Learning and Incentive Compatible Mechanisms for Public Goods Provision: An Experimental Study*

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Abstract

This is the first systematic experimental study of the comparative performance of two incentive compatible mechanisms for public goods provision: the Basic Quadratic mechanism by Groves and Ledyard ($\gamma = 1$ and $\gamma = 100$) and the Paired-Difference mechanism by Walker. Both mechanisms are Nash-efficient and balanced vvith the same dimensions of message space, and the latter has one advantage over the former in that in equilibrium it is individually rational. However, our experiments demonstrate that the actual Performance of the Basic Quadratic mechanism under a high punishment parameter is far better than the Basic Quadratic mechanism under a low punishment parameter, which, in turn, is better than the Paired-Difference mechanism, evaluated in terms of System efficiency, close to Pareto optimal level of public goods provision, convergence to stage game equilibrium and stability. From this we draw some lessons for mechanism design: Standard considerations. such as incentive compatibility, individual rationality and balanced budget, are not enough to guarantee that these desirable properties can actually be obtained in a dynamic process with human subjects. Other disequilibrium aspects, such as *deviation costs* which impose incentives for subjects to learn to play their equilibrium strategies, and *deviation sensitivity* which can either amplify or diminish noise in a system, are also important to induce good dynamics and stability. To understand principles of individual learning behavior, we estimated three static and four dynamic learning models. Variants of the Stimulus response models outperform the generalized fictitious play model. The comparative Performance of the three variants of the Stimulus response models are statistically indistinguishable.

Keywords: public goods, mechanism design

JEL Classification: C90; D70

1 Introduction

How to design decentralized institutions to facilitate cooperation in an environment with public goods has been a challenging problem for economists for a long time. Natural processes, such as the voluntary contribution mechanisms, have been shown both theoretically and experimentally, to be unable to solve the "free-rider" problem[1]. Therefore, since the 1970s, economists have been seeking decentralized mechanisms that are non-manipulable and achieve Pareto optimal allocation of resources with public goods.

By now it is well-known that it is impossible to design a mechanism for making collective allocation decisions, which is informationally decentralized, non-manipulable, and Pareto optimal[2]. There are many mechanisms which preserve Pareto optimality at the cost of non-manipulability, some of which preserve "some degree" of non-manipulability. In particular, some mechanisms which have the property that Nash equilibria[3] are Pareto optimal have been discovered. These can be found in the work of Graves and Ledyard (1977), Hurwicz (1979) and Walker (1981).

All these "next best" mechanisms have very similar theoretical properties, which leads one to consider properties, other than optimality of Nash equilibria, in an effort to distinguish among them. One important additional dimension of Performance is the dynamics induced by these mechanisms in a laboratory. Any actual implementation is necessarily a dynamic process, starting somewhere off the equilibrium path. The fundamental question conceming implementation of a specific mechanism is whether the dynamic processes will actually converge to one of the equilibria promised by theory, and once converged, whether the equilibrium will be stable. If the dynamic processes do not converge, then the nice properties in equilibrium cannot be achieved. Therefore, it is crucial to study the dynamic processes of a mechanism, and to extract the properties of static mechanisms that induce convergence and stability. This motivates the research reported in this paper.

We select two mechanisms to implement in a laboratory, whose Nash equilibria are Pareto optimal: the Basic Quadratic mechanism by Groves and Ledyard (1977) and the Paired-Difference mechanism by Walker (1981). While the Basic Quadratic mechanism has been studied in laboratories, the Paired-Difference mechanism has not been systematically studied in laboratories. A comparison of these two families of mechanisms has not been performed either. This comparison allows us to abstract the properties that induce convergence and stability when a mechanism is implemented among boundedly rational agents, i.e., to answer

[1]See Ledyard (1995) for a survey of the experimental literature on the voluntary contribution mechanisms.

[2]This impossibility has been demonstrated in the work of Green and Laffont (1977), Hurwicz (1975), Roberts (1979) and Walker (1980) in the context of resource allocation with public goods.

the question, what properties of a static mechanism can induce the subjects to learn to play an equilibrium strategy.

To study the dynamic learning processes induced by these mechanisms, we use two major families of learning mödels: three variants of the Stimulus response models and a generalized fictitious play model. Some clear conclusions emerge: the Basic Quadratic mechanism under a high punishment parameter converges the fastest and remains the most stable of all three. It is followed by the Basic Quadratic mechanism under a low punishment parameter, and then followed by the Paired-Difference mechanism. Reasons for this drastically different Performance of different mechanisms are discussed. in particular. the incentives to learn and deviation sensitivity are formalized.

The paper is organized as follows. In Section 2 we review the theoretical properties of the Basic Quadratic and Paired-Difference mechanism. Section 3 reviews previous implementation of these mechanisms. Section 4 goes over the experimental design. Section 5 summarizes the group level results. Section 6 introduces several learning models and uses them to analyze the data. Section 7 discusses two additional aspects of the mechanisms that induce good dynamics. Section 8 concludes the paper.

2 The Mechanisms - Static Properties

Two families of mechanisms are studied in the same environment: the Basic Quadratic mechanism (hereafter BQ) and the Paired-Difference mechanism (hereafter PD). These two mechanisms have very similar static properties. Both are Nash-efficient and balanced with the same dimensions of message space[4]. The PD mechanism is also individually rational in equilibrium. These properties are introduced in turn.

2.1 The Basic Quadratic Mechanism

The Basic Quadratic mechanism is the first mechanism in a general equilibrium model, in which through a government allocation-taxation scheme the behavioral equilibria (Nash) are Pareto optimal. That is, given the allocation-taxation scheme, consumers find it in their self-interest to reveal their true preferences for the public goods and the public goods are produced at an optimal level. Therefore the mechanism is incentive compatible, Pareto optimal, and it balances the budget both on and off the equilibrium path.

The BQ mechanism allocates each individual's share of the cost of public good by

$$T_i^{BQ}(x_i|\mu_{-i},\sigma_{-i}^2) = \frac{X}{I}\cdot q + \frac{\gamma}{2}[\frac{I-1}{I}(x_i-\mu_{-i})^2 - \sigma_{-i}^2],$$

where $\gamma > 0$ is the punishment parameter, I is the number of people in the economy, x_i is individual i's message, indicating her proposed addition to the total amount of public good provided, and $X = \sum_i x_i$ is the total amount of public good. Define $S_{-i} = \sum_{j \neq i} x_j$ as the sum of the proposed increments by all other members of the group except i, $\mu_{-i} = S_{-i}/(I-1)$ as the mean of others' messages, and $\sigma_{-i}^2 = \sum_{h \neq i}(x_h - \mu_{-i})^2/(I-2)$ as the squared standard error of the mean of others' messages. Production of public goods exhibits constant returns to scale, with q denoting the per unit cost of the public good.

Therefore, an individual's share of the cost is composed of three parts: the average cost of production, $X \cdot q/I$, plus a positive multiple, $\gamma/2$, of the difference between her own message and the mean of others' messages, $((I-1)/I) \times (x_i - \mu_{-i})^2$, and the squared standard error of the mean of others' messages, σ_{-i}^2. While the first two parts guarantee that Nash equilibria of the mechanism is Pareto optimal, the last part insures that budget is balanced both on and off the equilibrium path. Note that the free parameter, γ, determines the magnitude of punishment when an individual deviates from the mean of others' messages. Although it does not affect any of the static theoretical properties of the mechanism, as we will see from the experimental evidence that varying γ can induce very different dynamics.

The BQ mechanism has two drawbacks: it does not satisfy the individual rationality constraint, i.e., an individual can be worse off as a result of participating in the process; in a general environment multiple equilibria can exist[5]. The way we deal with the first problem is to give every subject an initial endowment. For the second problem, a quasilinear environment is used, in which there exists a unique Nash equilibrium. The equilibrium selection problem in a general environment is left for future research.

2.2 The Paired-Difference Mechanism

The Paired-Difference Mechanism implements the Lindahl equilibria in a public goods environment. Therefore, besides all the nice properties of the BQ mechanism, it is also individually rational in equilibrium, i.e., no individual will be worse off as a result of participating in the mechanism.

The PD mechanism allocates each individual's share of the cost of public good provision by

$$T_i^{PD}(x_i|S_{-i}, d_i) = (q/I + d_i)(x_i + S_{-i}) \equiv (q/I + x_{i-1} - x_{i+1})X,$$

where the level of individual i's tax, T_i^{PD}, depends upon her proposed addition, x_i, the sum of the proposed additions of other participants, S_{-i}, and the difference between the amounts proposed by her two neighbors, d_i. The amount, $q/I + x_{i-1} - x_{i+1}$, is i's Lindahl price.

Therefore, an individual's share of the cost is composed of two parts: the average cost of production, $X \cdot q/I$, plus an amount determined by one's two neighbors, $(x_{i-1} - x_{i+1})X$. The Nash equilibrium of the PD mechanism is Pareto optimal. It balances the budget both on and off the equilibrium path, i.e., $\sum_{i=1}^{I} T_i^{PD}(x_i|S_{-i}, d_i) = qX$. Individual rationality follows from the fact that the PD equilibrium coincides with the Lindahl equilibrium.

