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THE GEOGRAPHICAL IMPERATIVES OF THE BALANCE OF POWER IN 3-COUNTRY SYSTEMS

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Abstract

This essay extends a cooperative game-theoretic model of balance of power in anarchic international systems to include considerations of the asymmetry which geography occasions in the offensive and defensive capabilities of countries. The two substantive ideas which concern us are a formalization of the notion of a "balancer" and that of a "central power." What we show is that in stable systems, only specific countries (such as Britain in the 18th and the 19th centuries) can play the role of balancer, and that the strategic imperatives of a central country (e.g., Germany in the period 1871-1945) differ in important ways from those of "peripheral" countries.

The Geographical Imperatives or the Balance of Power in 3-Country Systems

> The geographic location of a state in the world is of basic importance in defining its problems of security. It conditions and influences all other factors ... [and] regional location defines potential enemies and allies and perhaps even the limits of a state's role as a participant in a system of collective security [1943: 22-3]
>
> Nicholas J. Spykman, *The Geography of the Peace*

> If a Soviet strategic planner could be granted one wish, it should be to move his country somewhere else [1987: 277]
>
> Stephen M. Walt, *The Origins of Alliances*

A fundamental difficulty with formulating a comprehensive theory of stability in anarchic international systems -- of systems devoid of exogenously imposed rules and institutions -- lies in the conceptualization of resources and military capability. In his early work on coalitions, the size principle, and the application of cooperative game theory to a formulation of balance of power, Riker [1962], for example, assumes that such systems are constant sum games in which winning coalitions are those which control a majority of resources. Much of the formal theorizing in the balance of power literature follows Riker's lead to the extent that it assumes that resources are additive across the members of a coalition, and that resources to attack one country can be used equally well to defend against an attack, or that those resources can be directed with equal effectiveness at any country (c.f. Zinnes, 1970; Wagner, 1986; Niou and Ordeshook, 1986, 1987). Clearly, though, such assumptions about resources provide, at best, an analysis of special cases. Common sense tells us that, although a land army may be effective for attacking a contiguous adversary, it may be useless against one with even a modest water barrier; and a tenant of today's strategic thinking is that modern technology fundamentally changes the implications of such barriers and distance. In short, "Even in the post-World-War-II technologically advanced environment, geographic distance seems to represent a cost in the movement of troops which all but the most powerful states are unable or unwilling to pay" (Pearson, 1974:455).

This argument's implication is that formulations of balance of power in which resources are treated symmetrically, although valuable for an initial understanding of the problem, may mislead us about the necessary and sufficient conditions for stability, and may be inappropriate for interpreting historical events and processes. Indeed, profoundly important questions cannot be answered adequately unless geography is taken explicitly into account. Should we, for example, attribute Britain's role as "balancer" in the 19th century to the motives and diplomatic skills of key decision makers or to its unique geographical position with respect to the Continental powers? Waltz correctly observes that "The notion of a balancer is more a historical generalization than a theoretical concept" (1979:164), but he goes further to assert that "Balance of power theory cannot incorporate the role of balancer because the playing of the role depends on such narrowly defined and historically unlikely conditions" (1979:164). We should ask whether this view is justified. What precisely are the advantages of geographical distance? For example, should we attribute the unification of China at the end of the Warring States Period in 221 BC at the hands of a geographically "peripheral" as against central power as mere happenstance brought about by the fortuitous combination of military and diplomatic skill, or is it reasonable to hypothesize that such peripheral powers have an advantage over central ones? Can that part of Germany's special, threatening role in European power politics from 1871 to 1945, which are seen as a consequence of its geographically central position, be understood theoretically and generally? What are the implications for international stability of any decline in geography's importance, as occasioned by advances in the technologies of weapons systems? Can the removal natural barriers to warfare disadvantage individual states, but make international systems more stable?

To answer such questions, this essay extends a game theoretic model of the balance of power (Niou and Ordeshook, 1986) to incorporate the asymmetries occasioned by geography in the use of resources. Section 1 of this essay reviews our earlier model, with emphasis on the necessary and sufficient conditions it establishes for the two forms of stability -- system- and resource-stability. The first stability concept concerns the ability of national leaders to secure the sovereignty of their countries or the survival of their regimes. Thus, a system is system-stable if we predict that no country will be eliminated from the game and its resources wholly absorbed by others. The second notion of stability concerns possible changes in the distribution of resources that do not threaten sovereignty or survival. A system is resource-stable if it is system-stable and if we predict that there will be no reallocation of resources whatsoever. Section 2 introduces the notation we require to generalize this model in its treatment of resources for three-country systems. Although we prefer a more general n-country formulation, such an analysis presently lies beyond our grasp; but an analysis of the three-country case does resolve essential conceptual issues. Section 3 offers our main theoretical results, and the necessary and sufficient conditions for both forms of stability which reveal the imperatives of geography. Section 4 interprets our results in terms of technological innovation and in the light of some key historical events, and, by focusing on the key notions of a balancing and a central power, it attempts to answer the questions about international stability which we pose earlier. An appendix gives the proofs of the results we introduce.

1. An Initial Model

Because the processes within even a three-country system are complex, we must proceed with analytic precision, which requires some mathematical notation. First, with respect to n-country systems, we let $S = \{1,2, ..., n\}$ denote the set of all countries, we let $C \subsetneq S$ be a specific coalition of countries, and we let $r = (r_1, r_2, ..., r_n)$ be an n-tuple of resources, where r_i denotes the resources controlled by country i. Naturally, we suppose that $r_i \geq 0$ for all i, we let $R_i = \{ r'_i \mid 0 \leq r'_i \leq r_i\}$, and for convenience we suppose that $r_1 \geq r_2 \geq ... \geq r_n$. Thus, (S,r) denotes an outcome or a state of the world. We also make use of the notation in which $R = \sum_{i \in S} r_i$ denotes the total resources controlled by all countries, and $r(C) = \sum_{i \in S} r_i$ is the total resources controlled by the coalition C.

This notation appears to imply a simple formulation of a cooperative n-person "international relations game" with this characteristic function:

$$v(C) = 0 \quad \text{if } r(C) < R/2 \qquad (1a)$$

$$v(C) = R \quad \text{if } r(C) > R/2 \qquad (1b)$$

$$v(C) = R/2 \text{ if } r(C) = R/2 \qquad (1c)$$

The presumption of this formulation is that winning coalitions can secure any resource redistribution (1b), including those in which minority coalitions are eliminated, whereas minority coalitions can secure nothing (1a). Blocking coalitions -- those with precisely half the resources -- can ensure only the status quo (1c). Of course, the inevitable implication of such a formulation is that unless one country controls all of the resources or unless two countries each control half the resources, the anarchic system which v(C) models is inherently unstable. Because of its constant-sum character, such a game does not possess a core -- outcomes which cannot be displaced by some winning coalition. Instead, cooperative solution hypotheses such as the V-set, the strong bargaining set, and the competitive solution predict resource distributions in which coalitions with a majority of resources eliminate coalitions with a minority of resources (cf. Ordeshook 1986).

Expressions (1a)-(1c), however, misspecify the strategic character of anarchic international systems if we assume that national leaders must also be concerned with the games which ensue after other countries are eliminated. In Kaplan's words (1979), national leaders must be certain that they avoid possibilities such as this one: " ... the weakest player, by joining a nearly predominant strong player, only creates a condition in which he will be the next victim." Similarly, Wagner (1986) notes that "What is important for balance of power theory is whether states will find it in their interests to combine to eliminate other states. By reasoning backwards from endpoints of the (implicit) game tree, one can examine whether such choices are optimal ..." The model which this essay extends, which is itself an adaptation of Wagner's analysis, takes such strategic considerations explicitly into account.

