# SMALL VERSUS LARGE-SCALE FISHERIES: A MULTI-SPECIES, MULTI-FLEET MODEL FOR EVALUATING THEIR INTERACTIONS AND POTENTIAL BENEFITS

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## ABSTRACT

In this paper, we present a method for evaluating the economic losses and biological impacts of a lack of co-ordination of effort on the part of small versus large-scale fisheries. We illustrate our method using fisheries of the Gulf of Maine and the George's Bank (USA). There are several novel methodological components of this work. First, we use an approach for defining which fisheries are small and which are large on a scale that is specific to political units since gear that is large-scale in one country may be categorized as small-scale in another. Second, we present a multi-species, multi-fleet, yield-per-recruit model that incorporates gear selection curves for each gear type. This permits an evaluation of the economic benefits of trade-offs in effort between the two small and large-scale fleets. Optimal combinations of effort by the two fleets are identified by subtracting costs of fishing effort from the gross value calculated by the model. Third, we estimate the value of foregone profits by comparing the rents produced at such an optimum with those produced by the current fishery. Finally, we identify a Nash bargaining solution that would be obtained if both sectors chose to cooperate by coordinating their levels of effort.

## INTRODUCTION

Throughout the world, fishing fleets are becoming progressively larger in scale and fisheries are becoming serially depleted. Vessels are becoming larger and faster, are traveling farther and farther from their homeports, are using more sophisticated (and expensive) technologies and are catching fish in shorter periods of time. The economic incentives for this trend are well understood. The open access nature of past fisheries clearly invited overcapitalization (Gordon 1954). These incentives persist in most modern day 'regulated access' and 'regulated restricted access' fisheries (Wilen and Homans 1997) and even in fisheries with individual quotas (Maurstad 2000). In addition, present declines in fish abundance requires indebted fishers to search ever further for fish. Government subsidies, based on the presumption that large-scale operations enjoy greater economies of scale, further accelerates this trend (see Milazzo 1998 for a general discussion of subsidies). However, evidence for the greater economic efficiency of large-scale gears is inconclusive (P. Tyedmers pers. comm. with respect to fuel efficiency) and there are clear social costs to these trends that are borne both by individuals and by society as a whole. Few studies consider the full range of hidden costs when assessing the desirability of supporting one or the other fishing sectors. A more detailed treatment of the 'ecological footprint' with respect to fuel inputs of each type of fishery is presented in Tyedmers (2000).

In this paper, we compare the economic profitability of small and large-scale sectors by identifying what combination(s) of effort by these operations generates the highest gross and net revenues. Our method of analysis is a multi-species, multi-fleet, value-per-recruit model that has been developed expressly for this purpose (Figure 1) (see appendix for details). Our analysis indicates that optimal combinations of effort differ greatly depending on whether net or gross returns are considered and hence, it is critical to incorporate cost estimates when evaluating management plans. Second, our use of a bio-economic approach allows us to estimate the rents lost to society when non-optimal levels of effort are applied to the fishery, or in other words, when the small and large-scale sectors do not cooperate. Having determined the optimal combination of effort obtainable if sectors do cooperate, we are able to identify a Nash equilibrium with side payments[1] using a modified version of the method developed by Nash (1953) and refined by Munro (1979). Our method differs in that we consider two sectors of a fishery rather than two countries competing for a trans-boundary resource and we use a multi-species

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[1] A Nash equilibrium is one where each individual should *not* wish to change strategy even if the other player does. In cooperative games with asymmetrical payoffs, players may reach agreements whereby one player pays the other some portion of the benefits obtained though cooperation.

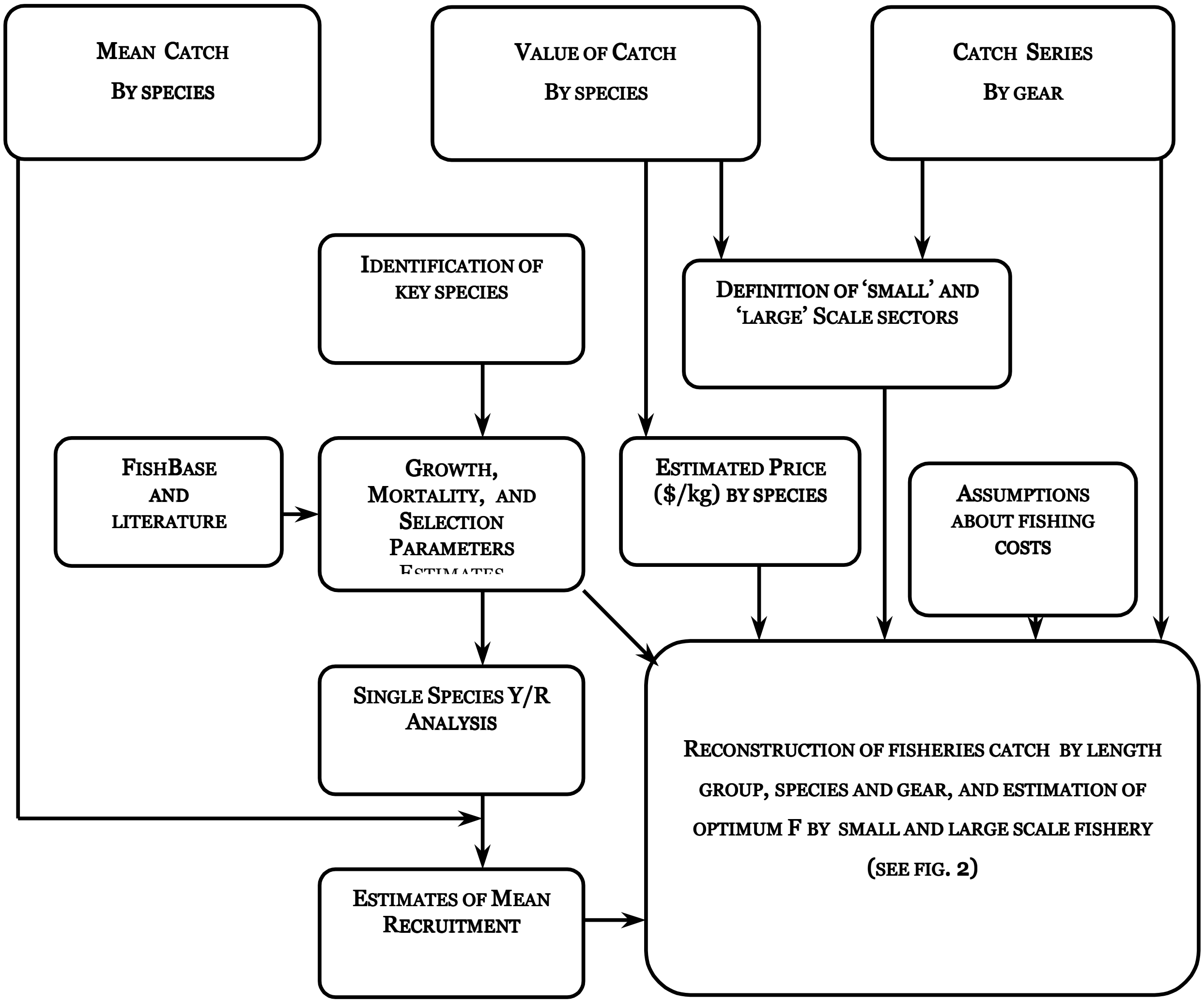

**Figure 1.** Flowchart of operations implied in the method described in this contribution.

value per recruit model rather than a surplus production model to predict optimal levels of effort (see also Sumaila 1997).

