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Understanding the Dynamics of Sustainable Social-Ecological Systems: Human Behavior, Institutions, and Regulatory Feedback

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Understanding the Dynamics of Sustainable Social-Ecological Systems: Human Behavior, Institutions, and Regulatory Feedback

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I present a general mathematical modeling framework that can provide a foundation for the study of sustainability in social ecological systems (SESs). Using basic principles from feedback control and a sequence of specific models from bioeconomics and economic growth, I outline several mathematical and empirical challenges associated with the study of sustainability of SESs. These challenges are categorized into three classes: 1) the social choice of performance measures, 2) uncertainty, and 3) collective action. Finally, I present some opportunities for combining stylized dynamical systems models with empirical data on human behavior and biophysical systems to address practical challenges for the design of effective governance regimes (policy feedbacks) for highly uncertain natural resource systems.

Keywords:
Dynamical Systems, Mathmatical Models and Sustainability.

1 Introduction

After more than 50 years of development in the academic literature, "sustainability" is now a mainstream concept. Yet I would guess that few people actually have a clear vision of how to operationalize the sustainability concept. For example, consumers are now confronted with sustainability information on many products but can't be sure about the implications of this information (Tejeda-Cruz et al., 2010; Golden et al., 2010). This raises an important question: can voluntary actions by multiple individuals and firms based on "sustainability concerns" affect properties of the global system in which they occur? Put simply, does individual sustainability "add up" to global sustainability? Alternatively, will sustainability require a top-down, centralized, command and control approach? Or a combination of the two? Such questions are clearly amenable to mathematical treatment.

Addressing these questions requires that we view "sustainability" as an attribute associated with emergent dynamics generated by individual actions in vast exchange networks operating at multiple levels of organization, embedded in a biophysical/technological context. Mathematics is very well suited to this kind of problem. In fact, studying "sustainability" requires that we align notions of sustainability with dynamical systems concepts to understand the effect individual actions have at the system level and our capacity to control outcomes at different levels of organization. There are three essential ingredients to this alignment: 1) a formal representation of the relationship between human decision making processes, capital stocks (including natural, human, and human-made) and sustainability, 2) analytical tools to study of non-linear feedback systems with uncertainty, and 3) a conceptual framework to connect 1) and 2). The first ingredient is well-developed; very simple mathematical bioeconomic models have long been used to explore the fundamentals of unsustainable resource use: the basic disconnect between individual and group welfare in the absence of effective governance (Gordon, 1954) and the challenge of inter-temporal valuation of capital stocks even with effective governance (Clark, 1973).

Although simple deterministic models with mild non-linearities generate important insights, they are insufficient for designing solutions. Real-world systems not only exhibit non-linear dynamics but also exhibit complexity of a different sort: the sheer number of interacting elements that comprise them. This type of complexity brings with it deep uncertainty that makes policymaking very difficult in practice. This fact underlies the need for the second ingredient which, like the first, is quite well developed. There is a range of specialized, powerful tools for the analysis of non-linear feedback systems with uncertainty (e.g. robust control and viability theory) that complement the basic tools of dynamical systems theory (e.g. stability and bifurcation analysis). What is lacking is the third ingredient that connects these powerful tools, typically used to analyze designed systems with tightly defined boundaries, to open, self-organizing systems of interest in the sustainability discourse.

In this paper we will explore how mathematical tools may contribute to connecting these two domains to develop design principles for robust governance of SESs comprised of both consciously designed and self-organizing subcomponents.

2 Moving the theory and mathematics sustainability forward

The mainstream nature of the "sustainability" concept brings with it imprecision; it means different things to different people. Lack of precision may ease contentious discourse (important because sustainability involves winners and losers) but severely impedes action. For example, in the common phrase "sustainable development" the term sustainability implies that something related to human welfare is maintained or increased over some temporal scale. However, this just raises questions about what is maintained for whom and for how long (Howarth and Norgaard, 1990; Howarth, 1995, 1997).

It is possible to define sustainability in sufficiently general terms to avoid semantic disagreements yet sufficiently precise to be of operational use. First, we must identify the system to which "sustainability" applies. Specifically, policies in the broadest sense are rules (institutions *sensu* Ostrom (1990, 2005)) that translate information (e.g biophysical information, information regarding actions of agents, etc.) about a system into action that feeds back into the system. That is, governance is a process of building feedback loops into social-ecological systems (SESs). This point is critical: most, if not all, SESs (at whatever scale is of interest) are simply feedback control systems. It is thus no surprise the mathematics of feedback control has figured so prominently in the study of SESs.

Given this definition of the system, a simple clean, definition of sustainability naturally follows: "sustainability" is simply a label for a certain class of feedback control problems where

1. the performance measure that drives the control system emphasizes inter-generational, intra-generational, and inter-species equity,

2. the "plant" is extremely complex, exhibits high levels of uncertainty and the potential for strong nonlinearities, critical thresholds, and irreversibility,

3. the "controller" exhibits internal dynamics involving "group" and "multilevel" data processing, and collective choice dilemmas (put simply, sensor data is processed by groups of people who disagree and controller actions are generated by majority rule!)

Mathematical approaches to the sustainability problem all involve some variation of this system. In some sense, the collection of the variations of mathematical treatments of this feedback control problem constitute a *theory* of sustainable SESs. In the next section, we will summarize past work in this area with a discussion of its limitations, especially in terms of linking with empirical data and moving from theoretical insights to practical application concerns. Finally, in Section 5 we will suggest some directions for future research.

3 Sustainability and feedback systems

There are, in fact, several precise definitions of sustainability based on optimal control approaches that relate to decisions about consumption and investment (Pezzey, 2004a,b). Early explorations of the sustainability concept were based on dynamic asset allocation problems in the context of economic growth models where the traditional decision set including consumption and investment in human-made capital was extended to include the option to invest in natural capital (see Common and Perrings, 1992, for an excellent overview). Despite these quite general yet concise characterizations, it seems that "[t]he considerable disagreement [regarding] the conceptual and operational content of [sustainability]" noted by Common and Perrings (1992) 20 years ago remains with us today.

