

*Center for the Study of Institutional Diversity*

CSID Working Paper Series

#CSID-2014-007

**The Effect of Infrastructure on Social-Ecological System Dynamics:  
Provision Thresholds and Asymmetric Access**

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August 27, 2014

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For several millennia, humans have created built environments to harness natural processes for their benefit. Today, human-environment interactions are mediated extensively by physical infrastructure in both rural and urban environments. Yet studies of social-ecological systems (SESs) have not paid sufficient attention to how infrastructure influences coupled natural and social processes. This misses an important point: critical infrastructure is often a public good that depends on cooperation of the agents who share it. Using a model of an irrigation system (the most ancient of public infrastructure systems) as a testing ground, we found that two properties of infrastructure, threshold of provision and asymmetric access to benefits, greatly influence SES sustainability. Asymmetric access to benefits induces regimes of economic inequality. High thresholds suppress economic inequality, but at the cost of increased likelihood of system collapse. Low thresholds help to avoid system collapse, but may make the system more vulnerable to economic inequality and socioeconomic stresses. Understanding how small scale irrigation SESs may respond to such globalization-related stresses is relevant for agricultural policy and our results provide some general guidance in this regard.

## Keywords:

Social Ecological Systems, Infrastructure, Irrigation Systems, Collective Action, Coupled Infrastructure Systems

# The effect of infrastructure on social-ecological system dynamics: Provision thresholds and asymmetric access

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## Abstract

For several millennia, humans have created built environments to harness natural processes for their benefit. Today, human-environment interactions are mediated extensively by physical infrastructure in both rural and urban environments. Yet studies of social-ecological systems (SESs) have not paid sufficient attention to how infrastructure influences coupled natural and social processes. This misses an important point: critical infrastructure is often a public good that depends on cooperation of the agents who share it. Using a model of an irrigation system (the most ancient of public infrastructure systems) as a testing ground, we found that two properties of infrastructure, threshold of provision and asymmetric access to benefits, greatly influence SES sustainability. Asymmetric access to benefits induces regimes of economic inequality. High thresholds suppress economic inequality, but at the cost of increased likelihood of system collapse. Low thresholds help to avoid system collapse, but may make the system more vulnerable to economic inequality and socioeconomic stresses. Understanding how small scale irrigation SESs may respond to such globalization-related stresses is relevant for agricultural policy and our results provide some general guidance in this regard.

Studies of social-ecological systems (SESs) have typically focused on an ecosystem as their primary non-social component (9, 19, 32). The implicit assumption is that humans interact with natural systems without much mediation. This assumption may be true for some types of systems, like wildlife reserves. But it obviously is not always true. In many sustainability problems of great importance, a third element—built environment (or infrastructure)—heavily mediates between human-environment interactions. The importance of infrastructure in SESs was recognized early on in the pioneering work of Clark and Munro on how features of infrastructure such as the irreversibility of investment in fishing boats affect the dynamics of SESs (10). With globalization, the importance of built infrastructure will likely become ever more important. For example, the fossil-fuel-based infrastructure systems are the medium through which human actions cause climate change. Global food security depends on irrigation infrastructure through which farmers obtain water. And the resilience of urban systems to natural hazards often depends on engineered structures such as levees, roads, or buildings. Infrastructure also often requires inputs of collective human effort to be operational; in return, it allows humans to reap benefits from natural processes. In mediated systems, infrastructure fundamentally shapes the endogenous dynamics of coupled natural and social processes (3, 23). What are the characteristics of infrastructure that may facilitate or constrain the sustainability of SESs? This study examines that question using a simple model of an

irrigation system—an exemplary SES in which infrastructure is the key interface between natural and social processes.

Irrigation systems provide an excellent testing ground for exploring how infrastructure affects SESs. Farmers need a reliable supply of water to produce food and often move water from its source through production infrastructure (weir) and distribution infrastructure (canals). But such systems are prone to two types of social problems. First, farmers must mobilize enough collective labor to repair the infrastructure each year (a threshold public good dilemma). Secondly, farmers must coordinate for fair water distribution, which can be undermined by upstream-downstream asymmetry stemming from the canal layout (an asymmetric commons dilemma) (17). Because of the tight links between livelihoods, inequality, ecosystems, social dilemmas, and infrastructure, it has been suggested that farmer-managed irrigation systems are to the study of SES sustainability what fruit-flies are to the study of evolutionary biology (15). Further, exploring the role of infrastructure can deepen our understanding of the vulnerability of small-scale agricultural systems under different built environments. Nearly 90% of farms worldwide support and are operated by small-holder farmers who cultivate less than 2 hectares of land (20). Most of these small-holders practice irrigated agriculture, which consumes an estimated 70% of global developed water supplies and produces nearly 40% of global agricultural outputs (8, 34). An important, policy-relevant question is to what extent these small-scale systems can continue to maintain cooperation and, with it, critical infrastructure and productivity of their systems in the face of global social, economic, and climate change.

We address the question of how infrastructure influences SES dynamics in two stages. First, we compared the effects of two types of distribution infrastructure, one with and one without upstream-downstream asymmetry. Secondly, we evaluated to what extent different threshold behaviors of infrastructure influence system behaviors and their robustness to an exogenous economic shock. Our results suggest that these properties can greatly influence SES sustainability. Asymmetric access induces multiple regimes with different degrees of economic inequality. High thresholds can suppress economic inequality, but at the cost of increased likelihood of system collapse. Low thresholds help to avoid system collapse, but may make the system more prone to economic inequality. Low thresholds also cause system outputs to be more sensitive to shocks in wage rates, i.e. an increase in the attractiveness of outside wage labor options.

What emerges from our analysis is the need to re-conceptualize SESs as coupled infrastructure systems (CIS) in which the role of public infrastructure is clearly present. As suggested by (5), this can be achieved by conceptualizing ecologies of SESs more broadly to include both built components and self-organizing natural systems, as in industrial ecology (14). We suggest that this broader view would foster more integrative SES research, help cross-fertilize knowledge with other disciplines, such as architecture and engineering, and has practical policy implications for addressing globalization challenges to SES sustainability.

## **The study system**

### **The nature of irrigation systems.**

In general, irrigation systems increase crop production and stabilize production levels despite uncertainties in annual water supply (8). To obtain these benefits, farmers must invest in infrastructure, and create and enforce governing rules to coordinate the infrastructure-provision and water-distribution processes (25). Without effective governing rules and their enforcement, farmers can free-ride on others' efforts during the provision process, i.e., take irrigated water without contribut-

ing labor to infrastructure provision. Likewise, upstream farmers can use their location advantage to over-appropriate water, passing an unfair share of water to downstream users during the distribution process. Interestingly, these two processes often become interdependent because provision of infrastructure often requires a critical mass of human labor, i.e., upstream farmers usually cannot carry out the task without help from downstream farmers (25). This interdependency reduces the likelihood that upstream farmers will over-appropriate water, because downstream farmers who do not get enough water can retaliate by reducing their inputs to the infrastructure (16).

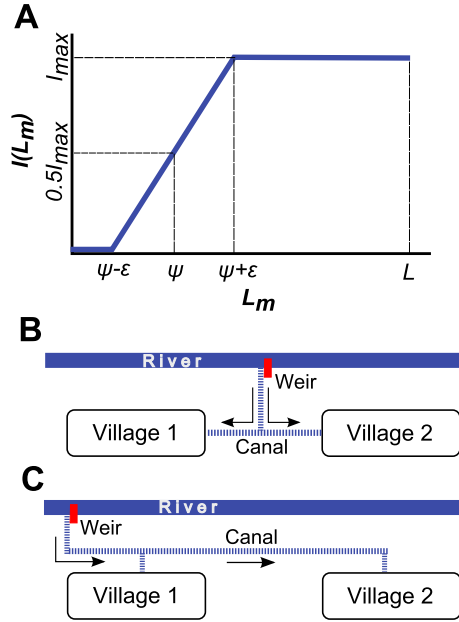
Despite the many challenges facing farmers, field studies have found irrigation communities that have successfully maintained their infrastructure and achieved fair water distribution for hundreds of years (29, 33). These long-lived, successful systems typically have well-tuned rules to govern user behavior, and they enforce those rules to achieve conformance (21). Behavioral experiments have followed up on field-study findings by demonstrating that individuals in small groups (5 players) can endogenously solve relatively complex commons dilemmas associated with irrigation if allowed to communicate (4, 17). Some interesting insights emerged in these studies, i.e., players were willing to tolerate some inequality in the amount of appropriated water, as long as there was some proportional equivalence between investments in and benefits from shared infrastructure (17). This tolerance went down with external shocks, i.e. inequality may make systems more sensitive to exogenous variability (4).

