Distributing the Benefits from the Commons

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1. Introduction and informal summary of the results

The common-property literature has often focused on the question of efficiency. Here we consider distributional issues instead. Closely related work includes Martin Weitzman (1974), John Roemer and Silvestre (1987, 1993) and Roemer (1989).

We use the fishery in the lake as a metaphor for the standard common-pool situation. The analysis can, in fact, be extended to a large class of externalities where both generators and recipients produce a good for the market.

We depart from a large portion of the literature on fisheries and adopt a static model. See Silvestre (1993) for a comparison with the more usual dynamic, steady-state formulation found, e.g., in Colin Clark (1976, 1990).

1.1. The question

How should the benefits of the commons be distributed? We approach the question from the viewpoint of <u>ownership</u>, rather than <u>income redistribution</u>. In other words, we do not contemplate using the benefits from the commons for equalizing incomes or for helping disadvantaged groups. Think of middle-class fishers supplying food to middle-class households.

As a preliminary task, we define the benefits from the commons as the valuation of the natural resource at an efficient allocation (Sections 2.3 and 2.5). Essentially the same valuation appears in different scenarios. The valuation

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* The author is indebted to John E. Roemer. He has also benefited from conversations with Marc Fleurbaey. The usual *caveat* applies in full force.

is only implicit in the case of spontaneous cooperation among a well-defined group of fishers, or in the case of a fishery regulated by nontransferable fishing quotas, but it is explicit in other instances.

First, if the fishery is operated as a competitive, private firm, then the valuation is simply the level of profits. This magnitude would also coincide with the revenue obtained by auctioning off the resource. Second, let the fishery be regulated by means of transferable permits. Then the valuation coincides with the amount of permits issued multiplied by the price of a unit permit reached at the resale market for permits. Finally, imagine that the fishing activity is subjected to Pigovian taxes (or subsidies). Then the valuation equals the total receipts of the tax bureau (or the total cost of the subsidy program). We shall simply refer to the "distribution of the benefits of the resource" without explicitly relating to a particular institution. (The reader can think of alternative distributions of the benefits as alternative profit shares in a firm or, perhaps, as alternative initial distributions of transferable fishing permits.)

1.2. Improving upon the "tragedy of the commons"?

A first set of issues (analyzed in Section 3 below) is the welfare comparison of an initial, inefficient status quo, called the open access solution, and various final, efficient allocations reached by regulation or privatization. We have in mind the following scenario. A resource has been exploited in the past by a group of people, albeit in an inefficient manner, so that a "tragedy of the commons" occurs. A regulation or privatization policy is implemented that eliminates the "tragedy," and, thus, it increases the size of the social pie. But, at the same time, it alters the way in which the pie is distributed. Who wins and who loses in the process? Depending on the distributional effects, the policy may violate the historical property rights of traditional users.

It turns out that the presence or absence of two features in the underlying data affect the conclusions, namely (i) whetner the productivity of labor is always positive or, on the contrary, crowding effects became so strong as to induce negative marginal products, and (ii) whether the supply-of-labor function by fishers is increasing or, on the contrary, backwards bending. Let us define the "conventional case" by the properties that (i) the marginal product of labor is always positive, and (ii) income effects are absent, which implies, in particular, that the supply of labor is increasing in the wage.[1]

Start from the open access solution and consider a move towards granting all fishing operations to a single, competitive firm: this we call the privatization solution. A first result is that, in the conventional case, the overall catch goes down and so do the returns to labor. The ultimate welfare effect on fishers and consumers depends on the distribution of the benefits. Consider first the case where all profits go to an outsider, i.e., not a fisher or consumer. [2] Then fishers and consumers alike end up worse off than at the open access solution.

The intuition goes as follows. Because the marginal product of labor is positive, the inefficiency of the open-access solution manifests itself as overproduction. Reducing production to optimal levels has a clear negative effect on the welfare of consumers, which can only be neutralized by channeling some of the profits towards the consumers. Fishers, on the other hand, receive the

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[1]Empirical studies on labor supply offer instances of both types of behavior. The conventional, positive sloped curve appears more often, but mildly backwards-bending curves are, by no means, empirically exceptional. See, for instance, Pencavel (1986).

[2]This is the case considered by Weitzman (1974)

marginal product of a small catch instead of the average product of the large, open-access catch. The net effect turns out to be negative. Again, only by channeling some of the profits towards the fishers can their welfare improve in the efficient situation.

We can conclude that, by the same argument, consumers, as a group, end up better off if all profits go to them, but fishers end up worse off, and the converse is true if all profits go to fishers. The last situation would obtain if the inefficiency of the open-accesss solution is eliminated by the cooperation among fishers or by nontransferable quotas.

Is there a distribution of the profits that makes each individual fisher and consumer better off than at the open access? An affirmative answer is provided by Proposition 3.3. The resulting allocation can be viewed as a social improvement that respects acquired rights on the resource.

The analysis is more complex outside the conventional case. First, let the the marginal product of labor be negative at the open access solution. Because the marginal product must be positive at any efficient solution, this means that employment is lower at the efficient solution than at the open access solution. Yet output may be higher or lower and, therefore, the effect on the welfare of consumers is ambiguous. If the labor supply curve is increasing then the lower employment must accompany a lower wage and, thus, fishers become worse off unless they share in the profits.

Assume now that the marginal product of labor is increasing, but the supply of labor is decreasing in the wage. Then the welfare of consumers and workers move in opposite directions, unless they share in the profits. It turns out that, if the privatization equilibrium is unique, then moving from open access hurts fishers and benefits consumers. If equilibrium is not unique then the converse outcome may occur.

1.3. Public ownership of the commons.

The previous discussion has postulated an initial, inefficient status quo, perhaps embodying historical property rights. Now we ask a different question. Suppose that no historical rights exist and, moreover, that inefficiency is not an issue: think, for instance, of a new fishing operation that is going to be efficiently carried. The resource is understood to be socially owned. What implications does the public ownership of the resource have for the distribution of its benefits?