Here we illustrate this method using data from the Gulf of Maine and George's Bank (USA). Numerous assumptions must be made, chief among them that there is constant recruitment, that costs scale with effort, and that rents are driven to zero, any of which may distort the results of any one analysis but may not be problematic when this method is applied to a large number of fisheries since overestimates in one may compensate for underestimates in another.

## MATERIALS AND METHODS

### *Gulf of Maine and George's Banks as a Study Area*

The Gulf of Maine is a deep and cold body of water bounded on the South and West by the US states of Massachusetts, New Hampshire, and Maine, on the North by the Canadian provinces of New Brunswick and Nova Scotia, and on the East by the George's Banks. The latter is a shallow water bank rising at the edge of the continental shelf and capable of very high productivity (Sissenwine et al. 1984). Nearly 140 species are landed in the US states bordering the Gulf bringing in a total value of close to $650 million per year during the 1990s However, the once abundant sea life in both areas has been progressively depleted and the average trophic level of catches is declining (Steneck 1997). According to Steneck, where predatory

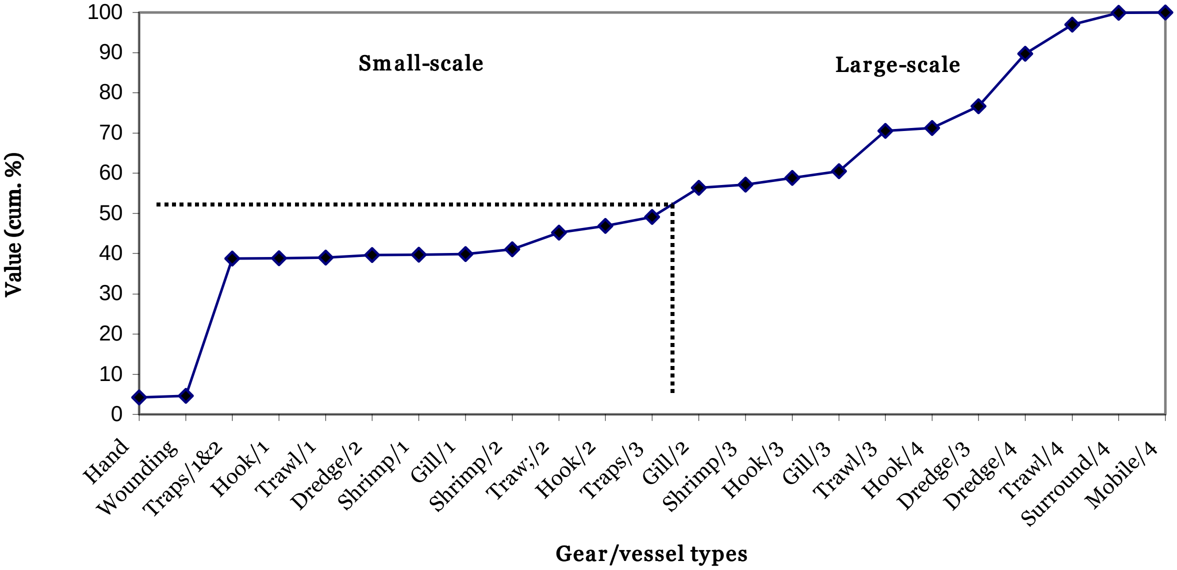

**Figure 2**. Cumulative % of landings value, by gear/vessel type first ranked from 'small' to 'large', with 50% line separating small from large-scale fisheries (see text).

groundfish were once the dominant species, now less valuable sculpins, dogfish and skates constitute the majority of the finfish catch. Data from the US National Marine Fisheries Service (NMFS) indicates that lobster, though always an important fishery, is now the largest fishery both in terms of value and tonnage and debate exists as to whether the current abundance is due to environmental changes or declines in the abundance of cod, one of its principal predators.

*Source of Landings Data*

Commercial and recreational landings recorded by gear and species from the states of Maine, New Hampshire and Massachusetts during the period 1989-98 were downloaded from the NMFS Office of Statistics and Technology website (http://www.st.nmfs.gov). The landings and their values were each averaged across the time period and then summed over the three states. These figures are henceforth referred to as 'total catches' or 'total values'. Undoubtedly a portion of these landings are caught in areas besides the Gulf of Maine and George's Banks. However, there is no simple method for apportioning them. It should be emphasized that no data on catch at length, or value of various size classes was available from the source we consulted, nor from other sources.

*Defining Fishing Scale*

There is no single definition of what is a small-scale fishery. However, the term is popularly used in reference to subsistence and artisanal fisheries, the latter typified by small, multi-species catches that are caught using small vessels taken on short fishing trips (Charles 1989, Munro 1980). Based on such a definition, we would find that most small-scale fisheries are found in the 'South', i.e. developing countries, while most large-scale fisheries are in the developed 'North'. On consideration it is clear, however, that many inshore fisheries in the developed world are much smaller in scale than the largest fisheries in those same areas. Thus we choose to categorize fisheries as small or large on a relative rather than absolute scale. The particular scale we use is catch per vessel per year. Our justification is that low catches are associated with smaller boats that travel shorter distances. Thus this scale captures the essence of 'smallness' with just one figure, although smallness also implies smaller crew, more limited range, etc.

In practice, we define particular fisheries as gear/vessel combinations. There are three steps. First, we categorized all fisheries as belonging to one of ten categories of gear and one of four categories of vessel size using the same tonnage categories as the NMFS (Table 1) and gear categories that are nearly identical to those used by Watson et al. (2000). The categories differ from theirs in that available data on catch rates necessitated aggregating bottom and mid-water trawlers, and permitted shrimp trawlers to be given their own categories; it was desirable to keep shrimp trawlers separate from other trawlers because they use different mesh size. Second, gear/vessel combinations were ranked in ascending order according to annual catch per vessel. Third, a cumulative percentage distribution is constructed with these ranked fisheries. The group of fisheries that provides the first 50% of landed value are then classified as 'small-scale' and the remainder as 'large-scale'

(Figure 2). The fisheries are divided into just two groups, large and small, using a cutoff point of 50% of cumulative landed value in order to provide a standard for comparison involving other variables such as employment, fuel consumption, etc. Finally, the cut-off point is examined to see whether all gears employed by the same size category vessels fall on the same side of the cut-off and the rankings adjusted if they do not. The justification for this is that many fishers, especially small-scale ones, use multiple gear types on the same boat.