This widespread evidence of self-organized governance regimes has baffled scholars because governance regimes are themselves public goods that are subject to free riding (24). If it is difficult for humans to solve infrastructure-provision and water-distribution problems, why would they solve the even more difficult problems of making governing rules and punishing non-conformers at a cost to themselves? Recent studies suggest that humans often overcome such social problems because they tend to have other-regarding preferences and social norms, and they use heuristics when making decisions (as opposed to acting purely rationally) (27). More importantly, these studies stress that contextual variables, such as the nature of a built environment, the availability of exit options, or power asymmetries, also influence human decisions in SES collective-action situations (2, 23). A related question that we address here is how do these factors combine to affect the robustness of SESs and their sustainability. Modeling can help answer questions about the SES dynamics in such a context: in our case, how threshold of provision and asymmetric access interact to influence outcomes and robustness in the long-run.

### **The basic model structure.**

Suppose that  $N$  farming households are spread across two villages (Village 1 and Village 2) that co-manage a single irrigation system. The number of households in each village is  $N_1$  and  $N_2$ , respectively, which satisfy  $N_1 + N_2 = N$ . Each farmer is endowed with the same amount of available labor ( $l$ ) each year and the same acreage ( $a$ ). A farmer may appropriate a volume of water ( $q$ ) from the system, and allocate labor among three kinds of work: farming ( $l_f$ ), maintaining infrastructure ( $l_m$ ), and outside employment ( $l_e$ ) with wage rate  $w$ , i.e.,  $l = l_f + l_m + l_e$ . Then, a farmer's income is  $\pi = f(l_f, q, a) + wl_e$ , where  $f(l_f, q, a)$  is the income function from farming and  $wl_e$  is the employment income. Similarly, the aggregate income of the two villages is given by  $\Pi = F(L_f, Q, A) + wL_e$  (upper case symbols represent aggregate-level quantities).

To receive irrigation water, farmers have to maintain the infrastructure each year (canals must be cleaned of debris and water diversion structures such as weirs must be repaired). If farmers' aggregate maintenance labor ( $L_m$ ) exceeds the threshold of provision, the infrastructure can start to deliver water. If too few farmers contribute labor, the infrastructure delivers no water. We use



**Figure 1:** Panel A shows infrastructure efficiency  $I(L_m)$  as a function of aggregate labor inputs  $L_m$ . The half-saturation point of labor ( $\psi$ ) and half-width of the threshold slope ( $\epsilon$ ) determine the threshold of provision ( $\psi - \epsilon$ ). When  $\epsilon = 0$ , slope is infinite and no water is generated until  $L_m = \psi$ . When  $\epsilon = \psi$ , the amount of water increases linearly with  $L_m$  until  $L_m = \psi + \epsilon$ . Panels B and C show two types of distribution infrastructure. In B, two villages have equal access to water. In C, Village 1 has advantage over Village 2 in water-access.

a piece-wise linear function  $I(L_m)$  (Fig.1A) to represent this threshold behavior. The parameter  $\psi$  is the half-saturation point of  $L_m$ , yielding half of the maximum infrastructure efficiency. The parameter  $\epsilon$  controls the slope of the threshold. It follows that the threshold of provision is  $\psi - \epsilon$ . Total irrigation water is given by  $Q = I(L_m)S(t)$ , where  $S(t)$  is the volume of water that can be transferred from ecological sources. In this study,  $S(t)$  is assumed to be constant ( $S(t) = S$ ).

Our model system is governed by the following rules. Maintenance labor to be contributed by a farmer is proportional to his or her acreage (which is assumed to be the same for all farmers in this study). Water allocated to a farmer is proportional to his or her acreage, but only among the water rights holders—only farmers who contributed labor to the infrastructure prior to the planting season obtain water rights. Reflecting Ostrom’s institutional design principles for long-lived commons (21), these rules ensure that the benefits and costs borne by a farmer are proportionate to each other.

Farmers choose between two strategies: group-conformist ( $G$ ) and opportunist ( $O$ ). The model tracks the fraction of  $G$ s in Village  $i$  denoted by  $X_i = N_i^G/N_i$ . We define the total number of  $G$ s as  $N^G = N_1^G + N_2^G$ . Accordingly, the fraction of  $O$ s in Village  $i$  is  $1 - X_i = N_i^O/N_i$  (we use the notational convention that subscripts and superscripts refer to village and agent type, respectively, on all variables throughout the remainder of the paper).  $G$ s follow and enforce the rules, and strive to maximize the total welfare of the two villages. Each  $G$  assumes everyone will contribute to the public infrastructure and contributes their proportionate share ( $1/N$ ) of the optimal total maintenance labor ( $L_m^*$ ), attempts to take only the allocated share ( $1/N^G$ ) of the total water ( $Q$ ), and allocates labor between farming and employment to maximize the total income.  $L_m^*$  is the optimal maintenance labor that would maximize the total welfare of the two villages via optimal

production of irrigated water, as would be prescribed by a village leader acting as a benevolent social planner. Further,  $G$ s monitor for rule violations in their own Village  $i$  and the other Village  $j$  ( $i \neq j$ ), and punish violators at a cost to themselves. The cost of enforcement for a  $G$  increases with the frequencies of opportunists (28), i.e.,  $\gamma_s(1 - X_i) + \gamma_o(1 - X_j)$  where  $\gamma_s$  and  $\gamma_o$  represent the maximum enforcement costs for the same village and the other village, respectively.

$O$ s, in contrast to  $G$ s, break the rules and attempt to maximize individual net income. They contribute zero maintenance labor ( $l_m = 0$ ), and thus do not hold water rights. Nevertheless, they take as much of other farmers' water as they can within the limits set by the penalties imposed, their capacity to compete for water relative to others, and the benefits to be gained from the outside employment.  $O$ s take an amount of water and allocate labor to employment to maximize their individual net income. The probability of being caught and punished increases with the average frequency of rule enforcers, i.e.,  $(X_i + X_j)/2$  (we are assuming here that  $N_1 = N_2$ ). The penalty varies by situation: it increases with the amount of water stolen ( $q^O$ ), but decreases with water abundance in the system. When water is abundant, rule violations are tolerated because farmers have little incentives to concern themselves with equity issues (1, 25). Empirical evidences show that resource users would be less likely to protect their resource when resource is abundant (2, 22). This effect is represented by  $\delta[1 - \sigma Q(L_m)/Q(L_m^*)]q^O$ ; where  $\delta$  is the maximum penalty,  $Q(L_m)/Q(L_m^*)$  is the proxy for water abundance, and  $\sigma \leq 1$  is the tolerance factor. (see the *Materials and Methods* for details on the payoffs for  $G$  and  $O$  in Village  $i$ ).

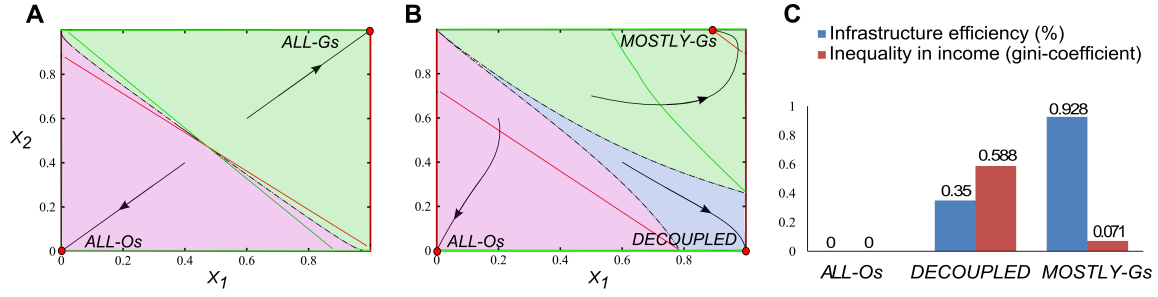
### The model with symmetric access to water.

We first consider the case in which upstream-downstream asymmetry is absent from the distribution infrastructure (Fig.1B). That is, the two villages have equal access to water. For simplicity, let us assume that *within* each village, farmers also have equal access. This symmetric case will serve as a benchmark to highlight the effects of asymmetric access.