A first option is to adopt a "land to the tiller" view and identify public ownership with the granting of property rights to the people who actually exploit the resource. This may be justifiable for several reasons, including the possible benefits of decentralization and the concentration of ownership, as well as potential improvements in the distribution of income. But the position is hard to defend if one leaves these issues aside: in fact, it assigns all property rights to a small group, in conflict with the idea that the resource belongs to everybody. Think, in the extreme case, of a large producer who exploits a publicly owned resource, say, mineral deposits or timber in federal lands.

A second option is to divide the benefits of the commons equally among the population: this option, under the name of <u>equal benefit solution,</u> is discussed to some extent in Roemer and Silvestre (1987). The basic idea is that, if there are one million people in society, then everybody gets one millionth of the benefits. For instance, every person would receive, at the end of the year, a check for one millionth of the profits of the fishing operation; alternatively, an efficiency-inducing amount of transferable fishing permits would be issued and distributed uniformly among the members of society..Another scheme in the same spirit would assign all fishing profits to the public treasury and, eventually, to the provision of public goods.

The equal benefit solution displays an unattractive feature, namely that it gives the same rights to users and nonusers of the resource, and, thus, it transfers income from the former to the latter. An example will illustrate. Suppose that it takes no effort to catch fish. Let there be two people in society: person one, who enjoys fish, and, thus, spends some time fishing, and person two, who does not. According to the equal benefit solution, person one must pay person two, because of two's property rights on the fish population, despite the fact that person two is in no way involved in the capture or consumption of fish. Person two plays the role of a shareholder of a privately owned corporation, rather than that of the coowner or a publicly owned resource. For this reason, the equal benefit solution reflects "equal private ownership" rather than the public ownership. Genuine public ownership of the resource should mean that person one, who has a use for it, freely enjoys the resource without having to compensate person two. The "private ownership" connotation of the equal-benefit solution is also apparent in the fact that there is no reason why ownership of natural resources should be restricted to a particular fraction of mankind. The equal-benefit solution presupposes that a well defined group of people are the members of society. But it is artificial to define "society" in terms of a particular nation-state.

This leads to a "usufruct" view of resource ownership, which, in its extreme form, implies that a consumer of the fruits of the publicly owned resource should end up paying exactly the average cost of production, whithout generating incomes for nonconsumers. Because the market price reflects the marginal cost of the product, higher than the average cost, this requires that the benefits of the fishery be distributed to consumers. The distribution must then in proportion to consumption, so that if a person consumes twice as much fish as another one, then the first one should receive a profit share twice as large. This idea leads to the proportional solution discussed in Roemer and Silvestre (1987, 1993).

In some sense, the proportional solution is the polar opposite to the "land-to-the-tiller" view mentioned above. All benefits accrue to fishers in the latter, and to consumers in the former. One could consider intermediate positions where a fraction σ of the benefits is distributed among consumers in proportion to their consumption, and the rest (i.e., the fraction $1-\sigma$) is distributed among fishers in proportion to their fishing effort. The "land-to-the-tiller" approach, then, corresponds to the value $\sigma = 0$, whereas the proportional solution sets $\sigma = 1$. A one-parameter family of solutions is obtained by letting σ range over the unit interval.

Section 4 below singles out a particular value for σ based on the following consideration. Consumers are the direct users of the fruits of the resource, but instead of directly contributing a productive input, they transfer numeraire (income) to the fishers. Suppose that the the ratios (value-of-return/value-of-contribution) are equalized across persons, with the understanding that a consumer contributes numeraire and obtains fish in return, while a fisher contributes time and obtains numeraire in return. In other words, the value of the good that one person (consumer or fisher) obtains is proportional to the value of the good that a person contributes. This means, in particular, that the ratio of the income obtained to the value of the input contributed by a fisher be equal, not only to the corresponding ratio of another fisher, but also to the value of a consumer's consumption of fish per unit of numeraire spent. We believe that this is an intuitively appealing condition. Let us call it the equal rate of return condition.

It turns out (Proposition 4.1 below) that the condition of equal rate of return induces a particular value for parameter σ, namely the ratio of the square root of the value of the output of fish divided by the sum of the square roots of the value of fish and the value of the fishers' labor. This would give consumers over one half of the benefits from the commons, reflecting the view that the social valuation of the product of the resource is higher than that of the input applied to it.

2. The model

2.1. Agents, goods, resources, technology.

Agents

Let there be F+C+1= M agents: F of them are fishers, indexed 1,....,F; C of them are consumers, indexed F+1,...., F+C; on occasion a role will be played by an outside agent (neither a fisher nor a consumer), indexed by M.

Goods

There are three goods. The first good is leisure time (or labor time), measured in hours; its individual final use (or supply) is denoted x_i (or L_i), and its aggregate supply denoted L. The second good is fish, measured in pounds; consumer i's individual consumption of fish is denoted y_i, and the aggregate quantity supplied and consumed is denoted y. The last good is gold, measured in ounces. Agent i's individual final holdings of gold are denoted m_i (i = 1,....,M).

Endowments

Labor time and gold are initially available in the amounts T and ω, respectively.

Technology:

Society's technological possibilities are described by a production function $y = f(L)$.

We maintain the following assumption on the technology.

<u>Concavity assumption</u>: $f: R_+ \to R_+$ is a strictly concave function; $f(0) = 0$.

We usually assume that f is an increasing function. On occasion, however, we will consider the case where strong "crowding" externalities occur, so that f has a maximum and decreases after it.

<u>Preferences</u>:

The preferences of fisher i (i = 1,...,F) are represented by a utility function $u_i(x_i, m_i)$, with quantities of leisure time and gold as arguments. The preferences of consumer i (i = F+1,...,F+C) are represented by a utility function $u_i(y_i, m_i)$, with quantities of fish and gold as arguments. The outsider (agent M) is only interested in gold.

The utility functions are assumed to be strictly quasiconcave and strictly monotonic in the interior of the relevant orthant. The discussion will be, on occasion, limited to the important special case where preferences are quasilinear or, in other words, where no income effects are present in the demand for fish or in the supply of labor. The following section explains.

2.2. An important special case: quasilinear preferences.

We will, on occasion and with explicit mention, specialize the model with the following assumption on preferences.