To specify a correspondingly appropriate cooperative game, we require assumptions about the motives of national leaders and about the "rules of the game." Ignoring considerations of geography altogether, we list our earlier model's assumptions so which we can highlight the extension which we offer in the next section. First, we suppose that countries or national leaders must choose one of several actions. Their alternatives are,

a1: negotiate to cede resources to other countries,
a2: aggressively act to secure resources from other countries,
a3: negotiate to secure resources from other countries,
a4: aggressively oppose the actions of other countries,
a5: act neither to secure or to cede resources.

With these alternatives in mind, our previous analysis makes the following assumptions:

A1: R is constant.

A2: R is infinitely divisible and transferable among nations.

A3: Country $i \in S$ is eliminated if $r_i = 0$.

A4: All decision makers have perfect and complete information, and the game's characteristics are common knowledge.

A5: i prefers r to r' if $r_i > r'_i$, provided that, as a direct consequence of r, it is not the case that i's resources can be reduced to zero.

A6: For C and $C' \subseteq S$, C can defeat C' if and only if $r(C) > r(C')$. If C defeats C', r(C') is transferred to C as specified by the members of C so that the resources now controlled by C equal $r(C) + r(C')$.

A7. If a2 and a3 lead to otherwise identical outcomes, a3 is preferred to a2.

A8. If, for disjoint C, C', and C", C attacks C' and C' attacks C", with $r(C) > r(C') > r(C")$, then C absorbs C', leaving C" unaffected.

There are, of course, a great many substantive implications buried in these assumptions which are not necessarily consonant with all of reality. Because we discuss these implications in our earlier essays, however, we comment on only two here. First, our notation supposes that we can unambiguously identify the countries relevant to the analysis. Naturally, this is not always the case since we may be uncertain about which countries are relevant and which countries are mere satellites of another, without true sovereignty in the sense that they cannot express an independent foreign policy. Despite such operationalization problems, we should be prepared to be flexible in empirical applications of our model. We may prefer, for example, to exclude certain countries as a temporary convenience, as when we ignore the role of the superpowers in the relations among smaller countries in studying regional economic matters, or when we exclude smaller countries to study superpower relations. Alternatively, we may decide that the superpowers have no conflicting interests in a situation, or any interest whatsoever, in which case we may choose to apply the analysis only to smaller countries. Or, to understand the constraints imposed by domestic politics, we might prefer to associate the members of S with key domestic political interests within a state. Second, the supposition that a country's resources can be represented by a single number is problematical. Certainly military capability, geographical advantages, population, size and robustness of an economy, and the degree of domestic political stability are all relevant resources; and each of these components even is difficult to measure by a single index (cf. Singer, Bremer, and Stuckey, 1972, as well as Organski and Kugler's argument for the use of GNP, 1980). Elsewhere, we offer a measure of r_i which depends on these specific factors: a country's military age male population, its ability to mobilize that population for conflict, its production of items that are key to any war effort -- coal, iron, and steel -- and its ability to project its military forces offensively and defensively (Niou, Ordeshook, Rose, 1988). And although such variables should correlate with the ones we use, the fact that we do not use economic measures such as indicators of GNP yields a bias for measuring direct military capability rather than potential value to an adversary of successful threats. This bias, in turn, reveals an ambiguity in our analysis. In what follows we implicitly equate military capability and the resources a country might lose to an adversary in a conflict. In reality, however, a country with an inferior economy but with a superior armed force, might overwhelm its adversary and force the transfer of economic resources which greatly exceed the costs of achieving the current disparity in strength. Nevertheless, our argument about the conditions for

stability and instability can be more clearly developed if we assume that the concept of resources is unidimensional and unencumbered by complex substantive qualification.

Even if we accept such qualifications as necessary compromises for the development of a formal analysis, it is important to realize how these assumptions preclude the full consideration of the implications of geography. Suppose, in particular, that countries i and j are separated by a natural obstacle (e.g., the English Channel). Then even if i's resources exceed j's, a system containing only i and j may nevertheless be stable (in violation of A6 and A8) -- neither country may possess the offensive resources to overcome the other's defensive capabilities. Further, neither i nor j may be able to transfer resources to each other without some diminution in the value of the transferred resources (in violation of A1 and A2). It is considerations such as these which our extension in the next section seeks to accommodate.

We should also comment on assumption AS because it incorporates our conceptualization of the sequential game that nations play. Suppose that we are at some initial state of the world (S,r) and that a subsequent state **(S',r')** is being contemplated by the decision makers in S. Moving from (S,r) to (S',r') may involve a voluntary transfer of resources, the formation of certain coalitions, or a war. Thus, the evaluation of **(S',r')** yields an evaluation of the actions leading to it from **(S,r);** and predicting an action necessarily requires that we know how each decision maker evaluates (S',r'). But we must accommodate the fact that (S',r') is not necessarily the "end of the game" -- that other states of the world may follow from (S',r'). Thus, how one evaluates **(S',r')** depends on how one evaluates its consequences. To model this evaluation, we envision the following sequential situation: Beginning with (S,r), nations are free to negotiate, war, transfer resources, make threats, etc. Each transfer of resources, and each war results, according to rules yet to be specified, in a new state (S',r'). If we ignore for the moment the complication that such a process might proceed indefinitely, and suppose instead that decision makers hold finite planning horizons, then, owing to the assumption that everyone shares the same information about the situation and that everyone knows that everyone shares this information, then each decision maker can predict (up to the determinism which game theory admits) the states of the world in the sequence -- including the prediction that certain states lead to its eventual elimination -- and no decision maker has any advantage in making such predictions. It also follows that, from any initial state, each decision maker can predict (again up to the determinism which game theory admits) whether a successive state will lead to its eventual elimination.

The qualification "up to the determinism that game theory admits" accommodates the fact that if, for example, three persons, 1, **2,** and **3,** must divide some sum of money using majority rule, if these persons are identical except for their labels, **and** if all three are concerned solely with the amount that they possess (if they are each unconcerned about the welfare of anyone else), then we can say only that two persons will coalesce to divide the sum among themselves, excluding the third. We cannot say whether this coalition will involve persons 1 and **2,** or 1 and **3,** or **2** and **3.** The likelihood — indeed the certainty -- of this indeterminism means that neither the analyst nor any decision maker can predict with certainty the outcomes that follow from a particular state (S',r'). But the assumption that the properties of the game being played are common knowledge implies that all participants will make the same predictions, even if those predictions merely identify a set of states that a particular initial description makes feasible.

Suppose, then, that a decision maker is evaluating two alternative states

(S',r') and (S",r"). which he can block or bring about if he adopts certain actions or strategies. The indeterminism of which we speak implies that he cannot be certain what states follow from these two, but suppose that (S",r") makes feasible a state of the world in which the decision maker in question is eliminated, whereas (S',r') does not. That is, suppose that if (S",r") prevails and if everyone acts in accordance with the rationality principles yet to be specified, then the decision maker in question cannot preclude the possibility that he is eliminated at some point in the future if not in (S",r") itself — suppose the security level of (S",r"), denoted s(S",r"), is zero -- but if everyone acts rationally with (S',r') as the starting point, then our decision maker knows that if everyone else responds rationally to the actions of everyone else, he can ensure his continued existence -- suppose s(S',r') > 0. Then our first assumption about preferences is that the decision maker prefers (S',r') over (S",r"). Second, if s(S',r') = s(S',r') = 0, then the decision maker is indifferent. Finally, we assume that if neither s(S',r') nor s(S",r") are zero, then the decision maker prefers (S',r') to (S",r") if his resources in **r*** exceed his resources in **r'.** This is the essence of assumption A5.