In this setting, farmers are symmetric in their capacity for competing for water because they all have equal access to water and are endowed with the same levels of available labor, technology, and skills. It follows that if all farmers are to rush and compete for water, they will face, on average, a constraint of  $Q/N$  on the amount of water they obtain. Hence,  $O$ s in each village harvest water and allocate labor at levels that maximize their payoff subject to the conditions  $q_i^O \leq Q/N$  and  $l_f^O + l_e^O = l$ . The total amount of water taken by  $O$ s in each village is  $Q_i^O = q_i^O(1 - X_i)N_i$  and we define  $Q^O = Q_1^O + Q_2^O$ . It follows, then, that  $G$ s in each village obtain the amount of water given by  $q_i^G = [Q - Q^O]/N^G$ , which is less than what they are supposed to receive ( $Q/N^G$ ). Finally, we use replicator equations (30) to model the dynamics of the two strategies within each village (28, 35) (see the *Materials and Methods* for details).

### The model with asymmetric access to water.

We now consider the case in which upstream-downstream asymmetry is present between Villages 1 and 2 (Fig.1C), which is the likely scenario in most irrigation systems. In this setting, water is accessed sequentially—farmers in Village 1 access water before those in Village 2. All other features of the model remain the same as in the symmetric case. Because of their privileged access,  $O$ s in Village 1 are less constrained on the amount of water they can appropriate ( $q_1^O \leq Q/N_1$ ) than before ( $q_i^O \leq Q/N$ ). At the same time,  $G$ s in village 1 rely on their upstream position to bring their appropriated water as close as possible to allocated amount, i.e.,  $q_1^G = \min [Q/N^G, (Q - Q_1^O)/N_1^G]$ . It follows, then, that  $q_2^O \leq [Q - Q_1^G - Q_1^O]/N_2$  is the upper bound on the amount of water that  $O$ s in



**Figure 2:** Comparison of dynamics with and without upstream-downstream asymmetry. The  $x$  and  $y$  axes show the fractions of  $G$ s in Village 1 ( $X_1$ ) and Village 2 ( $X_2$ ), respectively. Red dots represent stable equilibrium points of the dynamics. Arrows represent the flows of dynamics from particular initial states. Red and green lines represent  $X_1$  and  $X_2$  nullclines, respectively. Panel A shows possible dynamics when asymmetry is absent. Two regimes are possible: *ALL-Os* and *ALL-Gs*. Income inequality does not exist in these two regimes. Infrastructure is fully provided at *ALL-Gs*, but the system collapses at *ALL-Os*. Panel B shows possible regimes under asymmetry: *ALL-Os* (light pink area), *MOSTLY-Gs* (light green area), and *DECOUPLED* (light blue area). At *ALL-Os*, the irrigation system collapses. At *MOSTLY-Gs*, most farmers follow the rules, water is almost fully supplied, and some income inequality exists between the villages. At *DECOUPLED*, farmers in Village 2 leave farming, and considerable inequality in total income exists between Villages 1 and 2. Panel C shows a comparison of the three regimes shown in B (model with asymmetry) in terms of infrastructure efficiency and income inequality between Villages 1 and 2. Income inequality is computed by deriving gini-coefficients of total income between Villages 1 and 2. The default parameter values are:  $w = 0.2$ ,  $\psi = 0.2$ ,  $\varepsilon = 0.125$ ,  $I_{max} = 1$ ,  $S = 100$ ,  $a = 1$ ,  $l = 1$ ,  $N_1 = N_2 = 50$ ,  $\gamma_s = 0.05$ ,  $\gamma_o = 0.1$ ,  $\delta = 1.4$ , and  $\sigma = 0.9$ . (see the *SI* for the details).

village 2 can take. Finally,  $G$ s in village 2 obtain the amount  $q_2^G = [Q - Q_1^O - Q_1^G - Q_2^O] / N_2^G$ .

## Analysis I: Effect of upstream-downstream asymmetry

Figures 2A and B are phase-space representations of the overall cooperation level of the system. Figure 2A suggests that, in the absence of asymmetry, two regimes typically emerge for most parameters explored: all  $O$ s (*ALL-Os*) and all  $G$ s (*ALL-Gs*). There is no inequality in total income between Villages 1 and 2 in these two regimes. With asymmetry, however, three regimes are possible: *ALL-Os*; a sustainable situation in which most are  $G$ s in Village 1, but all are  $G$ s in village 2 (*MOSTLY-Gs*); and a decoupled situation in which the two villages stop collaborating for a common goal, i.e.,  $G$ s dominate in Village 1 but  $O$ s prevail in Village 2 (*DECOUPLED*) (Fig. 2B).

Figure 2C compares the three regimes of the asymmetric case in terms of infrastructure efficiency and inequality in total income. At *ALL-Os*, no water is supplied and everyone is equally bad off. At *MOSTLY-Gs*, water is almost fully supplied and the two villages have a roughly equal total income (but the income of Village 1 is somewhat higher than that of Village 2). At *DECOUPLED*, some water is supplied but considerable income inequality exists at village-level because only farmers in Village 1 obtain irrigated water. Farmers in Village 2 leave agriculture and resort to outside employment.

Without asymmetry, two feedback loops drive system dynamics. If  $G$ s are few such that  $L_m \approx \psi - \varepsilon$ , so little water is generated and water theft is so rampant that it is better for all farmers to become  $O$ s in a positive feedback loop. The system converges to *ALL-Os* eventually and everyone is equally bad off. If enough  $G$ s exist in the beginning,  $G$ s likely prevail over  $O$ s because the



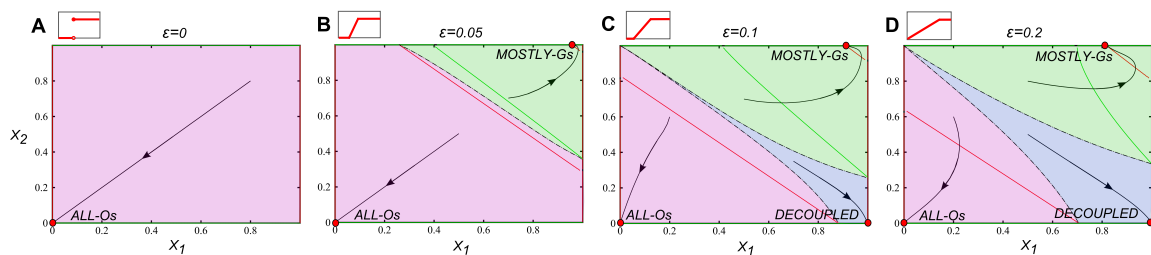
$G$ s increasingly obtain a moderate amount of water while  $O$ s are being punished in a positive feedback loop. The system, however, can stop short of reaching full cooperation if rule violations are sufficiently tolerated ( $\sigma$  close to 1.0) as water becomes more abundant.

With asymmetry, two regimes with different degrees of economic inequality can emerge: *MOSTLY-Gs* and *DECOUPLED*. The total income of Village 1 is higher than that of Village 2 in both regimes (but the inequality is much more severe in *DECOUPLED*). At *MOSTLY-Gs*, some  $O$ s exist in Village 1, but all are  $G$ s in village 2. This slightly unfair regime occurs because  $O$ s in Village 1 are less constrained in the amount of water they appropriate ( $q_1^O \leq Q/N_1$ ) than in the symmetric case ( $q_1^O \leq Q/N$ ) and than  $O$ s in Village 2 ( $q_2^O \leq [Q - Q_1^G - Q_1^O]/N_2$ ). This advantage enables some  $O$ s to survive in Village 1 in the asymmetric case. The system converges to *DECOUPLED* when enough  $G$ s exist in Village 1 but few  $G$ s exist in Village 2 at the outset. Because water rights holders ( $G$ s) are few in Village 2,  $G$ s in Village 1 pass down little water, which becomes subject to fierce competition between many  $O$ s and few  $G$ s in Village 2. Because of this competition for scant water, the amount of water that downstream  $O$ s get their hands on is decided by the physical limit of available water ( $q_2^O = [Q - Q_1^G - Q_1^O]/N_2$ ) rather than by the penalty they face. It follows, then, that  $G$ s and  $O$ s in Village 2 end up obtaining an equal amount of water, i.e.,  $q_2^G = [Q - Q_1^O - Q_1^G - Q_2^O]/N_2^G = [Q - Q_1^G - Q_1^O]/N_2$ . Eventually,  $O$ s prevail in Village 2 because they are penalized little (because they obtain only a meager amount of water), appropriate the same amount of water as  $G$ s', and free-ride on maintenance labor. In contrast to Village 2,  $G$ s triumph over  $O$ s in Village 1—a substantial amount of water is available in upstream and  $G$ s obtain more water than  $O$ s by means of rule enforcement. Hence, the system converges to *MOSTLY-Gs* in the long-run.