To accomplish the second step, data on annual catch per vessel were obtained from the Status of the Fishery Resources of the Northeastern United States (NOAA n.d.) which contained such information for the years 1994-96 and the entire Northeast region of the US (covering all states managed by the New England and the Mid-Atlantic Fishery Management Councils). The data for some categories of vessel size were not available from this source due to there being low numbers of vessels in these categories (and thus sampling and privacy issues). However, information on landed value and numbers of vessels was available for the missing categories. These latter figures were used to estimate annual catch per vessel for the missing vessel size categories.

There were five categories of gear for which no information on catch/vessel/year was available. Fortunately, two of these (hand gear and wounding/grappling gear) would clearly have the lowest catch rates and hence they were inserted into the beginning of the ranking. A third category, traps 1&2, was also assumed to be relatively small in scale and hence was inserted just after wounding gear. (Note that it was assumed that all trap vessels of size class 3 were offshore lobster vessels, a category for which catch/year/vessel was available). The other two categories (mobile seines and surrounding gear) can reasonably be assumed to have high if not the highest catches/vessel/year and thus they were inserted into the end of the rank order.

After ranking the gear/vessel combinations based on the 1994-96 data, each combination's total value for the 1989-98 period was calculated by partitioning the total value for a particular gear type among vessel categories in the following manner. Since total value for the 1994-96 period was available for both gear and vessel categories, it was possible to calculate the percentage of value that each vessel category produced for a given gear type. These percentages were then multiplied by the 1989-98 total values for each gear type. Once these figures were calculated it was possible to complete the third step of the process of defining which fisheries are small by constructing a cumulative percentage distribution using the ranked gear types and their associated total values. We used total value rather than total landings for pragmatic reasons. Since the smallest scale gears typically have small but highly valued catches, the use of tonnage would lead to two thirds of gears being classified as small scale. This is intuitively wrong. We therefore deliberately chose to use value and thereby minimize the numbers of gears that we consider to be small scale. In this particular case, the classification of the two boundary fisheries (traps/3 and gill/2) were switched so that all gears employed be size class 2 vessels are defined as small-scale and all size class 3 vessels are large-scale.

**Table 1**. Gear and vessel categories.

| **Code** | **Gears** |
|---|---|
| 11 | Shrimp trawl |
| 12 | Bottom or midwater trawl |
| 21 | Mobile seines |
| 31 | Surrounding nets (e.g. purse seines) |
| 41 | Gillnets and entangling nets |
| 51 | Hooks and lines |
| 61 | Traps and lift nets |
| 71 | Dredges |
| 81 | Grappling and wounding (e.g. harpoons) |
| 90 | Other gear (e.g. hand lines, hoes etc.) |
| | **Vessels (gross registered tons)** |
| 1 | <5 |
| 2 | 5 - 50 |
| 3 | 51-149.9 |
| 4 | 150+ |

### *Species Characteristics*

*Landings and Recruitment by species*

The species included in this model were chosen by ranking all species landed in the three states from highest to lowest in terms of total value of landings. A cumulative percentage distribution of total values for each species was calculated and the species generating the first 95% of the value of all landings were included in the initial sample (33 species). All sessile species as well as 2 species of worms were then removed. Three additional species are not included in the analysis due to a paucity of easily available information on their population dynamics (American eel, *Anguilla rostrata*; bay scallop, *Argopecten irradians*; and hagfish *Myxine glutinosa*) leaving a final sample of 21 species (Table 2).

Fifty-seven different gears were listed in the initial data set. These were aggregated into the ten gear categories described in Table 1.

**Table 2**. Total observed landings, value, US $/kg, fishing mortality (F, year $^{-1}$), mean length at first capture size ($L_c$, cm), and number of recruits for species included in the model.

| Common Name | Scientific Name | Landings (t) | Value ($10⁶) | $/kg | F | $L_c$ | Recruits (10⁶) |
|---|---|---:|---:|---:|---:|---:|---:|
| BASS, STRIPED | *Morone saxatilis* | 987.7 | 3.1 | 3.15 | 2.0 | 67.1 | 4.7 |
| COD, ATLANTIC | *Gadus morhua* | 30,723.1 | 53.7 | 1.74 | 2.0 | 103.6 | 22.7 |
| FLOUNDER,SUMMER | *Paralichthys dentatus* | 588.1 | 2.4 | 4.01 | 5.0 | 109.6 | 0.2 |
| FLOUNDER,WINTER | *Pseudopleuronectes americanus* | 4,689.9 | 13.6 | 2.64 | 3.0 | 35.2 | 42.4 |
| FLOUNDER,WITCH | *Glyptocephalus cynoglossus* | 2,009.8 | 7.5 | 3.45 | 2.0 | 32.8 | 157.8 |
| FLOUNDER,YELLOWTAIL | *Limanda ferruginea* | 4,475.5 | 11.3 | 2.59 | 2.5 | 35.0 | 87.0 |
| GOOSEFISH | *Lophius americanus* | 14,521.8 | 18.7 | 1.31 | 2.0 | 137.9 | 9.3 |
| HADDOCK | *Melanogrammus aeglefinus* | 1,618.1 | 4.2 | 2.62 | 3.0 | 65.6 | 5.0 |
| HAGFISH | *Myxine glutinosa* | 1,746.1 | 1.1 | 0.95 | 0.0 | | |
| HAKE, WHITE | *Urophycis tenuis* | 6,031.3 | 6.5 | 1.11 | 3.0 | 58.8 | 7.6 |
| HERRING, ATLANTIC | *Clupea harengus* | 65,211.7 | 8.0 | 0.24 | 3.0 | 25.2 | 5449.0 |
| LOBSTER, AMERICAN | *Homarus americanus* | 24,158.0 | 155.3 | 7.68 | 1.5 | 15.2 | 83.8 |
| PLAICE, AMERICAN | *Hippoglossoides platessoides* | 4,467.1 | 11.4 | 2.40 | 2.5 | 48.1 | 44.8 |
| POLLOCK | *Pollachius virens* | 8,953.8 | 11.9 | 1.30 | 2.5 | 85.6 | 11.4 |
| SCALLOP, SEA * | *Placopecten magellanicus* | 6358.0 | 69.4 | 13.07 | 1.5 | 12.8 | 578.7 |
| SCUP | *Stenotomus chrysops* | 23.9 | 0.1 | 4.11 | 4.0 | 29.7 | 0.5 |
| SEA URCHIN | *Strongylocentrotus droebachiensis* | 12,515.0 | 22.1 | 1.45 | 2.0 | 15.1 | 14342.7 |
| SHARK, SPINY DOGFISH | *Squalus acanthias* | 18,354.8 | 6.0 | 0.34 | 2.0 | 57.7 | 206.3 |
| SHRIMP, NORTHERN | *Pandalus borealis* | 4,730.2 | 8.9 | 2.89 | 3.5 | 12.2 | 6303.6 |
| SQUID, LONGFIN | *Loligo pealeii* | 1,706.4 | 2.1 | 1.35 | 5.0 | 26.8 | 398.9 |
| SWORDFISH | *Xiphias gladius* | 1,259.0 | 7.9 | 6.60 | 2.5 | 186.9 | 0.1 |
| TUNA, BLUEFIN | *Thunnus thynnus* | 929.6 | 16.9 | 12.38 | 2.0 | 280.8 | 0.0 |
| **Total** | | **209,700.8** | **442.2** | | | | **27,756.6** |