<u>Quasilinearity Assumption:</u> For i = 1,...,F,

$$u_i(x_i, m_i) = k_i(T_i - x_i) + m_i,$$

where $k_i: R_+ \to R$ is a decreasing and concave function, which satisfies: $k_i(0) = 0$; one interprets $-k_i(L_i)$ as i's <u>disutility of working</u> L_i <u>hours</u>.

For i = F+1,...,F+C,

$$u_i(y_i, m_i) = v_i(y_i) + m_i,$$

where v_i is increasing and concave, and satisfies: $v_i(0) = 0$; $v_i(y_i)$ is i's <u>willingness to pay for</u> y_i <u>pounds of fish.</u>

Define fisher i's <u>supply of labor</u> $\tilde{L}_i(w)$ by the equation:

$$w = - k'_i(L_i), \qquad i = 1,\ldots,F,$$

and consumer i's <u>demand for fish</u> $\tilde{y}_i(p)$ by the equation:

$$p = v'_i(y_i), \qquad i = F+1,\ldots, F+C.$$

<u>Lemma 2.1</u>. $\tilde{L}_i'(w) > 0$; $\tilde{y}_i'(p) < 0$.

<u>Proof</u>. Standard. Q.E.D.

Define the <u>social surplus function</u>:

$$s(p,w) = \Sigma_{i=1}^{F} k_i(\tilde{L}_i(w)) + \Sigma_{i=F+1}^{F+C} v_i(\tilde{y}_i(p)).$$

A standard argument shows that efficiency (under interiority) requires the maximization of surplus.

By adding and subtracting the expression $p\Sigma_{i=F+1}^{F+C} \tilde{y}_i(p) - w\, \Sigma_{i=1}^{F} \tilde{L}_i(w)$, and rearranging, we can decompose social surplus in the following manner:

$$s(p,w) = \Sigma_{i=1}^{F} s_i(w) + \Sigma_{i=F+1}^{F+C} s_i(p) + s_M(p,w),$$

where:

for $i = 1,\ldots,F$, $s_i(w) := k_i(\tilde{L}_i(w)) + w\tilde{L}_i(w)$ is i's <u>fisher's surplus</u>;[3]

for $i = F+1,\ldots,F+C$, $s_i(w) := v_i(\tilde{y}_i(p)) - p\tilde{y}_i(p)$ is i's <u>consumer's surplus</u>;

$s_M(p,w) = p\Sigma_{i=F+1}^{F+C} \tilde{y}_i(p) - w\, \Sigma_{i=1}^{F} \tilde{L}_i(w)$ is the <u>outsider's surplus</u>.

It is clear that a fisher's surplus is increasing in w, and that a consumer's surplus is decreasing in p.

2.3. Efficiency

We return to the general case. An allocation is a (2F+2C+1)-dimensional vector $(x_1,m_1,\ldots,x_F,m_F, y_{F+1},m_{F+1},\ldots,y_{F+C},m_{F+C}, m_M)$. An allocation is <u>feasible</u>

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[3] The symbol ":=" is read "defined as."

if: $\Sigma_{i=1}^{M} m_i = \omega$ and $\Sigma_{i=F+1}^{C} y_i = f(L)$, where $L = T - \Sigma_{i=1}^{F} x_i$. An allocation is interior if, for $i = 1,...,F$, $x_i > 0$, for $i = F+1,...,C$, $y_i > 0$, and, for $i = 1,..., F+C$, $m_i > 0$, i.e., the consumption vectors of all agents, except possibly the outside agent, have all their relevant components positive. At an efficient, interior allocation one must have that, for any pair i, h, where i is a fisher and h is a consumer,

$$\frac{\partial u_i/\partial x_i}{\partial u_i/\partial m_i} = \frac{\partial u_h/\partial y_h}{\partial u_h/\partial m_h} f'(L), \quad i = 1,...,F,\ h = F+1,...,C. \quad (2.1)$$

At an efficient, interior allocation, a valuation of the goods is given by the vector of efficiency prices (w, p, 1) defined as:

$$p = \frac{\partial u_h/\partial y_h}{\partial u_h/\partial m_h}, \text{ for any } h \text{ in } \{F+1,...,C\},$$

$$w = \frac{\partial u_i/\partial x_i}{\partial u_i/\partial m_i}, \text{ for any } i \text{ in } \{1,...,F\},$$

where gold is used as numeraire. We then say that the vector (1, w, p) supports the (efficient) allocation (i.e., it allows for profit maximization and for the constrained maximization of utility). Efficiency prices, in turn, provide a valuation of the lake and its fish as a productive resource, namely:

$$p\Sigma_{i=F+1}^{C} y_i - wL. \quad (2.2)$$

Note that the efficiency condition (2.1) can be written:

$$\frac{\partial u_i/\partial x_i}{\partial u_i/\partial m_i} = pf'(L), \quad i = 1,...,F. \quad (2.3)$$

2.4. Property relations.

We postulate that the technology, the aggregate endowments and the preferences are chosen by Mother Nature. Moreover, human history has defined some property relations.

Initially, the endowments of labor-time and gold are privately owned. Fisher i (i = 1,...,F) is initially endowed with T_i hours of labor time and zero ounces of gold.[4] Agent i (i = F + 1, ..., M) is initially endowed with a positive number ω_i of ounces of gold and zero labor time. Of course, $T = \Sigma_{i=1}^{F} T_i$ and $\omega = \Sigma_{i=F+1}^{F+C} \omega_i$.

The leisure-time x_i enjoyed, and the labor-time L_i supplied, by fisher i are related by the equality:

$$L_i + x_i = T_i. \qquad (2.4)$$

In is understood that the technology, on the contrary, is publicly owned.

2.5. Markets and prices.

Fish is bought and sold in a perfectly competitive fish market. We take gold as numeraire, and denote by p the price of fish (i.e., p ounces of gold are exchanged by one pound of fish in the market). The assumption of perfect competition means that both buyers and sellers of fish take the market price p of fish as given, as outside their control.

