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ELECTIONS WITH LIMITED INFORMATION: A FULFILLED EXPECTATIONS MODEL USING CONTEMPORANEOUS POLL AND ENDORSEMENT DATA AS INFORMATION SOURCES*

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ABSTRACT

This paper is one of several papers in which we develop and test models of 2 candidate elections under extremely decentralized and incomplete information conditions. We assume candidates do not know voter utility functions, and that most voters do not observe the policy positions adopted by the candidates. We assume that uninformed actors (voters and candidates alike) have "beliefs" about parameters of which they are uninformed, and that they attempt to inform these beliefs on the basis of readily observable variables endogenous to the system. Specifically, in this paper, we assume that uninformed actors inform their beliefs, and hence condition their behavior, on the basis of contemporaneous poll and (binary) endorsement data. An equilibrium is defined to be a set of strategies, together with a set of beliefs, such that all actors are maximizing expected utility subject to their beliefs, and such that no actor wants to revise his beliefs conditional on the information he does observe. This paper develops the above model only for the case of a one dimensional policy space with symmetric single peaked preferences.

When the electorate is modeled as being infinite, with the cumulative density of ideal points for both informed and uninformed voters being invertible, we show that regardless of the number of informed voters, in an equilibrium, the candidates behave exactly as if <u>all</u> voters had information. They respond to the preferences of the uninformed as well as the informed voters, ending up at the median ideal point of the <u>entire</u> electorate. Further, we show that <u>regardless</u> of candidate behavior, if voters are in equilibrium, their votes will extract all available information, in the sense that all voters, informed and uninformed alike, will vote as if they had perfect information about candidate positions. Finally, we give a dynamic for convergence of voting behavior, which shows that the model implies a "bandwagon" effect, with the speed of convergence depending on the ratio of the density of informed to uninformed voters at the true candidate midpoint.

In addition to the theoretical results, we run some experiments to test the implications of the model. The experiments show a moderate degree of support for the model.

I. INTRODUCTION

In the last 30 years or so, considerable effort has been expended at attempting to develop a formal theory of political systems and processes based on the economic paradigm of rational choice. Labeled variously positive political theory, public choice, or social choice, this effort encompasses a broad area of study, including spatial election models, coalition processes, voting rules and agenda manipulation. (See Riker and Ordeshook [1972] for a review of the early work in this area and Shepsle [1979], Kramer [1977], McKelvey et al [1978] for a sampling of the recent diretions of this literature). Generally, however, the models and theories that form the component parts of this effort are subject to a common and compelling criticism-they assume that political actors such as voters, candidates, legislators, etc., possess a level of knowledge of other people's preferences, candidate positions and the like that empirical investigation does not support. Thus, in these models, it is supposed typically that candidates adopt well defined positions on all issues and that voters know these positions and the issues (at least up to some well defined probability measure)-despite the well documented empirical fact that citizen-voters oftentimes do not even know the names of the candidates (cf Berelson et al [1954], Almond and Verba [1963], Converse [1975], and for an up to date review of this literature. Kinder and Sears [1982]). The effort of this paper can be thought of as an attempt to bring the informational assumptions of such models more in line with what we know empirically.

This paper is one in a series of papers in which we study election processes under limited and decentralized information conditions. Specifically, we develop models of policy formation in two candidate elections where most voters have little or no information about the policies or platforms adopted by the candidates, and where candidates have little or no information about the voter preference functions.

The key to understanding and modeling systems in which participants have limited access to information seems to us to be very similar to the ideas that have recently been applied successfully to similar situations in economics. When voters do not possess the perfect information assumed in earlier models, and when it is costly to obtain this information relative to the presumed expected benefits, we assume that voters take cues from other sources, endogenous in the system, that are easily observable and which they believe may convey useful information. Such sources may be other voters, interest groups, historical behavior of the candidates, or poll results. This paper concentrates on a model in which the observable data consists of poll results and interest group endorsements. Regardless of the source, we assume voters will condition their choices on such "low cost" information. Candidates, too, may condition their actions on poll results or on the choices of other candidates. Thus, there are

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variables, endogenous in the system, which carry information to the uninformed participants. When actors condition their behavior on the information from these endogenous sources, this, in itself will change the observed values of some of the endogenous variables. The system is in equilibrium only if all participants are acting optimally given available information, and further if the information generated when participants act in such a fashion does not change—i.e. it is stable, and consistent with this optimization behavior of all participants.

Several questions of considerable theoretical interest can be addressed by models of this sort. The most important and interesting of these questions is the extent to which the equilibria of systems with limited information correspond to the equilibria of systems with full information. That is, do the policy outcomes correspond to the outcomes that would prevail were all participants to have full information? As we show, in this and other papers, such a correspondence can be established. This gives rise to a second question; namely, how little information is needed for the system to have this correspondence property.