As mentioned in Section 2, this ambiguity can be significantly reduced by conceptualizing SESs as feedback systems (see Figure 1), or more broadly as regulatory feedback networks. Almost all "interesting" systems, e.g. those exhibiting biological or social complexity, can be usefully viewed in this way (Csete and Doyle, 2002; Anderies et al., 2007, 2013). The challenge is bringing together ideas from a disparate set of disciplines to study enormously complex regulatory feedback networks in SESs that involve biological, engineered, and human policy systems. Indeed, Csete and Doyle (2002) highlight opportunities and challenges associated with bringing together ideas from electrical engineering and control and biology. Readers of this journal are, of course, well aware of the long history of the interactions between mathematics, biology, ecology, and evolutionary theory that have spawned vigorous new research fields based on early pioneering work (e.g. Lotka, 1925; Hodgkin and Huxley, 1952; May, 1974; Smith, 1982; Edelstein-Keshet, 1988; Murray, 1993). Some will be familiar with work that combines the calculus of variations and optimal control with economics and management (Kamien and Schwartz, 1991). Finally, many will know the stream of work pioneered by Clark (1976) that combines both of these streams and founded the field of mathematical bioeconomics. Both of these research streams treat SESs as shown in Figure 1C, the implications of which are discussed in Section 3.1.

The essence of addressing sustainability of SESs lies in studying systems that are part self-organizing regulatory feedback networks (core of mathematical biology/ecology) and part designed regulatory feedback networks (core of dynamic optimization) as pioneered by Clark (1976). The key to moving the field forward involves addressing points 2 and 3 in Section 2, especially point 3. At the broadest level, the continued development of mathematics for sustainability requires an entirely different level of integration heretofore achieved. This integration must bring together mathematical biology/ecology with its focus on self-organization and the evolution of complexity, with feedback control, dynamic optimization and management/decision science with its focus on efficient dynamic resource allocation, with the very messy world of group decision making in hierarchical organizations and its attendent collective choice dilemmas that is the focus of political science and institutional analysis. In the remainder of this section, I will discuss some of the challenges and opportunities associated with this task.

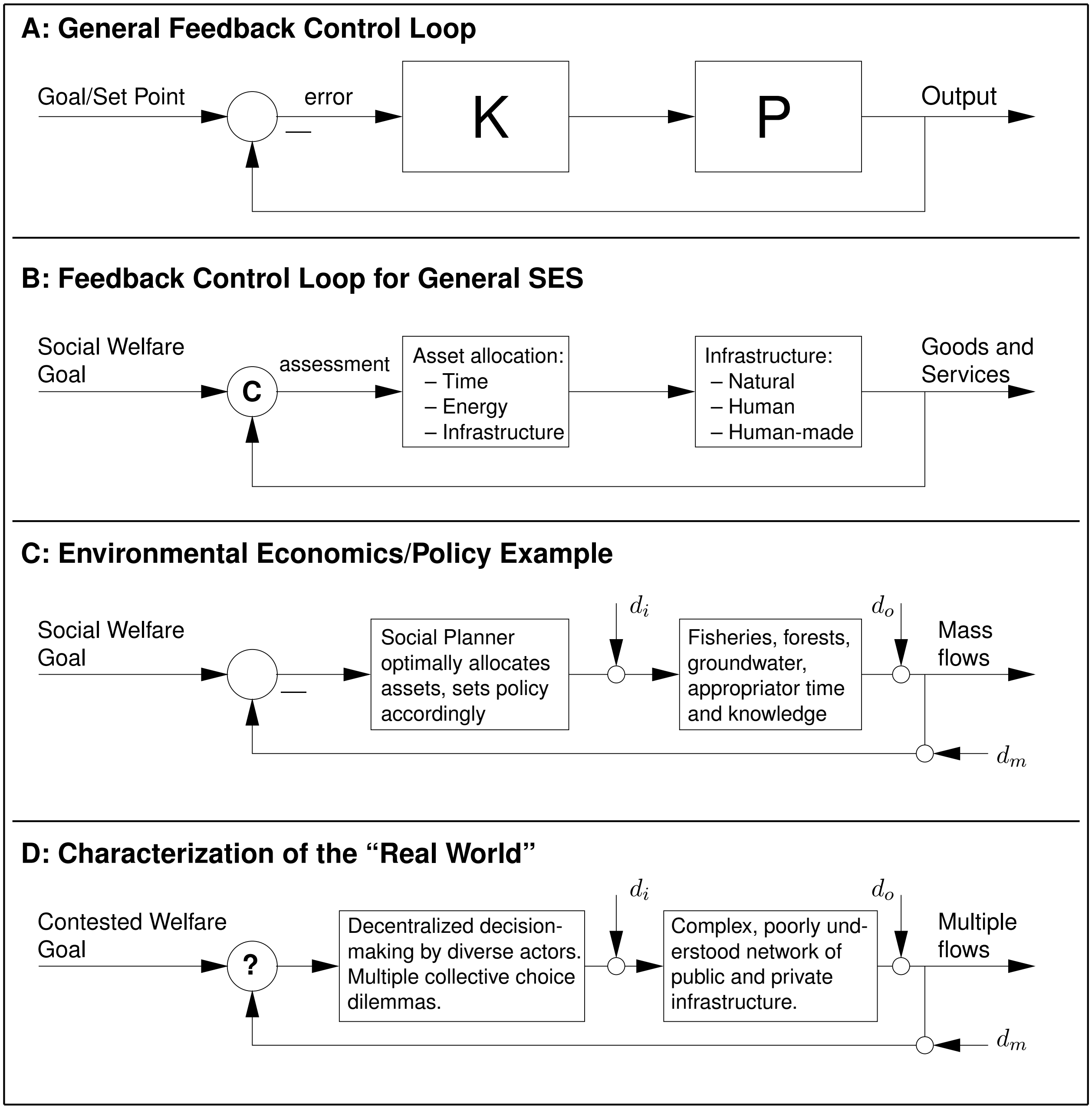


Figure 1: *Block diagram representation of sequentially more challenging "control" problems for sustainability. K and P are the traditional symbols for the "controller" and "plant". The symbols d_i, d_o, d_m represent disturbances to plant input (=controller output), plant output, and measurements, respectively. The small circles to the left represent "summing points". The symbols to the bottom right of the circles indicate whether the signals are added or subtracted. In A and C, the measured output is subtracted from the goal set point to compute the error upon which the controller acts. However, in SESs, the "summing point" idea must be generalized. This is depicted in B where instead of computing an error, some sort of comparative (thus the "c" in the circle) process is carried out, and an "assessment" is made. In the real world (D), it is not clear exactly how the comparison between social goals and system state may be carried out, thus the "?". See text for further discussion.*

3.1 The basic problem formulation

Here I will use basic block digrams to develop the basic models of coupled SESs that have been developed over time (Figure 1). Block diagrams A-C summarize the basic elements of how the SESs have been conceptualized as feedback systems. Block diagram C summarizes the basic elements of SESs typically included in state-of-the art bioeconomic models used in environmental economics and policy analysis. In the remainder of this section, I will quickly summarize the basic mathematics the have been developed around diagram C and, more importantly highlight its limitations regarding its utility as an actual policy making tool in the context of global change.