- landings converted from meat to shell weight using a 1:9 ratio (Caddy 1989)

**Table 3**. Parameters used in calculations of yield per recruit and sources of information.

| Common Name | $L_\infty$ (cm) | K ($yr^{-1}$) | $W_\infty$ (g) | a | b | M ($yr^{-1}$) | $L_r$ (cm) | $t_0$ (yr) | T °C | FishBase Population or References |
|---|---|---|---|---|---|---|---|---|---|---|
| BASS, STRIPED | 95.8 | 0.188 | 5,440 | 0.006 | 2.907 | 0.29 | 4.80 | 0.000 | 10.0 | Coos Bay |
| COD, ATLANTIC | 148.0 | 0.121 | 36,600 | 0.007 | 3.101 | 0.18 | 7.40 | 0.000 | 10.0 | Gulf of Maine/George's Banks |
| FLOUNDER, SUMMER | 137.0 | 0.843 | 29,949 | 0.007 | 3.117 | 0.65 | 6.90 | 0.000 | 10.0 | USA, Delaware Bay, 1966-71 |
| FLOUNDER, WINTER | 44.0 | 0.400 | 1,380 | 0.021 | 3.000 | 0.39 | 2.20 | 0.000 | 5.0 | Canada, East Coast |
| FLOUNDER, WITCH | 46.9 | 0.150 | 786 | 0.002 | 3.390 | 0.29 | 2.30 | 0.000 | 10.0 | Norway, Hekkingen, Malangen |
| FLOUNDER, YELLOWTAIL | 50.0 | 0.335 | 1,183 | 0.009 | 3.000 | 0.48 | 2.50 | 0.000 | 10.0 | USA, South New England |
| GOOSEFISH | 197.0 | 0.060 | 53,952 | 0.017 | 3.000 | 0.11 | 9.90 | -0.080 | 10.0 | Canada, Bay of Fundy |
| HADDOCK | 72.9 | 0.352 | 4,214 | 0.011 | 3.000 | 0.44 | 3.60 | 0.000 | 9.4 | Gulf of Maine |
| HAKE, WHITE | 84.0 | 0.218 | 13,685 | 0.004 | 3.147 | 0.17 | 6.80 | -0.280 | 0.0 | S. Gulf of St.Lawrence |
| HERRING, ATLANTIC | 36.0 | 0.210 | 350 | 0.008 | 3.000 | 0.35 | 1.80 | 0.000 | 8.0 | Norway, Atlanto-Scandian |
| LOBSTER, AMERICAN | 25.3 | 0.056 | 13,783 | 0.003 | 3.015 | 0.13 | 6.00 | -0.772 | 10.0 | Campbell (1986); Estrella &McKiernan (1989 p.7 & 13); Townsend (1986) |
| PLAICE, AMERICAN | 80.2 | 0.076 | 5,550 | 0.004 | 3.204 | 0.16 | 4.00 | 0.000 | 10.0 | ICNAF Res.Div.3L 1969-72 |
| POLLOCK | 107.0 | 0.190 | 11,634 | 0.008 | 3.000 | 0.21 | 5.40 | 0.000 | 6.0 | Norway, Norwegian Sea |
| SCALLOP, SEA | 14.2 | 0.317 | 47 | 0.016 | 3.000 | 0.10 | 1.80 | 1.385 | 10.0 | Caddy (1975 p.1316; 1989 p. 569) |
| SCUP | 42.4 | 0.170 | 1,723 | 0.023 | 3.000 | 0.32 | 2.10 | 0.000 | 10.0 | USA, Northwest Atlantic |
| SEA URCHIN | 18.9 | 0.122 | 12 | 0.081 | 2.905 | 0.13 | 4.50 | 0.050 | 10.0 | Longhurst and Pauly (1987); Russell et al. (1998 p. 46, 150); Swan (1958 p.512-13) |
| SHARK, SPINY DOGFISH | 96.1 | 0.067 | 3,580 | 0.004 | 3.004 | 0.09 | 4.80 | -5.000 | 10.0 | Georgia Strait, BC |
| SHRIMP, NORTHERN | 17.5 | 0.390 | 17 | 0.003 | 3.080 | 0.65 | 4.57 | -0.100 | 10.0 | Fournier et al. (1990 p.596); Haynes and Wigley (1969 p. 69, 74); Parsons and Frechette (1989 p. 74); Shumway et al. (1985 p.39) |
| SQUID, LONGFIN | 38.30 | 0.590 | 103 | 0.046 | 2.118 | 0.87 | 2.10 | 0.000 | 10.0 | Lange and Johnson (1981), Pauly (1985) |
| SWORDFISH | 267.00 | 0.120 | 274691 | 0.014 | 3.000 | 0.15 | 13.40 | -1.680 | 10.0 | USA, Atlantic Coast |
| TUNA, BLUEFIN | 468.00 | 0.050 | 1726165 | 0.037 | 2.870 | 0.15 | 23.40 | 0.000 | 12.0 | USA, Cape Cod-Long Island |

In the Gulf of Maine/George's Bank area this results in 23 different gear/vessel combinations. Since we had data on catch by gear type but not by vessel size, catches for each gear type were allocated to vessel categories by a method similar to that used to allocate values as described above (in *Defining Fishing Scale*). Most species had a category of 'uncoded' or multiple gears. These were given the same gear code as the most common gear used to catch that species. No dollar value was given for recreational fisheries and thus it was assumed that recreational and commercial fishermen using hook gear obtain the same price per kilogram.