In summary, upstream-downstream asymmetry induces regimes of economic inequality in our model system. It is important to note that these regimes are robust: the irrigation system remains functional even though income inequality persists in the system. Our findings are concordant with empirical studies. Behavioral laboratory experiments have shown that players in irrigation dilemma games may be willing to tolerate some degree of inequality, as long as there is some proportional equivalence between investments in and benefits from infrastructure (i.e., *MOSTLY-Gs*) (4, 17). Field studies also observed that downstream farmers may exit from co-managing a system if there is too much income inequality (i.e., *DECOUPLED*) (7).

## Analysis II: Effect of threshold behavior

Some systems are threshold public goods with a centralized point of services for users, e.g., a capital-intensive irrigation infrastructure that likely brings no water until some critical amount of maintenance inputs is invested each year. In contrast, a scalable system in which a collection of small-scale infrastructure provides services may start to bring water at much smaller maintenance inputs (i.e., a linear public good). The debate about which of the two approaches leads to more sustainable outcomes is an important on-going policy question (11, 12). While studies report superior performance of decentralized small-scale irrigation systems over centralized agency-managed ones in terms of efficiency of crop yield and collective action (18, 31), some studies also warn that decentralized systems are prone to capture by local elites who misappropriate benefits and cause inequity issues (13, 26). Therefore, we explored how the degree of threshold behavior affects system behavior in our model system. We restricted our analysis to asymmetric-access scenarios in which all of the three regimes (*ALL-Os*, *MOSTLY-Gs*, and *DECOUPLED*) are present at the default threshold ( $\psi - \varepsilon = 0.075$ ). This setup allowed us to focus on how the three regimes expand or shrink as the threshold is varied.



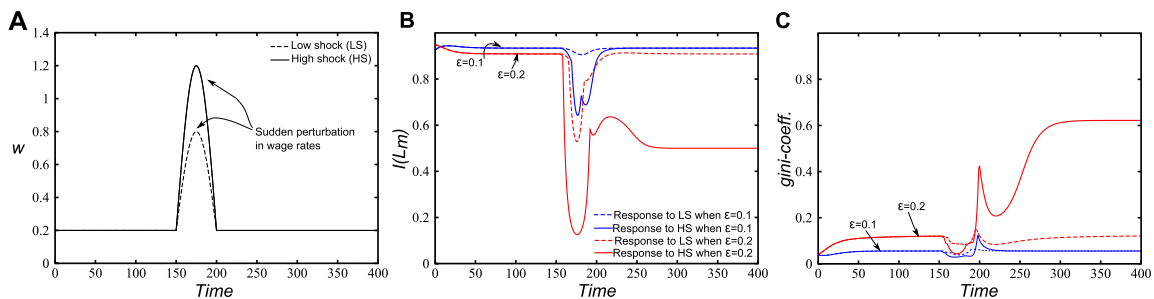
**Figure 3:** Effect of threshold of provision on system behavior when half-saturation point of labor  $\psi = 0.2$ . The three regimes shrink and expand as  $\epsilon$  is varied (Panels A to D). The small inset graphs next to the panel labels show the shape of the infrastructure-labor relation for each value of  $\epsilon$ . Except for the focal parameter, the same default parameter values were used as in Fig.2

Figure 3 shows that the threshold behavior determines the dominance of the three regimes. As the threshold gets lower and gentler in slope ( $\epsilon$  approaches  $\psi$ ), *ALL-Os* loses resilience, i.e., its basin of attraction shrinks. At the same time, *MOSTLY-Gs* emerges and expands. But this regime shift occurs at the cost of emergence of the *DECOUPLED* regime and more free riders in Village 1 of the *MOSTLY-Gs* regime (the red dot for *MOSTLY-G* moving to left as  $\epsilon$  approaches  $\psi$  in Fig.3). The opposite is true when the threshold gets higher and sharper in slope ( $\epsilon$  approaches zero). These results suggest that low-threshold scalable systems (i.e.,  $\epsilon \approx \psi$ ) are less likely to collapse, but are more prone to economic inequality. Conversely, centralized capital-intensive systems (i.e.,  $\epsilon \approx 0$ ) are less likely to have inequality issues, but are much more likely to collapse.

When the threshold is high and sharp ( $\psi - \epsilon \approx \psi$ ), little or no water is generated until most of the population is comprised of *Gs*. In such a situation, *DECOUPLED* cannot be stable because upstream farmers alone cannot maintain the infrastructure, i.e., strong interdependency exists between Villages 1 and 2. For the same reason, *MOSTLY-Gs* cannot be stable when the threshold is extremely sharp. The path toward sustainability is much narrower in high-threshold systems because most farmers need to participate in collective action each year just to make the system work.

In contrast, when the threshold is low and gentle in slope ( $\psi - \epsilon \approx 0$ ), water supply starts to increase almost linearly even at small labor contributions. Hence, *ALL-Os* has smaller resilience—as soon as some *Gs* are introduced, it is better for some farmers to cooperate and get some water than to quit farming (unless, of course, outside wage rates are sufficiently high). However, this departure from *ALL-Os* is accompanied by a higher likelihood of inequality in the system because of the introduction of the *DECOUPLED* regime. At *MOSTLY-Gs*, downstream farmers continue to cooperate despite some inequality because the infrastructure generates substantial benefits that trickle down to them. At *DECOUPLED*, however, the system functions poorly with considerable income inequality. Because low thresholds weaken the upstream-downstream interdependency, upstream farmers cooperate only among themselves to obtain enough water (i.e., they do not need labor contributions from downstream farmers).

We also explored how the combined factors of threshold behavior and asymmetry affect the *robustness* of SESs and their sustainability in this era of globalization. Field studies have found that, as globalization proceeds, rural communities tend to depend more on wage labor; suffer shortage of labor for maintaining shared infrastructure; and experience erosion of social norm for collective action, especially among younger generations (1, 6). Hence, we introduced into our model system a sudden rise in wage rates (Fig. 4A) to mimic the pressures of globalization processes. We tested how the *MOSTLY-Gs* regime respond under two different contexts of the infrastructure—one



**Figure 4:** Sensitivity of infrastructure efficiency and income inequality of the *MOSTLY-Gs* regime in response to a perturbation in wage rates. Panel A shows two shock-profiles of wage rates ( $w$ ) for outside employment. Solid line represents a high shock (HS) in which  $w$  jumps from 0.2 to 1.2. Dashed line represents a low shock (LS) in which  $w$  jumps from 0.2 to 0.8. The system is at *MOSTLY-Gs* when the shocks apply between  $T = 150$  and  $T = 200$ . Panel B shows the sensitivity of the infrastructure efficiency ( $I(L_m)$ ) to the wage shocks. Red and blue lines represent the system response when  $\epsilon = 0.2$  and  $\epsilon = 0.1$ , respectively. Panel C shows the sensitivity of the village-level income inequality (gini-coefficient) to the wage shocks. Except for the focal parameters, the same default parameter values were used as in Fig. 2.

with and one without threshold behavior. Figure 4B and C show the sensitivity of infrastructure efficiency and inequality in total income to the shocks. It turns out that the system without threshold behavior ( $\epsilon = 0.2$ ) is more sensitive or vulnerable to the shocks than is the one with threshold behavior ( $\epsilon = 0.1$ ). The reason is that downstream *Gs* are more disadvantaged under low-threshold infrastructure because of the weakened upstream-downstream interdependency. In such a situation, some increase in wage rates can easily tip the balance and cause *Os* to dominate in downstream because they skip maintenance work to earn better income from non-farming work. Given that economic inequality is higher in systems with low thresholds, our result is concordant with an empirical finding that inequality is associated with the reduced capacity of SESs to cope with external shocks (4).

## Conclusion

Farmer-managed irrigation systems contain all the basic features of complex social-ecological systems: natural infrastructure (watersheds and agricultural land), hard human-made infrastructure (water conveyance structures) and soft human-made infrastructure (institutional arrangements and organizational forms). Understanding how these infrastructures interact and respond to change is critical for maintaining food security for billions of people in the coming decades. Using a model of an irrigation system as a testing ground, we examined the effect of a key element of SES—physical infrastructure—whose influence has heretofore been under-emphasized in the SES approach. We have shown that infrastructure can greatly influence SES sustainability. When distribution infrastructure exhibits asymmetry, multiple regimes with different degrees of economic inequality can emerge. We also observed that the regimes of inequality and system collapse expand or shrink as the threshold of infrastructure-provision is varied. With a low-threshold scalable infrastructure, the likelihood of system collapse is low. The tradeoff is a higher likelihood that economic inequality will increase. With a high-threshold infrastructure, SESs can attenuate the possibility of economic inequality, but the tradeoff is that they become more prone to system collapse. The threshold behavior can also influence the robustness of SES to external economic shocks: under low thresholds, the *MOSTLY-Gs* regime is more sensitive to wage shocks.