I will introduce the Institutional Analysis and Development (IAD) framework which helps conceptualize how we can connect ideas from the mathematics of feedback control and dynamical systems to the complexities of social processes and collective choice. To begin, let's frame the mathematics associated with the most common instantiation of the feedback loops in Figure 1C: environment and resource economics. The most studied example is from fisheries, with the earliest treatments from Gordon (1954), Schaefer (1957), Smith (1969), and Clark (1973, 1976) and the model takes the form of an initial value problem:

$$\dot{x} = F(x) - H(x,u) \qquad\qquad x(0) = x_o. \tag{1}$$

where x is the fish population density (biomass) $F(x)$ is the biological growth function, u represents fishing activity, and $H(x,u)$ is the harvest (biomass per unit time) which depends on the stock size and human activity. Of course, u is the control variable in control theoretic terms and can be quite tricky to define. It is typically taken to be something along the lines of vessel-days per year. If $u = u(x)$, then we have our feedback loop in Figure 1C. Of course, several conditions on F, H, x, and u apply (Clark, 1976) so the model makes mathematical and biological sense.

The difficult part of the problem is actually determining what the growth and harvest functions actually are for a given system and coming up with the relationship between the stock and control variable (effort level). The zeroth order approach treats the fish and the fishers in a highly aggregated way - as homogenous "masses" of fish and effort that adjust smoothly and envisions the benevolent social planner of the flavor in Scott's *Seeing Like a State* 1998 adjusting total effort to maximize social welfare. Going the final distance of translating this into a formal mathematical problem requires a few more assumptions that stretch the imagination which I will get to in a moment, but doing so yields something along the lines of

$$\max_{u(t)} \quad \int_0^\infty e^{-\delta t}(pH(x,u) - C(u))\,dt \tag{2}$$

$$\text{s.t. Equation (1) and} \quad 0 \le u(t) \le u_{max}. \tag{3}$$

where p is the per unit biomass price of fish (or whatever is being harvested) and C(u) is the cost function. The assumption that fishing effort can be lumped assumes away important differences between fishers and defining the objective functional by summing "welfare" across fishers and over time (and discounting future income) is fraught with problems. Nonetheless, pressing further with this formulation it is often assumed that $H = qux$ where q, the so-called "catchability coefficient" is related to biophysical and technological features of the system in question and the catch is governed by a mass-action type relationship of interacting boats and fish. The assumption that total effort can be lumped is more problematic. We may (and often do) simply assume that all the fishers are identical, have an identical boat, and that total effort is simply the product of number of vessels employed times fishing days per year for each vessel - i.e. 50 boats all fishing 50 days per year is 2500 vessel-days per year of harvesting effort. It is often assumed that the cost of fishing is constant, c \$/vessel-day i.e. $C(u) = cu$. Obviously there are many ways to combine numbers of boats and vessel days for the same total effort, and although the fish may not notice a difference, the fishers certainly will.

Despite its many limitations this model has generated many important insights regarding resource exploitation decisions. The most important of these is the critical relationship between the resource growth rate and the discount rate (δ) in resource decisions (e.g. Clark (1973) critical work on the potential optimality of extinction), and the theoretical equivalence of different regulatory instruments such as taxes and quotas. Beyond such general insights, this model is of limited theoretical or practical value. Having said that, this model provides an extremely concise and comprehensive representation of the "sustainability problem" we face. It serves as an excellent road map to highlight key theoretical and practical challenges of the wise governance of shared resources ranging from a local groundwater system to the planet.

3.2 Theoretical Extensions

There have been some recent efforts to push the analysis of SESs beyond the view illustrated in Block Diagram C and move closer to D. The main story line in these efforts involves disaggregation of the lumped biological resource, $x(t)$, and lumped "control action", $u(t)$ to incorporate at least some features of the of the extraordinarily complex ecological and social systems that constitute the biological resource and control action, respectively. This disaggregation process proceeds along two lines: first, the "biological" resource must be re-envisioned as a biophysical resource comprised of both human-made and biological components and the feasible set of control actions expanded to include two broad classes of action: consumption (harvesting) and construction (investment).

There are many variations of the model given by Equations (1)- (3) that go some distance in this disaggregation process. There are treatments that replace the scalar state variable $x(t)$ with a state vector in which the i^{th} element represents population density in the i^{th} patch (e.g. Sanchirico and Wilen, 1999, 2001, 2005), in the i^{th} age class (e.g. Gurtin and Murphy, 1981a,b; Swartzman et al., 1983; Solberg and Haight, 1991; Tahvonen, 2009; Quaas et al., 2013), or of the i^{th} species (e.g. Silvert and Smith, 1977; Chaudhuri, 1988) to capture spatial structure, age structure, trophic interactions, or any combination of these. These extensions significantly improve how biological/ecological realities may change the policy implications of the basic model. Better understanding of the "plant" (P in Figure 1A) narrowly defined as the biological resource likely plays a relatively small role in improving our understanding of resource governance theory and practice; disaggregating control action is likely more important.

The reason for this lies in the fact that in most modern contexts we humans don't simply act on the environment. Rather, we act on the environment *through* built infrastructure[1] and we must account for the special feature of this infrastructure: it takes considerable time and effort to construct and can limit flexibility. Fishers don't just make decisions to fish, they first make a decision whether to invest in a boat and gear *necessary to fish*. Another element necessary to fish is knowledge. The "action" of harvesting fish is the result of a complex interaction between three types infrastructure: the hard human-made infrastructure of the boat and gear, the soft infrastructure of human knowledge, and the natural infrastructure of the food web that produces the fish. This complex assemblage does not behave like a "harvest dial" that a benevolent social planner can turn. Of course, Clark et al. (1979) recognized this and have demonstrated its implications in one early extension of the basic model.

The essential point here is that most mass, energy, and information flows relevant to human welfare are generated by a collection of different types of infrastructure. Thus, control action in SESs is concerned, for the most part, in investment in these sets of infrastructure. Thus, we must replace the scalar control variable $u(t)$ with a vector in which the i^{th} element represents human effort, energy and or material flows directed and producing infrastructure or coordinating interactions between different types of infrastructure. There have, in fact, been a considerable number of treatments of SESs with this view under the rubric of "economic growth and the environment" that take the initial steps in this disaggregation of $u(t)$ and acknowledges the complexity of the production process. In the simplest case, there are two types of infrastructure: labor, L, and capital (the term used in the economic growth literature for generalized productive physical assets) K so that $x(t) = (K(t), L(t))^T$. These stocks produce a generalized stream of "output" $Y(t) = f(K(t), L(t))$ where the function $f : \mathcal{R}^2 \to \mathcal{R}$ describes how these stocks interact (are coordinated) to produce output. The feedback control in this case consists of two variables: how much of the output Y to consume, $c(t, Y(t))$ and how much to reinvest in building up the capital stock, $b(t, Y(t))$ so that $u(t) = (c(t), b(t))$. In this case the model in equation (1) has been extended to a two-state two-control variable feedback control problem. In the simplest case, the model is