Most growth parameters for finfish species were taken from FishBase 99 (Table 3). In cases where asymptotic weight ($W_\infty$) was not available from FishBase (*Paralichthys dentatus*, *Stenotomus chrysops, Thunnus thynnus*) length and weight records were taken from Bigelow and Schroeder (1953) and used to calculate parameters a and b of length-weight relationships. These were then used to estimate $W_\infty$ from the asymptotic length ($L_\infty$). Mesh selection factors were determined by calculating each species' depth ratio from drawings available in FishBase 99 and in Bigelow and Schroeder (1953). The selection factor was then estimated using a nomogram in Pauly (1984, p. 11). Similar parameters for invertebrate species were gathered from the literature. Natural mortality estimates were obtained from FishBase 99 either as values associated with selected sets of growth parameters, or via the built-in estimation procedure based on the empirical equation of Pauly (1980), which uses $L_\infty$, K and mean water temperature to estimate M. A value of 10°C was used as an input for all such estimates.

The numbers of recruits were obtained using Beverton and Holt yield per recruit analysis built into the spreadsheet software (Table 2). Inputs include the growth parameters described above as well as estimates of fishing mortality, F, and the mean length at first capture, $L_c$. The latter two parameters were, in turn, obtained from the FishBase 99 yield per recruit module. This module provides a graphical interface permitting one to easily identify the values of F and $L_c$ associated with stable recruitment and the highest yield per recruit; these values were chosen since our goal is compare current rents against maximal possible rents.

### *Selection Characteristics*

Each species/gear combination was assigned a mean length at first capture ($L_{50}$, in cm) that was either equal to the minimum legal size of capture (if available) or calculated using a known mesh size of that particular gear and the species' selection factor. It was assumed that the $L_{50}$ of all non-mesh gear would be equal to the minimum legal size (e.g. hook gear, traps and pots, etc.). For some species, there is no minimum legal size and thus, for two of these (*Thunnus thynnus* and *Xiphias gladius*) an initial estimate of $L_{50}$ was based on the minimum size caught in length-frequency data available from FishBase 99.

Selection and de-selection curves were calculated. On the selection side, values of $L_{75}$ were calculated for finfish as equal to $L_{50}*1.25$ for all non-selective gear and equal to $L_{50}*1.10$ for all selective gear (here selective gears refers to size selectivity and include gillnets, hooks and lines, and traps; all other gears were considered non-selective). For invertebrates, $L_{75} = L_{50}*1.01$, the justification for this much steeper selection curve being that either the animals are hand picked out of the gear and there are minimum legal size limits (e.g. lobster, sea urchins and sea scallops) or that the nature of invertebrate body form justifies a steeper curve (e.g. shrimp and squid).

On the de-selection side, we set the $D_{50}$ to be equal to $L_\infty*0.95$ for non-selective gears and equal to $L_\infty*0.90$ for selective gears. One exception is for lobster (*Homarus americanus*) where the $D_{50}$ was set to be equal to the maximum legal size. Another exception was for cases where such a $D_{50}$ ended up being smaller than the $L_{50}$. In these cases, the $L_{50}$ was adjusted downwards. The $D_{75}$ for all species is equal to $D_{50}$ - 0.1cm.

### *Computation of Gross and Net Values*

Finally, these species parameters, catches, recruitment values, classifications of gear as large or small, and selection curves for species/gear combinations were entered into a multi-species, multi-fleet spreadsheet solution (Figure 3). The gist of the model is to get around the lack of catch-at-length data by first constructing combined, weighted, selection curves for each sector's catch of each species. From natural mortality rates and using a selection curve, the relative distribution of population length can be estimated. This distribution is then 'raised' to allow for both natural mortality and the observed landings, which provides the corresponding pattern of fishing mortality at length ('F-pattern'), similar to Jones (1984). Then, effort applied to each species by each fleet can be varied systematically with an effort multiplier, the f-factor (see Appendix).