Thus far, our assumptions, although specifying the rules of a cooperative n-person game, are not sufficient for defining a characteristic function, v(C), to which we might apply some solution hypothesis and render a prediction. In particular, we need to specify each country's security value, given what its leaders believe will be the game that results if certain countries are eliminated. To specify v(C), then, we must also model bargaining and specify the conditions under which countries can ensure their sovereignty. Our approach is to modify the perspectives of a particular solution hypothesis, bargaining set theory (Aumann and Maschler, 1964), so that it fits the problem at hand.

Our modifications incorporate the following: First, unlike solution theory, we are not necessarily identifying a set of "stable" payoffs to the players in the game. Rather, we are ascertaining whether specific countries or coalitions of countries will find it in their interest to upset a particular state of the world, the status quo (S,r). Thus, we are identifying the outcomes which can be reached from a particular starting point. Second, in the context of defining system-stability which arises exogenously, countries are not required to defend what they get in r; rather, they are defending their sovereignty. Thus, to say that (S,r) is system-stable does not require that country i defend its share of resources, r_i; instead, system-stability requires merely that *i* defend some nonzero payoff. It is in the separately considered context of resource-stability which we look at i's ability to defend the particular amount r_i. Third, countries prefer resources that are secured through "negotiation" rather than through conflict (assumption A7). Finally, no country should, if possible, allow another to secure a majority of resources since this implies the elimination of all but the predominant country.

With these considerations in mind, we offer the following notation and definitions: Letting W denote the set of winning coalitions (coalitions which control more than half the total resources), W* denote the set of minimum-winning coalitions, and $E(\mathbf{r}) = \bigcup_{C \in W^*} C$, be the set of essential countries in S, given **r** (a country is essential if it is a member of at least one minimal winning coalition). Further, let $\mathbf{C} = (C, \ldots)$ be a coalition structure which partitions the members of S into exhaustive and disjoint coalitions (the empty coalition, /, is always an element of **C**), and let (r,**C**) be a proposal consisting of a resource distribution and a particular coalition structure, then (ignoring geography) we define,

Threat: (r',**C**') is a threat by C against C' with respect to (S,r), current status quo, if and only if (i) $C, C' \in \mathbf{C}'$; (ii) $r(C) > r(C')$; (iii) $r'_i = 0$ for all $i \in C'$; and (iv) $r'_j > r_j$ for all $j \in C$.

And, in particular, (r',**C**') is a threat against i if $i \in C' \in \mathbf{C}'$. Condition i requires that C and C' both be disjoint coalitions in the coalition structure **C**'. This is only reasonable since if i attacks j, we can hardly say that i and j have coalesced to coordinate their strategies. Condition ii is borrowed from the idea that countries will attack others only if they anticipate being able to win; hence, C's resources must exceed the resources of C'. Condition iii requires that a threat is a proposal to eliminate attacked countries. Finally, condition iv states that the members of C coalesce to attack others only if, individually, each anticipates some gain in terms of increased resources from such an act.

Counter Threat: (r",**C**") is a counter to (r',**C**') by $K \subseteq C' \cap C''$ if and only if: (i) either $C \subseteq C^*$ or $C \cap C'' \neq O$, where $C'', C^* \in \mathbf{C}''$; (ii) (r",**C**") is a threat to C^*; and (iii) r''_i preferred to r'_i for all $i \in C''$

A counter threat by the collection K, then, is, according to conditions i and ii, a proposal in which K is in both C' (the coalition which is being attacked) and C" (the coalition which is formulating the counter) that either threatens all the members of C (the originally threatening coalition) or that coopts one or more members of C. In addition, condition iii requires that all countries in the counter coalition, C", prefer the counter to the original threat.

Viable Counter Threat: The counter threat (r",**C**") is viable for $i \in K$ if and only if there is no $C^o \subseteq C''-\{i\}$ such that C^o has a threat, $(r^o, \mathbf{C}^o)$, against C^* or $C^* + \{i\}$, with r^o_j preferred to r''_j for all $j \in C^o$.

A counter threat is viable for one of the threatened members of C' if and only if i's coalition partners in the counter have some incentive to coalesce with i in the sense that whenever they exclude i (to form C"-{i}), they cannot make a counter threat which they all prefer to the counter which they can make with i. Our final assumption now is the following:

> A9: $i \in S$ will not be eliminated from the game if and only if it has a viable counter threat to every threat.

It follows by definition that (S,r) is system stable if and only if, for all $i \in S$ and for every threat against i, i has a viable counter threat.

Our earlier analysis supposes that national leaders are free to negotiate for the transfer of resources among themselves or to threaten alliances for the forced reallocation of resources. But in taking such actions, each nation must make certain, if possible, that it does not permit a reallocation from the status quo in which it, at some future stage of the process, becomes a victim (as when some other nation secures over half of the available resources). From this perspective, we prove the following in the original presentation of this model with respect to a country's ability to ensure its survival:

Theorem 1: (S,r) is system-stable if and only if S = E(r).

To illustrate, consider the distribution (120,50,50,40,40), in which everyone is essential. For example, regardless of what threats and counters are made, country 4 or 5 can always transfer 30 units of resources to country 1. Our assumptions imply that no nation will secure more than half the resources, and because nations prefer receiving resources "peacefully" rather than "aggressively," an allocation such as (150,50,50,40,10) represents an ideal point for 1. Notice, in particular, then, that the security levels of countries 4 and 5 are not zero (as represented in the usual simple-game characteristic function representation of this

situation), but are 10 instead.

For the special case which is especially germane to this essay, we have the following corollary for three-country systems:

> **Corollary**: If $r_i < R/2$ for all i, then all three-country systems are system-stable.

Briefly, the mechanism of system-stability here is straightforward since, with three countries, if no country is predominant, then every country can form a viable counter threat by offering to transfer enough resources to the largest country (or to the second largest if it is the largest which is being threatened) so as to render that country near-predominant. And with one country near-predominant, then, by assumption A6, no country will threaten and act aggressively towards another. Thus, because any country can be rendered near-predominant in a three-country system by any other country, system-stability is assured.

Theorem 1 and the definitions of threats and viable counter-threats permit us to specify a more appropriate characteristic function for the analysis of resource stability. Briefly, if C is "winning" (if $r(C) > R/2$), and if ϵ is any arbitrarily small positive number, then the characteristic function of a system-stable game is as follows:

$$v(C) = r(C) + (R/2 - \max_{i \in C} [\, r_i \,]) \tag{2a}$$

if $r(S-C) > R/2 - \max_{i \in C} r_i$, otherwise $v(C) = R - \epsilon$; and

$$v(S-C) = r(S-C) - (R/2 - \max_{i \in C} [\, r_i \,]) \tag{2b}$$

if $r(S-C) > R/2 - \max_{i \in C} r_i$, otherwise $v(S-C) = \epsilon$.

Resource-stability, now, means that there exists an allocation of resources which, given the preceding characteristic function, cannot be upset by any country or coalition. In the lexicon of game theory, this means that the game has a core. Our second result establishes that a non-empty core requires a special circumstance:

Theorem 2: The cooperative game defined by expressions (1a) and (1b) has a non-empty core if and only if $r_i = R/2$ for some i.

Hence, resource stability is possible, but only if one country controls precisely half the resources. Otherwise, the game has no core and countries can cycle indefinitely, negotiating and renegotiating agreements (but without threatening the sovereignty of any player).

This model is extended elsewhere (Niou and Ordeshook, 1987) so that it permits the resources of countries to grow at differential rates, and so that national leaders are allowed to invest their resources. Such an extension accommodates the criticisms of balance of power theory which Organski and Kugler [1980], for example, offers, and it permits us to consider some of the imperatives of preventive wars. Although we appreciate that a fully general model should take as many possibilities into account as is analytically feasible, we do not consider these extensions in this essay. Instead, we turn to a consideration of the implications of geography when the total resources in a system are fixed.