In some, but not all, institutional arrangements, fisher's labor is also bought and sold in a competitive labor market. The market wage is denoted by w (again, using gold as numeraire: working one hour puts w ounces of gold in your pocket.)

If both fish and labor are sold in competitive markets and if the resulting allocation is efficient, then the valuation of the resource given in (2.2) coincides with the profits of the firm that hires labor and sells fish. The McKenzie construction interprets profits as the valuation of an implicit input, i.e., a productive factor not directly traded in the market. Any decreasing

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[4] Or a positive amount, as long as other assumptions, say, on consumption sets, guarantee that she never becomes a seller of gold.

returns-to-scale technology with N inputs can be viewed as a constant-returns technology with N + 1 inputs (simply define $\varphi(L,z) = zf(L/z)$), where z is available in one unit. McKenzie called the extra input "entrepreneurial input", but here it should be interpreted as the natural resource.

The (marginal product) valuation of the resource coincides with the (competitive) profits from operating the tehcnology. The valuation of the natural resource in this paper can be seen as:

(i) the profits of a firm that operates all fishing operations in the lake, a la McKenzie,

but also as:

(ii) the market value of transferable fishing permits, issued by the public authority in an efficiency-inducing amount,

or as:

(iii) the total receipts of a Pigovian tax policy that taxes fishers in accordance with their catches (or with the time spent fishing).

This paper illustrates the fact that, when the final allocation is efficient, the distributional implications of different institutional arrangements can be viewed as different forms of distributing the value of the resource among the members of society.

3. Comparing the distribution of welfare in open-access and in efficient allocations.

3.1. The open access solution

We consider a *status quo ante* without labor market: each fisher goes out and sells her catch in the fish market. We assume that the amount of fish caught by fisher i is proportional to the amount L_i of time that she spends fishing, i.e., all fishers are equally skilled and equally lucky, so that i's output is

$$\frac{f(L)}{L} L_i,$$

where $L = \Sigma_{h=1}^{F} L_h$. Thus, if the market price for fish is p, and i spends $L_i = T_i - x_i$ hours fishing, then her final consumption vector (x_i, m_i) is

$$(T_i - L_i, \frac{pf(L)}{L} L_i).$$

How does i choose L_i in the open-access situation? We adopt the following formulation.

Definition: A vector $(L_1,\ldots,L_F, y_{F+1},\ldots,y_{F+C}, \bar{p}, \bar{w})$ is an open-access equilibrium if, writing $L = \Sigma_{i=1}^{F} L_i$, $y = \Sigma_{i=F+1}^{F+C} y_i$:

(i) $y = f(L)$;

(ii) for $i = 1,\ldots, F$, L_i maximizes $u_i(T_i - L_i, \bar{w}L_i)$;

(iii) for $i = F+1,\ldots,F+C$, y_i maximizes $u_i(y_i, \omega_i - \bar{p}y_i)$;

(iv) $\bar{w} = \frac{\bar{p}f(L)}{L}$.

In words, a fisher assumes that the average social catch, f(L)/L, is not affected by her actions: this assumption is justified if each fisher is small. [5]

Note that the first order condition of i's maximization problem ($i = 1,\ldots,F$) is:

$$\frac{\partial u_i/\partial x_i}{\partial u_i/\partial m_i} = \frac{\bar{p}f(L)}{L}, \qquad (3.1)$$

with derivatives evaluated at $(T_i - L_i, \frac{\bar{p}f(L)}{L} L_i)$.

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[5] Appendix I offers an alternative formulation based on the Nash noncooperative equilibrium concept. The two formulations are approximately equivalent when there are many small fishers.

The discrepancy between (2.3) and (3.1) is at the root of the inefficiency of the open access solution, or, if you wish, of the tragedy of the commons. But (2.3) is satisfied at a continuum of allocations, each embodying a different pattern in the distribution of the benefits of the resource. The social pie is larger when (2.3) is satisfied than when (3.1) is, but (3.1) and the private property rights on gold and labor time define a particular distribution of the smaller pie, whereas (2.3) is compatible with a continuum of distributions. At some of these allocations some agents may be recieve a small share of the larger pie and end up worse off than at the "tragic" open-access allocation. To this we turn now.

3.2. Equilibria

Let a regulation or privatization policy be implemented that eliminates the inefficiency of the open access. Perhaps the fishers establish an efficiency-inducing system of restraints. Or perhaps a profit maximizing, price-and-wage taking, firm assumes the harvesting and selling of fish. Alternatively, the government issues an efficient number of transferable permit. We abstract from the specific insitution and postulate that an efficient allocation is reached where the benefits from the commons, defined by (2.2) above, are distributed in a particular manner among the population. This motivates the following definition. Its wording suggests the operation of a profit maximizing firm, but one can show that alternative efficiency-inducing schemes yield allocations that satisfy the same definition.

<u>Definition</u>: Let the vector $(\theta_1,\dots,\theta_M)$, where $\theta_i \geq 0$, $i = 1,\dots,M$, and $\Sigma_{i=1}^{M}\theta_i = 1$, be given. A vector $(L_1,m_1,\dots, L_F,m_F, y_{F+1},m_{F+1},\dots,y_{F+C},m_{F+C},m_M, w, p)$ is a

<u>Walrasian equilibrium for the benefit distribution</u> $(\theta_1,\ldots,\theta_M)$ if, writing $L = \Sigma_{i=1}^{F} L_i$, $y = \Sigma_{i=F+1}^{F+C} y_i$, and $\Pi = py - wL$,

(i) for $i = 1,\ldots, F$, $(T_i - L_i, m_i)$ maximizes $u_i(x_i, m_i)$ subject to

$$wx_i + m_i = wT_i + \theta_i\Pi;$$

(ii) for $i = F+1,\ldots,F+C$, (y_i, m_i) maximizes $u_i(y_i,m_i)$ subject to

$$py_i + m_i = \omega_i + \theta_i\Pi\ ;$$

(iii) $m_M = \theta_M\Pi$;

(iv) $y = f(L)$;

(v) $pf(L) - wL \geq pf(L') - wL'$, for all $L' \geq 0$;

(vi) $\Sigma_{i=1}^{M} m_i = \omega$.