So far the two families of mechanisms have very similar static properties. The PD mechanism has one more advantage over the BQ mechanism in that it is individually rational in equilibrium. An interesting question is whether they will induce similar dynamics when implemented in a laboratory. Before describing our experimental design, we review what had been done by previous researchers regarding the experimental studies of these two mechanisms.

3 Previous Implementation

There have been three groups of experiments[6] with mechanisms motivated by the Basic Quadratic mechanism.

First, Smith (1979) did two sets of experiments, using a simplified version of the BQ mechanism, which only balanced the budget in equilibrium. The process used was the Smith process, where all the subjects need to repeat the same choice three times in a row to finalize the production of public goods, and they were only paid when agreements were reached.

Secondly, Harstad and Marresse (hereafter shortened as HM) (1981, 1982) had run experiments using the complete version of the BQ mechanism. The Seriatim process used in HM also requires unanimity of the subjects to produce the public good, but it differs from the Smith process in that subjects only need to repeat their messages once for an iteration to end.

Neither Smith nor HM studied the effects of the punishment parameter, γ, on the performance of the mechanism. More recently, Chen and Plott (hereafter shortened as CP) (1996) did the first set of experiments to assess the performance of the BQ mechanisms under different punishment parameters, $\gamma = 1$ and 100. The two treatments were run sequentially in each session: two sessions with the order of $\gamma = 1$ proceeding $\gamma = 100$, and the other two with the reversed order. The Periodic process used by CP implemented the public goods

Our experimental design for the BQ mechanism closely resembles the CP experiments in that we also consider two treatments, $\gamma = 1$ and $\gamma = 100$, but differs in both the environments and the process used to facilitate the study of individual learning processes. Most importantly, we have run seven independent sessions for each treatment, which allows us to perform analysis that requires Statistical independence. Note that one Session is only one independent observation due to the intrinsic strategic interaction among subjects within each session. Therefore. compared with previous experiments on the BQ mechanisms, this is the first time some clear statistical results are presented. Indeed, we will see some sharp Statistical contrasts across treatments in the following sections. To compare the dynamic path induced by the mechanisms. we used much longer iterations (100 rounds per session) than any of the previous implementations. Detailed differences in the environments are explained in Section 4.

Compared with the BQ mechanism, there has been little experimental work on the PD mechanism. Robin Hanson ran a pilot experiment testing the PD mechanism with the Smith process, and found nonconvergence. So far there has been no published experimental work on the PD mechanism. One of the reasons for the lack of systematic experimental study of the PD mechanism might be due to the fact that theoretically it does not converge under either Bayesian or Cournot dynamic behavior. However, experimental evidence strongly rejects Bayesian and Cournot learning in favor of other models.[7] Because of its simplicity and good theoretical properties, we think it is worthwhile to actually implement it in a laboratory, in an environment which gives it a best shot, to study the actual dynamics it induces. If, in that case, we find no convergence with real human subjects after all, we need to ask why and draw some lessons for mechanism design.

4 Experimental Design

The experimental design reflects both technical and theoretical considerations. The economic environment and the experimental procedures are discussed in the sections below.

4.1 The Economic Environment

We are interested in an environment where theoretically the voluntary contribution mechanism predicts zero provision, while the BQ mechanism and PD mechanism predict Pareto efficient provision of public good. A second consideration is the influence of the punishment parameter in the BQ mechanism on the convergence of the dynamic processes.

[7]See, e.g., Boylan and El-Gamal (1993), Cox, Walker and Shachat (1996).

The parameters chosen for the experiments involve five individuals, $I = 5$. In all experiments a simple constant unit cost, q, is used to produce the public good, which is set to 100. Preferences are induced on units of the abstract public good by an individually specified value function, $V_i(X)$, which indicates the amount of money an individual will receive if the group choice of the public good is X and if the individual pays nothing for it. For simplicity and for comparison of our results with previous experiments, Smith (79), HM and CP, the valuation functions are set to be quadratic,

$$V_i(X) = A_i X - B_i X^2 + \alpha_i.$$

Given the size of the economy, the punishment parameter, γ, defines a family of BQ mechanisms. To study the effects of the punishment parameter on the dynamics and learning processes of the BQ mechanism, we set $\gamma = 1$ and 100.

In implementing the PD mechanism, one problem is the selection of one possible mechanism from an entire family. Given an economy with I individuals, there are $|I|!$ different possible circles and hence $|I|!$ corresponding equilibria. Of all the 5! circles that correspond to different PD mechanisms in this environment, we let the computer randomly pick a circle, 1-2-4-3-5, to implement.

[Table 1 about here.]

Table 1 lists the parameters of individual subject's valuation functions (A_i, B_i and α_i), their equilibrium proposals (x_i^e) and payoffs (π_i^e) under the three different mechanisms. The particular values of the preference parameters, A_i, B_i and α_i, are chosen for the following reasons: (1) In a voluntary contribution mechanism, the theoretical equilibrium is zero contribution for all subjects, while the three incentive compatible mechanisms predict Pareto efficient level of public goods, $X = 5$, as shown in the last row. (2) $\{B_i\}_i$ varies among subjects to induce diverse tastes for the public good. (3) Initial endowments, $\{\alpha_i\}_i$ were set such that the equilibrium payoffs[8] of all subjects, π_i^e, tabulated in the last three columns of Table 1, are approximately the same in all three mechanisms. (4) the equilibrium contributions for BQ1 and BQ100 are all integers, and the PD equilibria mutiplied by five are also integers[9]. To avoid fractions, the subjects actually chose $5x$, and all formula were adjusted accordingly. Therefore, in all three mechanisms, subjects could choose any integer from -20 to 30, which includes all the stage game equilibria.

[8]The payoff to each subject, π_i, is rounded up to the nearest integer, but due to the large conversion rate, it should have negligible effects. For example, 206.45 francs to rounded to 207 francs. With the smallest

4.2 Experimental Procedures

Seven independent computerized sessions for each mechanism were conducted in March, April and May 1996: four at Caltech, and three at the University of Amsterdam. All sessions were conducted in English by the first author[10]. Our subjects were students from the two universities. No subject was used more than one session. This gives us a total of 105 subjects and 21 independent sessions. Each session consisted of 100 rounds with no practice round[11], which lasted between one and two hours, with the first half an hour being used for instructions.

[Table 2 about here.]

Table 2 summarizes session numbers, dates and places experiments were conducted, and the conversion rates of these experiments. The conversion rates were set such that the expected average earning per hour was approximately the same as that of other experiments in each lab. The PD mechanism took longer time than the BQ mechanisms, therefore the conversion rate was set lower.

Subjects who participated in an experimental session randomly drew an I.D. number. Then each of them was seated in front of the corresponding terminal, with a folder containing a set of instructions, payoff chart(s) and record sheets. After the instructions (see Appendix A) were read aloud, subjects were required to finish the Review Questions, which were designed to test their understanding of the instructions[12]. Afterwards, the experimenter checked answers individually and answered questions. After this, subjects signed the Financial Agreement[13]. Then the experimenter read the computer instruction.

Implementation of the mechanisms was based on a periodic process, as in CP (1996). At round t, a subject submitted her proposed addition, $x_i(t)$. After everyone submitted their proposals, the following information appeared on i's screen:

$$\{x_i(t), S_{-i}(t), \sigma^2_{-i}(t), \pi_i(t), \textstyle\sum_{s=1}^t \pi_i(s)\}, \quad \text{(in BQ1, BQ100), or}$$
$$\{x_i(t), S_{-i}(t), d_i(t), \pi_i(t), \textstyle\sum_{s=1}^t \pi_i(s)\} \quad \text{(in PD).}$$

[10]Thus, the problems with the experimenter effect and the language effect are circumvented in our experiments. See, e.g., Roth *et. al.* (1991) for detailed discussions on such effects.

[11]Robin Hanson suggested that from his experience of running the pilot experiment on the PD mechanism, one should let it run as long as the subjects can take to see whether it will converge. Since it took about two hours to run 100 rounds, we decided to run 100 rounds.

[12]In one session [04/05/96(glcr)] one subject did not seem to understand the instructions and answered all Review questions incorrectly, which was only discovered by the experimenter after the session. This was the only session where a subject gave wrong answers to the Review Questions. We decided that it was not

Subjects were then required to record $x_i(t)$ and $\pi_i(t)$ on a record sheet, space was also provided for them to record $S_{-i}(t)$ and $d_i(t)$, but these were optional[14].

During the experiment, subjects could access past history at any time by hitting the History key. When subject i used the History key at round t, the following information was available on i's screen:

$$\{x_i(s), S_{-i}(s), \sigma^2_{-i}(s), \pi_i(s)\}_{s=1}^t \quad \text{(in BQ1, BQ100), or}$$
$$\{x_i(s), S_{-i}(s), d_i(s), \pi_i(s)\}_{s=1}^t \quad \text{(in PD).}$$

Some subjects did use the History page. Since most subjects had the above information recorded, they did not use the History page that often[15].

The process was repeated for 100 rounds, which was announced at the beginning of the instructions. At the end of a session, the subjects recorded their total earnings (in fictitious currency[16]) for all rounds and converted them to dollar (or guilder) payments. The conversion rate was announced in the instructions and was written on the board for their attention.