The model developed in this paper assumes the information source for uninformed voters is poll data and interest group endorsements. Our model is a model of a <u>single</u> election, so no historical information is available. There are two classes of participants: voters and candidates. However, the voters are further partitioned into informed and uninformed voters. All voters have single peaked preferences over a one dimensional issue space, X.

Strategies available to candidates are to adopt positions in the policy space, and strategies available to voters are to vote for one candidate or the other. The candidates do not know voter utility functions, and the uninformed voters do not know the candidate positions. The only source of information for the uninformed agents is "interest group endorsement" information and the results of a "Gallup poll" of all voters. In addition, uninformed voters know where their ideal point is in the distribution of total ideal points--i.e. they know how liberal or conservative they are with respect to the remaining population.

In this model an important question is whether the informed voters, by virtue of their better information, have a disproportionate impact on the final outcome. We find that the equilibrium outcome extracts all available information, with the outcome reflecting the preferences of the uninformed as well as the informed voters.

Our approach in this and companion papers parallels the development of rational expectations models in economics. (cf, Muth [1961], Lucas [1972], Radner [1972, 1979] and Grossman [1978] for development and references to some of this extensive literature). In that literature, the actors are buyers and sellers, and the information that is of concern is the future state of the world—a state that effects the future market value of the commodity being traded. The question addressed by the rational expectations literature is what will happen to the market price of a commodity when only a few specialized participants, called <u>insiders</u>, have information regarding the future state of the world. In those models, agents are able to condition their choices, and derive information from endogenous variables such as the price or historical market data. The principal result to emerge is called the efficient markets hypothesis, which asserts that the market will behave as if everyone had information, since the relevant information about the state of the world is itself conveyed to the other participants through the price.

In this and related papers, then, we explore how the rational expectations view might be applied to models of political processes. The correspondence between the models we develop and the rational expectations models in economics is that in our models interest groups or informed voters perform the same function as the insiders in the rational expectation market models: their choices provide signals to the other participants (voters and candidates alike) that convey information about the relevant properties of the election system. Namely, they convey some sketchy, but useful information about the relative positions of the candidates.

Further, an election or poll outcome serves the same role as the price in the market models. Just as the price elicits demand information in market models, so the election outcome conveys information to the candidates about the preferences of the voters. And in the models developed here, the true distribution of voter preferences is, to the candidates, the unknown state of the world, so the price conveys information about the true state of the world. Only when no participant wants to change his behavior given the information

that is being revealed by the behavior of the interest groups and the outcome of the election can the system be in equilibrium. This is the type of equilibrium we search for in these models.

Our investigations are both theoretical and experimental. That is, we extend the rational expectations view to certain political models. But, since these extensions necessarily entail assumptions about people's abilities to process and use information, we also to design several experiments that test these extensions in a laboratory setting.

2. The Formal Development

We are given a set N of <u>voters</u>, which may be finite or infinite, a set $X \subseteq \mathbb{R}^m$ of <u>alternatives</u>, and a <u>utility function</u> $u_\alpha: X \to \mathbb{R}$ for each $\alpha \in N$. We assume the population, N, of voters can be partitioned into two subgroups, I and U, representing the <u>informed</u> and <u>uninformed</u> voters, respectively. Let μ be a measure on the measurable subsets, $\underline{N}$, of N. For N finite, we take $\mu(C) = |C|/|N|$ for all $C \subseteq N$.

In addition to the voters, there is a set, $K = \{A,B\}$, of two <u>candidates</u>. We assume, in this paper, that the policy space, X, is one dimensional and convex, so $X \subseteq R$ is an interval, and that all voters have symmetric single peaked preferences: Voter α is said to have <u>symmetric single peaked</u> preferences if $\exists\, y_\alpha \in X$ such that $u_\alpha(x) = f(||x-y_\alpha||)$, where f is strictly monotone decreasing. So, with symmetric single peaked preferences, all voters can be completely characterized by their ideal point, y_α. For any measurable $C \subseteq N$, we use the following notation:

$$L_C(x) = \mu(\{\alpha \varepsilon C | y_\alpha < x\})$$
$$G_C(x) = \mu(\{\alpha \varepsilon C | y_\alpha > x\}) \qquad (2.1)$$
$$E_C(x) = \mu(\{\alpha \varepsilon C | y_\alpha = x\}).$$

So L_C and G_C can be thought of as cumulative density and reverse cumulative density function for the distribution of ideal points in C, except note that $L_C(x)$ does not include the measure of voters with ideal points at x.