It is clear, from the first fundamental theorem of welfare economics, that such an equilibrium is efficient, and that the allocation is supported by the price vector $(w, p, 1)$.

3.3. The welfare effects of privatization

Suppose that all fishing rights are given to the outsider, who then operates as a profit-maximizing, price-and-wage taking firm. We assume the resulting outcome is a Walrasian equilibrium for the distribution of benefits $(\theta_1,\ldots,\theta_F,\theta_{F+1},\ldots, \theta_{F+C},\theta_M) = (0,\ldots 0,0,\ldots,0,1)$. We call it a <u>privatization equilibrium.</u>

Given $p > 0$, consumer i's demand $(i = F+1,\ldots,F+C)$ is obtained from the maximization of $u_i(y_i,m_i)$ subject to: $m_i + py_i = \omega_i$. This defines i's demand function: $\tilde{y}_i: (0,\infty) \rightarrow R$. Define $\tilde{y}: (0,\infty) \rightarrow R$: $\tilde{y}(p) = \Sigma_{i=F+1}^{F+C} \tilde{y}_i(p)$. Let $\overline{p}$ be the supremum of the set $\{p \in (0,\infty) : \tilde{y}(p) > 0\}$. (Perhaps $\overline{p} = \infty$). We postulate that $\tilde{y}$ is decreasing in the interval $(0, \overline{p})$ (in other words, fish is not a Giffen good) and, moreover, that $\tilde{y}$ is differentiable on $(0, \overline{p})$, with negative derivative.

It follows that $\tilde{y}$ is invertible, i.e., there exists a function $\tilde{p}: (0,\overline{y}) \to (0,\overline{p})$, where $\overline{y} = \lim_{p\to 0} \tilde{y}(p)$ (perhaps $\overline{y} = \infty$) such that $\tilde{p}(\tilde{y}(p)) = p$. Moreover $\tilde{p}' < 0$.

Given $w > 0$, fisher i's supply of labor ($i = 1,\dots,F$) is obtained from the maximization of $u_i(x_i,m_i)$ subject to: $m_i + wx_i = wT_i$. This defines i's supply-of-labor function: $\tilde{L}_i: (0,\infty) \to R$. Define $\tilde{L}: (0,\infty) \to R$: $\tilde{L}(p) = \Sigma_{i=1}^{F} \tilde{L}_i(p)$.

Lemma 3.1. (a) The wage rate w is a component of an interior privatization equilibrium if and only if

$$\psi(w) := \tilde{p}(f(\tilde{L}(w))).f'(\tilde{L}(w)) - w = 0;$$

(b) The wage rate w is associated with an interior open-access equilibrium if and only if

$$\alpha(w) := \tilde{p}(f(\tilde{L}(w))).[f(\tilde{L}(w))/\tilde{L}(w)] - w = 0.$$

Proof: standard. Q.E.D.

Definition: A privatization equilibrium is regular if, at the associated wage rate w, $\psi'(w) \neq 0$.

The following proposition generalizes the main result in Weitzman (1974). Note that its language implicity assumes that a privatization equilibrium exists: existence is indeed guaranteed under our assumptions.

Proposition 3.1. Postulate the boundary condition $\lim_{w\to\infty} \psi(w) < 0$ (possibly $-\infty$). Assume that the privatization equilibrium is unique and regular, and denote by w^* (resp. $\overline{w}$) the wage rate corresponding to the privatization equilibrium (resp. to a given open access equilibrium.)

(a) $\overline{w} > w^*$;

(b) every fisher is worse off at the privatization equilibrium than at any open-access equilibrium.

Proof. Let w^* be the wage rate of the unique privatization equilibrium, i.e., w^* is the unique solution to the equation "$\psi(w) = 0$." Note that, if one had $\psi'(w^*) > 0$, then one would have that $\psi(w^* + \varepsilon) > 0$, for small ε, which implies, by the boundary condition, that $\psi(w) = 0$ for some $w > w^* + \varepsilon$, contradicting the uniqueness of equilibrium. Thus, $\psi'(w^*) \leq 0$, and hence, because of regularity, $\psi'(w^*) < 0$. This implies that $\psi(w^* - \varepsilon) > 0$, for small ε, which, again by the uniqueness of equilibrium, implies that $\psi(w) > 0$ for all w in $(0,w^*)$. The concavity of f implies that, for all $L > 0$, $\frac{f(L)}{L} > f'(L)$. This in turn implies that, if $\tilde{L}(w) > 0$, then $\alpha(w) > \psi(w)$, and, therefore, $\alpha(w) > 0$ for all w in $(0,w^*)$. Thus, "$\alpha(\overline{w}) = 0$" implies that $\overline{w} > w^*$, i.e., using Lemma 3.1(b), a wage rate $\overline{w}$ associated with an open access equilibrium must be higher than the privatization wage rate w^*. This proves (a): (b) follows immediately. Q.E.D.

Lemma 3.2. If $\tilde{L}'(w) > 0$ (in the interval $(\inf\{w > 0: \tilde{L}(w) > 0\}, \infty)$), then $\lim_{w\to\infty} \psi(w) < 0$ and the privatization equilibrium is unique and regular.

Proof. Easy. Q.E.D.

Lemma 3.3. If $\tilde{L}'(w) > 0$ (in the interval $(\inf\{w > 0: \tilde{L}(w) > 0\}, \infty)$), and $f' > 0$ at a given open access equilibrium, then $\overline{y} > y^*$, where $\overline{y}$ (resp. y^*) is the level of output at the open access equilibrium (resp. at the unique privatization equilibrium.)

Proof. By Lemma 3.2, the conditions in the statement of Proposition 3.1 are satisfied. Thus, $\overline{w} > w^*$ and, thus, $\tilde{L}(\overline{w}) > \tilde{L}(w^*)$. The assumption that $f' > 0$ at the given open access equilibrium that f is increasing between the open access and privatization equilibria and, thus, that $y^* < \overline{y}$. Q.E.D.