These outcomes highlight the need to re-conceptualize SESs as coupled infrastructure systems (CIS) so that the role of public infrastructure is more central in SES research. This can be achieved by expanding ecologies of SES to include both built components and natural systems. We suggest that viewing SES as CIS provides a better reflection of the sustainability problem context in the current era. It would also facilitate more dialogue and help cross-fertilize research between natural and social science-oriented sustainability scholars and those based in applied sciences of architecture and engineering.

Our findings also have implications for policy. Development policy has run between the extremes of highly centralized, large infrastructure (low  $\epsilon$ ) from the 1950's to the 90's followed by a push for decentralization in governance and less centralized (higher  $\epsilon$ ) infrastructure. Our analysis supports empirical observations that neither is a panacea and provides more detailed insights into the trade-offs associated with one or the other development approach that are of potential value to policy. Policymakers must balance the increased robustness of maintaining cooperation and system performance that scalable infrastructure confers to increased vulnerability to emerging inequality and to external wage labor opportunities. Likewise, if biophysical conditions favor centralized infrastructure, policymakers must be aware of the increased propensity for system collapse and reduced incentives for opportunism.

Of course, our findings are preliminary and represent a first step in better understanding the capacity of CIS to cope with global change. Our study restricted the number of strategies to two: group-conformist and opportunist. Additional strategies could be considered: for example, farmers could invest to the infrastructure but take more water than their allocated share. We assumed that the depreciation rate of infrastructure is fast, i.e., no water is provided unless the infrastructure is repaired every year. An alternative approach would be to assume a slower rate of depreciation, such that some water is still provided even if no repair is done in a given year. These variations will further our understanding of the influence of infrastructure in SES dynamics. Finally, further case-study analysis and empirical field work, like that which motivated this modeling effort (see [csid.asu.edu](http://csid.asu.edu)) are essential to support theoretical and practical results.

## Methods

Based on the model description, the payoffs for  $G$  and  $O$  in Village  $i$  can be expressed as the following:

$$\begin{aligned}\pi_i^G &= f(l_f^G, q_i^G, a) + wl_e^G - \gamma_s(1 - X_i) - \gamma_o(1 - X_j) \\ \pi_i^O &= f(l_f^O, q_i^O, a) + wl_e^O - \delta \left(1 - \sigma \frac{Q(L_m)}{Q(L_m^*)}\right) q^O \left(\frac{X_i + X_j}{2}\right)\end{aligned}$$

where  $j$  denotes the other village ( $i \neq j$ ). We used replicator equations to model the changes in the fraction of  $G$ s ( $X_i$ ) in Village  $i$ :

$$dX_i/dt = X_i[\pi_i^G - \bar{\pi}_i]$$

where  $\bar{\pi}_i$  is the average payoff of a farmer in Village  $i$ , i.e.,  $\bar{\pi}_i = \pi_i^G X_i + \pi_i^O(1 - X_i)$ . Our results were obtained from numerical simulation of the above system of equations. See the *SI* for more details.

## Acknowledgements

All authors acknowledge financial support from the National Science Foundation, Grant number GEO-1115054. Kathryn Kyle and Marco A. Janssen also provided feedback to the early drafts of the article. David J. Yu also thanks Charles L. Redman for inspiring this research with his reflective questions on the importance of infrastructure in SESs.

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# The effect of infrastructure on social-ecological system dynamics: Provision thresholds and asymmetric access (Supporting Information)

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## 1 Dynamics of the strategies

$N$  farming households are spread across two villages (Village 1 and Village 2) that co-manage a single irrigation system. The number of households in each village is  $N_1$  and  $N_2$ , respectively, which satisfy  $N_1 + N_2 = N$ . The irrigation system is governed by the following rules. Maintenance labor to be contributed by a farmer is proportional to his or her acreage. Water allocated to a farmer is proportional to his or her acreage, but only among the water rights holders. Only farmers who contributed labor to the infrastructure prior to the planting season obtain water rights. Farmers choose between two strategies: group-conformist ( $G$ ) and opportunist ( $O$ ).  $G$ s follow and enforce the rules, and strive to maximize the total welfare of the two villages.  $O$ s break the rules and attempt to maximize individual net income.

Our model tracks the fraction of  $G$ s in Village  $i$  denoted by  $X_i = N_i^G/N_i$ . We define the total number of  $G$ s as  $N^G = N_1^G + N_2^G$ . Finally, the fraction of  $O$ s in Village  $i$  is  $1 - X_i = N_i^O/N_i$ . We used replicator equations below to track the fractions of the strategies in Villages 1 and 2.

$$\frac{dX_1}{dt} = X_1[\pi_1^G - \bar{\pi}_1] \quad (1)$$

$$\frac{dX_2}{dt} = X_2[\pi_2^G - \bar{\pi}_2] \quad (2)$$

Here,  $\pi_i^G$  is the payoff of  $G$  in Village  $i$ . The term  $\bar{\pi}_i$ , the average payoff of a farmer in Village  $i$ , is derived by  $\bar{\pi}_i = \pi_i^G X_i + \pi_i^O (1 - X_i)$ , where  $\pi_i^O$  is the payoff of  $O$  in the same village.

To explore the dynamics of our model system, we used *XPPAUT*, a software package specialized for studying non-linear dynamical systems. *XPPAUT* numerically derives local stability properties of equilibrium points, i.e., a set of system states  $(X_1, X_2)$  where the changes in  $X_1$  and  $X_2$  are zero as time is varied. Equilibrium points reveal the stable attractors of our model system.



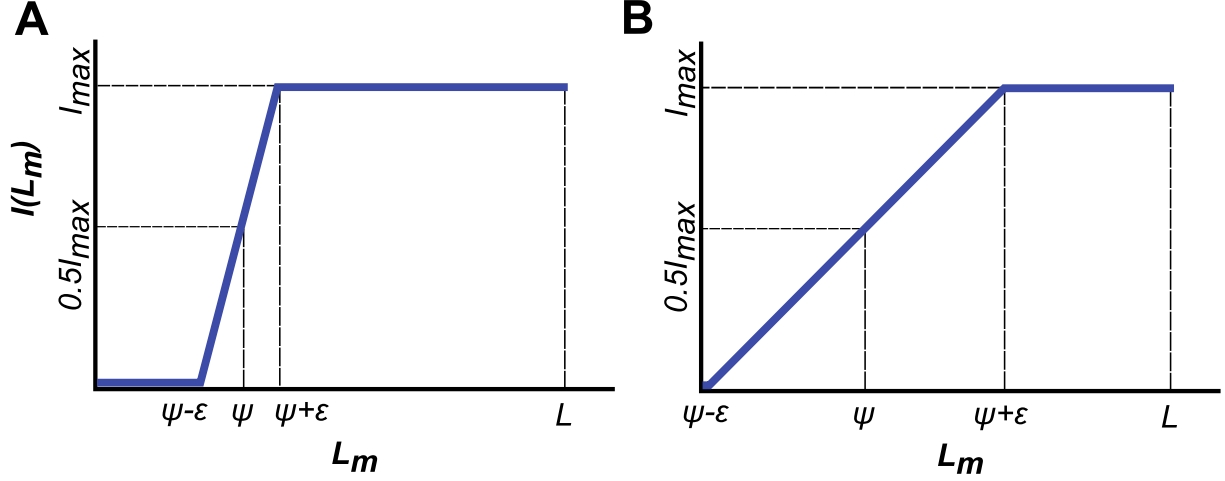


Figure 1: Panel A shows a high-threshold infrastructure. Panels B shows a low-threshold infrastructure (a linear public good).

