- Lindenberg, S. 2001. "Social rationality as a unified model of man (including bounded rationality)." *Journal of Management and Governance* no. 5 (3):239-251.
- Lindenberg, S. 2006a. "Rational Choice Theory." In *International Encyclopedia of Economic Sociology*, edited by Jens Beckert and Milan Zafirovski, 548-552. New York: Routledge.
- Lindenberg, S. 2006b. "Social Rationality." In *International Encyclopedia of Economic Sociology*, edited by Jens Beckert and Milan Zafirovski, 616-618. New York: Routledge.
- Moss, D. Timothy. 2012. "Spatial Fit, from Panacea to Practice: Implementing the EU Water Framework Directive." *Ecology and Society* no. 17 (3):2. doi: <http://www.ecologyandsociety.org/vol17/iss3/art2/>.
- Nash, John. 1951. "Non-Cooperative Games." *The Annals of Mathematics* no. 54 (2):286-295.
- Olson, M. 1971. *The logic of collective action: public goods and the theory of groups*. Cambridge: Harvard University Press.
- Ostrom, E. 2010a. "Beyond Markets and States: Polycentric Governance of Complex Economic Systems." *American Economic Review* no. 100 (3):641-672.
- Ostrom, E. 2010b. "The Institutional Analysis and Development Framework and the Commons." *Cornell Law Review* no. 95 (4):807-815.
- Ostrom, E. 2010c. "A Long Polycentric Journey." *Annual Review of Political Science* no. 13:1-23.
- Ostrom, E., and R. Gardner. 1993. "Coping with Asymmetries in the Commons - Self-Governing Irrigation

- Systems Can Work." *Journal of Economic Perspectives* no. 7 (4):93-112.
- Ostrom, E., R. Gardner, and J. Walker. 1994. *Rules, games, and common-pool resources*. Ann Arbor: University of Michigan Press.
- Ostrom, E., J. Walker, and R. Gardner. 1992. "Covenants with and without a Sword - Self-Governance Is Possible." *American Political Science Review* no. 86 (2):404-417.
- Raub, W., V. Buskens, and M. A. L. M. van Assen. 2011. "Micro-Macro Links and Microfoundations in Sociology." *Journal of Mathematical Sociology* no. 35 (1-3):1-25.
- Raub, W., and J. Weesie. 1990. "Reputation and Efficiency in Social Interactions - an Example of Network Effects." *American Journal of Sociology* no. 96 (3):626-654.
- Sally, D. 1995. "Conversation and Cooperation in Social Dilemmas - a Metaanalysis of Experiments from 1958 to 1992." *Rationality and Society* no. 7 (1):58-92.
- Scott, W. Richard, and John W. Meyer. 1994. *Institutional environments and organizations : structural complexity and individualism*. Thousand Oaks: California: SAGE Publications.
- Tan, F. F. 2008. "Punishment in a Linear Public Good Game with Productivity Heterogeneity." *Economist-Netherlands* no. 156 (3):269-293.
- Vandijk, E., and H. Wilke. 1995. "Coordination Rules in Asymmetric Social Dilemmas - a Comparison between Public Good Dilemmas and Resource Dilemmas." *Journal of Experimental Social Psychology* no. 31 (1):1-27.
- Wang, Raymond Y., Cho Nam Ng, and Vincent Buskens. 2012. *Social Embeddedness and Collective Cooperation in Water Pollution Interactions: A Game Theoretic Model of Information Effects*. The University of Hong Kong.
- Young, Oran R. . 2002. *The Institutional Dimensions of Environmental Change: Fit, Interplay, and Scale*. Cambridge, Massachusetts: MIT Press.