In this model, we assume the following strategy spaces:

Voter strategy space: $B_\alpha = K \cup \{\emptyset\} = \{A,B,\emptyset\}$.

Candidate strategy space: X

We let $\underline{B}$ denote the set of possible functions from N into $K \cup \{\emptyset\}$ Elements of $\underline{B}$ are denoted b, with $b(\alpha) \varepsilon B_\alpha$ representing the choice of strategy by $\alpha \varepsilon N$. Alternatively we will also write b_α for $b(\alpha)$. We call b_α voter α's ballot, with $b_\alpha = A$ denoting a vote for A, $b_\alpha = B$ a vote for B, and $b_\alpha = \emptyset$ an abstention.

A choice of strategies by all players yields a winner and a vote outcome. Thus, if A and B select strategies x_A, $x_B \varepsilon X$, respectively, and voters choose $b \varepsilon \underline{B}$, then this yields a vote proportion for A and B. We have

$$v_A(x_A,x_B,b) = \mu\{\alpha \varepsilon N | b_\alpha = A\}$$
$$v_B(x_A,x_B,b) = \mu\{\alpha \varepsilon N | b_\alpha = B\} \qquad (2.2)$$

and

$$y(x_A,x_B,b) = \begin{cases} A \text{ if } v_A(b) > v_B(b) \\ B \text{ if } v_A(b) < v_B(b) \\ \emptyset \text{ otherwise} \end{cases}$$

Also, we define the endorsement, $e(x_A,x_B)$ by

$$e(x_A,x_B) = \begin{cases} A \text{ if } x_A < x_B \\ B \text{ if } x_B < x_A \\ \emptyset \text{ otherwise} \end{cases} \qquad (2.4)$$

It is also convenient to define the anti endorsement, $\bar{e}(x_A,x_B) = e(x_B,x_A)$. I.e., $\bar{e} = B$ if $x_A < x_B$, etc.

The payoff function $M_\alpha(x_A,x_B,b)$, $M_A(x_A,x_B,b)$ $M_B(x_A,x_B,b)$ to voter α, candidate A and B respectively is

$$M_\alpha(x_A,x_B,b) = u_\alpha(x_{y(b)}) \qquad (2.5)$$

where it is understood that $u_\alpha(x_\emptyset) = 1/2\, u_\alpha(x_A) + 1/2 u_\alpha(x_B)$.

$$M_A(x_A,x_B,b) = \begin{cases} 1 \text{ if } y(x_A,x_B,b) = A \\ -1 \text{ if } y(x_A,x_B,b) = B \\ 0 \text{ otherwise} \end{cases} \qquad (2.6)$$

$$M_B(x_A,x_B,b) = \begin{cases} 1 \text{ if } y(x_A,x_B,b) = B \\ -1 \text{ if } y(x_A,x_B,b) = A \\ 0 \text{ otherwise} \end{cases} \qquad (2.7)$$

We assume the following information conditions. Given a choice of strategies by all participants, say x_A, $x_B \in X$ and $b \in \underline{B}$, we assume that only the candidates and informed voters know x_A and x_B. However, all voters know the summary statistics defined above, namely v_A, v_B, y and e. We assume that all agents have <u>beliefs</u> (to be defined) about certain parameters in the model of which they are uninformed. We summarize the information conditions:

<u>Information Conditions</u>

<u>General Assumptions</u> All actors assume that everyone else except possibly themselves has perfect information about any parameter of which they are uninformed. They also assume that other actors use this information rationally.

<u>Voters</u> Each voter $\alpha \in N$ has a belief $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ about candidate positions which he uses in deciding which candidate to vote for (see below). The belief, $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ is simply a probability measure on $X \times X$. In equilibrium, each voter's belief must be consistent with the information he observes. The informed and uninformed voters differ only in the information they have to inform this belief.

<u>Informed Voters</u>: Informed voters observe x_A and x_B, so in equilibrium, their beliefs are degenerate probability distributions located at the actual candidate positions.

<u>Uninformed Voters</u>: Uninformed voters do <u>not</u> observe candidate positions x_A and x_B. Rather, they only observe aggregate information, namely $v_A(x_A, x_B, b)$, $v_B(x_A, x_B, b)$ and $e(x_A, x_B)$. They also know $L_N(y_\alpha)$ and $G_N(y_\alpha)$. They do <u>not</u> know the distribution of preferences of the rest of the voters, i.e. they do not know $L_N(x)$ or $G_N(x)$ for $x \neq y_\alpha$, however they do know that all voters have symmetric single peaked preferences. In equilibrium, the uninformed voter's beliefs $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ must be consistent with the above observed data. Exactly what this implies is made explicit below.