## 2 Production infrastructure

Let  $I(L_m)$  be the efficiency of production infrastructure (water-diversion structure), where  $L_m$  is the sum of maintenance labor provided by all farmers each year, i.e.,  $L_m = \sum l_m$ . A threshold behavior is assumed for the provision of infrastructure, i.e., little or no water is generated until the annual maintenance labor reaches a certain threshold (see Fig.1 and Eq.3). We use a piece-wise linear function to represent this threshold behavior (Eq. 3). Here,  $I_{max}$  is the maximum infrastructure efficiency. The half-saturation point of labor ( $\psi$ ) and half-width of the threshold slope ( $\epsilon$ ) determine the threshold of provision ( $\psi - \epsilon$ ). When the maintenance labor is in the range of  $\psi - \epsilon \leq L_m \leq \psi + \epsilon$ , the efficiency increases by  $I_{max}/2\epsilon$  per unit of  $L_m$ . Below this range, zero efficiency exists. Above this range, no further efficiency is generated by adding more labor. Total irrigation water is given by  $Q(L_m) = I(L_m)S$ , where  $S$  is the volume of water that can be transferred from ecological sources.  $S$  is assumed to be constant in this study.

$$I(L_m) = \begin{cases} 0 & 0 \leq L_m < \psi - \epsilon \\ \frac{I_{max}}{2\epsilon} (L_m - \psi + \epsilon) & \psi - \epsilon \leq L_m \leq \psi + \epsilon \\ I_{max} & \psi + \epsilon < L_m \leq L \end{cases} \quad (3)$$

## 3 Income

Let  $f = f(l_f, q_i, a_i)$  be a farmer's farming income in village  $i$ , which depends on the production inputs of farming labor ( $l_f$ ), irrigated water ( $q_i$ ), and acreage ( $a_i$ ) (lowercase symbols are individual-level quantities). We specify  $f$  using a standard production function,  $f = pb(l_f)^j(a_i + r_i)^k a^{(1-j-k)}$ , where  $p$  is price per unit of agricultural yield,  $b$  is productivity coefficient for inputs of production,  $r_i$  is the amount of freely-available alternative water (e.g., rainfall) that a farmer receives, and  $j$  and  $k$  represent output elasticity of farming labor and irrigated water, respectively. The income from outside employment is given

by  $wl_e$ , where  $w$  is the wage rate for outside employment. A farmer total income in village  $i$  is then given by:

$$\pi_i = pb(l_f)^j(q_i + r_i)^k a_i^{(1-j-k)} + wl_e. \quad (4)$$

Similarly, the aggregate income of the two villages is given by:

$$\Pi = pb(L_f)^j(Q + R)^k A^{(1-j-k)} + wL_e \quad (5)$$

where the upper case symbols represent aggregate-level quantities.

## 4 Optimal provision of production infrastructure

We used the method of Lagrange multipliers for constrained optimization to calculate the optimal values of total labor for maintenance  $L_m^*$ , farming  $L_f^*$ , and employment  $L_e^*$ , that would maximize the aggregate income ( $\Pi$ ) of the two villages. This problem can be express as follows:

$$\max_{L_f, L_m} \Pi = pbL_f^j [I(L_m)S + R]^k A^{1-j-k} + wL_e; \quad \text{subject to: } L_f + L_m + L_e = L, \quad (6)$$

where  $L$  is the total available labor of the farmers in the two villages, i.e.,  $L = (N_1 + N_2)l$ . We assumed that each farmer is endowed with an equal amount of labor each year ( $l$ ), and  $N_1$  and  $N_2$  represent the number of farmers in Villages 1 and 2, respectively. Note that we can rewrite the expression for  $\Pi$  replacing  $L_e$  by  $L - L_m - L_f$ .

The optimization is performed for the three different regions of the production-infrastructure efficiency  $I(L_m)$  expressed in Eq.3. In the following, we present the different optimal values of  $L_f$ ,  $L_m$ ,  $L_e$ , for these different regions.

- **Region I:**  $0 \leq L_m \leq \psi - \varepsilon$ ;  $Q(L_m) = I(L_m)S = 0$ .

Clearly,  $L_m^* = 0$ . This is because any maintenance labor in this region would not exceed the threshold ( $\psi - \varepsilon$ ). As a result, the optimization problem becomes:

$$\max_{L_f, L_e} \Pi_I = pbL_f^j R^k A^{1-j-k} + wL_e; \quad \text{subject to: } L_f + L_e = L, \quad (7)$$

Using the constraint, we can rewrite  $\Pi_I = pbL_f^j R^k A^{1-j-k} + w(L - L_f)$ . We then solve  $d\Pi_I/dL_f = 0$  to obtain the following optimal values:  $L_f^* = (jpbA^{1-j-k}R^k/w)^{\frac{1}{1-j}}$ , and  $L_e^* = L - L_f^*$ . The maximum value of the aggregate income is:

$$\Pi_{I,max} = pbA^{1-j-k}(L_f^*)^j R^k + w(L - L_f^*).$$

When the return for wage labor is zero ( $w = 0$ ), the optimal values become:  $L_m^* = L_e^* = 0$ ,  $L_f^* = L$ , and  $\Pi_{I,max} = pbA^{1-j-k}L^j R^k$ .

- **Region 2:**  $\psi - \varepsilon < L_m \leq \psi + \varepsilon$ ;  $Q(L_m) = I(L_m)S = \frac{Q_{max}}{2\varepsilon}(L_m - \psi + \varepsilon)$ , where  $Q_{max} = I_{max}S$   
The optimization problem in this region becomes:

$$\begin{aligned} \max_{L_f, L_m} \Pi_{II} &= pbL_f^j \left[ \frac{Q_{max}}{2\varepsilon}(L_m - \psi + \varepsilon) + R \right]^k A^{1-j-k} + wL_e; \\ \text{subject to: } &L_f + L_m + L_e = L, \quad \text{and} \quad L_m \leq \psi + \varepsilon. \end{aligned} \quad (8)$$

Using the equality constraint to set  $L_e = L - L_f - L_m$ , we define the Lagrangian as follows:

$$\begin{aligned} \Lambda(L_f, L_m, \lambda) &= pbL_f^j \left[ \frac{Q_{max}}{2\varepsilon}(L_m - \psi + \varepsilon) + R \right]^k A^{1-j-k} \\ &\quad + w(L - L_f - L_m) + \lambda(\psi + \varepsilon - L_m), \end{aligned} \quad (9)$$

where  $\lambda$  is the lagrange multiplier for the inequality constraint.

To calculate the optimal values  $L_f^*$  and  $L_m^*$ , we solve the following set of equations:

$$\frac{\partial \Lambda}{\partial L_f} = \frac{\partial \Lambda}{\partial L_m} = \frac{\partial \Lambda}{\partial \lambda} = 0;$$

with the complementarity condition  $\lambda(\psi + \varepsilon - L_m) \geq 0$  and  $\lambda \geq 0$  to obtain the following values.

When  $\lambda = 0$  (i.e.,  $L_m < \psi + \varepsilon$ ):

$$\begin{aligned} L_f^* &= A \left[ \frac{jpb}{w} \left( \frac{kQ_{max}}{2j\varepsilon} \right)^k \right]^{\frac{1}{1-j-k}}, \\ L_m^* &= \psi - \varepsilon - \frac{2\varepsilon R}{Q_{max}} + \frac{2\varepsilon A}{Q_{max}} \left[ \frac{jpb}{w} \left( \frac{kQ_{max}}{2j\varepsilon} \right)^{1-j} \right]^{\frac{1}{1-j-k}}, \quad \text{and} \\ L_e^* &= L - L_f^* - L_m^*. \end{aligned} \quad (10)$$

The maximum value of the aggregate income can be calculated by substituting the optimal values:

$$\Pi_{II,max} = pbA^{1-j-k} (L_f^*)^j \left[ \frac{Q_{max}}{2\varepsilon}(L_m^* - \psi + \varepsilon) + R \right]^k + w(L - L_f^* - L_m^*).$$

When  $w = 0$ , the optimal values become:

$$\begin{aligned} L_m^* &= \frac{1}{j+k} \left[ kL + j(\psi - \varepsilon) - \frac{2j\varepsilon R}{Q_{max}} \right], \quad L_f^* = L - L_m^*, \quad L_e^* = 0, \quad \text{and}, \\ \Pi_{II,max} &= pbA^{1-j-k} (L - L_m^*)^j \left[ \frac{Q_{max}}{2\varepsilon}(L_m^* - \psi + \varepsilon) + R \right]^k. \end{aligned}$$

When  $\lambda > 0$  (i.e.,  $L_m = \psi + \varepsilon$ ); the optimal values will be equal to those for the region III that are presented next.

- **Region III:**  $\psi + \varepsilon \leq L_m \leq L$ ;  $Q(L_m) = I_{max}S = Q_{max}$ .

The optimal value of the maintenance labor is  $L_m^* = \psi + \varepsilon$ . The optimization problem then becomes:

$$\max_{L_f, L_e} \Pi_I = pbL_f^j [Q_{max} + R]^k A^{1-j-k} + wL_e; \text{ subject to: } L_f + L_e = L - \psi - \varepsilon, \quad (11)$$

Using the constraint, the aggregate income becomes:  $\Pi_{III} = pbA^{1-j-k} [Q_{max} + R]^k + w(L - L_f - \psi - \varepsilon)$ . We then solve for  $d\Pi_{III}/dL_f = 0$  to obtain the following optimal values:

$$L_f^* = \left[ \frac{jpbA^{1-j-k}(Q_{max}+R)^k}{w} \right], \quad L_e^* = L - \psi - \varepsilon - \left[ \frac{jpbA^{1-j-k}(Q_{max}+R)^k}{w} \right],$$

and  $\Pi_{III,max} = pbA^{1-j-k}(L_f^*)^j [Q_{max} + R]^k + w(L - \psi - \varepsilon - L_f^*)$ .