$$\dot{K} = b(t, Y(t)) - \delta K \tag{4}$$

$$\dot{L} = rL \tag{5}$$

$$K(0) = K_0, \quad L(0) = L_0 \tag{6}$$

[1] A note on terminology: the objects to which I refer with the term infrastructure are often called "capital". *Capital* is often used to refer to private productive assets (land, buildings, machinery) where as *infrastructure* is typically used to refer to public productive assets (roads, bridges, dams, fiber optic cable), i.e. the common phrases *private capital* versus *public or shared infrastructure*. In fact, both refer to exactly the same thing so I adopt the term infrastructure to refer to all instances of productive assets regardless of their ownership status

where δ is the rate at which capital depreciates (wears out) and the population grows exponentially at rate r. Note that if society does not waste its output, it either consumes or reinvests it so that $Y(t) = c(t, Y(t)) + b(t, Y(t))$ so the problem can be reduced to a single control variable. It is assumed that society benefits from consumption so that the control problem becomes

$$\max_{c(t)} \quad \int_0^\infty e^{-\delta t} c(t)\, dt \tag{7}$$

$$\text{s.t. Equations (4)} - \text{(6) and} \quad 0 \le c(t) \le Y(t). \tag{8}$$

If we dispense with the dynamic optimization for the moment and assume society devotes a constant proportion of its productive capacity to consumption, then $c(t, Y(t)) = cY(t)$ and $b(t, Y(t)) = bY(t)$ where $c \in [0, 1]$ and $b \in [0, 1]$ are parameters and add to 1 and we obtain the well known Solow-Swan growth model. In this case, if b is sufficiently large, K and L blow up - i.e. the economy grows forever (however the capital-labor ratio does converge to a constant which was the main intent of this model).

This basic economic growth model was not intended to address sustainability questions and thus lacks any connection to the physical world. This leads to the unbounded growth behavior of the model. As you can imagine, it is an easy matter to connect it to reality by introducing "natural capital stocks" of different flavors depletable (oil, coal) replenishable (ground water), and renewable (fisheries, forests, etc.). There have been several extensions of the basic growth model to incorporate a depletable stock. Define K_d as the depletable stock, then the model in (4)-(6) can be extend by adding the dynamics of the depletable resource, i.e.

$$\dot{K_d} = -c_d(t, K_d) \tag{9}$$

where $c_d(t, K_d)$ is the consumption rate of the depletable resource required to generate output, i.e. $Y = f(K, L, c_d)$. This introduces a connection to the real world - you can't go on making output with machines alone - they need fuel and raw inputs, i.e. iron ore and petroleum which are "finite". Hotelling (1931) provides one of the earliest analyses of this model amidst concerns over overexploitation of exhaustible resources in the early 20th century, leading to the famous Hotelling rule for optimal exploitation. Of course, "finite" here is used in an economic sense: a resource is used up when *known reserves* become *too expensive* to extract. Of course, investment in new technology can find new reserves and reduce costs so the notion of an "exhaustible resource" becomes rather vague. This leads to mathematical results illustrating that consumption can be maintained forever despite exhaustible resources (Cass and Mitra, 1991).

The next obvious extension of the this model of economic-resource dynamics is to add renewable resources and link human population dynamics to resources as in Anderies (2003). The model in this paper includes a stock of human made capital, $K_h(t)$, renewable natural capital, $K_r(t)$, and human population, $h(t)$. There are two sectors, manufacturing and food production. Outputs from these sectors feed back on birth and death rates through proxies for nutrition, health care, and changing norms (increasing wealth changes values regarding family size). This model abstracts away from notions of optimal investment and assumes a constant savings rate as in the classic Solow-Swan growth model (see Barro and Sala-i Martin, 2003). The model is used to explore the dynamics of transitions toward "sustainability" given the double-edged sword of economic growth and technological change that can push both birth and death rates down while increasing extractive capacity and, with it, the potential for resource overexploitation. The analysis illustrates the delicate balance that must be achieved between a number of interacting processes to avoid overshoot and collapse type dynamics. As in the case of the basic fishery model, there are dozens of extensions of this basic growth model to address sustainable growth in the 1990's (see Carraro and Siniscalco, 1997, for a nice overview) but work in this area has fallen off precipitously in the last decade.

The take-home message from these explorations of extensions of the basic resource model is that moving the mathematical framework for studying sustainable social-ecological systems forward requires disaggregation of lumped capital stocks. The discussion above provided examples of such disaggregation processes, but only scratch the surface. In what remains of this paper, I will try to sketch out some ideas for the next steps in this process.

3.3 Toward a General Theoretical Framework

If we accept the characterization of SESs as instances of general feedback systems depicted in Figure 1, we need a systematic approach to move from depiction B to D (around the very special case of C). To actually accomplish this task, we need to draw on theories from the social sciences (political science, economics, policy, anthropology, etc), ecology, design, engineering, and mathematics. Organizing our thoughts around such complexity requires a framework and, fortunately, the IAD framework (Figure 2) is up to the task. The IAD is organized around the "Action Situation", defined as a space where agents come together (center box, top, Figure 2) to exchange information, materials, and make decisions (interactions). Action situations are structured by the biophysical context (a football stadium), attributions of the community (the football teams), and the rules-in-use (the rules of the game. You can easily think of many examples of action situations related to sustainability.

To relate this framework to the models above, we must abstract what the developers of the IAD called "external variables" which for dynamic models is the underlying structure of the system, or the "slow" subsystem. To do this we, as above, use the notion of infrastructure. Up to now I have used the terms "capital" and "infrastructure" interchangeably but from here forward, I will use only the term infrastructure. "Biophysical Conditions" consist of natural infrastructure (I_N), and "hard" human-made infrastructure (I^h_{HM}). An example of I_N is a marine ecosystem that supports a valuable fish species. Examples of I^h_{HM} are boats, docks, roads, etc. Note that I^h_{HM} includes both public and private infrastructure.

