Non-binding Agreements and Fairness in Commons Dilemma Games

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Abstract

Usually, common pool games are analyzed without taking into account the cooperative features of the game, even when communication and non-binding agreements are involved. Whereas equilibria are inefficient, negotiations may induce some cooperation and may enhance efficiency. In the paper, we propose to use tools of cooperative game theory to advance the understanding of results in dilemma situations that allow for communication. By doing so, we present a short review of earlier experimental evidence given by Hackett, Schlager, and Walker 1994 (HSW) for the conditional stability of non-binding agreements established in face-to-face multilateral negotiations. For an experimental test, we reanalyze the HSW data set in a game-theoretical analysis of cooperative versions of social dilemma games. The results of cooperative game theory that are most important for the application are explained and interpreted with respect to their meaning for negotiation behavior. Then, theorems are discussed that cooperative social dilemma games are clear (alpha- and beta-values coincide) and that they are convex (it follows that the core is "large"): The main focus is on how arguments of power and fairness can be based on the structure of the game. A second item is how fairness and stability properties of a negotiated (non-binding) agreement can be judged. The use of cheap talk in evaluating experiments reveals that besides the relation of non-cooperative and cooperative solutions, say of equilibria and core, the relation of alpha-, beta- and gamma-values are of importance for the availability of attractive solutions and the stability of the such agreements. In the special case of the HSW scenario, the game shows properties favorable for stable and efficient solutions. Nevertheless, the realized agreements are less efficient than expected. The realized (and stable) agreements can be located between the equilibrium, the egalitarian solution and some fairness solutions. In order to represent the extent to which the subjects obey efficiency and fairness, we present and discuss patterns of the corresponding excess vectors.

Key words: Fairness, Kernel, Commons Dilemma, Tu Games

1 Introduction

Involved in **agreements**, people often ask: Will the agreed actions be executed? Will the agreement hold? Or will it be unstable? Indeed, there are also agreements where implementation is not a problem, because technical conditions or adequate institutions guarantee the fulfillment of the contract. But here we focus on **non-binding agreements**, simply referred to as agreements. With this, the question of **compliance**, just mentioned, is crucial.

An often expressed **hypothesis** on compliance is that people reduce compliance if they feel that they have been treated unfairly. It is worthwhile to examine the role fairness can play in explaining the evolution of the **commons**. An important step forward in the research on commons was the idea to combine field research, theoretical work and experiments and its realization. This kind of research was initiated by a group of researchers which has been referred to as Bloomington group, mainly members of the Workshop in Political Theory and Policy Analysis and members of the Faculty of Economics of the University of Indiana at Bloomington. Especially Elinor Ostrom's book "Governing the Commons" ([see 34]) can be seen as starting point of the modern analysis of commons.

Here, we are going to deal with a standard **experimental** commons à la Ostrom, Gardner, and Walker (**OGW** [35]) in a repeated game framework that is enriched by explicit communication breaks. Commons observed **in the field** often allow for communication; often they provide meetings for their members. It is more than natural that these meetings are also used for reflection on the state and process of the commons and to coordinate strategies, as well as to resolve conflicts.

The more formalized the settings that experimentalists do examine, the more the **bargainings** for agreements on future action become a central task. Traditionally, those settings have been analyzed with concepts known from **non-cooperative game theory**. In the hypothesis of cheap talk, it was erroneously assumed that, in the absence of binding contracts, only non-cooperative concepts should be considered. Moreover, the literature did not recognize that the introduction of communication in an experimental setup will change the game theoretical framework in which subjects are involved. Communication allows subjects to make proposals about which group they want to belong to and to coordinate their strategies. But this means that, at this stage, concepts from **cooperative game theory** have to be used to analyze the underlying situation.

Since, in our context, the non-cooperative solutions lack efficiency, subjects usually correct their fate by arranging to reach or at least to approach some **welfare optimum**. In such a way, welfare optima come into play not only in measuring the (in)efficiency of Nash-equilibria but also as targets for agreements. This stands in contrast to a purely non-cooperative approach. It is common practice to measure efficiency as the ratio between the aggregated payoffs and the welfare optimum. A consequence of such a limited investigation of efficiency is that the sacrifice of the subjects is not properly quantified. We will propose alternative, additional measurements.

A more crucial limitation of most experimental settings lies in the assumption that the group of actors is homogeneous. The OGW setting (1994) and its clones are symmetrical with respect to actors: all single-point solutions of cooperative game-theory prescribe the unique symmetric welfare optimum as a cooperative solution. Egalitarian and fairness solutions coincide, and both an explicit cooperative analysis and fairness considerations seem to be unnecessary. Things change if we turn to asymmetric settings: concepts from cooperative game theory as well as fairness concepts gain more power. A derivative of the OGW setting that introduces some heterogeneity was given by [12]. This HSW setting and the corresponding games will build the main part of our analysis.

Fairness refers to non-discriminatory and upright standards that even apply in situations with unequal partners. Imagine a situation in which there is a rich and powerful individual and a poor one. Equality rules would hurt the rich, and the extensive use of power would hurt the poor – it hurts their feelings, harms their individual rights, and does them material damage. Fairness rules establish respectful behavior and provide a solution that can be freely accepted by both sides – say a **fair compromise**.

In communication, not only proposals and claims can be exchanged, but also supporting and demanding **arguments** that may motivate the opponent to move. The exchange of proposals and arguments creates a common **virtual** world beside the basic relations found in reality. If the partners agree that a specific compromise is justified and binds them, then they change the situation into one in which compliance is not a problem.

In accord with fairness standards, the compromise should be reached by fair means, and fraud is to be excluded. If partners agree on fair play, they may agree to disagree – but they will not accept a proposal they intend to obstruct. Beyond fair play, not only a selfish motive, but also the feeling of being treated unfair, may cause obstruction.

In the forthcoming analysis, we investigate how communication may induce a fair compromise and stable outcome between actors in common pool situations. Analyzing solely the incentive structure of non-cooperative games and non-cooperative solution concepts like the Nash- and sub-game perfect equilibria can neither satisfactory explain that agreements are more efficient than expected nor the extent to which actors obey fairness standards. That non-cooperative solution concepts fail to explain observed behavior is mainly due by taking not into account the actor's incentives to cooperate. Moreover, non-cooperative analysis provides no instrument to quantify dissatisfaction of agents with respect to a proposed payoff distribution in order to judge if a proposal may be significantly considered as unfair. Having now the opportunity to bargain offers an environment where the prospects of cooperation can be discussed. But then, the arguments of power and fairness must be related to solution concepts, which are well known from cooperative game theory, to fill the gap left open by non-cooperative game theory. By doing so, we review in a first step certain important game properties which are important to obtain attractive and stable agreements in common pool situation. Then, in a second step, we introduce new concepts based on excess vectors to measure the extent to which subjects follow efficiency and fairness considerations. Finally, we conduct an experimental test in order to reanalyze the HSW-data set [12] in a game-theoretical analysis of cooperative common dilemma games. Thus, we propose to add constructs of **cooperative game theory** to our toolkit in order to advance the understanding of results in dilemma situations that allow for communication. The **commons dilemma game** noted in the title refers to the cooperative game induced by the commons dilemma situation.

We will proceed as follows: In preparation of the formal construct to be discussed, we have to introduce some notation. Specific games are defined; solution concepts and properties of the games are reported. As examples, two games are constructed that allow for a reanalysis of the asymmetric commons examined by [12]. After introducing **fairness solutions** that are provided from cooperative theory, the experimental data of [12] are reconsidered.

2 Commons Dilemma Situation and Games

In a common pool situation [11, 13], individuals have access to a natural or man-made resource, from which it is difficult to exclude somebody from use and where we observe rivalry in the consumption or yield. Individuals cannot be excluded from using the resource, or, for cases where it is feasible to exclude individuals, it is not economically meaningful to do so, because the costs of exclusion are prohibitive. For these kinds of economic situations, rivalry in consumption or yield means that a unit which has been consumed (extracted) by one individual cannot be consumed (extracted) by others. Thus, the consumption of the resource imposes negative externalities on the other individuals.[1] Examples of common pool situations are a fishery, ground water extraction or crude oil exploration to mention just some important real-life managerial problems.

It is widely assumed that rational agents who jointly manage a common pool resource (CPR) are confronted with a commons dilemma problem if they completely neglect the negative externalities they impose on other users. Pursuing only one's individual rational interest yields inefficient outcomes, and the common pool resource could be endangered because of an overuse by selfish individuals. Judgments like those are strongly supported by non-cooperative game theory, since the incentive structure induces an inefficient Nash equilibrium (cf. for instance with [5, 37, 10]). Therefore,

[1] Notice that the rivalry in use is the main difference from what are known as public good problems, where we observe no rivalry. Some experimental surveys related to public good games are given for instance by [18, 16]).

under the assumption of individual rationality, the corresponding model prescribes overuse, or even the destruction of the jointly used resource.

These results are widely challenged by field and experimental studies of situations in which an allowance is made for costless communication among the agents in the bargaining process, although the communication process has no effect on the incentive structure of the underlying game. The observed efficiency gains of allowing for communication in experiments are simply achieved because the individuals can now coordinate their strategies. This allows agents to extricate themselves from the commons dilemma situation. But when rational agents are able to extricate themselves from a commons dilemma situation, then non-cooperative game theory does not take into account the incentive for collective decision-making. In addressing this point, we have to rely on properties and solution concepts from cooperative game theory. In contrast to non-cooperative game theory, cooperative game theory allows agents involved to communicate with each other to make binding as well as non-binding agreements to coordinate their strategies. Moreover, cooperative game theory can also capture the fact that group members can be compensated with side payments, for instance, to achieve some internal fairness standards. This might happen in cases where subjects of equal strength have to contribute different amounts to achieve the maximal group payoff, and the group has agreed that subjects of equal strength should be treated equally by obtaining the same payoff. Therefore, whenever subjects allow for side payments, they can better solve the conflict between efficiency and fairness. As a consequence, for common pool situations with the opportunity of side payments, one might expect an enhancement of the efficiency compared to the situation in which side payments are not allowed, especially when individual heterogeneities that concern the endowment and cost situations have to be considered. Game properties and solution concepts from cooperative game theory provide us with clear ways of addressing the circumstances under which it is worthwhile for selfish agents to coordinate their economic activities in one entity or in sub-entities. In addition, concentrating on stylized cooperative bargaining arguments offers some insight into how agreement points can be stabilized. Theoretical results from cooperative game theory that foster the hypothesis – obtained from field studies – that the commons can be successfully managed by selfish individuals have been provided by [7, 8, 9], and by [22, 23].