When  $w = 0$ , the optimal values become:  $L_m^* = \psi + \varepsilon$ ,  $L_e = 0$ ,  $L_f^* = L - \psi - \varepsilon$ , and  $\Pi_{III,max} = pbA^{1-j-k}(L - \psi - \varepsilon)^j [Q_{max} + R]^k$ .

## 5 Strategy payoffs under upstream-downstream asymmetry

To reiterate, our model system is governed by the following rules. Maintenance labor to be contributed by a farmer is proportional to his or her acreage. Water allocated to a farmer is proportional to his or her acreage, but only among the water rights holders. Only farmers who contributed labor to the infrastructure prior to the planting season are given water rights. Farmers choose between two strategies: group-conformist (*G*) and opportunist (*O*). *G*s follow and enforce the rules, and strive to maximize the total welfare of the two villages. *O*s break the rules and attempt to maximize individual net income.

Each *G* assumes everyone will contribute to the public infrastructure and contributes their proportionate share ( $1/N$ ) of the optimal total maintenance labor ( $L_m^*$ ), attempts to take only the allocated share ( $1/N^G$ ) of the total water ( $Q$ ), and allocates labor between farming and employment to maximize the total income. *G*s also monitor for rule violations in their own Village  $i$  and the other village  $j$  ( $i \neq j$ ), and punish violators at a cost to themselves. The cost of enforcement for a *G* increases with the frequencies of opportunists, i.e.,  $[\gamma_s(1 - X_i) + \gamma_o(1 - X_j)]$  where  $\gamma_s$  and  $\gamma_o$  represent the maximum enforcement costs for the same village and the other village, respectively.

*O*s contribute zero maintenance labor ( $l_m = 0$ ), and thus do not hold water rights. Nevertheless, they steal as much of other farmers' water as they can within the limits set by the penalties imposed, their relative capacity for competing for water in comparison to others, and the benefits to be gained by using the outside employment options. *O*s steal an amount of water and allocate labor to employment so as to maximize their individual net income. The probability of being caught and punished increases with the frequency of rule enforcers, i.e.,  $(X_i + X_j)/2$ . The penalty varies by situation: it increases with the amount of water stolen ( $q^O$ ), but decreases with water abundance in the system. When water is abundant, rule violations are tolerated because farmers have little incentives to concern themselves with equity issues. This effect is represented by  $\delta[1 - \sigma Q(L_m)/Q(L_m^*)]q^O$ ; where  $\delta$  is the maximum penalty,  $Q(L_m)/Q(L_m^*)$  is the proxy for water abundance, and  $\sigma \leq 1$  is the tolerance factor.

The payoffs of the two strategies in Village 1 are given by the following two equations.

$$\pi_1^G = c_1(l_f^G)^j(q_1^G + r_1)^k + w(l_e^G) - [\gamma_s(1 - X_1) + \gamma_o(1 - X_2)] \quad (12)$$

$$\pi_1^O = c_1(l_f^O)^j(q_1^O + r_1)^k + w(l_e^O) - \delta \left(1 - \sigma \frac{Q(L_m)}{Q(L_m^*)}\right) q_1^O \left(\frac{X_1 + X_2}{2}\right) \quad (13)$$

Likewise, the payoffs of the two strategies in Village 2 are given by the following two equations.

$$\pi_2^G = c_2(l_f^G)^j(q_2^G + r_2)^k + w(l_e^G) - [\gamma_s(1 - X_2) + \gamma_o(1 - X_1)] \quad (14)$$

$$\pi_2^O = c_2(l_f^O)^j(q_2^O + r_2)^k + w(l_e^O) - \delta \left(1 - \sigma \frac{Q(L_m)}{Q(L_m^*)}\right) q_2^O \left(\frac{X_1 + X_2}{2}\right) \quad (15)$$

Here,  $c_i = pb(a_i)^{1-j-k}$ , and  $Q(L_m^*) = I(L_m^*)S$  is the optimal total amount of water desired by the two villages, and  $Q(L_m) = I(L_m)S$  is the total amount of water actually available in the system. Note that  $\pi_i^G$  could be negative if enforcement cost is larger than the combined farming and wage income. In such cases, we impose the condition that  $\pi_i^G = 0$ . Finally, we assume  $\gamma_o > \gamma_s$  because a farmer's cost for monitoring own village should be cheaper than that for monitoring other village.

For the analysis, we used a set of default model parameter values (Table 3). Under this default setting, three basins of attraction exist in the dynamics of  $X_1$  and  $X_2$ : *MOSTLY-Gs*, *ALL-Os*, and *DECOUPLED*.

In the following sub-sections, we specify the decisions of *Gs* and *Os* regarding their labor allocations and the amount of water they appropriate.

### 5.1 Case 1 ( $L_m \geq \psi - \varepsilon$ )

When  $L_m \geq \psi - \varepsilon$ , the optimal maintenance labor ( $L_m^*$ ) is derived by solving the following maximization problem:

$$\max_{L_f, L_m} \Pi = pbL_f^j[I(L_m)S + R]^k A^{1-j-k} + wL_e; \quad \text{subject to: } L_f + L_m + L_e = L.$$

A special case occurs when  $\psi + \varepsilon \leq L_m \leq L$ . In this situation, the optimal maintenance labor becomes  $L_m^* = \psi + \varepsilon$ .

Water is accessed sequentially when upstream-downstream asymmetry exists—farmers in Village 1 access water before those in Village 2. Note that the total amounts of water appropriated by *Gs* and *Os* in Village  $i$  are given by  $Q_i^G = q_i^G(X_i)N_i$  and  $Q_i^O = q_i^O(1 - X_i)N_i$ , respectively. Because of their privileged access, *Os* in Village 1 are less constrained by the higher upper bound on the amount of water they can steal ( $q_1^O \leq Q/N_1$ ). At the same time, *Gs* in Village 1 rely on their upstream position to extract water to bring their actual amount as close as possible to allocated amount, i.e.,  $q_1^G =$

$\min [Q/N^G, (Q - Q_1^O)/N_1^G]$ . It follows, then, that  $q_2^O \leq [Q - Q_1^G - Q_1^O]/N_2$  is the upper bound on the amount of water that  $O_s$  in Village 2 can steal. Finally,  $G_s$  in village 2 obtain the amount  $q_2^G = [Q - Q_1^O - Q_1^G - Q_2^O]/N_2^G$ .

The actions stipulated by the rules and the actual actions of  $G_s$  and  $O_s$  in Villages 1 and 2 (when  $L_m \geq \psi - \varepsilon$ ) are given in the following tables:

Table 1: The decisions of  $G_s$  and  $O_s$  in Village 1 (left) and Village 2 (right) for Case 1

Var.	Rule	Actual	Var.	Rule	Actual
$l_m^G$	$L_m^* (\frac{1}{N})$	$L_m^* (\frac{1}{N})$	$l_m^G$	$L_m^* (\frac{1}{N})$	$L_m^* (\frac{1}{N})$
$q_1^G$	$\frac{Q}{N^G}$	$\min \left[ \frac{Q}{N^G}, \frac{Q - Q_1^O}{N_1^G} \right]$	$q_2^G$	$\frac{Q}{N^G}$	$\frac{Q - Q_1^O - Q_1^G - Q_2^O}{N_2^G}$
$l_f^G$	n/a	$\left[ \frac{j c_1 (q_1^G + r_1)^k}{w^G} \right]^{\frac{1}{1-j}}$	$l_f^G$	n/a	$\left[ \frac{j c_2 (q_2^G + r_2)^k}{w^G} \right]^{\frac{1}{1-j}}$
$l_e^G$	n/a	$l - l_f^G$	$l_e^G$	n/a	$l - l_f^G$
$l_m^O$	$L_m^* (\frac{1}{N})$	0	$l_m^O$	$L_m^* (\frac{1}{N})$	0
$q_1^O$	$\frac{Q}{N^G}$	$\left[ c_1 \left( \frac{k}{\Omega} \right)^{1-j} \left( \frac{j}{w^O} \right)^j \right]^{\frac{1}{1-j-k}} - r_1$	$q_1^O$	$\frac{Q}{N^G}$	$\left[ c_2 \left( \frac{k}{\Omega} \right)^{1-j} \left( \frac{j}{w^O} \right)^j \right]^{\frac{1}{1-j-k}} - r_2$
$l_f^O$	n/a	$\left[ c_1 (j/w^O)^{1-k} (k/\Omega)^k \right]^{\frac{1}{1-j-k}}$	$l_f^O$	n/a	$\left[ c_2 (j/w^O)^{1-k} (k/\Omega)^k \right]^{\frac{1}{1-j-k}}$
$l_e^O$	n/a	$l_e^O = l - l_f^O$	$l_e^O$	n/a	$l_e^O = l - l_f^O$

where  $\Omega = \delta \left( \frac{X_1 + X_2}{2} \right) \left( 1 - \sigma \frac{Q(L_m)}{Q(L_m^*)} \right)$ . Note that the actual values of  $q_1^O$  and  $q_2^O$  should be below the specified limit, i.e.,  $q_1^O \leq Q/N_1$  and  $q_2^O \leq (Q - Q_1^G - Q_2^G)/N_2$ .