To summarize the information conditions, apart from the above information on their screen, each subject knew her own valuation function, the BQ cost function, or the PD cost function. They knew that other subjects in the same group might have different valuation functions, but everyone faced the same cost function. They do not know the distribution of preferences.

5 Group Level Results

Results on the aggregate performance of the mechanisms are summarized in Result 1 to Result 4. Two questions are of overriding importance. The first is related to the actual performance of the BQ and PD mechanism in general. The second is related to the underlying principles of individual behavior. We address[17] the first question in this section. A more detailed examination of individual behavioral models is reserved for the next section.

[14]We required the subjects to record $x_i(t)$ and $\pi_i(t)$ to prevent the loss of data in case of a computer crashdown. In the pilot experiments, $x_i(t)$ and $\pi_i(t)$ were the only two columns on the record sheets, but most subjects also recorded $S_{-i}(t)$ and $d_i(t)$ on the margin, which we subsequently added to the record sheets with a mark of [optional], in all the formal sessions. We found that all subjects recorded $x_i(t)$ and $\pi_i(t)$, about 80% also recorded $S_{-i}(t)$ and $d_i(t)$.

[15]The program did not record the number of times a subject accessed the History page. These were the impression of the experimenter.

[16]At Caltech, the fictitious currency is francs. In Amsterdam, the fictitious currency is pesos. The names

Group efficiency, mean and standard deviation of public good level, and the average absolute deviation from the optimal level of public good for each session are tabulated in Table 3.

[Table 3 about here.]

Group efficiency is calculated by taking the ratio of the sum of the actual earnings of all subjects in a session and the Pareto optimal earning of the group. As a benchmark case, if no public good is produced, the system efficiency is

$$E_0 = \frac{\text{Total initial endowment in private good}}{\text{P.O. value of the group}} = \frac{525}{1035} = 50.73\%$$

Result 1 : *The ranking of group efficiency is highly significant: BQ100 > BQ1 > PD.*

SUPPORT. The third column in Table 3 lists the sessional group efficiency under the three mechanisms. Permutation tests[18] show that BQ100 > BQ1 at a significance level of 0.23% (one-tailed), BQ100 > PD at a significance level of 0.03% (one-tailed), BQ1 > PD at a significance level of 0.20% (one-tailed). □

Result 1 shows that BQ100 generates the highest group efficiency, followed by BQ1, and then PD mechanism.

As can be seen from the fourth column of Table 3, and confirmed by permutation tests, the average levels of public good provision are not significantly different across mechanisms, since the mean averages out the over- and under-provision of public good across different rounds. However, the standard deviations from the average public good levels are significantly different.

Result 2 : *The ranking of the standard deviation from the average level of public good is highly significant: BQ100 < BQ1 < PD.*

SUPPORT. The fifth column of Table 3 shows the standard deviation of the average level of public good. Permutation tests show that BQ100 < BQ1 at a significance level of 0.17% (one-tailed), BQ100 < PD at a significance level of 0.03% (one-tailed), BQ1 < PD at a significance level of 0.12% (one-tailed). □

[18]The permutation test, also known as the Fisher randomization test, is a nonparametric version of a difference of two means t-test (See, e.g., Siegel and Castellan (1988), p.95-100.). The idea is simple and intuitive: by pooling the 14 independent observations, the p-value is obtained as the exact probability of

Result 2 shows that BQ100 induces the least amount of dispersion in the level of public good provided from period to period. It is followed by BQ1, and then by PD.

To assess how successful each mechanism is in providing close to Pareto optimal level of public good, we define a measure

$$D = \sum_{t=1}^{100} |X(t) - 5|/100,$$

as the average absolute deviation of the total level of public good each round from the Pareto optimal level of 5.

Result 3 : *The ranking of average absolute deviation from the Pareto optimal level of public good is highly significant: BQ100 < BQ1 < PD.*

SUPPORT. The last column of Table 3 shows the average absolute deviation from the Pareto optimal level of public good. Permutation tests show that BQ100 < BQ1 at a significance level of 2.10% (one-tailed), BQ100 < PD at a significance level of 0.03% (one-tailed), BQ1 < PD at a significance level of 0.26% (one-tailed). □

Result 3 states that BQ100 produces the closest to Pareto efficient level of public good, followed by BQ1, followed by PD.

Since two subject pools were used for the experiments, we also check whether there exist statistically significant differences between the subject pools.

Result 4 : *The difference between the Caltech sessions and the Amsterdam sessions for each mechanism is not statistically significant.*

SUPPORT. For BQ100, between the four sessions from Caltech and three sessions from Amsterdam, permutation test gives us a lower-tail probability of 14.3% for the sessional group efficiency, 14.3% for the standard deviation from the average level of public good, and 34.3% for the sessional average absolute deviation from the Pareto optimal level of public good. None of the probabilities fall under the usual one-sided 5% significance level to reject the "no difference" null hypothesis. Similar results are obtained for BQ1 (17.1%, 14.3% and 28.6%, respectively) and PD (14.3%, 20.0% and 28.6%, respectively). □

The aggregate results indicate that the performance of BQ100 is far better than BQ1, and both are better than PD. Since the three mechanisms have very similar static properties, it is clear that individual behavior is important in understanding the dynamics that leads to the above results. In the next section, we evaluate several learning models in an attempt

6 Learning

"Learning" can be viewed as any systematic change of behavior due to experience accumulation. A learning model, following the probabilistic approach of Bush and Mosteller (1955), is a mathematical system which predicts the probabilities of available choices or feasible actions at the next occurrence. There are many learning models attempting to capture the principles of human learning behavior[19]. Since it might be misleading to claim which model is the "true" description, we evaluated two major classes of models to see which one tracks the data better under different institutions/mechanisms, i.e., which one is closer to truth. Comparisons of the performance of different learning models can also lead to the discovery of results which are robust across different specifications of individual learning behavior.

To evaluate the accordance between model predictions and the experimental data, one can measure the deviation of the model predictions from the actual choices by the quadratic deviation measure (QDM), which is a proper scoring rule[20]. We also evaluated all models by two other scoring rules, the absolute deviation measure (ADM) and the proportion of inaccuracy (POI) scores. All qualitative results hold under all three scoring rules, although only the QDM scores are reported[21] in this paper, due to limited space.

Recall that subjects can choose any integer, $5x_i \in \{-20, 30\}$, namely, each has 51 stage-game strategies under each mechanism. We reduce the 51 strategies to eleven choice intervals, by dividing a choice number by 5 and rounding it up to the nearest integer in order to have mutiple observations for each strategy interval[22]. Note that under this treatment, all equilibria in each mechanism are still treated equally, even if they are not integers, since choices in the neighborhood of radius 0.5 of an integer are also given credit.

Let $j = 1, \cdots, 11$ correspond to the strategies of choosing the number

$$\{-4, -3, \cdots, 5, 6\}.$$

Let $\vec{c}_i(t) = (c_{i1}(t), c_{i2}(t), \cdots, c_{i11}(t))$ denote the indicator vector of subject i's contribution at round t,

$$c_{ij}(t) = \begin{cases} 1, & \text{if alternative } j \text{ is chosen in round t,} \\ 0, & \text{otherwise.} \end{cases}$$

[19]See Tang (1995, 1996b) for a summary of 18 different types of learning models, which were applied to the experimental data on three 3×3 bimatrix games.

[20]A scoring rule is "proper" if it does not give the forecaster any incentive to "ignore the verification system", or even worse, to "play the system". See Brier (1950) and a recent survey by Yates (1990).

Let $\vec{p}_i(t) = (p_{i1}(t), p_{i2}(t), \cdots, p_{i11}(t))$ denote the predicted choice probability vector for subject i at round t. Then the quadratic deviation for subject i at round t is

$$QDM_i(t) = \sum_{j=1}^{11} [c_{ij}(t) - p_{ij}(t)]^2.$$

It follows that the average quadratic deviation for an entire session is

$$QDM = \sum_{i=1}^{5} \sum_{t=1}^{100} QDM_i(t)/5.$$

And the overall average quadratic deviation measure for each mechanism is the average QDM scores over all seven sessions. Apparently, the smaller the QDM score a model produces, the better its prediction is.

The models we evaluated on our data sets can be classified into three categories: first, three static benchmark models; second, three variants of a basic class of individualistic Relative-Payoff-Sum (RPS) models; third, a class of population learning models, which we call a Generalized Fictitious Play model. The reason we focus on the last two categories of learning models is two-fold: the RPS-type models emerge as the top performers among the 18 different models evaluated in Tang (1996b); however, the fictitious play model, which revived a lot of theoretical attention lately, is too influential to be ignored. In the subsequent parts, we present all the learning models we have tested, and discuss the implication of the results.

6.1 Three Static Benchmark Models

The three static models are not really dynamic learning models. We use them as benchmarks to evaluate the real dynamic learning models.

As a baseline, we calculate the QDM for the **Random Choice Model**, i.e., each individual randomly chooses one of the 11 different alternatives with equal probability for all rounds. This model does not use any information about the particular mechanism and the relevant payoffs. It is a completely universal model.