The Attributes of the Community consists of social infrastructure (I_S) and human infrastructure (I_H). Social infrastructure is the web of relationships among agents that allow them to connect to one another to exchange material and information. Human infrastructure refers to the capacities of individual agents process information and make effort allocation decisions. Rules-in-use consists of soft, public, human-

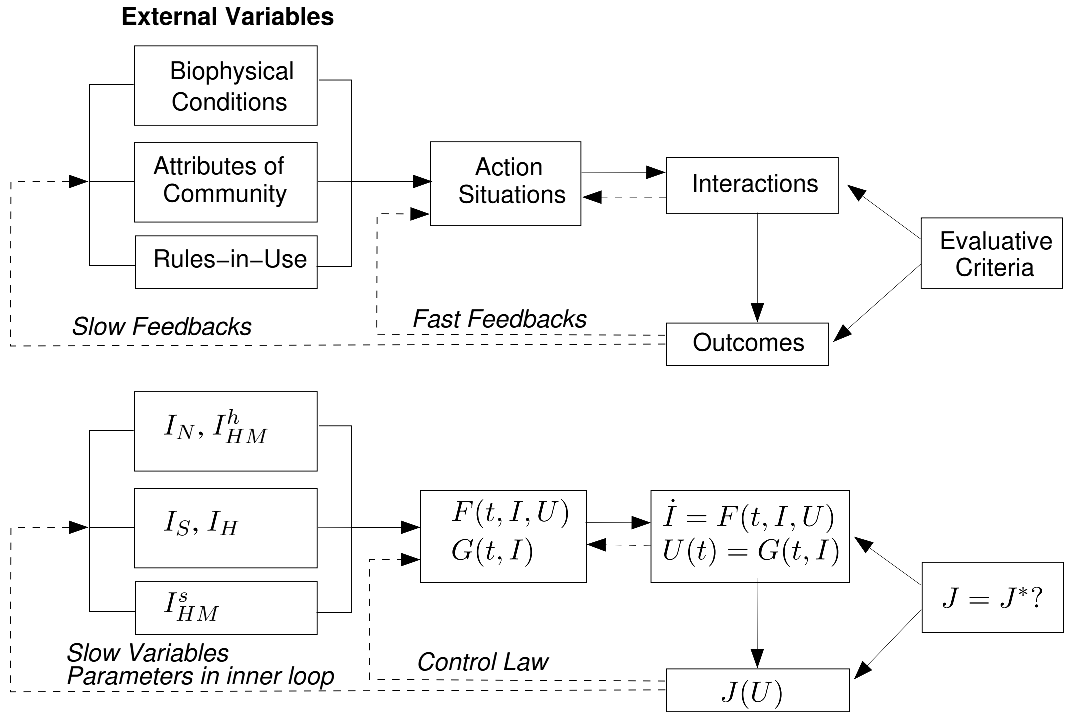


Figure 2: Top: The IAD Framework (adapted from Ostrom, 2011). The IAD Framework is a general feedback control system. Bottom: The IAD Framework expressed in mathematical terms. $I = (I_N, I^h_{HM}, I_S, I_H, I^s_{HM})^T \in \mathbb{R}^m$, $U \in \mathbb{R}^k$, $F : \mathbb{R}^{m+k+1} \mapsto \mathbb{R}^m$, $G : \mathbb{R}^{m+1} \mapsto \mathbb{R}^k$, $J : \mathbb{R}^{m+k+1} \mapsto \mathbb{R}$. J is the objective functional/performance measure, and J^* denotes the performance goal. Note, mathematical rigor is not the goal here. The actual mathematical implementation only maps approximately onto the conceptual model. The intent is to illustrate compactly the correspondence between conceptual and formal models.

made infrastructure, I^s_{HM}. I use the term "soft" to refer to the fact that, unlike the "hard" infrastructure of a bridge, the rules and norms that govern repeated human interactions are not tangible. They are only instantiated through the minds of the agents that share them. However, it makes perfect sense to refer to them as infrastructure because they are very costly to maintain (consider the cost of legal and legislative systems and rule enforcement) and generate real income (by reducing transaction costs of exchange and coordination). The same is true of social infrastructure which is also "soft". There is a direct analogue here to software and hardware in computing.

The IAD Framework was developed in the context of public policy and institutional analysis. Although its developers, most notably Ostrom (2011), fully understood that these sets of infrastructure are intimately connected, their studies focused on a subset of human-made infrastructure, I^s_{HM}, and aspects of I_S, and I_H that relate to solving collective action problems. This was already an enormous undertaking to which Ostrom made Nobel-Prize winning contributions. The next step is to connect this work to the physical world, included in the IAD framework but heretofore under explored.

The challenge in connecting the social and biophysical elements lies in making explicit the interactions between the "external variables" that are implicit in the IAD framework. The basic examples of renewable resource management and economic growth discussed above are suggestive as to how to proceed. First, they emphasize the need to adequately define the *production structure* of the SES that involves inputs beyond labor and capital - the subtle interaction between fishing effort and boats in the fishery model is an example. It isn't sufficient to consider fishing "action" as a lumped output. Related to this point, the examples emphasize that the notion of an exogenous controller that directs action (through $U(t)$) is insufficient to capture the notion of governance. Specifically, governance should not be conceived as just a sequence of decisions and actions. Rather, it should be understood as an output from a production structure and, further, as an emergent property of an SES.

The notion that "governance" is not something we do but, rather, something that emerges as a system feature may seem strange at first glance. Upon closer inspection, however, it becomes evident that most outputs of human activities are "emergent" in the sense that they involve inputs that are taken for granted, not a design consideration, or may even be unrecognized in the production process. This is the result of the simple fact that all production is "joint production" (Baumgärtner et al., 2001). Any productive activity generates multiple output streams, both the desired outputs along with unintended outputs. These unintended outputs (i.e. externalities or spillovers) may be negative (pollution) or positive (ideas and innovation). So, it may be that "good governance" is not the result of clever policy makers and administrators, but rather of some aspect of the biophysical environment that helps solve a governance challenge for free (a positive spillover).

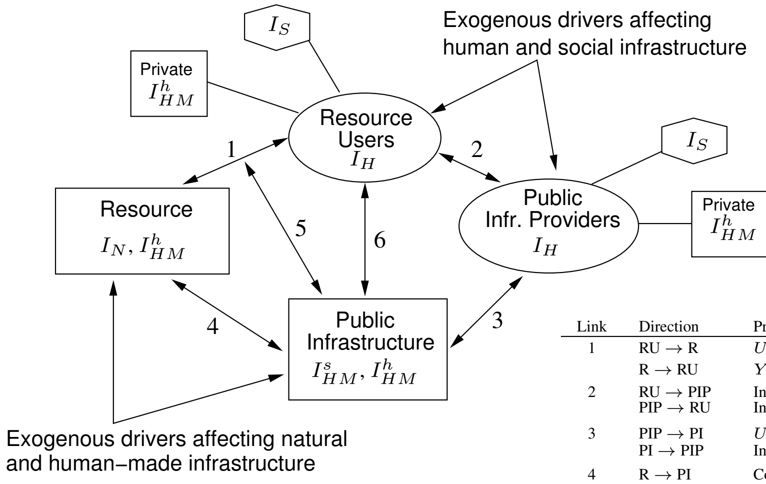
This recognition of the importance of spillovers suggests that we need to move from SESs to Coupled Infrastructure Systems (CISs). This will first require that we expand the definition of "infrastructure" as alluded to above and further detailed below. Next, moving from SESs to CISs points to a general reformulation of the resource management problem given by the feedback system shown in Figure 1C, formalized by equations (1)-(3) where the distinction between the "social" and "ecological" is crisp to that in Figure 1D. The basic problem structure can be cast in the IAD framework as shown in the bottom loop of Figure 2. However, one key distinguishing feature of social-ecological systems is the distinction between activities that produce private and public infrastructure that can generate significant conflict. Further, the notion of the "action situation" is too broad to make the issue of joint production explicit. Thus, researchers have recast the IAD framework to address these issues as shown in Figure 3, highlighting the web of interactions in a CIS.