<u>Candidates</u>

Each candidate, $k \in K$ is assumed to have a belief L_{Nk} of L_N which reflects his belief of the expected voting behavior of the electorate. L_{Nk} is assumed to be a monotone increasing, lower semi continuous function from X into [0,1]. Given an admissible L_{NA}, we define

$$E_{Nk}(x) = \lim_{h \to 0} [L_{NA}(x + h) - L_{NA}(x)]$$

and (2.8)

$$G_{Nk}(x) = \mu(N) - L_{Nk}(x) - E_{Nk}(x)$$

We assume that the candidate uses his beliefs L_{Nk}, E_{Nk} and G_{Nk} to make inferences about the payoff function $M_k(x_A, x_B, b)$. Specifically, candidate $k \in K$ assumes, for any (x_A, x_B, b), that

$$v_A(x_A, x_B, b) = L_{Nk}\left(\frac{x_A + x_B}{2}\right)$$

and (2.9)

$$v_B(x_A, x_B, b) = G_{Nk}\left(\frac{x_A + x_B}{2}\right)$$

Equilibrium Conditions

We now formally develop the notion of equilibrium we use here. We treat each class of actors in turn, to define the conditions that must be satisfied for that set of actors in equilibrium. We start with the voters.

Voters

For a standard Bayesian equilibrium, each voter would try to maximize his expected payoff, given his beliefs about the strategies of the other players. Thus, applying (2.5) voter $\alpha \varepsilon N$ would

$$\max_{b_\alpha \varepsilon B_\alpha} E[u_\alpha(x_{y(b)})] \tag{2.10}$$

where the expectation is taken with respect to α's belief of (x_A, x_B, b). However, since the aggregation procedure is positively responsive, it follows that given any beliefs of the candidate positions, $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$, voter α has a dominant strategy, regardless of the value of b. Namely, voting for the candidate with the highest expected utility can never hurt that candidate and might sometimes help. Therefore, in this analysis, we assume that voters adopt this dominant strategy. Thus, we can dispense with voter beliefs about b,

and assume that voter α will choose b_α to

$$\max_{b_\alpha \varepsilon B_\alpha} E[u_\alpha(x_{b_\alpha})] \tag{2.11}$$

where the expectation is now with respect to the voter's belief $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ of (x_A, x_B).

It should be pointed out that by assuming (2.11) directly instead of (2.10), we avoid an embarrasing difficulty for the infinite voter case: In the case where N is infinite, no one voter can have any impact on the outcome, so <u>any</u> strategy is equally good if we assume (2.10). By assuming (2.11), instead, we insure that even in the infinite voter case, voters will vote for the candidate whose policy position gives them the highest utility.

Our definition of equilibrium requires not only that voters maximize expected utility (2.11) with respect to their beliefs, but also that their beliefs be consistent with the information they receive. Here, we must differentiate between the informed and uninformed voters.

Informed Voters

The informed voters have perfect information, i.e. given x_A, $x_B \varepsilon X$, and $\alpha \varepsilon I$, for $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ to be in equilibrium,

$$\Pr[(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha}) = (x_A, x_B)] = 1 \tag{2.12}$$

So informed voter's beliefs of candidate positions must coincide with what the candidates actually decide to do. Putting this together with (2.10), it follows that the informed voter's strategy, b_α must solve

$$\max_{b_\alpha \varepsilon B_\alpha} u_\alpha(x_{b_\alpha}) \tag{2.13}$$

Since he has symmetric, single peaked preferences, this can be further simplified to

$$b_\alpha = \begin{cases} e(x_A, x_B) & \text{if } y_\alpha < \frac{x_A + x_B}{2} \\ \bar{e}(x_A, x_B) & \text{if } y_\alpha > \frac{x_A + x_B}{2} \end{cases} \tag{2.14}$$

(In case $y_\alpha = \frac{x_A + x_B}{2}$, any strategy is admissible). Thus, for the informed voters to be in equilibrium, it must be that the total vote for e is at least $L_I(\frac{x_A + x_B}{2})$, and that for $\bar{e}$ is at least $G_I(\frac{x_A + x_B}{2})$.