2. Incorporating Geography

The essential feature of geography is that resources may be especially advantageous for defense as against offense, and they may be especially advantageous or disadvantageous when used to attack one country as compared to another. Germany's army in 1940 might be well suited for overwhelming France but not Britain even though France certainly had a more powerful army than Britain. But even if ill

suited for attacking across the Channel, that army was sufficient for repelling a counter-invasion by Britain.

To accommodate such facts, let us reinterpret r as a summary of the defensive capabilities of countries, which can be applied offensively against another country only after they are discounted by some parameter. Let $r_i d_{ij}$ denote i's offensive capabilities with respect to j, where $0 \leq d_{ij} \leq 1$. Thus, as a modification of assumption A6, i can defeat j if and only if $r_i d_{ij} > r_j$, and j can defeat i if and only if $r_j d_{ji} > r_i$. Thus, even if resources as represented by the r's are not equal, neither country may be able to defeat the other if the d's are sufficiently small. Notice an immediate important implication of this fact. If all d's are equal to 1 in three-country systems, either one country is predominant (controls more than half the resources) or all two-country coalitions are winning in the sense that they can overwhelm the third. But now the set of two-country **winning coalitions**, W*, is defined by $\{C \mid C \subset S, |C| = 2, \sum_{i \in C} r_i d_{ik} > r_k, k \neq i, j\}$. So if the d's are sufficiently small, then W* is empty, whereas with an appropriate selection of discount factors only one or two coalitions may be winning. Conversely, we can interpret $d_{ij} = d_{ji} = 1$ as meaning that i and j are contiguous, at least from the perspective of the technology of resources. If resources are ICBM's or are readily converted into such weapons, then even if countries are not geographically contiguous, they are contiguous to the extent that a distance of 500 or 1000 miles is of little consequence. If all d's are 1, then our analysis should reduce to the model we offer in the previous section.

At this point we make a simplifying assumption which reduces the complexity of the conditions for stability that we offer later, but which does not impose any new conceptual constraint. Briefly, any asymmetry in the d's of the form $d_{ij} > d_{ji}$ means that i has a technological advantage over j -- that i's resources are more effective for attacking j than are j's resources for attacking i. If no such asymmetric superiority exists, then $d_{ij} = d_{ji}$ for all i and j in S. We make the assumption that, in this sense, the d's are symmetric (keeping in mind that our assumption implies nothing about the relation between d_{ij} and d_{ik} or between d_{ij} and d_{jk}).

To see how our earlier analysis is altered with this reformulation of resources, consider the three-country system with the initial resource distribution (140,120,40). If all d's equal 1, this system is system- but not resource-stable. Suppose, however, that $d_{12} = 1$, $d_{13} = d_{23} = .2$. Then 2 cannot absorb 3, nor can 1 absorb 3. But 1 can attack 2 without 3 being able to assist 2 in a viable counter -- 3 cannot transfer sufficient resources to 2 so that 2 can defend against 1 nor can 3 divert sufficient resources from 1 by attacking it. Thus, 1 absorbs 2, and with 260 units of resources, it subsequently absorbs 3. Hence, even though it initially controls less than half the resources, country 1 is in fact predominant. Alternatively, (160,80,60) is wholly stable if $d_{12} = d_{13} = 2/3$ and $d_{23} = .7$. To see this, notice that if 1 threatens to absorb 2 and become predominant, then 3 can joint 2 in a viable counter by attacking 1. If 3 attacks 1, 1 must hold $d_{31}r_3 = 40$ resources in reserve to counter 3's attack, leaving it with insufficient resources to threaten 2. And because 2 cannot threaten 3, (160,80,60) is resource- and thus system-stable. **So with geography taken into account, countries can become predominant even if they do not control a majority of resources, and countries with a majority of resources need not be predominant.**

To proceed further requires one additional assumption about how a country can uses resources secured from other countries. In our earlier model, we assume that if i defeats j and absorbs all of j's resources, then i can target $r_i + r_j$ resources at k. But now we must decide whether i can target $r_id_{ik} + r_jd_{jk}$ resources or $(r_i + r_j)d_{ik}$ resources at k.

This is equivalent to asking whether i can absorb j's resources into its own territory (in much the same way as Russia absorbed Germany's captured industrial plant after World War II) or whether it is merely the sovereignty of those resources which are transferred In the same way as the United States established military bases in Japan and Germany after that same war). If resources refer to territory, then the first assumption is appropriate, whereas if resources are physically transportable, then we should impose the second assumption. Thus, both assumptions are plausible, and we would prefer not having to choose between them. Indeed, a fully general model would give each country the choice. But the second assumption yields the simpler analysis (although it is merely algebraic and not conceptual complexity which distinguishes between them). Because this is our initial foray into accommodating considerations of geography, we abide by it.

With this ambiguity resolved (by assumption) we turn to the issue of when a country can become predominant -- can threaten the sovereignty of others without confronting any viable counter threats. Without considering geography, the corollary to Theorem 1 tells us that in three-country systems, any attack by one country on a smaller one threatens the third (unless the attacking country already controls over half the resources in the system), and, correspondingly, any attack by a country on another can be countered by a viable counter, in which case all three-country systems are system-stable. But with geographical considerations, a country, say i, can become predominant if, first, upon the absorption of, say j's resources, overcome k. Second, if k cannot effectively assist j by transferring resources to j. And third, if k cannot assist j by attacking i directly (thereby causing i to divert some share defending against k while attacking j). Formally, this yields the following revised definition of predominant:

Country i is **predominant** if and only if neither j nor k, acting alone or in concert, can formulate a viable counter threat to a threat by i that renders i predominant -- if and only if there exists a $j \in S-\{i\}$ such that: (1) $(r_i + r_j)d_{ik} > r_k$; (2) $r_i d_{ij} > r_k d_{kj} + r_j$; and (3) $r_i d_{ij} > r_k d_{ki} + r_j$.

Correspondingly, i is **near-predominant** if with r_i, i is not predominant, whereas i is rendered predominant if its resources are increased to $r_i + \epsilon$, where ϵ is any number greater than zero (in our earlier model, i is near predominant if $r_i = R/2$).

To generalize the model from Section 1, it is useful first to verify that a two-country (bipolar) system can be system- and resource-stable. In our earlier model, bipolar systems are stable if and only if both countries control an equal share of resources. Admittedly, such a "knife-edged" stability condition is a byproduct of our mathematics and of our failure to consider the uncertainty which confronts real decision makers in their assessments of resources. We should make certain, however, that incorporating geography does not destroy so fragile a condition, and the following remark covers this case:

Remark 1: In two-country systems either one country is predominant or the system is both system- and resource-stable.

To prove this remark, first suppose that country i has no threat against j (i cannot defeat j), in which case we must have $r_i d_{ij} \leq r_j = R - r_i$, or equivalently, $r_i \leq R/(1 + d_{ij})$. Second, if j cannot defeat i, then $r_j d_{ji} = (R - r_i)d_{ji} < r_i$, or equivalently, $r_i \geq Rd_{ji}/(1 + d_{ji})$. Putting the two equalities together, then, we have $R/(1 + d_{ij}) \geq r_i \geq Rd_{ji}/(1 + d_{ji})$.

To see how this result relates to our earlier analysis, notice that if $d_{ij} = d_{ji} = 1$, then the inequality becomes $1/2 \geq r_i/R \geq 1/2$, or simply $r_i = R/2$. More generally, the inequalities in Remark 6.1 can be satisfied by an appropriate choice of r_i only if the first term is at least as great as the third; otherwise we have a contradiction and system stability is impossible. Some simple algebraic manipulation shows, however, that $R/(1 + d_{ij}) \geq Rd_{ji}/(1 + d_{ji})$ implies $1 \geq d_{ji}d_{ij}$, which is an inequality that is necessarily satisfied, given the constraints on the d's that they not exceed 1. Thus, system-stability is possible in bipolar systems (as is resource-stability since both forms of stability are equivalent in bipolar systems). **And not only is system stability possible in bipolar systems after we take geography into consideration, but Remark 6.1 establishes that stability no longer requires a "knife-edged" equality of resources.** For example, even if country 1 has twice the resources of country 2, the system ({1,2},(200,100)) is stable so long as $d_{12} < .5$.