To formulate this mathematically, we can extend the simple growth model, letting

$$I = (I_N, I_{HM}^h, I_S, I_H, I_{HM}^s)^T \in \mathbb{R}^m \tag{10}$$

$$U \in \mathbb{R}^k \tag{11}$$

and defining the production structure as $Y(t) = f(I, U)$ where U defines the protocols (asset allocations/asset coordination) to animate the productive capacity of infrastructure stocks. Because the interaction



Link	Direction	Processes
1	RU → R	$U_e(I_N, I^h_{HM}, I_S, I_H, I^s_{HM})$
	R → RU	$Y_e(I_N, I^h_{HM}, I_S, I_H)$
2	RU → PIP	Information sharing, resources
	PIP → RU	Inclusion in decision making, resources
3	PIP → PI	$U_{pi}(I_N, I^h_{HM}, I_S, I_H, I^s_{HM})$
	PI → PIP	Information
4	R → PI	Contributes to $Y_g(I_N, I^h_{HM}, I_S, I_H, I^s_{HM})$
	PI → R	Contributes to $Y_e(\cdot)$
5	PI → Link 1	$Y_g(\cdot)$ Restricts $U_e(\cdot)$, affects distribution of $Y_e(\cdot)$
	Link 1 → PI	Potential spillovers affect PI
6	PI → RU	$Y_{pi}(I_N, I^h_{HM}, I_S, I_H, I^s_{HM})$
	RU → PI	Potential spillovers affect PI

Figure 3: The robustness framework (adapted from Anderies et al., 2004) as an extension of the action situation in the IAD framework. The large rectangles represent the biophysical or "ecological" subsystem while the large ovals represent the "social" subsystem of the SES. Note that the large rectangles depict **public infrastructure**. The small rectangles refer to **private infrastructure** that the actors in the system can mobilize. Note, social infrastructure, I_s can be both private and public at the same time. Individuals have relationships that they themselves can mobilize toward some end (priviate). Individuals may also have the capacity to mobilize the "community" at times (i.e. volunteer fire departments, neighborhood watches, etc.) The table provides a description of the interaction processes associated with the links.

between public and private infrastructure is at the core of sustainability challenges, we partition the production output and protocol vectors into economic production, public infrastructure production, and governance production, i.e. $Y(t) = (Y_e(I, U_e), Y_g(I, U_g), Y_{pi}(I, U_{pi}))^T$. Economic production refers to the output familiar to all of us directed at either private consumption or investment in infrastructure. Public infrastructure production is self-explanatory (visits to a national park). Governance production, on the other hand is more subtle. As mentioned above, governance is not something we just decide to do. It is, rather, the actual monitoring, sanctioning, conflict resolution, information sharing, deliberation, and coordination processes that are necessary for institutions (soft, human-made, public infrastructure) to function. This is a critical point not widely appreciated in the literature, especially the fact that Y_g depends on all infrastructure types, not just soft, human-made infrastructure, and may benefit significantly from spillovers.

Given this characterization of the CIS (or SES), we can now write down the feedback control problem. Let $(J_t)_{t\in[0,T]}$ be a sequence of socially determined and acceptable performance goal sets indexed on t. Let

U be the set of admissible controls[2]. Then the problem is

$$\underset{U(t)\in U}{\text{Ensure}}\; J(Y(t)) \in J_t \tag{12}$$

$$\text{Subject to } Y(t) = Y(I(t), U(t)) \tag{13}$$

$$\dot{I} = F(t, I(t), Y(t), U(t)) \tag{14}$$

$$U(t) \in U,\;\; t \in [0, T],\;\; I(0) = I_o. \tag{15}$$

The "ensure" operator is meant to convey a general adherence to a multidimensional definition of performance. If society chooses to maximize a notion of social welfare, the J_t are singletons and

$$\underset{U(t)\in U}{\text{Ensure}}\; J(Y(t)) \in J_t \Leftrightarrow J(Y(t)) = J_t, \tag{16}$$

i.e. $U(t)$ and $I(t)$ follow optimal paths. Alternatively, society might choose to set a minimum welfare level that all individuals must exceed and a maximum level of inequality (minimum Gini Coefficient or variant thereof), minimum output streams (i.e. set $Y_g i(t) = I_N^j$ so that the i^{th} public good output stream represents the "existence value" of the j^{th} natural asset to preserve a particular species or ecosystem), etc. This general feedback control problem presents many interesting challenges in the "mathematics of sustainability" that constitute the content of the next section.

4 Mathematical Challenges

To begin, I want to clarify what I mean (do not mean, actually) with the term "challenges". I am not referring to exploding computational complexity as we disaggregate and move from the model given by Equations (1- 3) to that in (12- 15) and the related distinction between "traditional dynamical systems" and "agent based" approaches or deterministic versus stochastic representations (I have stated the problem as a continuous-time, deterministic problem simply as a matter of convenience). The distinction between these approaches is technological rather than epistemological, and the challenge, regardless of the choice of analytical technology remains: how do we construct a version of (12- 15) with the *right* level of complexity that is useful for development of policy that respects the fact that SES are partially self-organizing and partially designed.

This claim is based, in part, on the empirical observation that there seem to be two classes of efforts in modeling/computation in sustainability science:

1. Theory driven: simple models that capture the fundamental dynamics of natural resources (ground water, fisheries, forests, rangelands, agriculture), individual decision processes (resource allocation

[2] Note, technological progress will cause U to change over time, so we might replace U with U_t to represent a sequence of control sets analogous to J_t.

decisions - i.e. how much ground water do I use, how many days per year do I devote to fishing, etc.), and the commons dilemmas associated with these decision problems.

2. ”Big data” driven: complex models that try to make sense out of large data sets. Such modeling can help make some sense of patterns in the data, but it offers little help in understanding the underlying drivers of those patterns.