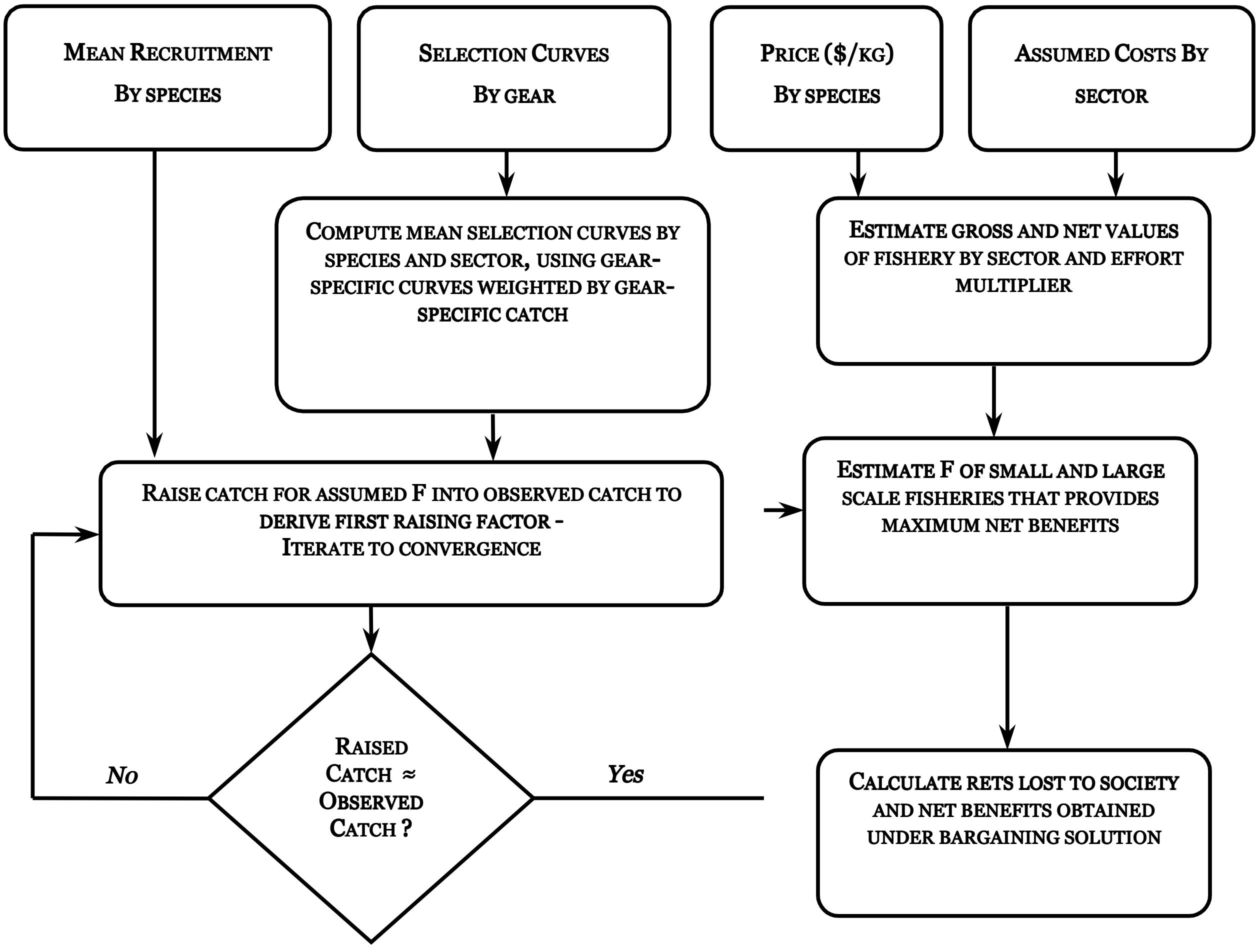

**Figure 3.** Procedures for calculating net returns for different fleet configurations (see text and Appendix)

The result is a matrix containing the aggregate gross value of the fishery for all combinations of effort on the part of small and large-scale fisheries. From this matrix we identify the combination(s) of effort leading to highest gross value(s). Net values are then calculated by subtracting costs from the gross values across the full range of f-factors of both sectors. In the absence of accurate data on fishing costs, we estimate them by assuming that the fishery is currently at its bio-economic equilibrium, i.e., that rents have been driven to zero (see below). If this is the case, the aggregate gross value (GV) of the current fishery is equal to the aggregate costs. By definition, the current gross value of the fishery is found at the point where the small and large scale f-factors are equal to one. Then, assuming that fishing costs scale linearly with fishing effort (see below), the net value (NV) for every combination of effort is estimated for the small scale sector as follows (where is l = large f-factor, and s = small f-factor):

$$\text{Small NV}(l,s) = \text{Aggregate GV}(l,s) - \text{SmallGV}(1,1)^{*}\,(s) \qquad \ldots 1)$$

Similar calculations can be made to determine the aggregate and large-scale net revenues. From the matrix of net values we identify the combination of f-factors yielding the highest aggregate net returns.

### *Cost Sensitivity Analysis*

Because accurate data on the costs of fishing effort are difficult to obtain, we have made simplifying assumptions: that rents are currently equal to zero and that costs scale linearly with effort. These assumptions are especially problematic if: a) rents deviate from zero in opposite directions in each sector and/or b) costs scale differently in each sector. We perform a cost sensitivity analysis to analyze what would happen to the optimal levels of effort if either of these two scenarios were the case. Four variants of our cost assumptions are analyzed. In the first variant, we ask what would happen if current small-scale rents are actually positive while large scale ones are negative. Specifically, we let the current small

NV = 5% of the small GV and the current large NV = -5% of the current large GV. The formula for determining net revenues in the case of the small sector is:

Small NV(l,s) = Aggregate GV(l,s) –
(SmallGV(1,1) – SmallNV(1,1))* (s) ...2)

The second variant is simply the reverse of the first; small net revenue is less than zero while large net is greater than zero. In the third variant, we let small scale costs scale at 95% of the f-factor while large scale costs scale at 105% of the f-factor. The formula for the small sector is:

Small NV(l,s) = Aggregate GV(l,s) –
(SmallGV(1,1)* (s)*0.95) ...3)

The fourth case is the reverse of the third case.

### ***Nash Bargaining Solution***

Two particularly interesting pieces of information can be drawn from the results. First, the value of rents foregone through non-cooperation by the large and small-scale fleets is equal to the value of the aggregate maximum since, by definition, the current rent is equal to zero. Second, Nash equilibria can be identified from among these points. In game theoretic terminology, the current state of the fishery serves as a threat point of a cooperative game (Nash 1953, in Munro 1979). The threat point gives us the payoff that each player can expect to take home if they do not cooperate. Assuming that the aggregate rents generated at our optimum are higher than at present (and thus Pareto optimal) and there is a single optimum point of highest aggregate net values, then, if the two sectors do choose to cooperate and co-ordinate their effort, and if side-payments are an acceptable solution, there is a Pareto frontier constituting the set of possible profits after various levels of side-payments have been made (imagine each axis of a graph as representing each sector's profits given that particular allocation of the 'extra' rent generated from cooperation). In this set of points, if the large [small] sector obtains a profit of x [y] without cooperating but obtains a proft of a [b] if they cooperate and side-payments are made, then the Nash bargaining solution for such a cooperative game is determined by choosing point (a,b) on the Pareto frontier so as to maximize the product of the difference between the payoffs received under cooperation and those received at the threat point:

$$\max(a-x)(b-y) \qquad \text{...4)}$$

Since by definition, x = y = 0 in our case, it can be shown that the values of x and y that maximize the product of a*b occurs when the extra benefits above the sum of threat point payoffs are shared equally between the participants, i.e. a = b (see also Luce and Raiffa 1967 in Munro 1979).

## RESULTS

Table 4 presents the results of the yield per recruit analysis. Levels of the f-factor for each sector, and the gross and net revenues obtained in aggregate and by sector are given for the three points; the current scenario, the point at which gross revenues are maximized and the point at which net revenues are maximized. In no case are there multiple optima. Present effort is by definition at the point where f-factor large = f-factor small = 1, and again, by definition, net values are equal to zero.

**Table 4**. Results of yield per recruit analysis. Values are US$ ($10^6$).

| | Aggregate | Small | Large |
|---|---|---|---|
| **Current** f-factor | | 1.00 | 1.00 |
| Gross Value | 235.37 | 62.68 | 172.69 |
| Net Value | 0.00 | 0.00 | 0.00 |
| | | | |
| **Gross Max** f-factor | | 2.50 | 0.92 |
| Gross Value | 251.52 | 111.55 | 139.97 |
| Net Value | -64.06 | -45.16 | -18.90 |
| | | | |
| **Net Max** f-factor | | 0.11 | 0.35 |
| Gross Value | 174.89 | 18.27 | 156.60 |
| Net Value | 107.56 | 11.38 | 96.18 |
| | | | |
| Payments | | 42.40 | -42.40 |
| **After bargaining** NV | | 53.78 | 53.78 |

We see that when only gross revenues are considered, the highest revenues are achieved when small-scale effort is at least 2.5 times the current effort and when large-scale effort is 0.92 times its current effort. This is not surprising given that the small-scale sector catches most of the highly valued invertebrates, e.g. lobster, sea urchins. When cost, and thus net values, are considered the optimal levels of effort change considerably; small-scale effort drops to 0.11 of its current value and large-scale effort drops to 0.35 of its current level. At this point the aggregate rents lost to society from over-fishing are equal to $107.56 million dollars. By coordinating effort levels, the small scale sector as a whole would stand to gain $11.38 million and the large scale sector $96.18 million. By co-ordinating effort and then in addition bargaining to share the proceeds from effort coordination, each sector could gain $53.78 million as a whole.

In table 5, we present the results of the cost sensitivity analysis. As described earlier, there are four different cases that we analyze. In the first two, we essentially change the intercept of the f-factor (x) versus cost (y) function. In variant 1 the intercept is lowered for the small-scale sector and increased for the large-scale one while the reverse is true for variant 2. In variant 3, the slope of the of the same function is decreased for the small–scale sector and increased for the large-scale sector. Variant 4 is the reverse of 3.