Next we introduce the formal game-theoretical model from which the HSW experimental design was drawn.

2.1 Endowments, actions, payoffs

A popular formal definition of a **commons dilemma situation** consists of a quadruple:

1. a set $N := \{1, 2, \ldots, n\}$ of individuals,

2. an endowment $\omega_k \in \mathbb{R}_+$ for every member $k \in N$,
3. an individual (private) cost function $C_k(x_k)$, where $x_k \in \mathbb{R}$ for every member $k \in N$, and
4. a joint production function f, which is concave, that is defined by

$$f(s) := s \cdot (a - b \cdot s), \tag{2.1}$$

with $f(0) = 0$ and $f'(0) \geq \max_{k \in N} C'_k(0)$. Whereas $a, b \in \mathbb{R}$ are parameters of the joint production function and $s := \sum_{k \in N} x_k$, denotes the joint action, where $x_k \in \mathbb{R}$ for all $k \in N$.

2.2 Games

The induced **non-cooperative base game** can be formally written as a triple:

1. players set N,
2. the individual strategy space $X_k := [0, \omega_k]$ for all $k \in N$,
3. an individual payoff function U_k, defined by

$$U_k(\vec{x}) = u_k(x_k, y_k) = \frac{x_k}{s} \cdot f(s) - c_k \cdot x_k \qquad \text{for all } k \in N. \tag{2.2}$$

Notice that if $s = 0$, then it holds in the limit $U_k(\vec{0}) = 0$ for all $k \in N$.

The set of all possible strategy combinations is defined by $X := \prod_{k \in N} X_k$ and a strategy combination by $\vec{x} = (x_k)_{k \in N}$. A player i can select a strategy x_i from his strategy space $[0, \omega_i]$, whereas the strategy of the opponents is defined by $\vec{x}_{-i} = (x_k)_{k \in N \setminus \{i\}} \in X_{N \setminus \{i\}}$. The induced joint action of the opponents, denoted by y_i, is given by $y_i := \sum_{k \in N \setminus \{i\}} x_k$, thus we can specify the joint action by $s = x_i + y_i = \sum_{k \in N} x_k$. The vector of payoff functions $\vec{U} = (U_k(\vec{x}))_{k \in N}$ assigns an n-dimensional vector of individual payoffs for each strategy combination $\vec{x} \in X$. Formally, we identify a non-cooperative game by $\Gamma := (N, X, \vec{U})$.

The most popular solution concept of the theory of non-cooperative games is the **Nash equilibrium** [30]. It can be characterized as "the individual's best response to itself". This characterization can be interpreted by the following stability property: once known and propagated, no individual has an incentive to deviate from it. Defined more precisely: a strategy combination $\vec{x}$ is a Nash equilibrium if, for all $i \in N$, the action x_i maximizes i's payoff in situations in which the partners' actions $\vec{x}_{-i}$ are fixed. Since in our case the payoff only depends on total amounts, we can speak of a best response to y_i, or we can state that a best response is a maximizer of the function $u_i(\cdot, y_i)$. Denoting the set of best responses by $b(y_i)$, a Nash equilibrium fulfills

$$(x_i, y_i) \in (b(y_i), y_i) \quad \text{for all } i \in N.$$

In contrast, a **cooperative** or **transferable utility game** is a pair of

1. the players set N, and
2. a characteristic function which assigns to every subset S of N a value $v(S)$, representing the worth attainable for the coalition S by mutual cooperation.

Formally, we identify a cooperative game by the vector $v := (v(S))_{S\subseteq N} \in \mathbb{R}^{2^{|N|}}$, if no confusion can arise, whereas in case of ambiguity, we identify a game by $\langle N, v\rangle$.

Here, we consider the case in which the worth can be expressed by a single number $v(S) \in \mathbb{R}$. Such a value is interpreted as a payoff that the coalition can guarantee to itself and that can be freely distributed among the members of a coalition, i.e. $u(S) = \sum_{k\in S} u_k \leq v(S)$.

The non-cooperative payoff function is easily derived; the payoff is equal to the negative of the individual cost, plus a proportional share of the commons' product (2.2); whereas the definition of cooperative values that correspond to the commons situation is a little bit more sophisticated.

3 Coalitions, values

To get a simple formalism allowing for the definition of the corresponding cooperative game, we replace the n-dimensional vector of payoff functions $(U_k(\vec{x}))_{k\in N}$ with a single function called u, which maps two numbers x and y to a payoff $u(x, y)$. This will be achieved by two projections, which map the vector $\vec{x}$ to its $|S|$- and $|N\backslash S|$-coordinates respectively. By these projecting mappings, we will obtain two vectors, $\vec{x}_S = (x_k)_{k\in S}$ and $\vec{x}_{N\backslash S} = (x_k)_{k\in N\backslash S}$, respectively. The corresponding vector of payoff functions is $(u_k(\vec{x}_S, \vec{x}_{N\backslash S}))_{k\in N} = (U_k(\vec{x}))_{k\in N}$. If we fix a coalition S, then we can specify x as the sum of actions chosen by the coalition, i.e. $x := x(S) = \sum_{i\in S} x_k$, whereas y is the sum of the actions chosen by its opposition; hence $y = x(N\backslash S) = \sum_{k\in N\backslash S} x_k$. We can now identify the payoff of a coalition S by the single function $u(x, y)$, instead of writing the payoff as $\sum_{k\in S} u_k(\vec{x}_S, \vec{x}_{N\backslash S})$. This can be done because the payoff does not depend on how the actions are distributed internally. As a consequence, for every coalition S, we can identify a feasible strategy set that is restricted by the sum of the individual endowments of its members, thus the extended strategy set of a coalition S can be written as

$$\bar{X}_S := \left\{ \vec{x}_S \in X_S \mid \sum_{k\in S} x_k \leq \sum_{k\in S} \omega_k \right\}. \tag{3.3}$$

It should be obvious that $X_S \subseteq \bar{X}_S$. In accordance with the previous discussion, we can now map the strategy space of coalition S into the real numbers. This simplifies the strategy space to

$$\bar{A}_S := \left\{ x \in \mathbb{R}_+ \mid 0 \leq x \leq \omega(S) \right\}. \tag{3.4}$$

We define the best response set of a coalition S as

$$\bar{B}_S(y) := \left\{ x^* \in \bar{A}_S \mid u(x^*, y) = \max_{x \in \bar{A}_S} u(x, y) \right\}. \tag{3.5}$$

Note that $\bar{B}_S(y)$ denotes the set of best responses by coalition S with respect to y, the sum of the actions chosen by its opposition $N \setminus S$. We can use this simplified definition instead of $B_S(\vec{x}_{N \setminus S})$, which denotes the set of best responses by a coalition S with respect to the vector of joint actions $\vec{x}_{N \setminus S}$ chosen by the opposition $N \setminus S$, since in [23, Proposition 6.1, p.126] it has been shown that considering strategy sets with and without transfers is without consequence for the best response sets, and therefore the enlarging of strategy sets has no effect on the maximal payoff a coalition S can guarantee to itself.

From the normal form game, we can model different types of arguing in the bargaining process. In the following, we focus on the α- and β-games (or characteristic functions), as introduced by [44] and further developed, especially by [1].[2]

Say coalition S will chose the joint action x, then the poorest result they can get is the payoff $\min u(x, y)$. Choosing a joint action x that maximizes this expression, we get a value that coalition S can guarantee to itself. This value is called α-value. By using α-values, the α-game is derived. More formally, we obtain the following definition:

$$v_\alpha(S) := \max_{x \in \bar{A}_S} \min_{y \in \bar{A}_{N \setminus S}} u(x, y). \tag{3.6}$$

If it is known that the opposition chooses action y, then the coalition may choose its best response, and it will get $\max u(x, y)$. Choosing an action y that minimizes this expression, we get a value that coalition S can not be prevented from. Whatever the opposition tries to do, the coalition may get at least this value. This is the way the β-value is derived from and the β-game is defined by

$$v_\beta(S) := \min_{y \in \bar{A}_{N \setminus S}} \max_{x \in \bar{A}_S} u(x, y). \tag{3.7}$$

In general, the β-value is equal to or greater than the α-value. This can be interpreted as indicating that there is weak incentive to react passively by awaiting the joint action of the opponents. Moreover, the most prominent solution concept in cooperative game theory, known as the core, will be different. This has some negative side-effects on stabilizing an agreement point inside of the α-core that does not belong to the β-core. In this case, we can expect that some bargaining difficulties will appear. But in 1994 the following theorem was proven in [32].

[2] In the literature, a third type of arguing was discussed, the s-types introduced by [26, 27] for two person games, and generalized for $n \geq 2$ in the γ-value by [31, 32]. For these games, the opposition does not rely on the complete strategy set to stabilize proposals as in the α- and β-game; it relies instead on the best response set. Strategies that hurt a coalition are not selected by its members.

Theorem 3.1 ([32]). *Common pool games are clear games, i.e.* $v_\alpha = v_\beta$*.*

The notion of **clear games** was introduced to the literature by [15]. Common pool games, as well as public good games, are clear games, i.e. α- and β-values, coincide. That means that agreement points inside the α-core that can be stabilized by using α-arguments cannot be countered by relying on β-arguments. No bargaining problems will appear.