So we are left with two disparate points in space of models and data: simple, data poor, theory rich models and complicated, theory poor, data rich models. A key challenge for “mathematical sustainability science” is to locate critical linkages between these two classes of models so that the more complex models that are essential for institutional design and practical governance action are better informed by critical theory insights. As a corollary, a second challenge is to make better use of available data. Using the general representation in Equations (12- 15), we can parse the problem along the lines of 3 challenges: 1) how do we choose $J(U)$, 2) uncertainty about the vector-valued functions F and Y, and 3) what is the feasible control set, U. In the remainder of this section, I take each of these in turn.

4.1 Practical Governance for Sustainability: Choosing J

Suppose that we have perfect knowledge of F, Y, U, and I_o. In theory, the sustainability problem can be solved if we knew $J(Y(t))$ and J_t. Unfortunately, understanding how output streams or system states (e.g. Y can be the identity map for some infrastructure classes) map into well being is non-trivial. More challenging is the fact that J_t must be determined by and for a large, heterogeneous group of individuals. This is a core challenge of “governance”: generate a set of coupled action situations (as in Figure 2) that respect notions of distributional and procedural justice that support deliberative processes to arrive at J_t. Mathematical tools based on game-theoretic notions can be used to design a “mechanism” to achieve a particular social choice (Maskin, 2008; Hurwicz and Reiter, 2006; Hurwicz, 1994; Myerson and Satterthwaite, 1983; Myerson, 1983, 1982; Hurwicz, 1973). This is an interesting set of tools that we will revisit in Section 4.3, but choosing J is much more than a “social choice” and involves a much deeper question of values. It is not clear how mathematics can contribute in this arena.

4.2 Practical Governance for Sustainability: Uncertainty

Suppose that society could, in fact, choose J_t. Suppose further that loop C in Figure 1 applies - i.e. that $J(Y(t)))$ is known and there exists a social planner who can unilaterally execute a feedback control program $U(t)$. Suppose, however, that there is uncertainty regarding $F(t, I, Y, U)$. This uncertainty may be due to measurement limitations (i.e. parametric uncertainty) or lack of knowledge about the fundamental form of F. What design methodology should the social planner employ to choose $U(t)$? Options include stochastic optimal control and dynamic programming. These methods, however, entail assumptions about the nature of uncertainty and J. Given that we only get the play the "civilization game" once, the common practice of using expectation to define J is clearly not appropriate. If such methods are used, maximin approaches are probably more appropriate.

In cases where distributions of random variables are not known and uncertainty is, in general, less structured, notions from robust control may be a better option. Several studies have applied robust control techniques to resource management problems (Rodriguez et al., 2011; Cifdaloz et al., 2010; Anderies et al., 2007). These studies focus on navigating robustness-vulnerability trade-offs inherent in feedback systems. The emphasis is not defining $U(t)$ as a fixed feedback rule (e.g. a PID controller[3], etc.), but rather, structuring a learning process based on dynamically evolving robustness-vulnerability trade-offs to adaptively determine $U(t)$ (Anderies et al., 2007) . Of course, the difficulty of the analysis escalates very quickly with model complexity. Thus, the core challenge emerges again: generating models that are at once quasi-analytically tractable but provide sufficient richness to be of use in a design sense.

This challenge of developing models of *practical design value* is essential to operate in *sustainability design environments* characterized by high levels of complexity and uncertainty. Further, the value of these models likely will not lie in their direct use in design. Rather, the analysis of these models should be used to identify common features of SES's that can be used to build a *design methodology*, similar in spirit to the robust control literature in electrical engineering. For example, controls may be constructed using fundamental building blocks including proportional, integral, derivative feedback controls augmented with feed-forward commands, command pre-filters, integral anti-windup logic, and controller roll-off specifications, etc. The behavior of each of these elements is well understood for given plant structures and, as such, form a basis for a systematic design methodology. The same can't be said for policy design. The early science-based policy design methodology for SESs included two options: a top down environmental dictator (the benevolent social planner in Figure 1C) or privatization of natural infrastructure (e.g. Hardin, 1968; Ostrom, 1990). Ostrom's pioneering (and Nobel Prize winning) work on hundreds of SESs around the world showed that, in fact, communities have devised very clever and effective feedback protocols that involve neither of the science-based protocols (Ostrom, 1990). Rather, these feedback protocols (governance) are collections of rules, taken from seven rule classes (like the P, I, and D in PID controllers), that are well "fit" (like command pre-filters, anti-windup, roll-off) to their contexts through a trial-and error, self-organizing, evolutionary process.

Although Ostrom attempted to extract some regularities from the plethora of cases she studied, her objective was not necessarily to develop a design methodology for SESs. It was, rather, to provide evidence for the existence of and to describe alternatives to the two science-based policy approaches described above. The idea was to use this inductively-derived understanding from a number case studies to move beyond policy panaceas (Ostrom et al., 2007). Given the subtleties, complexity, and uncertainty associated with the

[3]PID controllers are constructed using a weighted (weights="gains") sum of three signals consisting of a (p)roportion of the error signal, the (i)ntegral of the error signal, and the (d)erivative of the error signal.

cases she encountered, she advocated a diagnostic approach for policy design (Ostrom, 2007) drawn from a parallel with medical disease diagnosis of the complex human organism. She did derive 8 design principles for managing the commons (Ostrom, 1993, 2009a, 1995) that are really precursors to effective collective decision making. In the context of the broader framework being presented here, these 8 principles relate to the proper functioning of the control block in Figure 1D. Although Ostrom recognized the need to embed institutional arrangements in the SES in which they operate(Ostrom, 1998, 1995) and proposed intellectual frameworks to do so (Ostrom, 2009b; Anderies et al., 2004), little work has been done to develop the theory to accompany that framework.

Developing such a theory based on ideas from regulatory feedback networks in general, and robust control in particular is what I a encouraging in this paper. Given the complexity and uncertainty associated with SES, we must work inductively to extract "archetypal" models of SESs from the case-study data available (see `http://.seslibrary.asu.edu`) for examples and use them to experiment with policy designs. Much in the same way that mathematical biologists have built archetypal models for ecological systems, e.g. Volterra and Kolmogorov type predator-prey systems and other variations based on numerical and functional response canonical forms based on the work of Holling; various competition/coexistence, host-parasitoid systems, and mutualism models, we must attempt the same for SESs. I have presented here two archetypes in the models in Equations (1- 3) and (4- (8)), but these are just skeletons of what is required. We must uncover the general features of Y and F for specific types of ecological systems such as rangelands, forests, irrigation systems, groundwater systems, and fisheries. We must relate the features of collective choice arrangements to U and understand the dynamics they generate through Y and F. Analogous to normal forms used to categorize bifurcations and, with them, qualitative behavior of models, we must work toward defining normal forms for SESs that will allow us to develop robust governance and policy design methodologies.