The vector of predicted choice probabilities is

$$\vec{p}_i(t) = (\frac{1}{11}, \frac{1}{11}, \cdots, \frac{1}{11}) \quad \forall i, \text{ and } \forall t.$$

Then the quadratic deviation measure for a session is

Since this model is based on totally random choices of each subject, a learning model that produces a QDM score greater than the above score perhaps does not capture the essense of individual learning processes.

Another benchmark model is the **Equilibrium Model**, which uses the stage game equilibrium as the prediction,

$$p_{ij}(t) = \begin{cases} 1, & \text{if alternative } j \text{ is a stage game equilibrium strategy for } i, \\ 0, & \text{otherwise.} \end{cases}$$

One important measure of the performance of a mechanism is whether it induces convergence to its stage game equilibrium. Since all three mechanisms are implemented as repeated games, presumably one could conduct a traditional equilibrium analysis of the repeated game. However, applying refinements to the repeated game that describes the entire learning process does not increase their explanatory power, and seems less plausible in this complicated setting. Therefore, throughout the paper we adopt the working hypothesis prevalent in the learning literature that subjects focus on the stage-game strategies (see, e.g., Crawford (1995)).

Note that while the QDM score of any learning model is an indicator of how well the learning model performs, only the QDM score of the equilibrium model is an indicator of how well a mechanism performs.

The third static model we evaluate is the **Individual-Observed Frequency (Mean) Model**, which uses the actual observed choice frequency of a subject as the prediction. It is well known that the constant probability vector that minimizes the quadratic deviation measure for any subject is the actual observed choice frequency of that subject, i.e.,

$$\vec{p}_i(t) = (f_{i1}, f_{i2}, \cdots, f_{i11}), \quad \forall t,$$

where $f_{ij} = \sum_{t=1}^{100} c_{ij}(t)/100$ is the actual frequency that subject i chooses alternative j. Note that it is constructed *ex post*, with too many parameters (10 for each subject). Nevertheless, evaluating these static models can give us some idea about the baseline for the comparison of genuinely dynamic learning models and their performance. Any serious descriptive learning model should be able to overtake (or at least challenge) the static yardsticks set by these models.

6.2 Individualistic Learning Models

This section contains variants of the Relative Payoff Sum (RPS) model. The basic idea of learning model or stimulus response model (Fudenberg and Levine (1996)). Learning models in this spirit have a long history in biology and psychology, but their systematic application in experimental economics seems to start from Roth and Erev (1995), where several variants of the basic linear form were used to construct computer simulations at the group or population level to track the ultimatum bargaining, best-shot and market game experimental data from a comparative study in Jerusalem, Ljubljan, Pittsburgh and Tokyo. They did not generalize their simulations to nonlinear functional forms. Thus little could be said about the comparative performance of various learning models.

Since each of our experimental session consists of 100 rounds, which is long enough for performing more detailed analysis than the possibility available to Roth and Erev (1995), we can compare the performance of various learning models in tracking the data down to the individual level.

The first real dynamic model we tested is the linear form **Relative-Payoff-Sum Model (RPS)**. The original RPS model deals with non-negative payoffs only. How to deal with negative payoffs has been in much discussion (see Kahnman and Tversky (1979) and its following literature). Erev and Roth (1996) constructed a reference-point-based RPS reinforcement learning model[23]. We take a much simpler approach here, as suggested by R. Selten, that if a negative payoff occurs as a result of playing a certain strategy, subjects might increase the probability of playing other strategies. A simple mathematical formula follows[24].

Define $M_{ij}(t)$ as the discounted payoff sum of individual i to choose strategy j, we have

$$M_{ij}(t) = \begin{cases} qM_{ij}(t-1) + c_{ij}(t)\pi_{ik}(t), & \text{if } \pi_{ik}(t) \geq 0; \\ qM_{ij}(t-1) + [c_{ij}(t) - 1]\pi_{ik}(t), & \text{if } \pi_{ik}(t) < 0, \end{cases} \tag{1}$$

where $q \in [0, 1]$ is the time/memory discount factor.

The predicted probability for subject i at round $t+1$ is

$$p_{ij}(t+1) = \frac{M_{ij}(t)}{\sum_{k=1}^{11} M_{ik}(t)}, \quad \forall i, j.$$

Note these predictions are based solely on the individual payoff gains of that specific subject. In that sense, it is an individualistic learning model.

Two nonlinear variants of the basic RPS model are also tested. The **Power-RPS** model uses the power transformation of the basic RPS model, which has the predicted probability

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[23]They also tracked the data down to the individual level when the individual choice sequence was available

of

$$p_{ij}(t+1) = \frac{[M_{ij}(t)]^r}{\sum_{k=1}^{11}[M_{ik}(t)]^r}, \quad \forall i, j,$$

where r is a non-negative constant.

Note that when $r = 1$, we obtain the RPS model immediately. When $r = 0$, the model degenerates to the Random Choice model. The power parameter, r, helps to scale up (when $r > 1$) or scale down (when $r < 1$) the relative weights of the discounted payoff sums. This variant was independently discovered by Tang (1995, 1996) and Chen, Fiedman and Thisse (1996)[25].

Another nonlinear variant of the basic RPS model is the **Exponentialized-RPS** model, where the predicted probability is

$$p_{ij}(t+1) = \frac{e^{\lambda \cdot M_{ij}(t)}}{\sum_{k=1}^{11} e^{\lambda \cdot M_{ik}(t)}}, \quad \forall i, j,$$

where $\lambda \geq 0$ can be interpreted in a similar way as the power parameter, r. When $\lambda = 0$, again, we obtain the Random Choice model.

This model is also called the Quantal Response Learning Model (Mookherjee and Sopher (1996)), which is a dynamic learning version of the Quantal Response Equilibrium model of McKelvey and Palfrey (1994). This approach originated in the multinomial logit framework used in the econometric models of discrete choice (See, e.g., McFadden (1984)). It has also been applied to explain the evolution of market organization using field data (Weisbuch, Kirman and Herreiner (1996)).

One advantage of the Exponentialized-RPS model is that negative payoffs can be treated the same as positive payoffs, since the exponential function gives a positive number whether the discounted payoff sum is positive or negative. Therefore, this model gives us a wonderful opportunity to check the effects of modelling techniques of negative payoffs. We ran simulations in both ways: one using Eq. (1), the other using $M_{ij}(t) = qM_{ij}(t-1) + c_{ij}(t)\pi_{ik}(t)$ as the discounted payoff sum for both positive and negative payoffs. We found that in both cases the numerical results of the simulation are exactly the same until the eighth digit after the decimal point. This is due to the fact that negative payoffs occurred so rarely that their effects in overall simulation results of 100 rounds are negligible. This finding seems to indicate that specific modeling techniques of negative payoffs do not play an important role in this data set.

The initial value we used for all three RPS-type models is

$$M_{ij}(0) = 200, \text{ for all } i, j,$$

since the first-round payoffs for most of our subjects were around 200. We have also tried various other initial values, ranging from 10 to 500, which produced little difference. It seems that due to the long sequence of play, as long as the initial values are not set too large or too small, the performance of the RPS-type learning models is hardly affected[26]. Furthermore, these initial values resulted in probability predictions around the centroid, $(1/11, \cdots, 1/11)$, for the first round, where no history information is available, a somewhat "natural" starting point.

For all three models, we have searched the discount factor $q \in [0, 1]$ at a grid size of 0.05. We searched for the power-parameter, r, at a grid size of 0.01 for the power-RPS model, and the λ parameter at a grid size of 0.001 for the exp-RPS model, until the minimum QDM scores are obtained.

6.3 Population Learning Model - Generalized Fictitious Play

Compared with the individualistic learning models, where an individual subject bases her decision on her own past payoff information only, population learning models allow an individual to base her decision on some summary statistics of the population as well. We use a generalized version of the fictitious play model to analyze the data.

It is straightforward to calculate the best response for both mechanisms. For the BQ mechanism, a player's best response to some predicted population characteristics is

$$x_i = a_i S_{-i} + b_i,$$

where

$$a_i = \frac{(\gamma/I) - 2B_i}{\gamma(I-1)/I + 2B_i}, \; b_i = \frac{A_i - q/I}{\gamma(I-1)/I + 2B_i}.$$

For the PD mechanism, the best response function is

$$x_i = m_i - n_i d_i - S_{-i},$$

where

$$m_i = \frac{A_i - b/I}{2B_i}, \quad n_i = \frac{1}{2B_i}.$$

The dynamics of the decision process is according to a retrospective learning rule. For some discount factor, δ, we assume that players predict the S_{-i}, d_i at round $t+1$ according to[27],

$$S_i(t+1) = \frac{S_i(t) + \sum_{u=1}^{t-1} \delta^u S_i(t-u)}{1 + \sum_{u=1}^{t-1} \delta^u}, \quad d_i(t+1) = \frac{d_i(t) + \sum_{u=1}^{t-1} \delta^u d_i(t-u)}{1 + \sum_{u=1}^{t-1} \delta^u}.$$

Note that this model is quite general. When $\delta = 0$, it yields the Cournot model of expectations $S_i(t+1) = S_i(t)$ and $d_i(t+1) = d_i(t)$. When $\delta = 1$, it yields the fictitious play model. The usual adaptive expectations model assumes $0 < \delta < 1$, so all observations influence the expected state but more recent observations have greater weight.