Before we explain the solution concept core, let us stress the fact that, according to **experimental** research on cooperative games based on a normal form game, these two properties of the game ease agreements and stability. The first point is caused by the availability of attractive proposals, namely elements of the core. The latter, the stability, can be explained in reference to the experience known from negotiating, that waiting or reaction does not pay extra.

4 Equilibrium, core, and convex games

Let us recall some definitions of the most popular solution concepts. For non-cooperative games, it is the Nash equilibrium, which is defined by the property of being an individual best response to itself. No individual has an incentive to deviate from the equilibrium if it is arranged for. From the best response property, we can derive that individual payoffs in equilibrium can not exceed the β-values of the corresponding single-player coalitions. For commons dilemma situations, the usual finding is that equilibria are inefficient. Cooperative solution concepts are efficient because they distribute the optimum $v(N)$, i.e. the payoff vectors $\vec{u} \in \mathbb{R}^N$, satisfying the principle $u(N) = \sum_{k \in N} u_k = v(N)$.

Consider now the excess of a coalition S that is expressed by the formula $e(S, \vec{u}) := v(S) - u(S)$. Members of S can argue that no proposed distribution $\vec{u} \in \mathbb{R}^N$ is acceptable if the coalition S gets less than $v(S)$, which means that the excess for the coalition S is positive, i.e. $e(S, \vec{u}) > 0$. From the viewpoint of members in S, the value $v(S)$ is a sure gain; a lower gain from accepting $\vec{u}$ can be seen as a loss. Therefore, we can say that the smaller the excess of a coalition S, the better off the coalition will be at an allocation $\vec{u}$.

Now, the core [3, 42] is the set of all efficient payoff vectors such that no coalition suffers a loss, that is

$$\mathcal{C}(v) := \{\vec{u} \in \mathbb{R}^n \mid e(N, \vec{u}) = 0,\ e(S, \vec{u}) \leq 0 \quad \forall S \in 2^N \backslash \{\emptyset, N\}\}. \tag{4.8}$$

For every payoff vector inside the core $\mathcal{C}(v)$, no coalition can improve upon or block an agreement; that is, multilateral cooperation is better than acting unilaterally. For an non-empty core, the grand coalition can distribute the highest profit to its members; we have a weak incentive for merging economic activities into a monopoly.

Games may exhibit an empty core. But for common dilemma games, the core can be expected to be large. This can be derived from the convexity property of such games, proven by [22].

Theorem 4.1 ([22])**.** *Common pool games are convex games.*

A game v is said to be **convex** or **supermodular** if the value added by an individual increases with respect to the coalition size to which the individual is merged, i.e. $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$ for all $S \subseteq T \subseteq N \backslash \{i\}$. Convexity or supermodularity expresses non-decreasing marginal returns with respect to the coalition size; that can be interpreted as large scale cooperation. Intermediate coalitions are relatively weak in providing relative small profits to their members compared to the profit the grand coalition can make available to the whole member set. Joining a larger coalition yields an over-proportional surplus, which imposes a strong incentive for mutual cooperation on the grand coalition. In this case, we observe a strong incentive to merge economic activities into a cartel or monopoly ([cf. 8, 9]).

4.1 Example (HSW1)

Our theoretical results discussed in the previous sections are valid for a large class of games (cf. [23, 7, 8, 9]). Empirical results are reexamined only for two representative games of the Bloomington type. This is why we restrict our attention in the following analysis to common dilemma situations, as reported by the Bloomington group. Their standard experimental commons use a repeated game framework. Our attention is concentrated on a setup enriched by explicit communication breaks (cf. [36]). Moreover, we are going to deal with the asymmetric setup examined by [12].

In a non-cooperative setting, finitely repeated games can be "solved" by backward reasoning: the (dynamic) equilibria consist in a series of one-shot equilibria. Nevertheless, acting "beyond equilibrium" may involve strategies that are dependent on history. It is known from experiments that some of these strategies – the most famous one is tit-for-tat in the context of prisoners' dilemma games – can be a rather successful substitute for explicit cooperation. Since the settings considered here provide for communication breaks after each round, we assume that, as a first approximation, we can abstract from history dependency. Whenever we detect some history dependency, we will report on this. The cases of history dependency we observe are of two different kinds: first, there are bargained agreements explicitly made for more than one stage game; second, in case of broken agreements, these "non-cooperative" events can destroy the trust in future agreements.

In **experiments** with communication (and without binding agreements), referring to symmetric situations, [35] found evidence that results are far from the inefficient (non-cooperative) equilibrium, and communication often leads to near-optimal results. Moreover, most agreements found during the communication phase are stable in the sense that from a (private) decision once made, nobody deviates from it. Nevertheless, it can be argued that, in such symmetric situations, it is easy to establish cooperative solutions because, according to all normative standards, single valued solutions result in equal shares, even without referring to any strategic considerations. This changes when there is asymmetry. In such a case, actors, respectively, subjects, favored by the conditions have to find convincing arguments in support of the larger share they would like to get. An experimental study with an asymmetric common is reported in [12].

In the **asymmetric HSW experiment**, a group of eight subjects participates in two consecutive 10-round sequences. In the first 10 rounds (not to be consider here), the subjects are not allowed to communicate, while in the last 10 rounds subjects have the opportunity to communicate with each other. Asymmetry is captured by two different values for the endowment corresponding to two different roles – say the "very rich" and the "very poor". Half of the group gets high endowments; the other half gets low ones.

Let us now reconsider the **non-cooperative game** Γ used in the experimental setup by [12]. Hackett, Schlager, and Walker decided to choose the following parameters for the CPR setting:

$$N = \{1, 2, 3, 4, 5, 6, 7, 8\},\ \vec{\omega} = \{8, 8, 8, 8, 24, 24, 24, 24\},$$
$$a = 33,\ b = .25,\ c_k = 5 \quad \forall k \in N.$$

For this particular setup, the joint production function (2.1) becomes

$$f(s) = s \cdot (33 - .25 \cdot s). \tag{4.9}$$

The two different types consist of four players of the "very poor" type, with an endowment of 8 tokens per round and player, and four players of the "very rich" type, with 24 tokens per round and player. The unique **Nash equilibrium** of the game prescribes that the rich players choose 16 tokens, an amount smaller than their endowment, while the poor players have to allocate their total endowment, that means 8 tokens. The Nash equilibrium produces the following payoff vector for the agents:

$$u^* = (32, 32, 32, 32, 64, 64, 64, 64).$$

The aggregate payoff for the "rich" corresponds to 256, and the respective aggregate payoff for the "poor" amounts to 128. In contrast, the welfare optimum with equal shares prescribes an individual allocation of 7 tokens, resulting in a 98 units payoff for each player.

The corresponding cooperative common pool game v is derived from the normal form game by applying formula (3.6) for each coalition. We can proceed in this way, since, due to Theorem 3.1, cooperative CPR games are clear. In light of the two different types of players, the number of coalitions we need to consider reduces from 256 to 25. The values of the reduced set of coalitions are given by Table 4.1. Note that the extreme four-person coalitions get 16 and 400 respectively. The latter value is much larger than 256, the coalition's payoff in equilibrium, which is two third of the total equilibrium payoff, since 256/384. Observe, in addition, that the egalitarian allocation, i.e. the 98 units that every player gets as payoff, is not a core element, since the extreme coalition of all rich players $S = \{5, 6, 7, 8\}$ can block the proposal. The rich players coalition can guarantee to itself a payoff of 400 units, that is, its coalition value, in contrast to a payoff of 392 units, which it can obtain if their members would accept the egalitarian division (cf. Table 4.1 which is taken from [22]).

In the last part of the paper, a more complex setting is added, which splits the formerly rich into two pairs: the "very rich" and the "moderately rich". It also splits the formerly poor into two pairs: the "very poor" and the "moderately poor".

Table 4.1: HSW1 Coalitional Values for Different Profiles

No. of rich, poor	0	1	2	3	4
0	0	0	0	4	16
1	4	16	36	64	100
2	64	100	144	196	256
3	196	256	324	400	484
4	400	484	576	676	784

The commons dilemma situation, as discussed in [12], is derived by introducing asymmetry with respect to endowment into the [35] standard setting. There are four rich individuals and four poor ones. Table 4.1 shows the values of the cooperative game.

4.2 Example (HSW2)

As in the previous scenario, the **more complex HSW experiment** consists of a group of eight subjects participating in two consecutive 10-round sequences, one with no communication, and one with communication. Now, asymmetry is boosted by introducing four different values for the endowment, corresponding to four different roles – say the "very rich", "moderately rich", "moderately poor" and the "very poor". This experiment departs from the previous setup, in which the endowments are completely randomly assigned to the players (cf. [12, pp. 120-121]). The endowment vector changes to

$$\vec{\omega} = \{8, 8, 12, 12, 20, 20, 24, 24\},$$

whereas the other parameters remain unchanged. The unique **Nash equilibrium** prescribes that the poor types allocate the full endowment with 8 and 12 tokens, while the rich types allocate 14.4 tokens in the CPR. The Nash equilibrium induces the following payoff vector to the agents:

$$u^* = \{28.8, 28.8, 43.2, 43.2, 51.84, 51.84, 51.84, 51.84\}.$$

In the Nash equilibrium, the poor types obtain an aggregate payoff of 144, and the total payoff of the rich types is about 207.36. For the HSW2 experiment, the tokens were not divisible, which implies that the Nash equilibrium is not unique, and the rich types could here have made decisions to invest 14 or 15 tokens in the CPR (cf. [12, p.121]).