We might be led to infer from this discussion of the bipolar case that the discounting of offensive resources occasioned by geography make both system- and resource-stability more likely -- that the constraints on the r's required to ensure a country's sovereignty are weakened. But this implication is an illusion. To see this we proceed by proving two lemmas. Our first lemma concerns only the issue of technological possibilities in the context of the viable counter-threats that countries can make, and it does not constitute a prediction about final outcomes. Letting $S = \{i,j,k\}$, and letting $C = \{i,j\}$ be any two-country coalition in W^*, we define the function $g_i(C)$ to equal the maximum amount of resources which i can gain either from k alone or from k and j, such that i is not rendered predominant (the proof of this lemma and other formal results are presented in the appendix).

Lemma 1: For $\{i,j\} \in W^*$, $g_i(\{i,j\}) = (r_j + r_k - r_i d_{ij})/(1 + d_{ij})$

$$= R/(1 + d_{ij}) - r_i$$

To see that lemma 1 is consistent with our earlier model, let the d's equal 1, in which case $g_i(\{i,j\}) = (r_j + rk - r_i)/2$. So, for example, if $r = (120,100,80)$, then $g_1(\{1,2\}) = g_1(\{1,3\}) = 30$, which is to say that country 1 can expect to win at most 30 units of resources, since 2 and 3 can block 1 from becoming winning more and becoming predominant. Similarly, $g_2(\{1,2\}) = g_2(\{2,3\}) = 50$ and $g_3(\{1,3\}) = g_3(\{2,3\}) = 70$. (Notice that Lemma 1 does not depend on the assumption that $d_{ij} = d_{ji}$ for all i and j.) This lemma, however, is not sufficient to establish necessary and sufficient conditions for system- and resource-stability in general, but with it we can establish the following result:

> **Remark 2**: If there are only two winning coalitions, say $\{i,j\}$ and $\{i,k\}$ in a three-country system, if $d_{ij} > d_{ik}$, and if i is not predominant when it absorbs either j or k, then the system cannot be system-stable. In particular, a stable two-country system will emerge without j.

Suppose i is not predominant over j even if i controls $r_i + r_k$ resources and that it is not predominant over k even if i controls $r_i + r_j$ resources. Notice from Lemma 1 and the definition of g_i that i gains something from each winning coalition. Since i is the sole pivot, it follows that there cannot be any counter to a threat by $\{i,j\}$ to eliminate k or by $\{i,k\}$ to eliminate j. It also follows from Lemma 1 that $g_i(\{i,j\}) > g_i(\{i,k\})$ if and only if $d_{ik} > d_{ij}$, so, in particular, i prefers a coalition with k if and only if $d_{ik} < d_{ij}$. Thus, in this instance, the core of the corresponding cooperative game is an outcome in which j is eliminated.

This remark, in effect, states that if one country, i, holds a superior position over the other two in the sense that those two cannot coalesce to defeat i, then i will either become near-predominant or it will join in a coalition with the

country which is the more difficult for it to overcome which eliminates the remaining country. To illustrate this remark with a numerical example, suppose $r = (140,90,70)$, and, to simplify matters, let $d_{ij} = d_{ji}$ for all i and j (so no country possesses a technological superiority over another in its ability to translate a unit of resource into an offensive capability). In particular, let $d_{12} = .5$, $d_{13} = .25$, and $d_{23} = .5$. Hence, $\{1,2\}$ and $\{1,3\}$ are winning, but $\{2,3\}$ is not winning. From Lemma 1, $g_1(\{1,2\}) = 60$, whereas $g_1(\{1,3\}) = 100$. Thus, 1 prefers a coalition with 3, and the final outcome is (240,0,60) -- 2 is eliminated.

The preceding remark deals with a special case; moreover neither it nor Lemma 1 informs us as to the circumstances under which a country can become near-predominant. Without considering geography, we know that a country is near predominant if and only if it controls precisely half the resources. Our next lemma establishes necessary and sufficient conditions for a country to be near-predominant when the d's in our model are taken into account:

> **Lemma 2**: For any $i \in S$, i is near-predominant if and only if (1) $r_i d_{ij} = r_i d_{ik} = r_j + r_k$; and (2) either $d_{kj} = 1$ or $d_{ki} = d_{ij} = 1$.

In our earlier model, countries can ensure their existence and, hence, the stability of systems by forming viable counter-threats in which one country is rendered near predominant. If, for example, the initial resource distribution is (120,100,80), then, because no country has any incentive to let another become predominant, countries will act to block any threat which gives anyone more than half the resources. Indeed, country 3 can counter any threat by transferring 30 units of resources to 1, which gives 1 its most preferred feasible alternative -- half the resources without war. With the new distribution (150,100,50), 1 cannot threaten 2 or 3 since 2 and 3 would coalesce to forestall 1 from becoming predominant. And 2 cannot threaten 3, since this only leaves open the door for 1 to attack 2 or 3 and become predominant (assumption A6). Thus, (120,100,80) is system-stable because one country can be rendered near-predominant, whereas (150,100,50) is both resource- and system-stable because one country is near- predominant. Lemma 2 reveals the circumstances under which a country is near- predominant or can be rendered near predominant when geography is taken into consideration. Thus it is an important step towards establishing conditions for system- and resource-stability.

The conditions of the lemma are also substantively important. If we suppose that $d_{kj} = 1$ means that k and j are contiguous (and therefore that $d_{jk} = 1$) then a country is near-predominant only if the other two countries in the system are contiguous ($d_{kj} = 1$) or if i is contiguous to both other countries ($d_{ki} = d_{ij} = 1$). To the extent, then, that system- and resource-stability depend on the ability of countries to form viable counter-threats by rendering another country near-predominant, the stability of systems depends directly on geography or on the technology of resources.

3. System and Resource Stability

We can now establish necessary and sufficient conditions for both types of stability. Keep in mind that without geographical considerations, all three-country systems are system stable (provided that no country controls more than half the resources). This stability is assured because every country necessarily controls enough resources to render someone near-predominant. Our lemmas permit us to establish a necessary and sufficient condition for system-stability when the d's are less than 1, but now this condition gains in complexity. But, following the same logic as Theorem 1, those conditions reduce to the following: either geography renders the three countries irrelevant to each other's security, or one country is near-predominant, or each country can, as a viable counter-threat, render another

near-predominant by the transfer of some appropriate sum of resources.

Theorem 3: A three-country system is system-stable if and only if (1) $W^* = \emptyset$; (2) for some $i \in S$, i is near-predominant; or (3) for every $i \in S$ such that $\{j,k\} \in W^*$, there exists a $C = \{i,j\} \in W^*$ such that (a) $d_{ij} < d_{jk}$, or (b) $(r_i + r_j)d_{jk} > r_k$ and either $d_{ij} = d_{jk} = 1$ or $d_{ik} = 1$

With Theorem 3, as well as the definitions of near-predominant and predominant in mind, we can now redefine the characteristic function for the cooperative 3-person game among countries thus: Assuming that all coalitions denoted by C contain two members, then $v(S) = R$ and,

$$v(i,j) = r_i + r_j \text{ and } v(k) = r_k \text{ if } W^* \text{ is empty or if i or j are near-predominant;} \quad (3a)$$

Clearly, no threats are possible if W^* is empty, so trivially, there is a valid counter to every threat, in which no country needs to cede or otherwise transfer resources to another. Similarly, if some i is near-predominant, then as in our earlier model, j and k cannot threaten each other for fear that i can become predominant (assumption A6), nor will i join in any coalition to threaten a third country since it knows that it will not be allowed to gain any resources in that coalition. Next,

$$v(i,j) = r_i + r_j + \delta \text{ and } v(k) = r_k - \delta \text{ if } \{i,j\} \in W^* \text{ and if either i or j is near-predominant if i or j controls } \delta \text{ additional resources, } \delta < r_k; \quad (3b)$$

If j is not near-predominant, but if it can join with k to eliminate i (if $\{j,k\} \in W^*$), then by Theorem 3, i can ensure that it loses no more than δ by ceding δ to j.