For the same reason as with the RPS-type models, we have used the centroid $(1/11, \cdots, 1/11)$ as the starting probability prediction vector for the first round for the Generalized Fictitious Play model. The discount factor, $\delta \in [0, 1]$, was searched at a grid size of 0.01. The major results from evaluating the different learning models are summarized in the next section.

6.4 Results from Evaluating the Learning Models

[Tables 4 to 9 about here.]

The numerical results are summarized in Tables 4 to 9, where the "best" or minimum QDM scores are tabulated for each model. Tables 4, 6 and 8 present the initial values, the estimated parameter values and the average QDM scores over all seven sessions for PD, BQ1 and BQ100 respectively. Tables 5, 7 and 9 give the QDM scores of each individual session, using the same estimated parameter values as those listed in Tables 4, 6 and 8. The first column of each table lists the category of the models, with "S" representing Static Benchmark, "I" representing Individualistic, and "P" representing Population. Recall that a QDM score is the sums of 100 rounds of quadratic deviations between model predictions and actual choices. Therefore, the smallest possible score is 0 if a model gives completely correct predictions, and the largest possible score is 200 if every prediction is wrong.

One striking result is that the static Equilibrium model, which uses stage game equilibria as predictions, produces extraordinarily small QDM scores under BQ100, but very large QDM scores under the other two mechanisms.

Result 5 : *Individual players under BQ100 followed their stage game equilibria at an extraordinarily high frequency, much higher than under either BQ1 or PD. Individual players under BQ1 followed their stage game equilibria at a higher frequency than under PD.*

SUPPORT. The "Equilibrium" row from Tables 4, 6 and 8 shows the sessional average QDM scores for PD, BQ1 and BQ100 are 183.09, 169.71, 13.03 respectively. On the sessional level, the "Equilibrium" row from Tables 5, 7 and 9 shows that QDM(BQ100) < QDM(BQ1), or QDM(BQ100) < QDM(PD) is so obvious that any statistical test is superfluous. Permutation test shows that QDM(BQ1) < QDM(PD) at a weak significant level of frequency of choosing stage game equilibrium strategies. As an extension of the above result, we would like to see whether a mechanism induced convergence to its stage game equilibrium. Theoretically, convergence implies that no deviation will ever be observed once the system equilibrates. In an experimental setting with long iterations, even after the system equilibrates, subjects sometimes experiment by occasional deviation. Therefore, it is necessary to have some behavioral definition of convergence: a system converges to an equilibrium at round t, if $x_i(s) = x_i^e \forall i$ and $\forall s \geq t$, except for a maximum of n rounds of deviation for $s > t$, where n is small. For our experiments of 100 rounds, we let $n \leq 5$, i.e., there could be a total of up to 5 rounds of experimentation or mistakes after the system converged. Admittedly, the requirement of $n \leq 5$ is to some extent arbitrary. However, it is necessary to have

some criterion in order to distinguish between sessions that converged and those that did not converge, and to have a measure of the speed of convergence.

Result 6 : *Every session of BQ100 converged to its stage game equilibrium, most of which converged fairly quickly. BQ1 and PD never converged to their stage game equilibrium.*

SUPPORT. The seven sessions of BQ100 (Session No. 15 to 21) converged to its stage game equilibrium on the following round: 22, 9, 76, 44, 9, 44, 60. In all seven sessions, every deviation after convergence was made by a single subject while all other subjects still chose their stage game equilibrium strategy. The other two mechanisms never converged to their stage game equilibria. Moreover, stage game equilibrium under BQ1 and PD was not even reached by all subjects simultaneously at any round in any session. □

Another interesting observation is that under BQ100 none of the dynamic learning models we evaluated exhibits smaller overall QDM scores than the static individual-observed-frequency (mean) model, in contrast to the observation that the RPS-type dynamic learning models produce much smaller overall QDM scores than all the static models under BQ1 and PD. This sharp contrast indicates that BQ100 has been very successful in inducing the subjects to play stage game equilibria and in stablizing the choices around the equilibria. Therefore, a simple constant probability vector becomes the "best" predictor for their behavior.

Result 7 : *BQ100 is the most stable of the three mechanisms.*

SUPPORT. Comparing the QDM scores of each learning model across mechanisms in

While Results 5. 6 and 7 provide further evidence in ranking the Performance of the mechanisms, Results 8 and 9 compare the performance of the learning models evaluated on this data set. Since BQ100 induces fast and stable convergence to its stage game equilibria, all learning models perform very well under this mechanism. It is the relatively volatile dynamic paths of BQ1 and PD that provide a sharp separation of the performance of the RPS-type model and the Generalized Fictitious Play model.

Result 8 : *The individualistic RPS-type learning models fit the BQl and PD data much better than Generalized Fictitious Play model.*

SUPPORT. Tables 4 to 9 show that the Generalized Fictitious Play model produces much larger (almost double) QDM scores than any of the three individualistic RPS-type learning models, not only at overall averages but also at independent sessional averages. The difference is so obvious that Statistical tests are superfluous. Either permutation test or Wilcoxon test can give a clear-cut statistic Separation at 1% significance level (one-tailed).
•

One might argue that these two types of learning models are not entirely comparable, since the generalized fictitious play model is a deterministic model which makes extreme predictions of 0 or 1, while all the RPS-type models make stochastic predictions. To correct for this bias, we also evaluated all learning models under the absolute deviation measure[28] and proportion of inaccuracy[29] scores. All the results still hold under these two scoring rules.

The next result shows that the performance of the three RPS models is statistically indistinguishable.

Result 9 : *The comparative performance of the three individualistic learning models, the linear form RPS, power-RPS and exp-RPS is statistically indistinguishable. However, considering that the linear form RPS model has only one parameter while the power-RPS and exp-RPS model each have two parameters, one can say that the linear form RPS model tracks our experimental data reasonably and robustly well, across sessions and across mechanisms.*

[Table 10 about here.]

[28]The absolute deviation measure is calculated as ADM $= \sum_{i=1}^{5}\sum_{t=1}^{100}\sum_{j=1}^{11}|c_{ij}(t) - p_{ij}(t)|/5$.

[29]The proportion of inaccuracy score "returns the value of 0 if the subject made the most likely choice under the model, the value of 1 if the subject choose a strategy that differs from the most likely prediction, and 1 - 1/(the number of equally likely predictions) if the model predicts that few strategies are equally likely and the subject choose one of them. (Thus the POI score judges all the models on the basis of their

SUPPORT. Table 10 tabulates the results from permutation tests comparing the performance of these three individualistic learning models, which are statistically indistinguishable. Both the upper-tail and the lower-tail probabilities are much larger than the ususal one-sided 5% significance level, i.e., we cannot reject the "indifference" null hypothesis in either direction. □

The group level and individual behavioral level results suggest that the performance of the three mechanisms can be ranked as $BQ100 >> BQ1 > PD$, in terms of efficiency, close to Pareto optimal level of public goods provision, less dispersion of public goods provided, speed of convergence to stage game equilibrium, frequency of stage game equilibrium strategies being used, and stability. Recall that the static theoretical properties of the three mechanisms are very similar, with PD having one advantage over the BQ mechanism in that it also satisfies the individual rationality constraint in equilibrium. Therefore, from the mechanism design perspective, it is important to understand what aspects of a mechanism can induce desirable dynamic properties. For example, why is it that BQ100 has by far the best dynamic properties among the three. In the next section, we analyze two additional disequilibrium aspects of incentive compatible mechanisms.

7 Incentives to Learn and Deviation Sensitivity

Since implementation of a static mechanism usually starts somewhere off the equilibrium path, disequilibrium aspects of a mechanism are especially important in inducing convergence to the equilibrium. We define *deviation cost*, DC^{ε}, as a subject's net utility loss when she deviates ε from the equilibrium strategy, i.e.,

$$DC^{\varepsilon} = \pi(x^e) - \pi(x^e + \varepsilon),$$

where x^e is the equilibrium strategy of a player. Subscripts are suppressed for simplicity. It is straightforward to calculate the deviation costs for the two mechanisms:

$$\begin{aligned} DC^{\varepsilon}(BQ) &= [V(S + x^e) - T^{BQ}(x^e|\mu, \sigma^2)] - [V(S + x^e + \varepsilon) - T^{BQ}(x^e + \varepsilon|\mu, \sigma^2)] \\ &= (B + \frac{I-1}{2I}\gamma)\varepsilon^2; \end{aligned}$$

and

$$\begin{aligned} DC^{\varepsilon}(PD) &= [V(S + x^e) - T^{PD}(x^e|S, d)] - [V(S + x^e + \varepsilon) - T^{PD}(x^e + \varepsilon|S, d)] \\ &= B\varepsilon^2. \end{aligned}$$

punishment is, the higher an incentive she has to learn to play an equilibrium strategy. Th incentive is captured by the possible increase in utility (or monetary payoffs). Since

$$DC^{\varepsilon}(BQ100) >> DC^{\varepsilon}(BQ1) > DC^{\varepsilon}(PD),$$

the punishment for deviation from equilibrium strategies is much higher in BQ100 than i either BQ1 or PD. This partly explains why convergence was so fast in BQ100, and why tl frequencies that the subjects play their stage game equilibrium strategies follow the san ordering. When a mechanism is implemented and a player is not playing her equilibriu strategy, she should "know" that she is not doing her best. Under BQ100 it can really resu in big losses if one is not doing one's best, but not so much under BQ1 or PD.