The coalition values of the cooperative CPR game v are derived from (3.6). For the game HSW2, there are $3^5 = 243$ different profiles. As in the previous setup, we are able to reduce the number of coalitions by relying on the symmetry property of players. Due to four different types of players, we can reduce the number of different profiles from 243 to 81. Nevertheless, because keeping the overview of 81 different types of coalitions might be quite inconvenient, we have delegated the coalitional values for the different profiles in the Appendix and they are listed in Table A.4 and Table A.5. We summarize the results of the calculations for all profiles in a table similar to that used for HSW1. This is done by joining the two poorer types on the one side and the two richer types on the other side. Correspondingly, we get 4 crude pseudo-profiles. As a consequence, some entries in Table 4.2 show intervals to which the value of a corresponding coalition belongs.

Table 4.2: HSW2 Coalitional Values for Different Pseudo-profiles

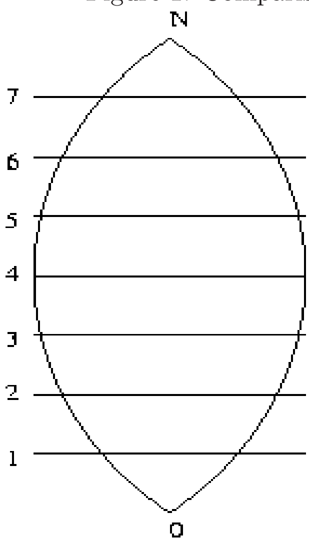
No. of rich, poor	0	1	2	3	4
0	0	0	0-4	9-16	36
1	1-4	9-25	25-64	64-100	121-144
2	36-64	64-121	100-196	196-256	256-324
3	144-169	196-256	256-361	361-441	484-529
4	324	400-441	484-576	625-676	784

4.3 Example (HSW1 vs. HSW2)

In both games, equilibrium prescribes that the four poor individuals are restricted by their endowment. Compared to the HSW1 setup, the total payoff of the Nash equilibrium is lowered from 384 to 351.36. In both setups, the welfare optimum is achieved at 784, and this implies that the overall efficiency measured in terms of the the Pareto-optimum is lowered as well. In addition, we observe that the rich types suffer a loss in this new setup by obtaining an aggregate payoff of 207.36 instead of 256, while the poor types gain in the new setup by obtaining in total 144 instead of 128. Nevertheless, the payoff share of the rich at the Nash equilibria, measured in terms of the total payoff that the rich coalition can assure in both settings, 400 and 324 respectively, remains unchanged at 64%.

Raising the diversity has lowered the equilibrium payoff and changed the values of middle-sized coalitions, as can be seen from Figure 1. If we have a closer look on the data given for the extreme four person coalitions in the HSW2 setting, this reveals that the poor types have realized a gain, while the rich types have suffered a loss. More precisely, the poor types obtain 36 and the rich types get 324, instead of 16 and 400 as in the HSW1 setup. In general, we observe that the poor (or rich) types benefit (or lose) from the redistribution of endowments when they belong to a middle-sized coalition. As a consequence, the value spread for the middle-sized coalition has decreased. One interesting point that is worth mentioning here is that the additional entitlement of endowments for the poor types does not have any impact on some coalitions at the margin.[3] The increase in the endowment for these coalitions does not increase their power to extract larger parts from the yield or to detain the opposition in its harvesting activity.

Figure 1: Comparison of the values for different HSW setups



HSW1	$\#S$	HSW2
784	8	784
484 - 676	7	484 - 676
256 - 576	6	256 - 576
100 - 484	5	**121 - 441**
16 - 400	4	**36 - 324**
4 - 196	3	**9 - 169**
0 - 64	2	0 - 64
0 - 4	1	0 - 4

5 Fairness solutions

In the previous discussion, we have introduced the core concept that specifies all efficient and coalitional rational payoff distributions. We argued that agreements in the core are favorable outcomes, which are self-enforcing. Agreements might be reached by exchanging claims and arguments in some bargaining process. But so far we have not been able to judge if an agreed upon core allocation is really a fair outcome. If one is only concerned about its own favorable outcome, it is relatively simple to decide on the approval or disapproval of some proposal. It is sufficient to judge if this proposal provides a higher share than the outcomes one can attain in various coalitions

[3]For more detailed information, consult Table A.4 and Table A.5 in the Appendix.

to which one can belong. No information about the payoffs of the opponents is really needed to evaluate if this has improved one's own situation or not. In contrast, if one is also concerned about some fairness standards, then in order to judge the fairness of a proposal, one needs information on one's own payoff share and the payoff shares of the opponents, as well as a set of subjective or objective principles to specify rules of arbitration. Cooperative game theory provides more than one fairness standard. Of course, micro-economic theory also provides some fairness standards like the competitive equilibrium with equal incomes or the virtual price solution that has been applied in the analysis of commons dilemma theory as a fair compensation rule (cf. [28, chap. 5.]). Such fair division rules will not be discussed here, since they base the notion of fairness on the indirect utility functions of agents, which do not take into the account either the non-cooperative or the cooperative structure of the game. The point of view from which we consider fairness is methodologically different: we want to base our fairness discussion on objective principles on which subjects can agree or disagree. We do not want to view fairness as the private property or a subjective feeling of an individual; rather, we want to deal with two well-known fairness solutions from cooperative game theory, which are conceptually based on a set of objective principles. These are the **Shapley value** and the **kernel**. The **Shapley value** [41] assigns a unique payoff vector to every game. We can classify this payoff vector as a fairness solution to the game by referring either to a set of principles or to a simple rule of distributive justice. The four principles (axioms) are described as follows:

1. (**efficiency**): the solution should distribute the maximal total payoff
2. (**symmetry**): any two players who contribute the same input should obtain the same payoff
3. (**dummy player**): any player who contributes nothing to any coalition should obtain his value
4. (**additivity**): adding the solution of two games together produces the solution of the sum of these games (in this sense, the solution is invariant against an arbitrary decomposition of the game)

The **Shapley value** is the unique scheme of the game v that satisfies these principles.[4]

It is easy to show that the **Shapley value** can be characterized by the following simple rule of distributive justice: every player should receive his/her mean contribution to the coalitional values (cf. [38]). In order to characterize the Shapley value, let us first define $s := |S|$ for all $S \subset N$ and $n := |N|$: we get the mean value added by considering the $n!$ different orderings in which the players may appear, one after another. There are

[4] An informal discussion of the axiomatic characterization of the Shapley value can be found in [28, pp. 159-162].

$(n-s-1)!\ s!$ orderings in which a coalition S is assembled and a specific player i comes next. Setting $\pi(S) = (n-1-s)!\ s!/n!$, the Shapley value is given by the following formula:

$$\phi_k(v) = \sum_{S \subseteq N\backslash\{k\}} \pi(S) \cdot (v(S \cup \{k\}) - v(S) \qquad \forall k \in N. \tag{5.10}$$

The weight $\pi(S)$ can be interpreted as the probability that S is already assembled; and in this sense the Shapley value ϕ is a vector of mean contributions. In addition, the Shapley value satisfies the balanced contributions property [29, Proposition 2], that is, for all $i, j \in N, i \neq j$, the Shapley value satisfies:

$$\phi_i(N, v) - \phi_i(N\backslash\{j\}, v) = \phi_j(N, v) - \phi_j(N\backslash\{i\}, v), \tag{5.11}$$

where $\phi_i(N\backslash\{j\}, v)$ and $\phi_j(N\backslash\{i\}, v)$ are the payoffs distributed by the Shapley value to player i and player j under the subgames $\langle N\backslash\{j\}, v\rangle$ and $\langle N\backslash\{i\}, v\rangle$ respectively. Note that $\phi_k(N, v) = \phi_k(v)$ for all $k \in N$. It should be apparent that the difference given in formula (5.11) must be positive for convex games. Thus, for convex games, players have no incentive to exclude one another in order to form smaller coalitions than the grand coalition.[5]

The axiomatic properties of the Shapley value, its probabilistic interpretation as the mean contribution and its balanced contributions property are reasons for its attractiveness as a fairness standard.

The other fairness solution that we want to discuss briefly is the **kernel**.[6] The kernel has the advantage of addressing a stylized bargaining process, in which the figure of argumentation is a **pairwise equilibrium procedure** of claims. The game theoretical independent principle of a pairwise bargaining process that equalizes the claims between each pair of agents was recommended in the Babylonian Talmud[7] to solve bankruptcy problems. The unique Talmudic rule is a generalization of the contested garment principle,[8] which satisfies a consistency and self-duality property that assigns surplus and loss in the same way (cf. [2]). One can interpret the Talmudic rule as a hybrid of uniform surplus and uniform loss solutions [28, p.58]. One interesting fact that is worth mentioning here is that the division suggested by the Talmudic rule for solving bankruptcy problems is not based

[5] In particular, the Shapley value qualifies as a solution concept which connects cooperative and non-cooperative games (cf. [39, 4]).

[6] The kernel as a solution concept in cooperative game theory was introduced by [6] to study general existence results and properties of the bargaining set. The kernel solution concept relies on the idea of excess and maximum excess.

[7] A 2000 year old document that forms the basis for Jewish civil, criminal, and religious law.

[8] The contested garment principle is a division rule from the Talmud on how a garment should be divided between two persons when one claims the whole garment and the other claims half of it. The principle says that the lesser claimant cedes half of the garment to his opponent and therefore the remaining half should be divided equally between them, since both of them can assert claims on the remaining half. According to this rule the first claimant gets 3/4 and the second claimant gets 1/4 of the garment (cf. [2]).

on the proportional rule to award the claims, which is often considered as a fair division rule. The unique numbers presented in the Talmud to solve particular bankruptcy problems coincides with the kernel solution or the nucleolus of the corresponding bankruptcy game (cf. [2], [43]). Therefore, the kernel solution must possess a particular consistency property (cf. [19]). Consistency is widely regarded as a desirable property of a solution in cooperative game theory. Whenever the solution to an appropriately defined reduced game yields to the same allocations that have been obtained in the original game, then it is viewed as consistent under this specific rule. No subgroup of agents has an incentive to deviate from the original proposal and to play their own game in order to improve their situation. Consistency can be regarded in some sense as the sub-game perfection of a cooperative solution concept. The **Shapley value** does not satisfy the described kind of sub-game perfection,[9] which we consider essential for stabilizing fair proposals.