**Table 5**. Cost sensitivity analysis results. Values are US$ ($10^6$).

| Variant | | Small | Large |
|---|---|---|---|
| 1 | f-factor | 0.18 | 0.32 |
| | Net Value | 18.55 | 86.57 |
| 2 | f-factor | 0.05 | 0.38 |
| | Net Value | 5.12 | 105.34 |
| 3 | f-factor | 0.19 | 0.32 |
| | Net Value | 18.7 | 88.6 |
| 4 | f-factor | 0.05 | 0.38 |
| | Net Value | 5.28 | 102.06 |

## DISCUSSION

The results indicate that given our inputs and assumptions, there is a single set of effort levels on the part of the large and small-scale fleets that maximizes aggregate rents, i.e., provides a Pareto efficient solution. A single Nash bargaining solution can thus be identified as occurring when the benefits from cooperation are shared equally through side payments. In this case, the flow of payments is from the large-scale fleet to the small-scale fleet.

One of the most intriguing findings from this analysis results from a comparison of the levels of effort needed to produce maximum gross as opposed to net returns. In the former case, a very large increase in small-scale effort above current levels is called for and there is a sizable decline in net revenues generated by the fishery as a whole ($-64 million). In contrast, a sizable reduction in small-scale effort is required for maximum net revenues to be generated but the result is an increase in rents to society of over $100 million dollars. Each sector is also better off than currently. These results highlight the need for fisheries managers to attend to net returns to fishing and not simply gross returns.

An analysis of the sensitivity of these results to our assumptions about costs supports our overall conclusion that current levels of effort need to be substantially reduced, in both sectors. The relative levels of effort reduction do vary, however, depending on the specific assumptions. What is especially notable is that the optimal effort level of the small-scale sector is much more sensitive to changes in cost estimates than is the large-scale sector. Very modest changes in the slope and intercept of the cost function were reported here (5%). When changes of 10% were examined for variant 1, the recommended effort level of the small-scale sector jumped up to 0.26 while the optimal large-scale effort level declined to only 0.29.

Two notes of caution should be taken regarding this analysis. First, the different behavior of the two sectors is driven entirely by differences in gear selectivity. We do not include any other differences between the two sectors, e.g. discount rates, harvesting costs, selling price differences, etc. Munro (1979) has considered a number of these in the context of a bio-economic model based on a surplus-production (Schaefer) model. He finds that each of these factors can greatly influence the equilibrium outcome. Although we consider the optimal behavior of two sectors of a fishery rather than the decisions made by two countries it might be suspected that these variables do differ between sectors, in particular discount rates. We justify our lack of such a detailed economic analysis by noting that the aim of this particular model is simply to demonstrate the amount of rent that is lost from a non-optimal allocation of effort. We hope that others use these results as the basis for a more sophisticated economic analysis of the entry and exit decisions of small and large-scale fishers.

This brings us to a second note of caution. With respect to the fact that our results pertain to two sectors rather than two nations, we have presumed a willingness on the part of fishers to make side-payments. While this has been an effective solution in the case of transnational resources where the two players, countries, act effectively as individuals (see Munro 1979 for examples), it is not clear to what extent side-payments would be an acceptable solution to the many individuals who comprise the small and large-scale sectors. One cannot treat them as individual players as easily as one would two countries. Yet in this particular case both sectors actually benefit from cooperation without needing to bargain. However, the small-scale sector is composed of many more individual fishers and thus, per capita increases in returns may be insignificant if there is no bargaining and no transfer of revenues from the large to the small-scale sector.

Overall, we find that both gross and net returns can be increased if the two sectors of the fishery co-ordinate their levels of effort. However, different levels of effort are required to increase net as opposed to gross revenue and furthermore, the direction of change is opposite for the small-scale sector. When net returns are considered, a sizable reduction in total fishing effort can generate sizable increases in revenues

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## APPENDIX

### *Key Equations and Assumptions*

Beverton and Holt (1957), following up on Baranov (1918) showed that the catch ($C_i$) from a population during a unit time period, i, is equal to the product of the population size at the beginning of the time period ($N_i$) times the fraction of the deaths caused by fishing, times the fraction of total deaths, which can be written

$$C_i / N_{i+1} = (F_i / Z_i) \cdot (e^{Z_i} - 1) \qquad \text{...5)}$$

where F/Z expresses the fraction of the mortality caused by fishing. This is the equation for the virtual population analysis (VPA) of Gulland (1965).

Given values of $C_i$ and an estimate of M, Equation (5) can be used to estimate (retroactively) the size of past cohorts (i.e. of groups of fish born at the same time and exposed to the same mortalities throughout their lives), given an estimate of $N_{i+1}$, from which to start the computation (Mesnil 1980). An approximation to (5) is given by

$$N_i = (N_{i+1} \cdot e^{-M/2} + C_i) \cdot e^{M/2} \qquad \text{...6)}$$

wherein fishing mortality, for which Equation (5) cannot be solved directly, does not occur as an explicit parameter.

If we work backwards in time, estimating a new population size (N) at each step, fishing mortality estimates can be then obtained from the successive N values, using:

$$F_{i+1} = \ln(N_i / N_{i+1}) - M \qquad \text{...7)}$$

When recruitment and new F for each length group (i) is given, the process can then be used to predict the catch. This predictive method is commonly called, 'Thompson and Bell' (1934) method.

While the same procedural flow is followed as with the age-structured Thompson and Bell model, some equations need to be altered to account for the conversion of length to age (or to relative age when $t_o$ is not known). Converting length to age requires the use of a mathematical expression of fish growth, here the VBGF (von Bertalanffy growth function; Bertalanffy 1934):

$$L_t = L_\infty \left[1 - e^{-K(t - t_o)}\right] \qquad \text{... 8)}$$

where

$L_\infty$ is the asymptotic length, that is the mean length the fish of a given stock would reach if they were to grow indefinitely;

K is the rate (of dimension time$^{-1}$) at which $L_\infty$ is approached; and

$t_o$ is the 'age of the fish at zero length' if they had always grown in the manner described by the equation (note that $t_o$ is generally negative).

Thus, any age $t_i$ pertaining to a length $L_i$ can be obtained from

$$t_i = (1/K) \cdot \ln\left[1 - (L_i / L_\infty)\right] + t_o \qquad \text{...9)}$$

and similarly for age $t_{i+1}$, pertaining to $L_{i+1}$. From the length–age relationships for $L_i$ and $L_{i+1}$, $\Delta t_i$ is obtained as the difference between $t_{i+1}$ and $t_i$, or after some rearrangement

$$\Delta t_i = (1/K) \cdot \ln\left[(L_\infty - L_i)/(L_\infty - L_{i+1})\right] \qquad \text{... 10)}$$

Recursively applying Equations (6) and (7), the catches can be computed for a change in the F-array (Jones 1984).