The values of  $l_{i,f}^G$  (where  $i = 1, 2$ , that refers to the village  $i$ ) are obtained by the following optimization problem:

$$\begin{aligned} \max_{l_{i,f}^G} : & \left[ c_i \left( l_{i,f}^G \right)^j \left( q_i^G + r_i \right)^k + w \left( l_{i,e}^G \right) \right] \\ \text{subject to } & l_{i,f}^G + l_{i,e}^G = l - l_{i,m}^G \end{aligned}$$

The values of  $l_{1,f}^O, q_1^O$  can also be obtained by the following optimization problem:

$$\begin{aligned} \max_{l_f^O, q_1^O} : & \left[ c_1 \left( l_f^O \right)^j \left( q_1^O + r_1 \right)^k + w l_e^O - \delta \left( 1 - \sigma \frac{Q(L_m)}{Q(L_m^*)} \right) q_1^O \left( \frac{X_1 + X_2}{2} \right) \right] \\ \text{subject to } & l_f^O + l_e^O = l \\ & q_1^O \leq \frac{Q}{N_1} \end{aligned}$$

and that for  $l_{2,f}^O, q_2^O$  from:

$$\begin{aligned} \max_{l_f^O, q_2^O} : & \left[ c_2 \left( l_f^O \right)^j \left( q_2^O + r_2 \right)^k + w l_e^O - \delta \left( 1 - \sigma \frac{Q(L_m)}{Q(L_m^*)} \right) q_2^O \left( \frac{X_1 + X_2}{2} \right) \right] \\ & \text{subject to } l_f^O + l_e^O = l \\ & q_2^O \leq \frac{Q - Q_1^O - Q_1^G}{N_2} \end{aligned}$$

## 5.2 Case 2 ( $0 \leq L_m \leq \psi - \varepsilon$ )

In this region, the maintenance provision labor is  $L_m^* = 0$ ; hence,  $Q = I(L_m^*)S = 0$ . This means that no irrigated water is produced in the system. As a result, water theft and rule enforcement do not exist. In this case, the payoffs of  $G$ s and  $O$ s in Village  $i$  are modified to:  $\pi_i^G = f(l_f^G, q_i^G, a) + w l_e^G$  and  $\pi_i^O = f(l_f^O, q_i^O, a) + w l_e^O$ , respectively.

The values of  $l_{1,f}^G, l_{1,f}^O, l_{2,f}^G, l_{2,f}^O$  can be derived by the following optimization (note that  $q_i^G = q_i^O = 0$  because there is no irrigated water):

$$\max_{l_{i,f}^I} : \left[ c_i \left( l_{i,f}^I \right)^j \left( r_i \right)^k + w^I \left( l_e^I \right) \right], \text{ subject to: } l_{i,f}^I + l_{i,e}^I = l$$

where  $i = 1, 2$  and  $I = G, O$ .

The actions stipulated by the rules and the actual actions of  $G$ s and  $O$ s in Villages 1 and 2 (when  $0 \leq L_m \leq \psi - \varepsilon$ ) are given in the following Tables:

Table 2: The decisions of  $G$ s and  $O$ s in Village 1 (left) and Village 2 (right) for Case 2.

Var.	Rule	Actual	Var.	Rule	Actual
$l_{1,m}^G$	n/a	0	$l_{2,m}^G$	n/a	0
$q_1^G$	n/a	0	$q_2^G$	n/a	0
$l_{1,f}^G$	n/a	$(j c_1 r_1^k / w^G)^{\frac{1}{1-j}}$	$l_{2,f}^G$	n/a	$(j c_2 r_2^k / w^G)^{\frac{1}{1-j}}$
$l_{1,e}^G$	n/a	$l - l_{1,f}^G$	$l_{2,e}^G$	n/a	$l - l_{2,f}^G$
$l_{1,m}^O$	n/a	0	$l_{2,m}^O$	n/a	0
$q_1^O$	n/a	0	$q_2^O$	n/a	0
$l_{1,f}^O$	n/a	$(j c_1 r_1^k / w^O)^{\frac{1}{1-j}}$	$l_{2,f}^O$	n/a	$(j c_2 r_2^k / w^O)^{\frac{1}{1-j}}$
$l_{1,e}^O$	n/a	$l - l_{1,f}^O$	$l_{2,e}^O$	n/a	$l - l_{2,f}^O$

## 6 Supplementary movie legends

### 6.1 Movie 1

**A video clip showing the optimal values of  $L_m$ ,  $L_f$ , and  $\Pi$  for different values of wage.** We ran a model simulation using a set of default model parameter values (Table 3). It illustrates how the optimal of  $L_m$ ,  $L_f$ , and  $\Pi$  change as the wage rate for the outside employment ( $w$ ) is increased from 0 to 1.5. We have  $L_f^* + L_m^* = L$  until  $w \approx 0.4$ . Beyond this point,  $L_f^*$  begins to drop and  $L_e^*$  begins to increase, while  $L_m^*$  remains fixed. From  $w \approx 1.0$ ,  $L_m^*$  begins to decrease as well. Then, when  $w \approx 1.28$ , both  $L_f^*$  and  $L_m^*$  suddenly drop to zero and  $L_e^*$  becomes  $L$ .

### 6.2 Movie 2

**A video clip showing the phase-portraits of  $X_1$  and  $X_2$  for different values of  $\varepsilon$**  We tracked how the dynamics of  $X_1$  and  $X_2$  change as  $\varepsilon$  (a proxy for the threshold behavior of infrastructure) is increased from 0 to 20 (note that the half-saturation point,  $\psi$ , is 20). The default model parameter values used are given in Table 3. When  $\varepsilon \approx 0$ , all phase-portraits converge to  $(0,0)$ —the *ALL-Os* regime. From  $\varepsilon \approx 0.6$ , the *MOSTLY-Gs* regime appears and begins to co-exist. Then, when  $\varepsilon \approx 8$ , the *DECOUPLED* regime emerges. As  $\varepsilon$  is increased further, the *ALL-Os* regime continues to shrink and the regimes of *MOSTLY-Gs* and *DECOUPLED* expand.

### 6.3 Movie 3

**A video clip showing the phase-portraits of  $X_1$  and  $X_2$  for different values of  $\varepsilon$  when wage is higher.** We re-tested the simulation shown in Movie 2 under a higher wage rate for outside employment ( $w = 0.5$ ). All other parameters remain unchanged. The notable difference is that when  $\varepsilon \approx 17$ , the *MOSTLY-Gs* regime disappears. From this point on, only the regimes of *ALL-Os* and *DECOUPLED* exist in the system.



Table 3: Default parameter values

Symbol	Definition	Values(s)
$w$	Wage for outside employment.	0.2
$\psi$	Half-saturation point of $L_m$ yielding $I_{max}/2$ infrastructure efficiency.	0.2
$\varepsilon$	Half-width of the threshold slope for infrastructure provision.	0.125
$I_{max}$	Maximum infrastructure efficiency.	1
$j, k$	Output elasticities of farming labor and irrigated water for agricultural yield, respectively.	$j = 0.3, k = 0.4$
$p$	Price per unit of agricultural yield.	1
$b$	Productivity coefficient for the inputs of production.	1
$a_i$	Acreage of farmer in Village $i$ .	$a_1 = a_2 = 1$
$l$	Available labor per farmer.	1
$N_i$	Number of farmers in Village $i$ .	$N_1 = N_2 = 50$
$R$	Total amount of freely-available alternative water (e.g., rainfall).	0
$r_i$	Amount of alternative water available to a farmer in Village $i$ . (e.g., rainfall).	0
$\gamma_s, \gamma_o$	Maximum enforcement costs for monitoring opportunists in the same village and the other village, respectively.	0.05, 0.1
$\delta$	Maximum penalty cost imposed on opportunists.	1.4
$\sigma$	Tolerance factor for opportunistic behavior shown by group-conformists when water is abundant in the system ( $\leq 1.0$ ).	0.9