To see how the pairwise bargaining process we have in mind works, let us first observe that an efficient allocation $\vec{u}' \in \mathbb{R}^N$ is more favorable than an efficient allocation $\vec{u} \in \mathbb{R}^N$ for a coalition S whenever $e(S, \vec{u}') < e(S, \vec{u})$ is satisfied. While for a certain player i – who could be member in coalition S_0 and S_1 – the coalition S_0 is more favorable than the coalition S_1 if $e(S_0, \vec{u}) < e(S_1, \vec{u})$ holds for a payoff vector $\vec{u} \in \mathbb{R}^N$. The dissatisfaction of an individual i, measured in terms of the excess she/he would obtain at the payoff $\vec{u}$, is greater if she/he belongs to coalition S_1 than if she/he is a member of a coalition S_0. In a pairwise discussion concerning some efficient payoff proposal $\vec{u}$, every individual can refer to the largest loss (measured against the excess function) she/he faces as member of a coalition to which his opponent does not belong, that is

$$s_{ij}(\vec{u}) := \max_{S \in \mathcal{G}_{ij}} e(S, \vec{u}) \qquad \text{where } \mathcal{G}_{ij} := \{S \mid i \in S \text{ and } j \notin S\}. \tag{5.12}$$

The expression $s_{ij}(\vec{u})$ describes the maximum loss at the efficient payoff vector $\vec{u}$ that player i will receive without relying on the cooperation of player j.

Now, we can consider a stylized pairwise bargaining procedure where agent j makes the proposal $\vec{u}$ to agent i. If, for a proposal $\vec{u}$, the largest loss of the agent i is greater than the largest loss of the agent j, i.e. $s_{ij}(\vec{u}) > s_{ji}(\vec{u})$, and agent i has better alternatives available, the proposal is refused for being unfair. Agent i can express his dissatisfaction with proposal $\vec{u}$ to his opponent j and enjoin him to render to him a share of his claim in order to balance their losses. Agent i can justify his claim on the grounds that there are proposals available to him that produce smaller losses for

[9] Note that the consistency or reduced game property satisfied by the Shapley value (cf. [14]) is different from the pre-kernel or pre-nucleolus in the sense that partners from outside will be paid according to the solution of the new game instead of the orginal game. Furthermore, the pre-kernel possesses in addition to the Shapley value the converse reduction game property. For a more thorough discussion, we refer the interested reader to [19].

him, without counting on the cooperation of agent j. The proposal $\vec{u}$ is contestable and therefore unstable. Certainly, by balancing the losses among each pair of agents, we allow an agent to obtain less than his value. This situation dismisses individual rationality. But for cooperative CPR games which, due to Theorem 4.1, satisfy the convexity property, there is no need to care about individual rationality, since each agent will receive at least his individual rational payoff when the losses among all pairs of agents are equalized. Furthermore, in cases of cooperative CPR games, the core is not empty, the largest losses are negative and thus the pairs compare their smallest gains in order to equalize them. For the case of zero-monotonic games,[10] as in the subcase of convex games, the set of all efficient allocations $\vec{u}$ that balances the maximal excesses for each distinct pair of players $\{i,j\}$ is called the **kernel** of the game v, and is defined by

$$\mathcal{K}^*(v) := \{\vec{u} \in \mathcal{I}^*(v) \mid s_{ij}(\vec{u}) = s_{ji}(\vec{u}) \quad \text{for all } i,j \in N, i \neq j\}, \tag{5.13}$$

where $\mathcal{I}^*(v) := \{\vec{u} \in \mathbb{R}^N \mid v(N) = u(N)\}$.

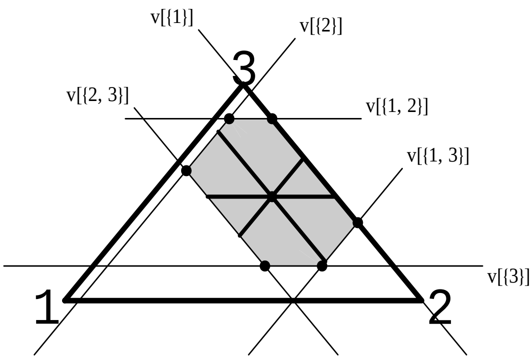
Now, let us rely on a pairwise equilibrium procedure of claims for each possible pair of players in discussing an intuitive interpretation of the kernel solution that was given in the literature.[11] [20, p.330] considered a payoff vector $\vec{u}$ that was an element of an non-empty core $\mathcal{C}(v)$. From the selected payoff vector $\vec{u}$, a certain bargaining range for a pair of agents $\{i,j\}$ inside the core can be specified. Assume that, for the proposal $\vec{u}$, the other agents who do not participate in the bargain receive their allocation in $\vec{u}$. Now, agent i can press agent j for an amount $\delta_{ji}(\vec{u})$, while agent j cannot find any coalition that contains him as a member and i as a non-member to resist the claim of agent i. If agent i tries to claim more than $\delta_{ji}(\vec{u})$, these can be countered by j, so that she/he is able to establish at least a coalition to block the claim of i. By pressing the amount $\delta_{ji}(\vec{u})$ from agent j, she/he is pushed against a wall (core boundary); trying to push agent j beyond the wall is not possible. Under such circumstances, she/he can present "best arguments", that is, the coalition that will support her/his claim. This endpoint of the bargaining range represents agent i's maximal claim against agent j that j cannot oppose by forming a coalition to block the claim. The kernel solution characterizes all those allocations in which all pairs of players $\{i,j\}$ are symmetric with respect to their bargaining range. The kernel consists of those elements that split the bargaining range in half for any pair of players. In this sense, the kernel can be understood as a multi-bilateral bargaining equilibrium.

The geometrical characterization of the kernel as a solution that splits the relevant bargaining ranges within the core in half is elucidated for a

[10] Note that a transferable utility game is called zero-monotonic if $v(S) \leq v(T) - \sum_{i \in T \setminus S} v(\{i\})$ whenever $S \subseteq T \neq \emptyset$ is satisfied.

[11] In the manuscript, we will use the term kernel instead of pre-kernel, although the definition given here is actually the pre-kernel definition. For zero-monotonic games and therefore for convex games, the kernel and the pre-kernel solution coincide, hence we can dismiss the prefix and we can speak of the kernel solution to simplify the notion.

three person HSW1 sub-game by the next figure. The sub-game is constructed from the original eight person HSW1 game by merging the players $\{1,2\}, \{4,5\}$ and $\{7,8\}$ to new types. In this case, the poor player 4 and the rich player 5 form a new type, a middle class type.



$$
\begin{aligned}
&v(\{1\}) = 0,\\
&v(\{2\}) = 16,\\
&v(\{3\}) = 64,\\
&v(\{1,2\}) = 64,\\
&v(\{1,3\}) = 144,\\
&v(\{2,3\}) = 256,\\
&v(\{1,2,3\}) = 400,\\
&\{80, 136, 184\} \in \mathcal{K}^*(v)
\end{aligned}
$$

Figure 2: Kernel solution splits the relevant bargaining ranges in half

For games that are convex, as in our case, the kernel is a single point in the core, and it coincides with the other prominent fairness solution discussed in cooperative game theory: the **nucleolus**.[12] It is a unique efficient payoff vector that simultaneously minimizes the losses of coalitions. The following theorem summarizes the important properties proven in [21] and [20], which are of particular relevance for cooperative CPR games.

Theorem 5.1. *For convex games, the kernel is a singleton that coincides with the nucleolus and belongs to the core.*

Now, we are in the position to introduce the kernel (nucleolus) solutions for the HSW1 and HSW2 setup. For symmetric games, the kernel solution is trivial, because it assigns $v(N)/n$ to each player. For asymmetric games, like ours, the task of computing the kernel solution is more cumbersome. Trying to solve the system of nonlinear equations given in the definition of the kernel is hopeless. Accordingly, the calculations for the kernel (especially of the second example game) had to be run on a computer.[13] Nevertheless, to check that our computed solutions make sense, we note that players of equal strength should receive the same payoff. In the HSW1 setup, we

[12] [40] was led to discover the nucleolus by studying the intersection of the kernel with the core. Notice that due to the convexity of the cooperative games, we can dispense without loss of generality from the nucleolus definition. In our case, it is just enough to refer to the pre-kernel definition. The exact definition of the nucleolus can be found in the Schmeidler article or in [19].

[13] The algorithm to compute the kernel of the game is described in [25], while the documentation of the corresponding computer program can be found in [24]. Moreover, the corresponding computer program can be downloaded from http://library.wolfram.com/infocenter/MathSource/5709/.

identified two different types, whereas in the HSW2 setting, four different types appeared. Thus, in the HSW1 setting, we can expect two different payoff values, and in the HSW2 setup, we can expect four different payoff values to capture the different types of players. Moreover, more powerful or more desirable types receive higher payoffs. In Table 5.3, we present the kernel ν, Shapley value ϕ and the Nash equilibrium u^* for the different HSW setups.

Table 5.3: Game theoretical solutions for the different HSW setups

	HSW1 versus HSW2
ν_1	$\{54, 54, 54, 54, 142, 142, 142, 142\}$
ν_2	$\{216, 216, 318, 318, 511, 511, 523, 523\}/4$
ϕ_1	$\{348, 348, 348, 348, 1024, 1024, 1024, 1024\}/7$
ϕ_2	$\{1389, 1389, 2073, 2073, 3421, 3421, 4093, 4093\}/28$
u_1^*	$\{32, 32, 32, 32, 64, 64, 64, 64\}$
u_2^*	$\{720, 720, 1080, 1080, 1296, 1296, 1296, 1296\}/25$

In the HSW1 or HSW2 setting, in trying to reach a fair agreement on how to split the benefits of mutual cooperation, we can rely on the payoff distribution assigned by the kernel or the Shapley value. The kernel and Shapley value belong to the core; that is, no coalition can block such proposals, nor can a player propose an objection against the kernel or Shapley value that cannot be countered by formulating a counter-objection. In this sense, both fairness standards can be stabilized in the HSW settings.