$v(C) = R$ and $v(i, i \in C) = 0$ otherwise. (3c)

Having thus defined the characteristic function for a constant sum 3-person game, and assuming, as before, that a country is resource-stable if and only if that 3-person game has a core, then,

> **Theorem 4**: A three-country system is resource-stable if and only if W^* is empty or if there is an $i \in S$ such that i is near-predominant.

Thus, a three-country system is resource stable if and only if the d's are sufficiently small so as to render each country essentially irrelevant to the others, or if the conditions of Lemma 2 are satisfied -- which excludes systems of three noncontiguous but nevertheless jointly relevant countries.

4. Implications: Balancers and Central Powers

Perhaps the most important implication we can draw from our analysis is that once we take geography into account, not all three-country systems are necessarily system-stable. Theorem 3 leaves open the possibility that whenever all d's are not equal to 1 -- whenever geography matters -- system-stability depends on the specific character of geographical dissimilarities. Indeed, that theorem tells us that **whenever there is discounting between all pairs of countries owing to geography, then as long as that discounting is not so great as to render each country a separate "system," system-stability among three countries is impossible.**

With this in mind, it is reasonable to speculate that World War II ended the applicability of the logic of the diplomatic imperatives which dictated great power international politics in the 18th and 19th centuries. The histories of those centuries, as well as the first half of ours, are commonly described as a constant process of negotiation and balancing, in which stability, if it existed at all,

existed solely because of the constant efforts and diplomatic skills of key decision makers. To the extent that negotiation and balancing, as well as the striving for advantage, were conscious objectives, we can surmise that participants perceived no natural stability to their systems. If there was a natural stability, then certainly it should not have been so difficult and skillful to achieve, nor so fragile that it lead to major wars. But modern technology should alter strategic considerations. We might reasonably conjecture that the threat of Russian dominance on the continent serves as the chief unifying factor -- that this threat submerges destabilizing competition. However, we can also speculate that to the extent that Britain, France, and Germany (West, East, or both) are now rendered contiguous by technology, system stability is assured with or without a Soviet threat.

With our attention focused on Europe, it is interesting to see also how our analysis explains why Britain in particular is credited with playing the role of balancer in earlier centuries. It is tempting to attribute this role to the goals of its leaders and its business elites. But if that is our explanation, then we must also explain those goals. Short of pursuing a complete theory of history, we note simply that Britain alone is separated from the continent by any meaningful natural barrier, whereas the remaining key actors at the several important historical periods are essentially contiguous. (Napoleon's drive on Russia was not with a contiguous adversary, as his defeat reveals, but it is only when he challenged Britain's security by challenging contiguous adversaries did Britain become a central actor). So if we look at Lemma 2, we see that only Britain can be a near-predominant country -- explicitly or implicitly ceding resources to any other country cannot ensure stability (Russia, which we might also conceptualize as being non-contiguous was simply too economically backward and too weak to play Britain's role of becoming near-predominant). This is not to say that other countries cannot become predominant through technology or by the miscalculation of others. However, Britain's special geography rendered it a particularly distinctive coalition partner: it alone could be rendered near-predominant (which in reality, owing to uncertainly and the like, is a less precise concept which our theory presumes) without fear that miscalculation could render it predominant.

This discussion, though, does not directly contradict Waltz's assertion, noted earlier, that the role of a balancer cannot be formally incorporated into the analysis. To see, however, that this assertion is incorrect consider the following definition of a balancer which incorporates two ideas: first, the country in question, i, must be capable of determining which of the two coalitions of which it can be a member is winning (it must be able to ensure the defeat of either potential aggressor); second, the system must be system-stable so as to preclude the possibility that i is not merely a potentially predominant country; and, finally, the system should not to be resource-stable in order to assure that there are some incentives for the formation of coalitions.

> Country i is a **balancer** in (S={i,j,k},r) if and only if {i,j} and {i,k} are winning and (S,r) is system- but not resource-stable.

Without geography as a consideration, of course, every country in a system-stable three-country system is a balancer. With the British case in the 18th and 19th centuries in mind as the phenomenon to be explained, what we are particularly interested in, though, are the circumstances, owing to geography, which place one country uniquely in this role. The following theorem, which follows largely as a corollary to the conditions for system stability which Theorem 3 establishes (see the appendix), tells us what we want to know:

> **Theorem 5**: Country i in S = {i,j,k} is the unique balancer in (S,r) if and only if (1) $d_{jk} = 1$ (countries j and k are contiguous); (2) $d_{ij} = d_{ik} = d < 1$ (country i is not contiguous to either j or k); (3) $d(r_j + r_k) < r_i$ (the coalition {j,k} is losing and cannot defeat i); and (4) $d(r_i + r_j) \geq r_k$ and $d(r_i + r_k) \geq r_j$ (coalitions {i,j} and {i,k} are winning).

The implications of this theorem are profound, because it reveals that Britain's role was not the mere accident of skillful diplomacy or the realization of a particular diplomatic stance which other countries could have adopted as well. Instead, Theorem 5 suggests that Britain was uniquely positioned to play the role of balancer, and thus, to contradict Waltz, it provides a theoretical explanation for that role (we say "suggests" since our analysis presumes three countries rather than the six or so major powers that played on the stage of 19th Century European politics). This theorem also permits us to respond to the speculation that Britain played its role merely because it was "a status quo power" or that a tradition begun by Cardinal Wolsey's policies towards Bourbon and Habsburg monarchies were mere accidents of leadership. Rather, Theorem 5 turns the "causal arrow" around and suggests that it was such a power and that it enjoyed such leadership as a result of its unique natural position. (We note parenthetically that the rise of Germany's navy and Britain's involvement in a continental war destroyed this position, but it established a new peripheral power -- the United States, which played its role in two successive world wars.)

Waltz (1979:164) argues that Britain's role of balancer requires meeting three conditions: "The first of these was that the margin of power on the side of the aggressor not be so large that British strength added to the weaker side would be insufficient to redress the balance ... the second condition was that Britain's ends on the continent remain negative, for positive ends help to determine alignments ...

Finally ... Britain required a status in power at least equal to that of the mightiest." Waltz's first condition corresponds to (4) in the theorem; the meaning of his second condition is unclear, except perhaps to require geographical remoteness -- condition (2); but Waltz's last condition is clearly incorrect. Morgenthau, however, comes closer to tapping the intuition behind Theorem 5: "Britain was capable of making its beneficial contributions to peace and stability only because it was geographically remote from the centers of friction and conflict, because it had no vital interests in the stakes of these conflicts as such, because it had the opportunity of satisfying its aspirations for power in areas beyond the seas which generally were beyond the reach of the main contenders of power" (1966:340) Although only the first of Morgenthau's three conditions concern Theorem 5 (his last two certainly contributed to British motives), his observations about European politics in general reach the other conditions of the theorem: "Both naval supremacy and virtual immunity from foreign attack for more than three centuries enabled great Britain to perform this function [condition (3)] ... Air power has ... put an end to [this] invulnerability ... " (p. 337). "France under Louis XIV and Italy in the decade before the First World War attempted to play this role ... but France was too deeply involved ... lacking in commanding superiority [in violation of conditions (2) and (4)]. Italy on the other hand had not enough weight to throw around to give it the key position in the balance of power [in violation of condition (4)]" (p. 188).