Proposition 3.2. Assume that $\tilde{L}'(w) > 0$ in the interval $(\inf\{w > 0: \tilde{L}(w) > 0\}, \infty)$.

(a) Any fisher is worse off at the unique privatization equilibrium than at any open-access equilibrium;

(b) Any consumer is worse off at the unique privatization equilibrium than at any open-access equilibrium where $f' > 0$.

Proof: (a) follows immediately from Proposition 3.1 and Lemma 3.2, while (b) follows from Lemma 3.3 and the fact that, because the demand function is decreasing, we must have that, in obvious notation, $p^* > \overline{p}$. Q.E.D.

The intuition in the proof of Proposition 3.2 (b) is that, as long as the labor supply curve is increasing and the marginal product is positive, the welfare of fishers and consumers moves in the same direction. The qualification of positive marginal product is needed because, otherwise, the open access equilibrium could entail lower output (at higher employment!) than the privatization equilibrium, privatization then becoming beneficial to consumers.

What if the labor supply curve bends backwards? Then multiple privatization equilibria are likely, and the comparative welfare analysis becomes more casuistic. One can, nevertheless, observe that, as long as we compare a privatization equilibrium and an open access equilibrium that are on the same, decreasing portion of the labor supply curve, then (assuming positive marginal products) the welfare of fishers and consumers move in opposite directions. If the privatization equilibrium is unique, then, by Proposition 3.1, fishers will lose from privatization, whereas consumers will gain. But moving from an open access equilibrium to a privatization one may well benefit fishers and hurt consumers when equilibria are multiple.

3.4. The welfare effects of attaining efficiency while fishers retain all benefits.

Consider now a situation where the fishers collectively agree on the efficient amount L^* of fishing. The outcome is an efficient equilibrium for the distribution of benefits of the type $(\theta_1,\ldots,\theta_F,\theta_{F+1},\ldots,\theta_{F+C},\theta_M) = (0,\ldots 0,\theta_{F+1},\ldots,\theta_{F+C},0)$. Assume that the utility functions of fishers are quasilinear, i.e., the supply-of-labor function is increasing and it does not depend on income and, hence, is the same whether fishers receive benefits or not. As far as consumers are concerned, the resulting equilibrium is, thus, undistinguishable from a privatization equilibrium and, by Proposition 3.2 (b), consumers lose when fishers abandon the open access equilibrium. <u>Cooperation among fishers does solve the tragedy of the commons, but, to some extent, at the expense of consumers</u>.

3.5. Pareto-dominating the open access allocation: the quasilinear case.

Assume quasilinearity (Section 2.2 above) and, for $i = 1,\ldots,F$, write s_i^* (resp. $\overline{s_i}$) for i's fisher's surplus at the unique privatization equilibrium (resp. at a given open-access equilibrium, assumed to be interior); for $i = F+1,\ldots,F+C$, write s_i^* and $\overline{s_i}$ for the corresponding consumer surpluses, and y^* and $\overline{y}$ for the corresponding level of output.

<u>Lemma 3.4. (a) For</u> $i = 1,\ldots, F$, $\overline{s_i} > s_i^*$.

(b) <u>If</u> $f' > 0$ <u>then</u> $\overline{s_i} > s_i^*$ <u>for</u> $i = F+1,\ldots, F+C$.

<u>Proof</u>. Immediate from Proposition 3.2. Q.E.D.

Write also:

$\overline{s^F} = \Sigma_{i=1}^{F}\overline{s_i}$ for the aggregate fishers' surplus at the open-access solution,

$\overline{s^C} = \Sigma_{i=F+1}^{F+C}\overline{s_i}$ for the aggregate consumers' surplus at the open-access solution,

$\overline{s_M} = 0$ for the owner's surplus at the open-access solution,

$\overline{s} = \overline{s^F} + \overline{s^C} + \overline{s_M}$ for the social surplus in the open access solution,

and s_i^*, $i = 1,\dots,M$, s^{F*}, s^{C*} and s^* for the corresponding magnitudes in the privatization solution.

Lemma 3.5. $s_M^* > \overline{s^F} + \overline{s^C} - s^{F*} - s^{C*}$.

Proof: Because the privatization solution yields an efficient allocation, whereas the open access solution does not, one must have $s^* > \overline{s}$, i.e., :

$$s^{F*} + s^{C*} + s_M^* > \overline{s^F} + \overline{s^C}. \qquad \text{Q.E.D.}$$

Proposition 3.3. Consider a given open access equilibrium where $f' > 0$. Assume that the utility functions of fishers and consumers are quasilinear, and write s_i^* (resp. $\overline{s_i}$) for i's fisher or consumer surplus at the unique privatization equilibrium (resp., at the given open access equilibrium, assumed to be interior). Then there is a (F+C-1)-dimensional continuum of benefit-distribution vectors of the form $(\theta_1,\dots,\theta_F,\theta_{F+1},\dots,\theta_{F+C}, 0)$ such that the allocation obtained at the Walrasian equilibrium for $(\theta_1,\dots,\theta_F,\theta_{F+1},\dots,\theta_{F+C}, 0)$ Pareto dominates the given open-access solution.

Proof. Define the benefit distribution shares:

$$\theta_i^* = \frac{\overline{s_i} - s_i^*}{\overline{s^F} + \overline{s^C} - s^{F*} - s^{C*}}, \quad i = 1,\dots, F + C,$$

and, of course, $\theta_M^* = 0$. By Lemma 3.4 (a)-(b), $\theta_i^* \geq 0$, $i = 1,\dots,M$, and $\Sigma_{i=1}^{M}\theta_i^* = 1$. For $i = 1,\dots,F+C$, agent i's utility in the efficient equilibrium for $(\theta_1^*,\dots,\theta_M^*)$ is:

$$\omega_i + s_i^* + \theta_i^* s_M^* = \omega_i + s_i^* + \frac{\overline{s_i} - s_i^*}{\overline{s^F} + \overline{s^C} - s^{F*} - s^{C*}} s_M^*$$

$$> \omega_i + s_i^* + \overline{s_i} - s_i^*,$$

by Lemma 3.5. But $\omega_i + \overline{s_i}$ is her utility in the open access solution. Slight perturbations of the vector $(\theta_1^*,\ldots,\theta_M^*)$ while preserving nonnegativity and the property of adding up to one will maintain the utility inequalitities. Q.E.D.