6 Hackett-Schlager-Walker data

In the previous section, we covered the game theoretical models that reflect the conflict situation behind the joint management of a common pool resource in both the non-cooperative and the cooperative view. We argued that in respective situations in which the actors involved can communicate and can negotiate for an agreement about how to coordinate actions and/or how to allocate payoffs, it is necessary to consider cooperative solution concepts, even if the agreements cannot be guaranteed to be binding. From the foregoing discussion, we know that the equilibrium is not efficient for a CPR and that the underlying cooperative game is convex, so there is a strong incentive for cooperation. The extent to which people dealing with such situations can establish cooperation and can enhance their results by arranging and implementing favorable agreements is now an empirical question.

In Section 4, we introduced and analyzed the two asymmetric games on which the experiments of the Hackett-Schlager-Walker study are based (cf. [12]). Results show, in an impressive way, that also in the asymmetric case, even just introducing communication enhances the efficiency considerably. In most cases, subjects find stable agreements realizing the gains of cooperation.

As noted in Subsection 4.1, in the asymmetric HSW experiments, a group of eight subjects participates in two consecutive 10-round sequences. In the first 10 rounds (not to be considered here), it is not allowed for the subjects to communicate, while in the last 10 rounds, subjects have the opportunity to communicate among themselves. After the communication break ends, with or without an agreement, subjects made their unobserved and independent decisions. That means that even when the agreements were attained, subjects had the freedom to follow or to deviate from the agreement, since they were not binding. Thus, subjects followed an agreement if they thought that it is in their own interest to do so. In the following, we consider the data from the rounds with communication.

In the HSW experiment based on the first game (cf. 4.1), half of the group gets high endowments, the other half gets low ones. There are two different modes of assigning the endowments. In the "random" condition, subjects are randomly assigned to roles. For the second condition, the authors install an auction, where subjects can bid for the right to become rich (cf. [12, pp. 110-112]). In the following, we refer to the 8 trials with 10 rounds within each communication included. This data set contains 4 random trials and 4 auction trials.

In the HSW experiment based on the second game (cf. 4.2), Hackett et al. speak of the "complex condition". There are four different endowments, each randomly assigned to two subjects of the group. We refer to the 30 rounds with communication reported from the three trials of the experiment.

To get a first impression about how payoff results are distributed, let us consider Figure 3 (see Appendix). In order to make the results based on the two different games comparable, we locate the results in a two-dimensional space by summing up (1) payoffs of the 4 richest subjects and (2) payoffs of the 4 poorest subjects. In this representation, in addition, we can locate the core vertices, the nucleolus, the Shapley value, the Nash-equilibrium, the egalitarian solution and the average payoff. In Figure 3, the star nearest to the origin represents the Nash-equilibrium. The declining line corresponds to the welfare optimum, otherwise called the Pareto-frontier or region of efficiency. The points at the lower right on the Pareto-frontier represent the vertices of the core, the nucleolus and the Shapley value. The two straight lines which emanate form the origin represent the equal payoff sharing and the payoff sharing, according to the endowment distribution. Most of the payoff data points lie in between the two straight lines, which emanate from the origin and are close to the Pareto-frontier.

In both experiments, most payoff results are located within the triangle between (the inefficient) equilibrium, the egalitarian and the fairness solution; but not a single data point lies within the core. Lying outside in this sense is more frequent in the 4-type scenario, and results seem to be more efficient in the 2-type scenario.

7 Agreements, compliance, efficiency of results

A closer look reveals that the poorer performance in the 4-type scenario is mainly caused by two facts: first, the frequency of the failure to reach an agreement is large (9 out of 30 rounds); and second, compliance is notably lower than in the 2-type scenario.

For the game with only two types (HSW1), failure to reach an agreement is concentrated to trial 144; similarly, defection is concentrated to trial 150, where a rotation scheme is used. (Efficiency 93,6%, failure rate 10%, individual defection rate 6,4%.)

For the second game with four types (HSW2), the efficiency is reduced to the level that was reached in the other experiments with defection (efficiency 83,4%). Compared to the first experiment, the failure rate triples (30%) and the individual defection rate doubles (12,9%). Two of the three trials use rotation schemes that may invite defection.

Figure 3, considered in Section 6, suggests that the subjects tend to have both egalitarian and efficiency considerations, since so many payoff data points are lying relatively close to the egalitarian solution and to the welfare optimum. For a clearer judgment of how close they come to normative solutions, we have to introduce exact measures. We are especially interested in getting to know the extent to which they follow fairness considerations as well. To come up with the respective judgments, we now rely on excesses of the results; we also report distances between the data-point and normative solutions.

In Section 4, we introduced the notion of excess. The vector of excesses

$$(e(S,\vec{u}))_{S\subseteq N} = (v(S) - u(S))_{S\subseteq N}, \tag{7.14}$$

at some proposed or realized payoff $\vec{u}$ contains the information what the members of the respective coalitions have (or had) to give up at the payoff distribution $\vec{u}$.

Thus the variable **excess for the grand coalition**

$$gcex = gcex(\vec{u}) = v(N) - u(N), \tag{7.15}$$

can be interpreted as an efficiency measure, in addition to the efficiency measure relating the aggregate payoffs to the welfare optimum (cf. [12, p.

116]). According to this measurement concept, we are able to quantify the sacrifice with respect to the welfare optimum resulting from the respective agreement or result. In that sense, the variable *gcex* **measures inefficiency**. Observe that, due to convexity, the grand coalition is the coalition that normally will show the largest excess or, say, loss, since the exhaustive potential of the grand coalition is larger than the intermediate coalitions.

Nevertheless, there are extreme cases in which the variable *gcex* of an agreement does not coincide with the **maximum excess**

$$mex = mex(\vec{u}) = \max_{S \subseteq N} \{v(S) - u(S)\}. \tag{7.16}$$

Consider, for example, the game HSW1, and resume the equally distributed welfare optimum with an individual share of 98. For such an agreement, as a maximizer, we get the coalition of the extreme rich $\{5, 6, 7, 8\}$ ($mex = 8, gcex = 0$), the unique coalition that would like to block the egalitarian proposal. For super-additive games (and convex games are super-additive) it holds that, whenever the grand coalition N is not a maximizer of excess, a maximizing coalition S subsidizes outsiders. Starting with the inequality

$$mex = v(S) - u(S) > v(N) - u(N), \tag{7.17}$$

and inserting $v(N - S) + v(S) \leq v(N)$, rising from convexity (respectively super-additivity), we get

$$v(S) - u(S) > v(N) - u(N) \geq v(N - S) + v(S) - u(S) - u(N - S), \tag{7.18}$$

and finally

$$0 > v(N - S) - u(N - S). \tag{7.19}$$

Hence, the opposition $N \backslash S$ of the coalition S gains from such an agreement $\vec{u}$. Compared with the former measurement concept, maximum excess gives us an indication of the maximal discontent with regard to the respective payoff allocation. In that sense, the variable, *mex*, measures **discontent**. In case that the excess of the grand coalition is not maximal, we can state that the corresponding proposal, agreement or result is significantly unbalanced or unfair. In our case, it is typical for such a payoff vector that some payoff to a poor player is larger than the payoff to some rich player.

In our HSW1 data set, in 66 of the 80 cases, maximum excess and grand coalition excess coincide (we get a similar result for the HSW2 data set, see Figure 5 in the Appendix). In the auction subset, there is only one exception to coincidence ($gcex = 56.25, mex = 64$). For the pooled set, the average difference for the 14 deviations is 70 ($s.d. = 38.6$). A closer look reveals that in the random subset there are four extreme cases of unfair results in which the aggregate income of the poor surpasses the aggregate income of the rich.

Figure 4 (see Appendix) shows the cumulative distributions of the variables *gcex* and *mex* for the random and for the auction groups. Whereas, for *gcex*, there is no significant difference between the two experimental conditions (random and auction), besides the few very inefficient payoffs (with corresponding extremely large excesses), the maximal excess *mex* exhibits a significant difference between the two conditions.

The missing significant effect on the inefficiency measure *gcex* showed by the Kruskal-Wallis test indicates that a location-shift is probably not the cause of the difference between the distributions, even when just looking at the mean ($mean$: 31 to 50.7, see for more details Table A.6) suggests otherwise. The random setting has a larger mean because of the small number of data points mentioned above, namely the three data points close to 300. We may ask for other potential causes of these unusual data points. In summary, we reject the usual presumption that market entitlements induce more efficient results in general.

In contrast, the statistical analysis of the variable *mex*, the measure of discontent, reveals a significant difference in distributions (Kolmogorov-Smirnov test, cf. Table A.6) and in location (Kruskal-Wallis test, cf. Table A.6). The cummulative distribution function of the discontent in the auction sample is first-order stochastically dominated by the one in the random sample. This result suggests that, in the random sample, subjects are not so much concerned about the arguments of the intermediate size coalitions. Means and medians for *mex* show a smaller value in the auction sample. Despite the remarkably smaller standard deviation for the auction group, the Kruskal-Wallis test indicates a significant difference in location. It may be that the lower discontent in the auction sample is based on the fact that subjects have had an opportunity to dissipate rents by bidding for the high entitlement. Or let us say they have had a better opportunity to learn what a role is worth, when they have had to solve the additional task of bidding.

7.1 Distances to normative solutions

Let us denote the distance variable between payoff results and normative payoff vectors by $D(\nu)$ for the nucleolus ν, $D(\phi)$ for the Shapley value ϕ, $D(u^*)$ to designate the distance variable for the Nash equilibrium u^*, and finally $D(E)$, to denote the distance variable for the egalitarian solution E, respectively.