To see further the advantages which a peripheral, balancing power enjoys, suppose $r = (150,75,75)$, let $d_{ij} = d_{ji}$ for all i and j, and in particular let $d_{12} = d_{13} = .5$, and $d_{23} = 1$. In this instance, 1 and 2 could threaten to divide 3's resources evenly, and 3's only viable counter is to transfer 50 units to 1 so as to make 1 near-predominant. Neither 2 nor 3, alone or together, however, can threaten

1. And if either coalesces with 1 to attack the third, the third should immediately cede 50 units to 1 since any gain by 2 renders 3 inessential, and vise versa. In this instance, then, the cooperative game has a core which yields the outcome (200,25,75) or (200,75,25). Thus, not only can geographically distinct countries such as Britain enjoy a greater level of resources without threatening the stability of systems, they are also the countries to which resources will be ceded when another must ensure its survival.

The position Britain enjoyed as a balancer, however, is not the sole asymmetric possibility which geography occasions. Consider the following observation by McNeill (1982:148):

> States located towards the margins of the European world -- Great Britain and Russia in particular -- were able to increase their control of resources more rapidly than was possible in the more crowded center. The rise of such march states to dominance over older and smaller polities located near the center where important innovation first concentrated is one of the oldest and best-attested patterns of civilized history.... (such) states conquered older, smaller polities at least three times in the ancient Near East: Akkad (ca. 2350 B.C.); Assyria (ca. 1000-612 B.C.); and Persia (550-331 B.C.). Mediterranean history offers a similar array of instances: the rise of Macedon (338 B.C.) and then of Rome (168 B.C.) in classical times followed in modern times by the Spanish Dominion over Italy (by 1557).... Ancient China (rise of Ch'in 221 B.C.) and ancient India (rise of Magadha, ca. 321 B.C.) as well as Amerindian Mexico (Aztecs) and Peru (Incas) all seem to exhibit a parallel pattern.

To see how this recurring historical pattern finds reflection in our analysis consider a system in which $d_{12} = d_{13} = 1$ but $d_{23} < 1$. Thus, country 1 is central -- it is contiguous to two non-contiguous countries, and again, from Lemma 2, it is the sole country which can enjoy a position of being near-predominant. But with Theorem 3 in mind, suppose $r = (100,100,100)$ and $d_{23} = .5$. Although {1,2} can threaten 3 (or, equivalently, {1,3} can threaten 2), 3 can form a viable counter by ceding 50 units of resources to 1 so as to make 1 nearly-predominant. But if 2 and 3 coalesce to attack 1 with the understanding that they will divide 1's resources evenly, I cannot form a viable counter: if I propose a coalition with 2 (or 3) which offers 2 more than 150 units of resources at the expense of 3, then 1 has failed to increase its security value since it can now be defeated by 2 alone, and 3 does not have sufficient resources to form a viable counter with 1 against 2 -- 2 is predominant. Nor can I propose to merely transfer 50 units of resources to 2, since such a transfer cannot make 2 near-predominant so as to render the system resource-stable -- such a transfer merely weakens 1 further.

In this example, then, the a central power must control more resources than either adversary in order to secure its survival. Remark 3, however, which follows from some simple algebraic manipulations on condition 3 of Theorem 3, shows that for the special case in which both such adversaries' (both "peripheral powers") resources are equal, the resources which the central power must control to ensure system stability depend on d, the discount between the two non-central powers.

> **Remark 3**: If $r_j = r_k = r$ and $d_{ij} = d_{ik} = 1$, then $(\{i,j,k\},r)$ is system stable if and only if $2r \geq r_i > r/d_{jk} - r$.

If, as in our example, $r = 100$ and $d = .5$, then r_i must exceed 100; if d is less than .5, then r_i must exceed something greater than 100; and if d is greater than .5, then r_i must exceed something less than 100. Thus, the greater the remoteness of the two peripheral powers, the greater are the resources which the central power must control to ensure that it is essential. If the resources of each peripheral power are nearly irrelevant to the other (if $d << 1$), then a central must do more than match the resources of its potential adversaries; otherwise that power is inessential and its two adversaries can coalesce to eliminate it from the game. The empirical manifestation of this example, then, could easily be the Bismark's Germany and the concern it felt about France and Russia. To the extent that we can

suppose that the resource discount between Russia and France was substantial (although not zero, as the Napoleonic invasion of Russia established), Germany was essential on the continent only if it controlled more resources than either Russia or France. This, in part, accounts for the necessity of an Austro-German alliance and for Germany's perceived need for military superiority. Within this framework, German interests were best served by a neutral Britain, or at least a Britain that did not side with a Franco-Russian alliance. Unfortunately, securing continental near-predominance endangers Britain's role as balancer: with Theorem 5 in mind, that role existed only insofar as a German continental hegemony did not threaten Britain directly. Thus, Germany was presented with the difficult task of forming alliances and increasing its resources so as not to undermine Britain's natural position and strategy. Whether this task was impossible or merely to difficult for the likes of a Tirpitz we cannot say. We suspect but cannot prove that the military armament policies which Germany pursued after Bismark were in fact a colossal blunder, and that, because Britain could no more tolerate a French or Russian hegemony that a German one, the extension of Bismark's policies to the 20th century would have done as much to ensured the survival of the German state. In any event, the outbreak of war in 1914 reveals that the task was not accomplished.

Our analysis does not apply, of course, if there are more than three essential actors, and thus we must leave it as a conjecture that the advantage of peripheral powers does not disappear as the number of countries increases. We suspect that Remark 3 (as well as Theorem 5) will generalize, but before we go too far in interpreting the historical events which McNeill summarizes as support for our analysis, we ought to keep in mind that just as there are a great many peripheral powers which supplant central ones, so there must be peripheral powers which have failed to do so. This too is consistent with our model in that, if a central power anticipates events, then even if it is the power with the fewest resources, it should be able to meet threats with viable counter threats. Thus, if our theory is essentially correct, the eventual dominations which history records must follow from miscalculation, and we suspect that our analysis shows that miscalculation is more disadvantageous to a central power. Our example shows that a central power can, by skillful formation of alliances, maintain system-stability with fewer resources than peripheral powers. But if it fails to act appropriately by misjudging its adversaries' resources, then its resource inferiority dooms it to defeat. On the other hand, if it miscalculates by failing to heed the imperatives of balance of power as uncovered by our analysis and tries to ensure against such a possibility by seeking resource parity with its neighbors, then those neighbors should view events as a threat to stability and to their sovereignty. In this case we can anticipate a preventive war, which the central power loses.

Appendix

Proof of Lemma 1: Since $\{i,j\} \in W^*$, i and j together can defeat k. But before i and j divide r_k, j must make certain the i does not control enough resources to defeat j subsequently. If the addition of r_k renders i predominant, suppose that i and j divide r_k α and $r_k - \alpha$ respectively. Then it must be the case that $(r_i + \alpha)d_{ij} \leq r_j + r_k - \alpha$. This implies that $\alpha \leq (r_j + r_k - r_i d_{ij})/(1 + d_{ij})$. So α is maximized when the statement of the lemma is true. The second inequality is established by setting $r_j + r_k = R - r_i$ and applying appropriate algebraic manipulations. QED.