A practical measure of system stability in actual implementation is how sensitive tl system is to deviation. When a system reaches equilibrium, if one person deviates fro equilibrium, what are the effects on the rest of the subjects? Does the noise get diminish or amplified? For simplicity, the following analysis assumes best response dynamics. O could easily carry out the same analysis with other models, e.g., generalized fictitious pla

Suppose player j deviates ε from her equilibrium message at time t, $x_j(t) = x_j^e + \varepsilon$, th for everyone else at time $t+1$, the summary statistics is changed to

$$S_i(t+1) = S_i(t) + \varepsilon, \forall i \neq j, \text{and}$$

$$d_i(t+1) = \begin{cases} d_i(t) - \varepsilon, & \text{if } i = j+1; \\ d_i(t) + \varepsilon, & \text{if } i = j-1; \\ d_i(t), & \text{if } i \neq j-1, j+1. \end{cases}$$

Under the BQ mechanism, player i's $(i \neq j)$ best response at time $t+1$ is

$$x_i(t+1) = x_i(t) + \left[\frac{\gamma/I - 2B_i}{\gamma - \gamma/I + 2B_i}\right]\varepsilon.$$

If the planner knows the distribution of preferences, then she can pick γ such that t *deviation sensitivity* coefficient

$$DS^{BQ} \equiv \left|\frac{\gamma/I - 2B_i}{\gamma - \gamma/I + 2B_i}\right| < 1.$$

With the parameters of our experiments, this is satisfied for both $\gamma = 1$ and 100. Therefc any noise in the system due to deviation or mistake of some player gets diminished un i.e., the noise gets diminished more, the larger the population is. This is because with the BQ mechanism, players react to the mean of everyone eise's message. In a large population. noise created by deviation gets averaged out.

This is not true with the PD mechanism. Under the PD mechanism,

$$x_i(t+1) = \begin{cases} x_i(t) - \varepsilon - \frac{\varepsilon}{2B_i}, & \text{if } i = j+1; \\ x_i(t) - \varepsilon + \frac{\varepsilon}{2B_i}, & \text{if } i = j-1; \\ x_i(t) - \varepsilon, & \text{if } i \neq j-1, j+1. \end{cases}$$

Therefore, the deviation sensitivity coefficient for the PD mechanism is

$$DS^{PD} = \begin{cases} |1 + \frac{1}{2B_i}|, & \text{if } i = j+1; \\ |1 - \frac{1}{2B_i}|, & \text{if } i = j-1; \\ 1, & \text{if } i \neq j-1, j+1. \end{cases}$$

So noise in the system from someone's deviation or mistake does not get diminished except possibly for $i = j - 1$, rather, it either remains the same ($i \neq j - 1, j + 1$) or gets amplified ($i = j + 1$), which can cause the system to unravel.

The above analysis suggests that the success of a mechanism depends not only on its properties in equilibrium, but also on its disequilibrium properties. The comparative performance of the BQ mechanism and the PD mechanism, as well as their disequilibrium properties, provides some lessons for mechanism design. Two aspects are identified, the incentives to learn, and deviation sensitivity. The deviation cost, DC^{ϵ}, imposes incentives for subjects to learn to play their equilibrium strategies by punishing deviations. With proper incentives, such as that of BQ100, a mechanism can successfully induce a subject to play equilibrium strategies. The deviation sensitivity coefficient affects whether noise in a system gets diminished or amplified. A mechanism that uses population characteristics, such as the mean of others' messages, can be designed in such a way that the noise gets diminished in the system. On the other hand, a mechanism that uses individual players' characteristics, such as the difference of one's two neighbors' messages, tends to get unstable because idiosyncrasies or mistakes of a single player can cause the entire system to unravel.

8 Concluding Remarks

The free-rider problem has been the corner stone of the problem of public goods provision. Many mechanisms promise a solution. Two of the most famous ones are the Basic Quadratic mechanism and the Paired-Difference mechanism. Both have very similar static properties:

we study the dynamic properties induced by these mechanisms in a laboratory. Our experiments show that they did induce very different dynamics. Despite all the perfect theoretical properties of the PD mechanism. the empirical evidence from our experiments indicates that in a simple quasilinear environment the BQ mechanism with a properly chosen punishment parameter has much better dyanmic properties.

Comparing the Performance of the BQ mechanism under a high punishment parameter (7 = 100). a low parameter (7 = 1). and the PD mechanism, we conclude that the Performance of BQ100 is far better than BQ1, which. in turn, is better than PD. in terms of system efficiency, close to Pareto optimal level of public goods provision. less dispersion, convergence to stage game equilibrium and stability. All rankings are statistically highly significant.

These results suggest that when we design a mechanism, Standard considerations, such as incentive compatibility, individual rationality and balanced budget, are not enough to guarantee that these desirable properties can actually be obtained in a dynamic process with real human subjects. Other disequilibrium aspects, such as *deviation costs* which impose incentives for subjects to learn to play their equilibrium strategies, and *deviation sensitivity* which can either amplify or diminish noise in a system, are also important to induce good dynamics and stability of a mechanism.

Individual learning rules are important for us to understand the dynamic properties of incentive compatible mechanisms. In an attempt to understand the principles of individual learning behavior, we estimated three static models and four dynamics learning models. Variants of the individualistic relative-payoff-sum (RPS) models outperform the population model of Generalized Fictitious Play on this data set. The comparative Performance of the three variants of the RPS model are shown to be statistically indistinguishable.

To abstract aspects of mechanisms that induce boundedly rational individuals to play equilibrium strategies is an important but difficult task, which requires experimental studies of many mechanisms. This study begins to give us some intuition from comparing two interesting mechanisms. Further experimental study of other mechanisms are needed to confirm the intuition obtained from this study.

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Parameter Subject ID	A_i	B_i	α_i	x_i^e (BQ1)	x_i^e (BQ100)	x_i^e (PD)	π_i^e (BQ1)	π_i^e (BQ100)	π_i^e (PD)
1	26	1	200	-3	1	1.8	202	205	225
2	104	8	10	5	1	3.8	224	230	210
3	38	2	160	-1	1	1.8	204	200	210
4	82	6	40	3	1	-2.2	204	200	190
5	60	4	100	1	1	-0.2	207	200	200
$\sum$	310	21	510	5	5	5			

Table 1: Parameters, Equilibrium Values and Payoffs (in fictitious currency)

Session	Date (Code)	Place	Mechanism	Conv. rate
1	03/31/96(wb)	Caltech	PD	400
2	03/31/96(wr)	Caltech	PD	400
3	04/01/96(wb)	Caltech	PD	400
4	04/01/96(wr)	Caltech	PD	400
5	05/08/96(wb)	Amsterdam	PD	400
6	05/08/96(wr)	Amsterdam	PD	400
7	05/08/96(wy)	Amsterdam	PD	400
8	04/04/96(glib)	Caltech	BQ1	800
9	04/04/96(glir)	Caltech	BQ1	800
10	04/05/96(glib)	Caltech	BQ1	800
11	04/05/96(glir)	Caltech	BQ1	800
12	05/06/96(glib)	Amsterdam	BQ1	700
13	05/06/96(glir)	Amsterdam	BQ1	700
14	05/06/96(gliy)	Amsterdam	BQ1	700
15	04/08/96(glcb)	Caltech	BQ100	800
16	04/05/96(glcr)	Caltech	BQ100	800
17	04/06/96(glcb)	Caltech	BQ100	800
18	04/06/96(glcr)	Caltech	BQ100	800
19	05/07/96(glcb)	Amsterdam	BQ100	700
20	05/07/96(glcr)	Amsterdam	BQ100	700
21	05/07/96(glcy)	Amsterdam	BQ100	700

Mechanism	Session	Efficiency	$\bar{X}$	(Std. Dev.)	D
PD	1	0.741	4.596	(3.566)	2.708
	2	0.787	4.840	(3.250)	2.344
	3	0.879	4.206	(2.329)	1.710
	4	0.746	4.188	(3.460)	2.624
	5	0.628	4.790	(4.302)	3.226
	6	0.878	5.322	(2.448)	1.294
	7	0.577	5.576	(4.552)	3.708
BQ1	8	0.968	4.906	(1.266)	0.826
	9	0.987	4.938	(0.839)	0.330
	10	0.945	4.670	(1.628)	1.026
	11	0.965	5.082	(1.338)	0.642
	12	0.961	4.710	(1.411)	0.474
	13	0.791	4.870	(3.242)	2.590
	14	0.965	4.976	(1.343)	0.684
BQ100	15	0.997	4.982	(0.390)	0.182
	16	0.995	5.010	(0.510)	0.206
	17	0.979	4.684	(0.978)	0.688
	18	0.985	4.946	(0.857)	0.490
	19	0.986	4.836	(0.825)	0.256
	20	0.973	4.910	(1.152)	0.630
	21	0.983	5.088	(0.911)	0.492

Table 3: Group Efficiency, Mean Level of Public Good (Standard Deviation), Avg Absolute Deviation from the Optimal Level of Public Good