Relying on a fairness consideration, we determine the Euclidean distances of the payoff vectors with respect to the nucleolus and to the Shapley value. Recall that the kernel (coinciding with the nucleolus) simultaneously equalizes and minimizes all losses for the players that the players can present by best arguments against each other. Fairness decreases as distance from the nucleolus increases and, according to Shapley standards, as distance to the Shapley value increases.

In Figure 6 of the Appendix, the HSW1 data on the Euclidean distances are shown. In the upper part, the auction trials are represented; in the lower part the random trials. The corresponding HSW2 data are visualized in Figure 7 of the Appendix. The overall result is:

$D(E) < D(\nu) < D(\phi) < D(u^*)$.

Most inequalities also hold in the sense of stochastic dominance. For the pooled data set, one can argue that the kind of fairness consideration provided by kernel or nucleolus or Shapley arguments is not the subjects main aim, since there is a relatively large distance from, say, the nucleolus. We would rather say that subjects behave so that payoffs are a compromise between the fairness and the egalitarian solution (cf. with the statistical tests given in Table A.7 and Table A.8).

In accord with the foregoing discussion, it should be apparent that the egalitarian solution performs better than the Shapley value, and the kernel solution is not the cause that subjects really do not have to work for the endowment they get. If this were the reason, then the cumulative distribution functions with respect to the Euclidean distance for the auction, as well as for the random scenario, could not be very close to each other. But our results indicate that they in fact are close together. A closer look on Figure 8 II reveals that there is no large difference between the cumulative distribution in the case of the random scenario, where the endowment of the token is randomly assigned, and the cumulative distribution in the case of an auction endowment, where subjects have to bid for the right to become rich. Only the performance of fairness solutions is worse under the random scenario than under the auction scenario (cf. Figure 6 and Figure 8 I-III).

Whereas in the HSW2 setting, the distribution of the distances to the fairness solutions is relatively close to the distribution in the egalitarian solution, in the HSW1 setup with random assignment, the distributions relating to fairness are close to the Nash equilibrium. A direct comparison of the random and auction conditions can again be found in Figure 8.

Both the higher variability in endowments and the bidding process enhance the fairness of results.

8 Summary and Discussion

The starting point for our considerations was the following: In the field, actors dealing with common dilemma tasks are often able to employ means of communication to foster cooperation. Investigations of the underlying incentive structure that are solely based on non-cooperative games and non-cooperative solution concepts like Nash- and sub-game perfect equilibria fail to explain the actor's behavior. This is not only due to wrong assumptions concerning the real actor's rationality. The non-cooperative solution concepts also fail because situations allowing for communication present an arena in which the incentives to cooperate can be discussed. In order to avoid the typical dilemma, in which one has to choose between instability and an inefficient solution, actors often reach near optimum results that are introduced and stabilized by communication, even if agreements are not binding. It is becoming clear that, in the presence of communication, where arguments can be exchanged and claims and blames can be pronounced, there is a need for an analysis of the prospects of cooperation. Such an analysis would fill the gap left open by a purely non-cooperative analysis. The bargaining process renders "cheap talk" to "real agreements", which, in case of "good agreements", are expected to be kept. We recommend that experimental studies should no longer neglect the tools of cooperative game theory when dealing with common dilemma situations that allow for communication.

It is known that the relation of non-cooperative and cooperative solutions is important for the availability of attractive proposals and the stability of agreements. Indeed, it can be shown that commons dilemma games exhibit adequate properties – they are clear and convex games. For clear games, coalitions have nothing to gain by reacting to the opposition's proposals. For convex games, the cooperative solution concept core is not empty and "large" (w.r.t. dimensionality); moreover, the solution concepts of kernel and nucleolus coincide.

In our game-theoretic analysis, we apply the formalism of transferable utility games (TU games). The general case, NTU games (non-transferable utility) are not discussed here. Indeed, NTU games are somewhat more natural in our context, but first, it can be shown that, in the case of commons dilemma games, the respective NTU-games differ only inessentially from the respective TU games (cf. [33]), and, second, the formal structure of TU games is much simpler.

Early laboratory experiments dealt with symmetric commons dilemma situations. They showed that allowing for communication resulted in strong efficiency gains. As for symmetric games, a fairness solution of any kind would propose nothing other than efficient and equal sharing (the "egalitarian solution"); the game theoretically interesting case is that of asymmetric games in which we observe a difference between fairness and equality. We propose considering the kernel as a fairness solution.

In reanalyzing the seminal experiments of [12], we calculated the respective cooperative solutions and showed that results are a compromise between the equilibrium, egalitarian, and the fairness solutions.

Further theoretical and experimental research dealing with asymmetric commons is needed. Here, fairness becomes a central feature for understanding behavior. It may be not enough to reduce fairness to a concept that can be described within cooperative game theory. Let us remember that fairness is a widely used notion, somewhat misty in its content. Thus it may be helpful to discuss the roots of fair action. Without doubt, fair action is seen as opposed to social action that devalues, debases or exploits the opponent. At the core of the concept of fairness is the comprehensive and non-discriminatory respect for and recognition of the other as partner, with autonomous rights and needs. An impressive example of the success of this – for its time; innovative idea in non-discriminatory terms is the humanism-inspired conduct of the Dutch in Far-East trade. In about 1650, the General Instructions of the East Indian Company recommended "to look to the wishes of the ... (other) nation and to please it in everything ... only modest, humble, polite, and friendly individuals should be sent out there" (quoted in [17, p.21]. Such attitudes, indeed, paid off. To offer just one example, the Dutch were the only ones allowed to remain in Japan during the isolationist period of the Tokugawa-shogunate (1640-1854) (cf. [17, p.22]).

Communication can be used to establish a climate of fairness in order to collect and share the gains of cooperation. Analyzing the game structures in cooperative terms can help to detect fair solutions and to understand the behavior of actors in the respective situations.

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A Appendix

Recall that the HSW2 setup consists of four different types of players. We call these types $1, 3, 5$ and 7. The first two numbers represent the poor types, while the rich types are characterized by 5 and 7. For example, the coalition $\{3, 3, 5, 5\}$ is made up by two types, a poor and a rich type. The original notation for this coalition was $\{3, 4, 5, 6\}$. By relying on this notation, we are able to reduce the number of coalitions to 81. The corresponding cooperative game for the HSW2 setting is given in the Table A.4 and Table A.5.

Table A.4: HSW2 Coalitional Values for Different Profiles (Part 1)

S	$v(S)$	S	$v(S)$	S	$v(S)$
{∅}	0	{1,1,7}	36	{1,1,5,5}	100
{1}	0	{1,3,3}	16	{1,1,5,7}	121
{3}	0	{1,3,5}	36	{1,1,7,7}	144
{5}	1	{1,3,7}	49	{1,3,3,5}	81
{7}	4	{1,5,5}	64	{1,3,3,7}	100
{1,1}	0	{1,5,7}	81	{1,3,5,5}	121
{1,3}	1	{1,7,7}	100	{1,3,5,7}	144
{1,5}	9	{3,3,5}	49	{1,3,7,7}	169
{1,7}	16	{3,3,7}	64	{1,5,5,7}	196
{3,3}	4	{3,5,5}	81	{1,5,7,7}	225
{3,5}	16	{3,5,7}	100	{3,3,5,5}	144
{3,7}	25	{3,7,7}	121	{3,3,5,7}	169
{5,5}	36	{5,5,7}	144	{3,3,7,7}	196
{5,7}	49	{5,7,7}	169	{3,5,5,7}	225
{7,7}	64	{1,1,3,3}	36	{3,5,7,7}	256
{1,1,3}	9	{1,1,3,5}	64	{5,5,7,7}	324
{1,1,5}	25	{ 1,1,3,7}	81	{1,1,3,3,5}	121

Table A.5: HSW2 Coalitional Values for Different Profiles (Part 2)

S	$v(S)$	S	$v(S)$
{1,1,3,3,7}	144	{1,1,3,3,7,7}	324
{1,1,3,5,5}	169	{1,1,3,5,5,7}	361
{1,1,3,5,7}	196	{1,1,3,5,7,7}	400
{1,1,3,7,7}	225	{1,1,5,5,7,7}	484
{1,1,5,5,7}	256	{1,3,3,5,5,7}	400
{1,1,5,7,7}	289	{1,3,3,5,7,7}	441
{1,3,3,5,5}	196	{1,3,5,5,7,7}	529
{1,3,3,5,7}	225	{3,3,5,5,7,7}	576
{1,3,3,7,7}	256	{1,1,3,3,5,5,7}	484
{1,3,5,5,7}	289	{1,1,3,3,5,7,7}	529
{1,3,5,7,7}	324	{1,1,3,5,5,7,7}	625
{1,5,5,7,7}	400	{1,3,3,5,5,7,7}	676
{3,3,5,5,7}	324	{1,1,3,3,5,5,7,7}	784
{3,3,5,7,7}	361		
{3,5,5,7,7}	441		
{1,1,3,3,5,5}	256		
{1,1,3,3,5,7}	289		

Table A.6: Comparison of "Auction" and "Random" (HSW1)

		gcex	mex	dnuc
mean	auction	31.0	31.2	175.5
	random	50.7	75.1	242.8
	pooled	40.8	53.1	209.2
s.d.	auction	29.1	29.3	49.8
	random	79.9	76.8	43.5
	pooled	60.1	61.8	57.7
median	auction	16.0	16.0	189.6
	random	18.1	46.1	242.2
	pooled	16.0	22.6	212.2
K-Sm (DN, sign.)		0.175, 0.57	0.4, 0.0033	0.7, 6 E-9
mean rank	auction	39.6	31.1	26.2
	random	41.4	49.9	54.9
K-W (Z,sign.)		0.13, 0.72	13.5, 2.4 E-4	30.8, 3 E-8

Legend: *dnuc*: Euclidean Distance w.r.t. the Nucleolus; *gcex*: Excess for the Grand Coalition; *mex*: Maximum Excess.
K-W: Kruskal-Wallis; K-Sm: Kolmogorov-Smirnov.