4. The public ownership of natural resources.

The reader is referred to Section 1.4 above for motivation and for an informal discussion of the concepts and issues.

4.1. The equal benefit equilibrium

Definition: A vector $(L_1,m_1,\ldots, L_F,m_F, y_{F+1},m_{F+1},\ldots,y_{F+C},m_{F+C},m_M, w, p)$ is an equal-benefit equilibrium if it is an efficient equilibrium for the benefit distribution $(\theta_1,\ldots,\theta_F,\theta_{F+1},\ldots,\theta_{F+C}, \theta_M)$ =

$$\left(\frac{1}{F+C+1},\ldots,\frac{1}{F+C+1},\frac{1}{F+C+1},\ldots,\frac{1}{F+C+1},\frac{1}{F+C+1}\right).$$

4.2. A one-parameter family of public ownership solutions

Definition: Given $\sigma \in [0,1]$, a vector

$(L_1,m_1,\ldots, L_F,m_F, y_{F+1},m_{F+1},\ldots,y_{F+C},m_{F+C},m_M, w, p)$

is a σ-public-ownership equilibrium if it is an efficient equilibrium for the benefit distribution

$$(\theta_1,\ldots,\theta_F,\theta_{F+1},\ldots,\theta_{F+C}, \theta_M) = \left((1-\sigma)\frac{L_1}{L},\ldots,(1-\sigma)\frac{L_F}{L}, \sigma\frac{y_{F+1}}{y},\ldots,\sigma\frac{y_{F+C}}{y}, 0\right).$$

Note that the case $\sigma = 0$ (resp. $\sigma = 1$) corresponds to the "land-to-the-tiller" approach (resp. the proportional solution) mentioned in the Section 1.4.

4.3. The equalization of the rate of return

Definition: An efficient, interior allocation $(x_1, m_1, \ldots, x_F, m_F, y_{F+1}, m_{F+1}, \ldots, y_{F+C}, m_{F+C}, 0)$ is an equal rate-of-return solution if for each pair (i,h), where i is a fisher and h is a consumer,

$$\frac{m_i}{wL_i} = \frac{py_h}{\omega_h - m_h}, \tag{4.1}$$

where $L_i = T_i - x_i$, $i = 1,\ldots,F$, and where the price vector (w, p, 1) supports the allocation.

As mentioned in Section 1.4, we view fishers as deriving a return in gold (numeraire) from a contribution of labor time, whereas consumers derive a return equal to the value of fish they consume from a contribution, in gold, paid to fishers. The equal-rate-of-return condition requires returns to be proportional to contributions, all valued at the supporting prices.

Proposition 4.1: The efficient, interior allocation $(x_1, m_1, \ldots, x_F, m_F, y_{F+1}, m_{F+1}, \ldots, y_{F+C}, m_{F+C}, 0)$, with support prices (w, p, 1), and with aggregate quantities $y = \Sigma_{i=F+1}^{F+C} y_i$ and $L = \Sigma_{i=1}^{F} L_i$, where $L_i = T_i - x_i$, $i = 1,\ldots,F$, is an equal-rate-of return solution if and only if $(L_1, m_1, \ldots, L_F, m_F, y_{F+1}, m_{F+1}, \ldots, y_{F+C}, m_{F+C}, m_M, w, p)$ is a σ^*-public-ownership equilibrium, for

$$\sigma^* = \frac{\sqrt{py}}{\sqrt{py} + \sqrt{wL}}.$$

Proof. The result will follow after showing that condition (4.1) obtains if and only if:

$$m_i = wL_i + (1-\sigma^*)(L_i/L)(py - wL), \ i = 1,\ldots,F, \tag{4.2}$$

and:

$$m_h = \omega_h - py_h + \sigma^*(y_h/y)(py - wL), \ h = F+1,\ldots,F+C. \tag{4.3}$$

First we prove that (4.1) implies (4.2). Write $B = \Sigma_{k=1}^{F} m_k$, the total amount of gold transferred from consumers to fishers. Note that (4.1) implies that:

$$\frac{m_i}{wL_i} = \frac{B}{wL}, \ i = 1,\ldots,F, \tag{4.4}$$

and:

$$\frac{py_h}{\omega_h - m_h} = \frac{py}{\omega - \Sigma_{k=F+1}^{F+C} m_k} \quad , \quad h = F+1,\dots,F+C. \qquad (4.5)$$

Because $\Sigma_{k=1}^{F+C} m_k = \omega$ at an efficient allocation, we have that $\omega - \Sigma_{k=F+1}^{F+C} m_k =$ B. Thus, (4.5) can be written:

$$\frac{py_h}{\omega_h - m_h} = \frac{py}{B} \quad , \quad h = F+1,\dots,F+C. \qquad (4.6)$$

From (4.1), (4.4) and (4.6), we obtain: $B^2 = pywL$, i.e., $B = \sqrt{pywL}$. Thus, (4.4) and (4.6) become:

$$\frac{m_i}{wL_i} = \frac{\sqrt{py}}{\sqrt{wL}}, \quad i = 1,\dots F, \qquad (4.7)$$

$$\frac{py_h}{\omega_h - m_h} = \frac{\sqrt{py}}{\sqrt{wL}} \quad , \quad h = F+1,\dots,F+C. \qquad (4.8)$$