Type	Model	Initial Values	Parameters	Parameter Values	QDM
S	Random	$(1/11, \cdots, 1/11)$	—	—	90.909
	Equilibrium	stage game equi.	—	—	183.09
	Mean	individual freq.	—	—	69.682
I	RPS	200	q	.75	62.906
	power-RPS	200	(r, q)	(.88, .75)	62.799
	exp-RPS	200	(λ, q)	(.007, .70)	62.145
P	Generalized Fictitious Play	$(1/11, \cdots, 1/11)$	δ	.90	134.39

Table 4: QDM (quadratic deviation measure) scores of Learning Models for PD - Average Over All Sessions

Type	Model	QDM_1	QDM_2	QDM_3	QDM_4	QDM_5	QDM_6	QDM_7
S	Random	90.909	90.909	90.909	90.909	90.909	90.909	90.909
	Equilibrium	182.800	174.700	179.200	189.200	183.600	197.600	174.800
	Mean	73.836	73.068	75.984	71.608	77.740	39.480	76.056
I	RPS	77.803	66.767	54.952	64.325	78.024	26.555	71.918
	power-RPS	76.905	66.141	55.162	64.788	77.864	26.865	71.869
	exp-RPS	79.369	67.677	53.904	61.092	76.189	25.887	70.898
P	Generalized Fict. Play	135.71	121.31	120.51	158.51	142.51	99.31	162.91

Table 5: QDM (quadratic deviation measure) scores of Learning Models for PD - Individual Sessions (parameter values same as the previous table)

Type	Model	Initial Values	Parameters	Parameter Values	QDM
S	Random	$(1/11, \cdots, 1/11)$	—	—	90.909
	Equilibrium	stage game equil.	—	—	169.71
	Mean	individual freq.	—	—	41.469
I	RPS	200	q	.75	32.918
	power-RPS	200	(r, q)	(.84, .70)	32.900
	exp-RPS	200	(λ, q)	(.007, .70)	33.053
P	Generalized Fict. Play	$(1/11, \cdots, 1/11)$	δ	.720	83.02

Table 6: QDM (quadratic deviation measure) scores of Learning Models for BQ1 - Average Over All Sessions

Type	Model	QDM_8	QDM_9	QDM_{10}	QDM_{11}	QDM_{12}	QDM_{13}	QDM_{14}
S	Random	90.909	90.909	90.909	90.909	90.909	90.909	90.909
	Equilibrium	156.800	194.000	170.000	150.400	198.800	180.400	137.600
	Mean	41.128	30.860	65.876	24.708	16.864	77.836	33.008
I	RPS	32.830	20.359	43.188	16.803	14.847	81.808	20.593
	power-RPS	33.056	20.075	42.771	16.967	14.972	81.894	20.565
	exp-RPS	34.272	19.838	43.668	17.444	14.222	81.002	20.926
P	Generalized Fict. Play	79.71	83.71	94.51	62.51	64.11	134.91	61.71

Table 7: QDM (quadratic deviation measure) scores of Learning Models for BQ1 - Individual

Type	Model	Initial Values	Parameters	Parameter Values	QDM
S	Random	$(1/11, \cdots, 1/11)$	—	—	90.909
	Equilibrium	stage game equil.	—	—	13.03
	Mean	individual freq.	—	—	11.801
I	RPS	200	q	.650	13.467
	power-RPS	200	(r, q)	(2.02, .80)	12.556
	exp-RPS	200	(λ, q)	(.006, .80)	12.457
P	Generalized Fict. Play	$(1/11, \cdots, 1/11)$	δ	$[.49, .52] \cup [.57, .64] \cup [.67, .78]$	13.423

Table 8: QDM (quadratic deviation measure) scores of Learning Models for BQ100 - Average Over All Sessions

Type	Model	QDM_{15}	QDM_{16}	QDM_{17}	QDM_{18}	QDM_{19}	QDM_{20}	QDM_{21}
S	Random	90.909	90.909	90.909	90.909	90.909	90.909	90.909
	Equilibrium	6.400	4.800	26.000	10.400	8.800	20.800	14.000
	Mean	6.068	4.644	22.852	9.936	8.392	18.108	12.604
I	RPS	8.726	6.277	25.642	11.201	8.439	18.273	15.707
	power-RPS	7.582	5.385	24.607	10.467	7.950	17.454	14.447
	exp-RPS	7.867	5.798	24.532	10.443	7.640	16.969	13.952
P	Generalized Fict. Play	6.910	4.510	26.510	10.910	8.510	21.710	14.910

Table 9: QDM (quadratic deviation measure) scores of Learning Models for BQ100 - Individual Sessions (parameter values same as the previous table)

PD	RPS	power-RPS	exp-RPS
RPS	–	0.51	0.54
power-RPS	0.49	–	0.53
exp-RPS	0.46	0.47	–

BQ1	RPS	power-RPS	exp-RPS
RPS	–	0.50	0.49
power-RPS	0.50	–	0.49
exp-RPS	0.51	0.51	–

BQ100	RPS	power-RPS	exp-RPS
RPS	–	0.60	0.61
power-RPS	0.40	–	0.52
exp-RPS	0.39	0.48	–

Table 10: Comparison of Three Variants of the RPS Model - Results from Permutation Tests

Appendix A. Experiment Instructions

The instructions for Mechanism A corresponds to BQ1. BQ100 is essentially the same as BQ1, with an adjusted formula, so it is not shown here. Mechanism W corresponds to the Paired-Difference mechanism. Only the parts of instruction for Mechanism W which are different from those of Mechanism A are included. Both instructions are for subject #1 at Caltech. Instructions for the Amsterdam experiments are the same except for the names of currencies, as explained in the paper. All other instructions are available from the authors upon request.

Experiment Instructions – Mechanism A *ID = 1*

Introduction

You are about to participate in a decision process in which one of numerous competing alternatives will be chosen. This is part of a study intended to provide insight into certain features of decision processes. If you follow the instructions carefully and make good decisions you may earn a considerable amount of money. You will be paid in cash at the end of the experiment.

Your final payoff will be determined by a project level which will be chosen by the group, and by your individual expenditure on the project. The decision process will proceed as a series of rounds during each of which a project level will be determined and financed. The "level" can be negative, zero or positive "units", the exact level of which must be determined.

In your folder, you will find a chart which describes the payoffs to you of various decisions, called the Payoff Chart, and a "Record Sheet", where you will record your decision and payoff each round. *You are not to reveal this information to anyone.* It is your own private information.

The Situation

The payoff each period, which is yours to keep, is the difference between the value to you of the project level which is chosen, and your individual expenditure on the project. All values are stated in francs and can be converted into cash at a rate of ___ francs per dollar at the end of the experiment. Note that in some cases your values can be negative. It is also possible that your expenditures can be negative (that is, rather than paying for the project you are paid.). These will be explained in turn.

Project level determination (Y) Each round each individual will choose a proposed addition *(x)* to the status quo of zero project level. This proposed addition can be any integer amount ranging from _-20_ to _30_. These amounts will be added together to get the total of proposed additions *(Y)*. This total is the project level that will be chosen. For example, if $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, then the project level is $Y = x_1 + x_2 + x_3 + x_4 + x_5 = 15$.

Individual Value Each individual has a different set of values for each different project level[1]. This value can be positive or negative. For your convenience the value for the various project levels is included in your payoff chart.

Level of individual expenditures The level of your individual expenditures depends upon your individual proposed addition (**x**), the sum of proposed additions of other participants (**S**) and the variability among the proposed additions of the other participants (**V**). For example, if $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, then for subject No. 1, the sum of others' proposed addition is $S_1 = x_2 + x_3 + x_4 + x_5 = 14$. (Notice that S_1 is also $Y - x_1$.) The variability measure reflects how scattered the additions of others are. For example if all of the other participants give the exact same number then there is no scatter at all and the variability is zero. Suppose that all of the other participants give a different number but all numbers differ very little, then the scatter is low as is the measure of variability. In the above example, the variability for subject No. 1 is $V_1 = [(x_2 - S_1/4)^2 + (x_3 - S_1/4)^2 + (x_4 - S_1/4)^2 + (x_5 - S_1/4)^2]/3 = 1.67$. Notice your proposed addition does not affect the calculation of your own S and V. The formula for your expenditure is somewhat cumbersome[2], so a Payoff Chart that summarizes all of the relevant information will be used instead.

Payoff Chart The payoff chart summarizes both the value of the level of the project chosen and the level of individual expenditures that you will incur depending upon the choices of additions that you and other participants make. The horizontal axis is the sum of others' proposed additions, S. The vertical axis is your payoff when the variability of others is $V = 0$. Each curve represents your payoff from a particular choice of proposed addition, x. The small box on the right hand side of the chart gives the color of the curve for the eleven different proposed additions charted. Since the chart would be difficult to read if all possible proposed additions were plotted, only eleven different ones equal distant from each other are given. The curves for proposed additions which are not given, lie between the given curves. For example, the curve for a proposed addition of $x = 11$ would be between those of $x = 10$ and $x = 15$ and slightly closer to the $x = 10$ curve than the $x = 15$ curve. For example, in the Payoff Chart on the screen, if you choose $x = -10$, which is represented by the yellow curve on the Chart as indicated in the small box, and the sum of others' proposed addition is $S = 35$, then your value on the vertical axis for this period is 100. Another example, if you choose $x = -5$, which is represented by the dark green curve on the Chart, and the sum of others' proposed addition is $S = 10$, then your value on the vertical axis for this period is 200.