Table A.7: Comparison of "Auction" and "Random" (HSW1)

		deq	dshap	dnash
mean	auction	82.73	199.5	265.7
	random	64.31	264.7	264.7
	pooled	73.52	232.1	265.2
s.d.	auction	47.7	49.8	22.8
	random	82.43	47.7	66.2
	pooled	67.6	58.5	49.2
median	auction	72.44	213.6	275.3
	random	24.02	265.2	283.4
	pooled	64.36	236.4	275.4
K-Sm (DN, sign.)		0.475, 0.00024	0.7, 6 E-9	0.475, 0.00024
mean rank	auction	47.2	26.5	47.7
	random	33.6	54.5	33.3
K-W (Z,sign.)		6.8, 0.0092	29.2, 7 E-8	7.7, 0.0055

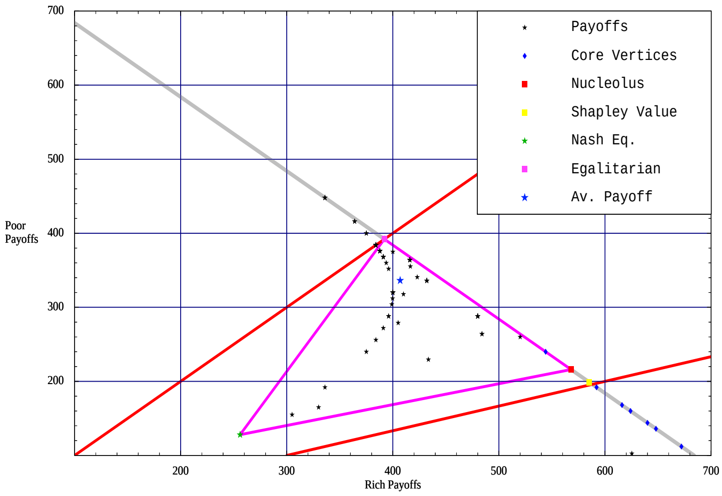
Legend: *deq*: Euclidean Distance w.r.t. the Egalitarian Solution; *dshap*: Euclidean Distance w.r.t. the Shapley Value; *dnash*: Euclidean Distance w.r.t. the Nash Eq.;
K-W: Kruskal-Wallis; K-Sm: Kolmogorov-Smirnov.

Table A.8: Basic Statistic of the HSW2 Setting

		gcex	mex	dnuc	deq	dshap	dnash
mean	pooled	130.3	142.3	162.7	148.7	180.6	248.3
s.d.	pooled	119.2	113.2	69.9	93.7	67.4	92.5
median	pooled	85.6	100	168.9	122.9	190.2	268.5

Legend: *dnuc*: Euclidean Distance w.r.t. the Nucleolus; *gcex*: Excess for the Grand Coalition; *mex*: Maximum Excess.

Figure 3: Payoff Data "HSW1" and "HSW2"



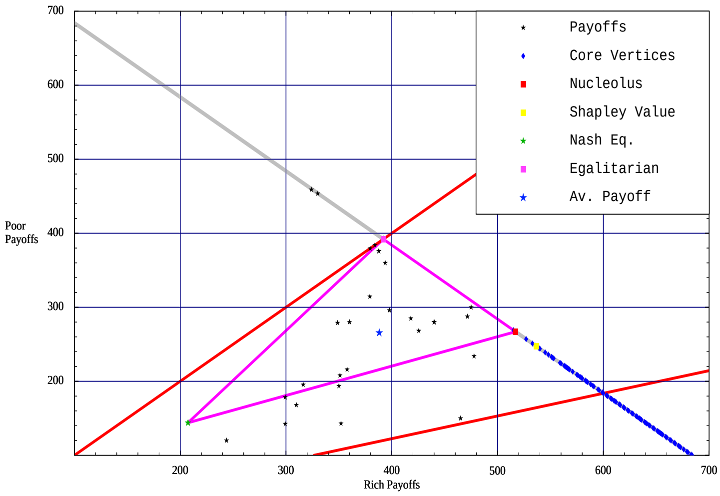


Figure 4: CDF of gcex and mex “Auction” and “Random” (HSW1)

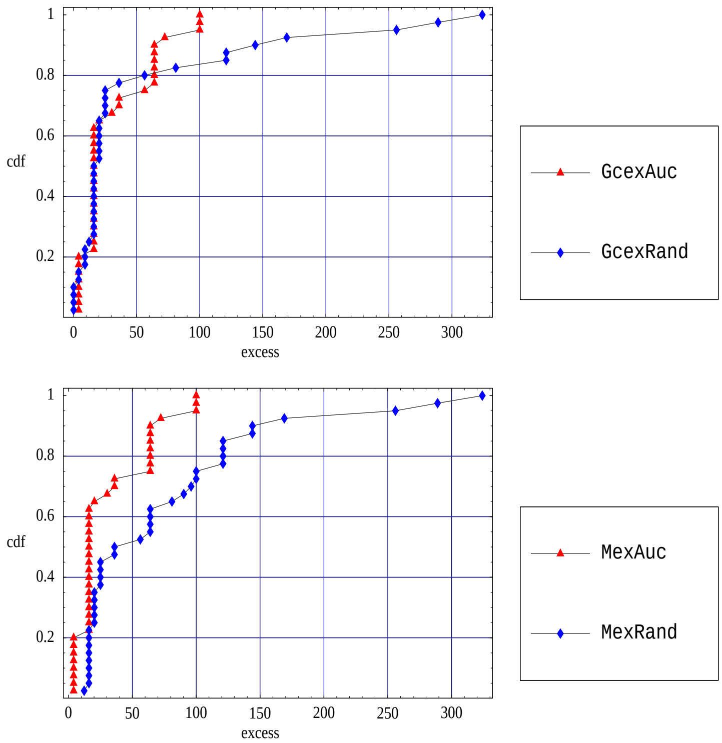


Figure 5: CDF of gcex and mex (HSW2)

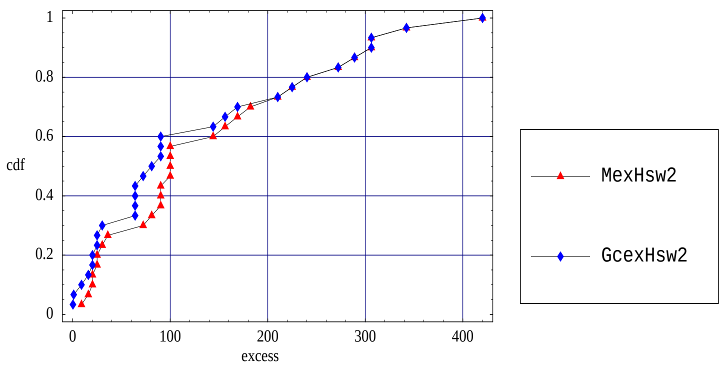


Figure 6: CDF of the Euclidean distance “Auction” and “Random” (HSW1)

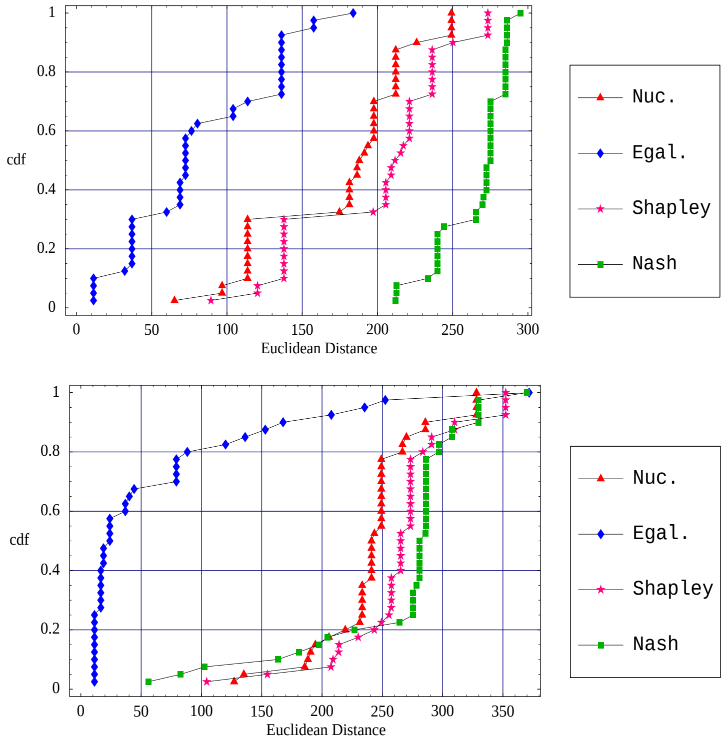


Figure 7: CDF of the Euclidean Distance of the HSW2 Setting

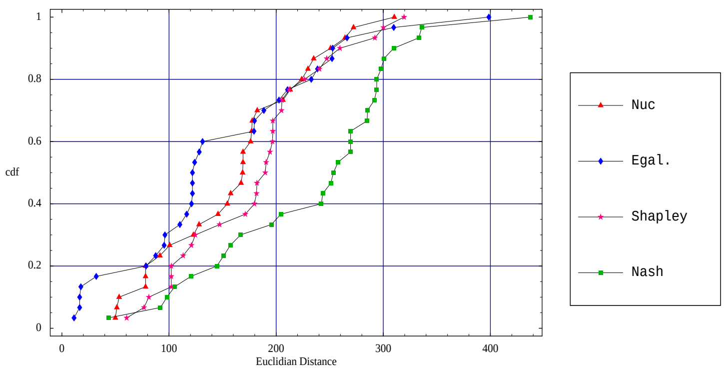


Figure 8: CDF of the Euclidean distance "Auction" and "Random" (HSW1)