Consider fisher i, i = 1,....,F. From (4.7),

$$\begin{aligned} m_i &= wL_i \frac{\sqrt{py}}{\sqrt{wL}} \\ &= wL_i + wL_i\left(\frac{\sqrt{py}}{\sqrt{wL}} - 1\right) \\ &= wL_i + wL_i\left(\frac{\sqrt{py} - \sqrt{wL}}{\sqrt{wL}}\right) \\ &= wL_i + wL_i\left(\frac{\sqrt{py} - \sqrt{wL}}{\sqrt{wL}}\right) \frac{\sqrt{py} + \sqrt{wL}}{\sqrt{py} + \sqrt{wL}} \\ &= wL_i + \frac{wL_i}{wL} \frac{wL}{\sqrt{wL}} \frac{py - wL}{\sqrt{py} + \sqrt{wL}} \\ &= wL_i + \frac{L_i}{L} (1 - \sigma^*)(py - wL), \end{aligned}$$

proving that (4.2) holds. In a parallel manner, consider consumer h, h = F+1, ...,F+C. From (4.8) we have that:

$$py_h \frac{\sqrt{wL}}{\sqrt{py}} = \omega_h - m_h \quad ,$$

i.e.,

$$\begin{aligned} m_h &= \omega_h - py_h + py_h\left(1 - \frac{\sqrt{wL}}{\sqrt{py}}\right) \\ &= \omega_h - py_h + py_h \frac{\sqrt{py} - \sqrt{wL}}{\sqrt{py}} \\ &= \omega_h - py_h + py_h \frac{\sqrt{py} - \sqrt{wL}}{\sqrt{py}} \frac{\sqrt{py} + \sqrt{wL}}{\sqrt{py} + \sqrt{wL}} \end{aligned}$$

$$= \omega_h - py_h + \frac{py_h}{py} \frac{py}{\sqrt{py}} \frac{py - wL}{\sqrt{py} + \sqrt{wL}}$$

$$= \omega_h - py_h + \frac{y_h}{y} \sigma^* (py - wL),$$

showing that (4.3) is satisfied. The support property of prices guarantees utility maximization and profit maximization. Thus, writing $\theta_i = \frac{L_i}{L}(1-\sigma^*)$, $i = 1,\ldots,F$, and $\theta_h = \frac{y_h}{y}\sigma^*$, $h = F+1,\ldots,F+C$, we conclude that $(L_1,\ldots,L_F, y_1,\ldots,y_M, p, w, \theta_1, \ldots, \theta_{F+C})$ is a σ^*-public-ownership equilibrium for $\sigma^* = \frac{\sqrt{py}}{\sqrt{py} + \sqrt{wL}}$.

Conversely, consider a σ^*-public ownership equilibrium for the stated σ^*. From (4.2), we have that

$$m_i = wL_i \left(1 + (1-\sigma^*)\left(\frac{py - wL}{wL}\right)\right)$$

$$= wL_i\left(1 + \frac{\sqrt{wL}}{\sqrt{py} + \sqrt{wL}} \frac{(\sqrt{py} + \sqrt{wL})(\sqrt{py} - \sqrt{wL})}{wL}\right)$$

$$= wL_i\left(1 + \frac{\sqrt{py} - \sqrt{wL}}{\sqrt{wL}}\right)$$

$$= wL_i \frac{\sqrt{py}}{\sqrt{wL}},$$

i.e.,

$$\frac{m_i}{wL_i} = \frac{\sqrt{py}}{\sqrt{wL}}, \text{ for } i = 1,\ldots,F.$$

In a parallel manner, from (4.3) we obtain:

$$\frac{py_h}{\omega_h - m_h} = \frac{\sqrt{py}}{\sqrt{wL}}, \quad h = F+1,\ldots,F+C.$$

Thus, the equal-rate-of-return equalities (4.1) are satisfied. Efficiency follows from the first fundamental theorem, and the supporting property of prices from utility maximization and profit maximization. Q.E.D.

APPENDIX

Write: $L_{-i} = \Sigma_{j=1, j\neq i}^{F} L_j$.

Definition: A vector $(L_1,\ldots,L_M, y_1,\ldots,y_M, p)$ is a Nash noncooperative equilibrium if

(i) for $i = 1,\dots, F$, L_i maximizes $u_i(T_i - L_i, \frac{pf(L_{-i}+L_i)}{L_{-i}+L_i} L_i)$, given p and L_{-i}.

(ii) for $i = F+1,\dots,F+C$, y_i maximizes $u_i(y_i, \omega_i - py_i)$, given p;

(iii) $\Sigma_{i=F+1}^{F+C} y_i = f(\Sigma_{i=1}^{F} L_i)$.

The first order condition of i's maximization problem ($i = 1,\dots,F$) is:

$$\frac{\partial u_i/\partial x_i}{\partial u_i/\partial m_i} = \frac{pf(L)}{L} + \frac{L_i}{L}\left(pf'(L) - \frac{f(L)}{L}\right)$$

$$= \frac{L_{-i}}{L}\frac{pf(L)}{L} + \frac{L_i}{L} pf'(L), \qquad \text{(A.1)},$$

with derivatives evaluated at $(T_i - L_i, \frac{pf(L)}{L} L_i)$. The right-hand side of (A.1) lies between those of (2.3) and (3.1) in the text. It, in particular, coincides with the one in (2.3) for the special case where $F = 1$. If, on the contrary, L_i is small relative to L (i.e., L_{-i} is close to L), then the right-hand side of (A.1) is approximately equal to the one in (3.1).

References

Clark, Colin W. (1976, 1990) Mathematical Bioeconomics: The Optimal Management of Renewable Resources, (first edition: 1976, second edition: 1990) New York: John Wiley.

Pencavel, John (1986) "Labor Supply of Men: A Survey," Chapter 1 in O. Ashenfelter and R. Layard, Handbook of Labor Economics, Vol.I, Amsterdam: North Holland.

Roemer, John E. (1989) "A Public Ownership Resolution of the Tragedy of the Commons," Social Philosophy and Policy, 6, 74-92.

Roemer, John E., and Joaquim Silvestre (1987) "Public Ownership: Three Proposals for Resource Allocation," Department of Economics, University of California, Davis, Working Paper Series No. 307.

Roemer, John E., and Joaquim Silvestre (1993) "The Proportional Solution for Economies with both Private and Public Ownership," Journal of Economic Theory, 59 (2), 426-444.

Silvestre, Joaquim (1993) "The Commons: Static and Dynamic Externalities," unpublished.

Weitzman, Martin (1974) "Free Access vs. Private Ownership as Alternative Systems for Managing Public Property," Journal of Economic Theory 8(2), 225-234.