Notice the values on the vertical axis of your payoff chart are your payoffs when the variability of others is $V = 0$. Your actual payoff is the value on the vertical axis plus $0.02V$. *Different participants might have different Payoff Chart.*

Procedure for Each Round At the beginning of each round, you will enter a proposal on the terminal. The central computer will then calculate the sum of others' proposals, the variability of others' proposals and your net payoff, and send this information back to you. At the end of each round, you should record your proposed addition, x, in the first column, and your payoff, P, in the second column of your **Record Sheet**. You can also record the sum of others' proposals, S, in the third column of your Record Sheet, but this is optional.

It is crucial that you check your Payoff Chart before and after each decision. From the Chart you can see your choice determine which curve you use, and others' choices determine the level of S and the amount of shift (due to V) in your payoffs.

There will be 100 rounds using this mechanism. There will be no practice rounds. From the first round, you will be paid for each decision you make.

Feel free to earn as much cash as you can. Are there any questions?

Review Questions

1. If each of you propose the following units (the subscripts correspond to your real ID numbers): $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, (therefore, $V_1 = 1.67$, $V_2 = 2.92$, $V_3 = 3.33$, $V_4 = 2.92$, $V_5 = 1.67$.)

   (1) The total level of the project. $Y =$

   (2) The sum of others' proposal, $S =$

2. Suppose all others have the same proposed addition as in Question 1, you alone raise your addition by 1 unit, then

   (1) The total level of the project, $Y =$

   (2) The sum of others' proposal, $S =$

   (3) The variability of others' proposed additions, $V =$

3. True or False: My payoff is determined by my own proposed addition only.

4. If your proposed addition is $x = -10$, the sum of others' proposed addition is $S = 45$,

   (1) Find your payoff from your Payoff Chart when $V = 0$: $P =$

   (2) If $V = 100$, your payoff is ____.

Financial Agreement

Should my earnings from the experiment be negative, I agree to work in the EEPS Laboratory at a rate of seven dollars per hour until the loss is repaid.

Name ____________ Signature ____________

Date ____________

Computer Instructions At the beginning of each round, you are free to enter any proposed choice is out of range and you need to change your selection. Now everybody please use the **Back Space** key to erase your choice, and then type in your first decision. Now please press the **S** key and then the **Enter** key. Once you type the **Enter** key, you cannot change your choice anymore. After everyone sends their choices, the computer will calculate the sum of others' proposals, S, the variability, V, and your corresponding payoff for this round, P, and send these numbers to your screen. This process will be repeated on each round. Now go ahead and record the result of the first round to the first row of your Record Sheet.

The **H** key allows you to review the history of your decisions and payoffs. Once you are in the history page, you can use the arrow keys to choose the period.

Key Function Summaries

S: prepare to submit your choice.
Back Space: erase your choices.
Enter: send your choice off to the central computer.
H: go to the history page.

Experiment Instructions - Mechanism W *ID = 1*

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Level of Individual Expenditures Each individual has been randomly assigned an I.D. number. These numbers have been randomly arranged into a secret circle. This circle will remain the same in all rounds, but its arrangement will not be disclosed to you. The level of your individual expenditures depends upon your individual proposed addition (**x**), the proposed additions of other participants (**S**), and the difference (**d**) between the amounts proposed by the individual to the left of you in the circle, and the individual to the right of you.

For example, if the circle is 1-2-3-4-5-1, and the proposed additions are $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$ respectively, then for subject No. 1, the sum of others' proposed addition is $S_1 = x_2 + x_3 + x_4 + x_5 = 14$. (Notice that S_1 is also $Y - x_1$.) The difference between his/her two neighbors for subject No. 1 is $d_1 = x_5 - x_2 = 3$, and the difference between his/her two neighbors for subject No. 2 is $d_2 = x1 - x3 = -2$. Notice that your proposed addition does not affect the calculation of S and d. Your expenditure[3] is your individual "price" for the project, multiplied by the project level, and then divided by 25. Your individual "price" for the project is a base price, 100, which is the same for everyone, plus the difference (**d**) between the amounts proposed by your two neighbors. For your convenience, some payoff charts that summarize all of the relevant information will be used.

additions that you and other participants make. The horizontal axis is the sum of others' proposed additions, S. The vertical axis is your net payoff. Each curve represents your payoff from a particular choice of proposed addition, x. The small box on the right hand side of the chart gives the color of the curve for the eleven different proposed additions charted. Since the chart would be difficult to read if all possible proposed additions were plotted, only eleven different ones equal distant from each other are given. The curves for proposed additions which are not given, lie between the given curves. For example, the curve for a proposed addition of x=11 would be between those of x=9 and x=14 and slightly closer to the x=9 curve than the x=14 curve. The payoff charts are ordered by the difference (d) between your neighbors' proposals. This value, which ranges from -50 to 50 with a step of 5, is located at the lower right corner of each chart.

For example, in the Payoff Chart on the screen, if the difference of your neighbors' proposed additions is $d = 25$, your proposed addition is $x = 4$, which is represented by the black curve, and the sum of proposed addition of all others is $S = -5$, then your payoff is 200. It is important that you check the different Payoff Charts ordered by d. Although they might seem to have similar shapes to you, your payoff can be significantly different when d is different. For example, in the current Payoff Chart on the screen, when $d = 25$, if $S = -5$, then choosing $x = 4$ (the black curve) will give you the highest payoff. In the next Payoff Chart, when $d = -50$, if the sum of others' proposed additions is still $S = -5$, choosing $x = 4$ (the black curve) no longer gives you the highest payoff. In this case, choosing $x = 14$ (the blue curve) will give you the highest payoff. With the two slides on top of each other, you can see that the curves shift both horizontally and vertically. *Different participants might have different payoff charts.*

... ...

Review Questions

1. If the circle is 1-2-3-4-5-1, if each of you proposes $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, then
   (1) The project level is $Y =$
   (2) The sum of others' proposal is $S =$
   (3) The difference of your neighbors' proposal (left - right) is $d =$

2. Suppose all others have the same proposed addition as in Question 1, you alone raise your addition by 1 unit, then
   (1) The total level of the project, $Y =$
   (2) The sum of others' proposal, $S =$
   (3) The difference of your neighbors' proposal (left - right) is $d =$

3. If you propose $x = -16$, which is represented by the purple curve, the difference of your neighbors' proposals is $d = 35$, the sum of others' proposals is $S = 10$, find your payoff from your

additions that you and other participants make. The horizontal axis is the sum of others' proposed additions, S. The vertical axis is your net payoff. Each curve represents your payoff from a particular choice of proposed addition, x. The small box on the right hand side of the chart gives the color of the curve for the eleven different proposed additions charted. Since the chart would be difficult to read if all possible proposed additions were plotted, only eleven different ones equal distant from each other are given. The curves for proposed additions which are not given, lie between the given curves. For example, the curve for a proposed addition of x=11 would be between those of x=9 and x=14 and slightly closer to the x=9 curve than the x=14 curve. The payoff charts are ordered by the difference (d) between your neighbors' proposals. This value, which ranges from -50 to 50 with a step of 5, is located at the lower right corner of each chart.

For example, in the Payoff Chart on the screen, if the difference of your neighbors' proposed additions is $d = 25$, your proposed addition is $x = 4$, which is represented by the black curve, and the sum of proposed addition of all others is $S = -5$, then your payoff is 200. It is important that you check the different Payoff Charts ordered by d. Although they might seem to have similar shapes to you, your payoff can be significantly different when d is different. For example, in the current Payoff Chart on the screen, when $d = 25$, if $S = -5$, then choosing $x = 4$ (the black curve) will give you the highest payoff. In the next Payoff Chart, when $d = -50$, if the sum of others' proposed additions is still $S = -5$, choosing $x = 4$ (the black curve) no longer gives you the highest payoff. In this case, choosing $x = 14$ (the blue curve) will give you the highest payoff. With the two slides on top of each other, you can see that the curves shift both horizontally and vertically. *Different participants might have different payoff charts.*

... ...

Review Questions

1. If the circle is 1-2-3-4-5-1, if each of you proposes $x_1 = 1,\ x_2 = 2,\ x_3 = 3,\ x_4 = 4,\ x_5 = 5$, then

   (1) The project level is $Y =$

   (2) The sum of others' proposal is $S =$

   (3) The difference of your neighbors' proposal (left - right) is $d =$

2. Suppose all others have the same proposed addition as in Question 1, you alone raise your addition by 1 unit, then

   (1) The total level of the project, $Y =$

   (2) The sum of others' proposal, $S =$

   (3) The difference of your neighbors' proposal (left - right) is $d =$

3. If you propose $x = -16$, which is represented by the purple curve, the difference of your neighbors' proposals is $d = 35$, the sum of others' proposals is $S = 10$, find your payoff from your Payoff Charts, P=

4. True or False: if you propose the same x, and the project level, Y, remains the same, you will get the same payoff.