Given the parameters (a, b) of a length–weight relationship and the computed catches per length group ($C_i$), the corresponding yield ($Y_i$) then can be estimated (Beyer 1987) from

$$Y_i = \overline{w}_i \cdot C_i \qquad \text{...11)}$$

where

$$\overline{w}_i = \left(\frac{1}{L_{i+1} - L_i}\right)\left(\frac{a}{b+1}\right) \cdot \left({L_{i+1}}^{b+1} - {L_i}^{b+1}\right) \qquad \text{...12)}$$

Similarly, multiplying the yield estimates to a mean value (e.g. commodity price) will provide an overview of the expected change in the total value of the return.

Multiplying an F-array (see below) by a factor (the f-factor) simulate a change in effort (f). Thus, it is straightforward to estimate the amount of effort that should be added to or removed from a fleet.

The method presented can be used straightforwardly in multi-species situations if two crucial assumptions are met:

(i) The fishing pattern has no influence on recruitment;

(ii) Biological interactions among species can be neglected.

Assumption (i) implies here not only that over a wide range, recruitment is not affected by changes in the effort level — as is also assumed for single-species Y/R analyses — but also that the relative strength of recruitment between species remains unaffected by fishing. Thus, it is assumed that if three species A, B and C recruit to the fishing ground with relative strengths of 0.1, 0.6 and 0.3 respectively, species B will remain dominant even if its adults are targeted by the fishery.

This assumption is not likely to be met in reality — at least not strictly. However, radical changes of the relative species composition of a multi-species stock take a while to manifest themselves, even when they are induced by a fishery. Also, there are configurations that are more stable than others, with certain species remaining dominant over decades. Finally, it must be recalled that yield per recruit analyses usually lead to advice that, when implemented, may be conducive to *stabilizing* recruitment to the stock, especially when these analyses consider spawning biomass per recruit.

Assumption (ii), that species do not interact biologically means, in terms of the multi-species version of the approach presented; that the species-specific M values do not change as a function of fishing mortality. Thus, it is assumed among other things that the natural mortality of small fish remains constant irrespective of the biomass of large fish, i.e., of actual and/or potential predators.

This assumption is evidently not likely to be met in any real stock. Models exist (e.g. multi-species VPA) in which M is explicitly made to vary with predator biomass and size (age) structure (Christensen 1995). However, even without variable natural mortalities, the multi-species version of the Thompson and Bell model represents an improvement over the single-species approach. Further, there is always the possibility of running the model several times, with different values of M such as to be able to assess the effects of changes of M on yields.

### *Estimating an F-array from Selection Data*

The model presented above requires estimates of mean size at first capture, i.e. ,the length at which 50 percent of the fish encountering a gear are retained if ($L_{50}$, or $L_c$). A common method to estimate $L_c$ is to fit selection data with a logistic curve of the form

$$P_i = 1 \Big/ \left[1 + e^{-r(L_i - L_c)}\right] \qquad \text{...13)}$$

where $P_i$ is the probability of capture at the midpoint of a length class i and r a constant whose value increases with the steepness of the selection curve; assuming the observed selection pattern to be symmetrical (or nearly so). Equation (13) may also be rewritten

$$P_i = 1 \Big/ \left[1 + e^{(S_1 - S_2 \cdot L_i)}\right] \qquad \text{...14)}$$

and $L_i$ is the length interval midpoint, $S_1$ and $S_2$ being constant (Paloheimo and Cadima 1964, Kimura 1977 and Hoydal, Rørvik and Sparre 1982). Equation (25) can be re-expressed as

$$\ln[(1/P_i) - 1] = S_1 - S_2 \cdot L_i \qquad \text{...15)}$$

which can be identified with a regression line, where $S_1 = a$ and $S_2 = b$ (note that Equation 15 is not defined for $P_i = 0$ or $P_i = 1$).

There is a one-to-one correspondence between $S_1$ and $S_2$ and $L_{25}$, $L_{50}$ and $L_{75}$, the lengths at which respectively 25, 50 and 75 percent of the fish are retained. The length range from $L_{25}$ to $L_{75}$, which is symmetrical around $L_{50}$, is called the *selection range*.

The formulae for calculating $L_{25}$, $L_{50}$ and $L_{75}$ are

$$L_{25} = [S_1 - \ln(3)]/S_2 \qquad \text{...16)}$$

$$L_{50} = S_1/S_2 \qquad \text{...17)}$$

$$L_{75} = [\ln(3) + S_1]/S_2 \qquad \text{...18)}$$

$S_1$ and $S_2$ can be derived from $L_{75}$ and $L_{50}$ using:

$$S_1 = L_{50} \cdot \ln(3)/(L_{75} - L_{50}) \qquad \text{...19)}$$

$$S_2 = \ln(3)/(L_{75} - L_{50}) = S_1/L_{50} \qquad \text{...20)}$$

Similarly, de-selection (ability of the fish to escape or avoid the gear) can also be evaluated from

$D_{25} = [D_1 - \ln(3)]/D_2$ ...21)

$D_{50} = D_1/D_2$ ...22)

$D_{75} = [\ln(3) + D_1]/D_2$ ...23)

$D_1$ and $D_2$ can be derived from $D_{75}$ and $D_{50}$ using:

$D_1 = L_{50} \cdot \ln(3)/(D_{75} - D_{50})$ ...24)

$D_2 = \ln(3)/(D_{75} - D_{50}) = D_1/D_{50}$ ...25)

The cumulative effects of selection and de-selection effects can be computed as

$$P_i = 1 \Big/ \left[1 + e^{(S_1 - S_2 \cdot L_i)}\right] \cdot 1 \Big/ \left[1 + e^{(D_1 - D_2 \cdot L_i)}\right] \quad \text{...26)}$$

where $P_i$ is the probability of capture for length group (i). When more than one gear (g) is used to exploit the given stock, the total probability of capture can be computed from

$$P_{i,g} = \sum \left(1 \Big/ \left[1 + e^{(S_{1,g} - S_{2,g} \cdot L_i)}\right] \cdot 1 \Big/ \left[1 + e^{(D_{,g1} - D_{2,g} \cdot L_i)}\right]\right)$$

...27)

The relative fishing mortality per length group (F-array) can be derived from Equation 27 by multiplying it by the catch (in number)

$$F'_i = \sum \left(C_i \Big/ \left[1 + e^{(S_{1,g} - S_{2,g} \cdot L_i)}\right] \cdot 1 \Big/ \left[1 + e^{(D_{,g1} - D_{2,g} \cdot L_i)}\right]\right)$$

...28)

The approximation of the F-array can then be computed by recursively applying Equation, (28) until the difference between the estimated total catch and the recorded total catch for a given gear is minimized.