4.3 Practical Governance for Sustainability: Collective Action

I conclude this section with a discussion of arguably the most important sustainability challenge: collective action. I will pick up two threads mentioned in Sections 4.1 and 4.2: the institutional design principles and mechanism design. As in the previous section, we highlight here just one class challenges associated with designing the feedback systems in Figure 1. Thus, suppose society can choose J_t and has complete information about $J(Y)$, F, and Y and the capacity for perfect measurements. We are left with the problem of diverse actors, diverse interests, and collective action dilemmas. How can mathematical tools contribute to this problem?

There are two threads here. First is the problem understanding of human decision making in a range of different contexts. This is largely an empirical question for behavioral economics. The second is that once relationships between decision-making behavior and the biophysical and social context in which it is embedded can be established (including some characterization of the uncertainty associated with these relationships), one of two social choice (=collective action) design methodologies can be used 1) mechanism design, 2) extension of the institutional design principles (see Table 1).

Recall my claim that a distinguishing feature of SES feedback systems is that they exhibit a high degree of self-organization in both the control and plant blocks. Of course, a major feature and virtue of feedback control (as opposed to open loop control) is "self-organization". Although parts of this self-organizing processes are designed, the point is that the designed components constitute only a few and most general features of the system. The system then self-organizes within the design constraints - this is a key benefit of robust feedback control. The question of control versus self-organization arises in all feedback systems, it is just a matter of degree across different systems. As Maskin (2008) notes "The theory of mechanism design can be thought of as the "engineering" side of economic theory.. [which] asks whether on not an appropriate institution (mechanism) can be designed to attain... [a desired social]...goal." Essentially mechanism design involves designing payoffs for games to achieve a desired equilibrium. In essence, mechanism design provides an approach to build a "self-organizing" controller by implementing lower level rules (rather than the controller input-output rules themselves). However, given its game-theoretic nature, classical mechanism design involves strong assumptions about the level of common knowledge available about agents' beliefs and suffers from the well-known sensitivity of game theoretic results to uncertainty about these beliefs. Although recent work by Bergemann and Morris (2005) and others has focused on robust mechanism design where the designer has limited or no information about agents' utility structure, in the case of dynamic SESs and dynamic systems of coupled SES, uncertainty extends well beyond agents utility structure and it seems unlikely that mechanism design, given the associated mathematical challenges, would be of practical use beyond very simple design problems.

This leaves us with Ostrom's institutional design principles as a starting point. These contrast significantly with mechanism designs as they do not stress optimality or a solution to a planner's problem, but rather offer heuristics to enable the "controller" to function in almost any circumstances. The design principles, in contrast to solutions to mechanism design problems which are seen as too complex and sensitive to implement, are easy to implement. The design principles are even lower-level rules than those we might generate from mechanism design (although the mechanism design problem could likely be so generally specified as to generate the design principles). Can formal mathematical representations of these principles provide useful building blocks for robust control design?

Although important, the discussion of "institutional design" misses the point. One of the main points of this paper is that "governance" is a product of a set of interacting infrastructure types, that is, governance services, i.e. $Y_g = Y_g(I_N, I^h_{HM}, I_S, I_H, I^s_{HM})$. To address governance in dynamic SESs we must move beyond conceptualizing governance as simply a product of soft public infrastructure comprised of formal rules and norms, i.e. assuming $Y_g = Y_g(I_S, I^s_{HM})$ is insufficient. Determining the form of Y_g for different SES contexts requires the extensive case-study analysis and model typology development mentioned above. Work in this area is underway. For example, Cifdaloz et al. (2010) use a dynamic model to study how

Ostrom's Institutional Design Principles

1. **Clearly defined boundaries.** The boundaries of the resource system and the individuals with use rights are clearly defined.
2. **Proportional equivalence between benefits and costs.** Rules specifying resource allocations are related to local conditions and to rules concerning inputs.
3. **Collective-choice arrangements.** Many of the individuals affected by harvesting and protection rules are included in the group that can modify these rules.
4. **Monitoring.** Monitors, who actively audit biophysical conditions and user behavior, are at least partially accountable to the users and/or are the users themselves.
5. **Graduated sanctions.** Users who violate rules receive graduated sanctions from other users, from officials accountable to these users, or from both.
6. **Conflict-resolution mechanisms.** Users and their officials have rapid access to low-cost, local action situations to resolve conflict among users or between users and officials.
7. **Minimal recognition of rights to organize.** Users' rights to devise their own institutions are not challenged by external governmental authorities, and users have long-term tenure rights.
8. **Nested enterprises.** Appropriation, provision, monitoring, enforcement, conflict resolution, and governance activities are organized in multiple layers of nested enterprises.

Table 1: Ostrom's Design Principles for stable, common-pool resource governance.

topography, community size, and the nature of irrigation infrastructure itself contribute to Y_g, enabling farmers to execute their rules to adaptively allocate water in a variable environment. These effectiveness of these rules (feedback control) depends critically on the structure of the SES itself, and likely would not function if simply transplanted to another SES. What is the design methodology to transplant these rules analogous to that of tuning a PID controller to a new plant? The SES Library `http://seslibrary.asu.edu` at the Center for the Study of institutional Diversity `http://csid.asu.edu` is an effort to build a database of case studies to try to begin to build such a design methodology.

5 Future Directions

The challenge we face is not convincing the research community or policy makers that complexity, hierarchy, and resilience-efficiency (or robustness-performance) trade-offs can (in theory) and should be integrated to address environmental challenges, but rather, developing tools to actually do it. How do we actually allocate scarce resources given massive uncertainty about how SESs operate and the potential for (possibly irreversible) regime shifts? This paper argues that addressing this practical issue should be a key research area in mathematical sustainability.

I have argued that one way the research program might proceed would be as follows:

1. Based on many SES case studies, build a typology of archetypal SESs based on Figure 3. What are

the forms of Y and F and what are the elements in I we see repeated in similar SES contexts. How do these vary across contexts.

2. Characterize the mathematical structure of the archetypal SESs and build a classification -e.g. analogous to the classification of different types of canonical bifurcations. The classification should also address biophysical regularities analogous to the classification of simple ecosystem models, e.g. predator prey models with building blocks - prey density dependence, Holling type 1-3 functional responses, predator-mediated coexistence, etc.

3. Based on this characterization of mathematical structure in SESs, build a design methodology analogous to those in robust control.

This research program should enable "governance engineers" to provide to communities basic building blocks which they may use through a participatory process (also informed by this research program), to implement robust feedback control systems to better govern their SES for sustainability.

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