Proof of Lemma 2: If we add $\epsilon > 0$ to r_i so that $(r_i + \epsilon)d_{ij} > r_j + r_k$, for all $j \in S\text{-}\{j\}$ and $k \in S\text{-}\{i,j\}$, then, by definition i is predominant. Thus, to prove sufficiency we must show that if the conditions of the lemma are satisfied, i is not predominant without the addition of ϵ -- that every threat by i can be countered with a viable counter threat -- whereas i becomes predominant with ϵ. Notice first that if $r_i d_{ij} = r_j + r_k$, then by lemma 1, $g_i(C) = 0$ for all C in W^*, $i \in C$. Thus i cannot gain resources by coalescing with either j or k in a threat against the third country, so i cannot be predominant. However, if we add ϵ resources to i, then $(r_i + \epsilon)d_{ij} > r_j + r_k$ and i is predominant. Second, if i threatens j and k together, let r^o_i be the resources i allocates against j, in which case $r_i d_{ij} = (r_i - r^o_i)d_{ij} + r^o_i d_{ij} = r_i d_{ij}$. But if $r_i d_{ij} = r_j + r_k$, for all $j \in S\text{-}\{i\}$ and $k \in S\text{-}\{i,j\}$, then $d_{ij} = d_{ik}$, in which case we can rewrite the previous equality as $(r_i - r^o_i)d_{ik} + r^o_i d_{ij} = r_i d_{ij} = r_j + r_k$. So i cannot overcome j and k simultaneously, nor can it overcome one without rendering itself vulnerable to the other. Thus, i is not predominant. But if we add ϵ resources to i, the previous equality becomes ">", and i becomes predominant, which means that it is near predominant without ϵ. Finally, suppose i threatens j alone. If $d_{kj} = 1$, then $r_j +$

$r_k d_{ki} = r_j + r_k = r_i d_{ij}$, k can join j in a counter by attacking i, and, thus, i is not predominant. But if ϵ is added to i's resources, the last equality becomes "<" and k cannot join j in a viable counter by attacking i. Nor can k form a counter by transferring its resources to j. Such a transfer yields a viable counter only if $r_j + r_k \geq r_i d_{ij} + \epsilon d_{ij}$. Since $\epsilon > 0$, this requires that $r_j + r_k > r_i d_{ij}$, which violates the assumption of the lemma. So again, i is near predominant. Suppose, on the other hand, that $d_{ki} = d_{ij} = 1$. Then if i threatens j, k can join j in a viable counter by attacking i. Country i must target more than r_j of its resources at j and at least r_k at k. If $r_i = r_j + r_k$, this is impossible, whereas if ϵ is added to i's resources, i has sufficient resources to defeat j and k. To prove necessity, we already know by definition that if $r_i d_{ij} > r_j + r_k$, then i is predominant; so suppose that "<" holds. Buy lemma i, $g_i(C) > 0$ for some C in W*, $i \in C$. But i cannot gain enough to eliminate its coalition partner subsequently since such a coalition agreement violates our model's rationality assumptions. Suppose, then that "=" holds. We are left with two cases. First, if $d_{kj} < 1$ and $d_{ki} < 1$ for some $j \in S-\{i\}$, then $r_i d_{ij} > r_j + r_k d_{kj}$ and $r_i d_{ij} > r_j + r_k d_{ki} d_{ij}$, in which case, by definition, i is predominant. Second, if $d_{kj} < 1$ and $d_{ij} < 1$ for some $j \in S-\{i\}$, then again $r_i d_{ij} > r_j + r_k d_{kj}$ and $r_i d_{ij} > r_j + r_k d_{ki} d_{ij}$. QED.

Proof of Theorem 3: It is evident that conditions (1) and (2) are sufficient. To show that (3a) is sufficient, suppose (r',{j,k},{i}) is a threat against i. From the definition of a threat, this requires that $\{j,k\} \in W^*$, $r(\{j,k\}) > r_i$, $r'_i = 0$, and $r'_j > r_j$ and $r'_k > r_k$. But if, as (3a) assumes, there exists a C" $\in$ W*, say {i,j}, such that $d_{ij} < d_{jk}$, then (r",{i,j},{k}) is a counter-threat to (r',{j,k},{i}) by i since C'∩C" = {j}, (r",{i,j},{k}) is a threat to k, and by Lemma 1, $g_j(\{i,j\}) > g_j(\{j,k\})$, so r" is preferred to r' by i and j. That (r",{i,j},{k}) is viable follows from the fact that if j, by attacking k alone, gains more than

$g_j(\{i,j\})$, then from Lemma 1 (see proof), j becomes predominant. So i will act to prevent j from gaining more than $g_j(\{i,j\})$, which is to say that j alone cannot do better than with the counter-threat. To establish the sufficiency of (3b), notice that if i, when threatened by (r',{j,k},{i}), is in some winning coalition, say (i,j) such that the conditions of (3b) are satisfied, then i can transfer $g_j(\{i,j\})$ resources to j. In this case, by Lemma 2, j becomes near-predominant and the transfer is a counter. That i can transfer $g_j(\{i,j\})$ resources to j without setting its resources to 0 follows from the fact that $(r_i + r_j)d_{jk} > r_k$, which implies that $g_j(\{j,k\}) < r_i$ -- otherwise, j is predominant when it secures $g_j(\{j,k\})$, in contradiction of Lemma 1. It is a viable counter since, by assumption A7, j prefers the transfer to securing an equivalent amount of resources by eliminating i, and since, by lemma 1, j cannot gain more than $g_j(\{i,j\})$. To prove that the conditions of the theorem are necessary, suppose (r,S) is not system stable, that W* is not empty, but i is not a member of any winning coalition. Then trivially, the winning coalition must be {j,k}, which can pose a threat to i that i cannot counter. Alternatively, if i is a member of some coalition in W*, say {i,j}, but $d_{ij} > d_{jk}$, then i cannot formulate a counter to the threat (r',{j,k},{i}), because i cannot offer j an amount that exceeds $r'_j = g_j(\{j,k\}) + r_j$. If $d_{ik} \neq 1$ and d_{ij} or $d_{jk} \neq 1$, then, by Lemma 2, i cannot render j near-predominant. Finally, if $(r_i + r_j)d_{jk} \leq r_k$, then i does not have enough resources to transfer to j to render j near-predominant. QED.

Proof of Theorem 4: That the conditions of the theorem are sufficient is trivial. To see that they are necessary, notice that if W* is not empty and if no i is near-predominant, then the corresponding constant sum game is essential. Referring to (3b), $v(i,j) = r_i + rj + \delta > v(i) + v(j) = r_i + r_j - 2\delta$. But if W* is not empty and no i is near predominant, then there exists a $\delta > 0$ that renders some i predominant, so the game is essential. It follows from the fact that only inessential constant sum n-person games have non-empty cores, that the core is empty for situations described by the characteristic function in (3b) and that the system is not resource stable.

Proof of Theorem 5: To prove sufficiency, notice that by Theorem 3 and the definition of a balancer, conditions (1), (2), and (4) imply that i is a balancer and the system is system-stable, whereas condition (3) implies that neither j nor k can be balancers. To prove necessity, notice, first, that if condition (4) is violated, then {i,j} and {i,k} cannot be winning an i cannot be a balancer. Second, if $d(r_j + r_k) > r_i$, then i is not the unique balancer. Specifically, {j,i} and {j,k} are winning: {j,k} is winning from the assumption that condition (3) is violated; and condition (4) implies that $r_j + dr_i > r_k$, so {j,i} is winning. Third, if condition (2) is violated and $d_{ij} = d_{ik} = 1$, then, ceteris paribus, by condition (3), $r_j + r_k \leq r_i$. In the case of strict inequality, i is predominant and the system is not system stable; if equality holds, then, by Theorem 2, the system is resource-stable and no country is a balancer. Finally, if condition (1) is violated and d_{jk} is not equal to 1, then, ceteris paribus, the system is not system-stable (by condition 3b of Theorem 3). QED.

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