<u>Uninformed Voters</u>

An uninformed voter's belief $(\tilde{x}_{A\alpha}, \tilde{x}_{B\alpha})$ of (x_A, x_B) must be consistent with v_A, v_B, e, and $L_N(y_\alpha)$. It turns out that the information available from these aggregate data is sufficient for the voter to be able to make fairly useful inferences about x_A and x_B. To see this, recall that the uninformed voter acts under the assumption that all other voters who are voting are perfectly informed and rational. Together with the symmetric single peaked assumption on

preferences, this allows the voter to make inferences about the midpoint $\frac{x_A + x_B}{2}$ between the candidates on the basis of the aggregate vote data. The voter infers that all votes for candidate $e(x_A, x_B)$ must come from voters with ideal points at or to the left of $\frac{x_A + x_B}{2}$, and all votes for $\bar{e}(x_A, x_B)$ must come from voters with ideal points at or to the right of $\frac{x_A + x_B}{2}$. It follows that the uninformed voter with ideal point at y_α can infer that if the number of voters (except for himself) who are voting for e equals or exceeds $L_N(y_\alpha) + E_N(y_\alpha)$ then $\frac{x_A + x_B}{2}$ must be to the right of y_α. Similarly, if the vote for $\bar{e}$ equals or exceeds $G_N(y_\alpha) + E_N(y_\alpha)$ then $\frac{x_A + x_B}{2} < y_\alpha$. Thus, for the voter's beliefs to be consistent with the observed information, we must have, in the first case

$$\Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \leq y_\alpha\right) = 0 \tag{2.15}$$

and in the second

$$\Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \geq y_\alpha\right) = 0 \tag{2.16}$$

We summarize the above discussion formally: For the case when $|N|$ is finite, we have, writing e for $e(x_A, x_B)$, and $\bar{e}(x_A, x_B)$,

$$\left.\begin{array}{l} v_e > L_N(y_\alpha) + E_N(y_\alpha) \\ \text{or} \\ v_e = L_N(y_\alpha) + E_N(y_\alpha) \\ \text{and } b_\alpha = \bar{e} \end{array}\right\} \Rightarrow \Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \leq y_\alpha\right) = 0$$

$$\left.\begin{aligned} & v_{\bar{e}} > G_N(y_\alpha) + E_N(y_\alpha) \\ \text{or} \quad & \\ & v_{\bar{e}} = G_N(y_\alpha) + E_N(y_\alpha) \\ & \text{and } b_\alpha = e \end{aligned}\right\} \Rightarrow \Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \geq y_\alpha\right) = 0 \tag{2.17}$$

In the first case, obviously, the voter should vote for candidate e, whereas in the latter he should vote for $\bar{e}$. But then, for uninformed voters to be in equilibrium, we have

$$\begin{aligned} L_N(y_\alpha) + E_N(y_\alpha) \leq v_e &\Rightarrow b_\alpha = e \\ G_E(y_\alpha) + E_N(y_\alpha) \leq v_{\bar{e}} &\Rightarrow b_\alpha = \bar{e} \end{aligned} \tag{2.18}$$

If case N is infinite, then the above equations simplify somewhat, since then any individual voter's vote is insignificant, and the voter cannot make any inference if $v_e = L_N(y_\alpha) + E_N(y_\alpha)$. For infinite N, we have, instead

$$\begin{aligned} v_e > L_N(y_\alpha) + E_N(y_\alpha) &\Rightarrow \Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \leq y_\alpha\right) = 0 \\ v_{\bar{e}} > G_N(y_\alpha) + E_N(y_\alpha) &\Rightarrow \Pr\left(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} > y_\alpha\right) = 0 \end{aligned} \tag{2.19}$$

The analogue of (2.18) then becomes

$$\begin{aligned} L_N(y_\alpha) + E_N(y_\alpha) < v_e &\Rightarrow b_\alpha = e \\ G_E(y_\alpha) + E_N(y_\alpha) < v_{\bar{e}} &\Rightarrow b_\alpha = e \end{aligned} \tag{2.20}$$

<u>Candidates</u>

The candidates will choose policy positions to maximize their expectations of winning the election, subject to the beliefs they have about the voter utility functions, and hence about the voting behavior of the electorate. These beliefs, summarized by their belief L_{Nk} of L_N, must be consistent with the information they have about v_A, v_B, y, and e. Now for a standard Bayesian equilibrium, candidate $k \in K$ should choose $x_k \in X$ to solve

$$\max_{x_k \in X} E[M_k(x_A, x_B, b)] \tag{2.21}$$

where the expectation is taken over his belief of b and of $x_{\bar{k}}$ for $\bar{k} \in K - \{k\}$. However here, we require that for any $(x_A, x_B) \in X \times X$, candidate k assumes that b (and hence also v_A, v_B, and y) are generated according to his belief L_{Nk} of L_N. It follows that M_k can be written as a function only of x_A, x_B, and L_{Nk}. Specifically, we write

$$\hat{v}_e(x_A, x_B | L_{Nk}) = L_{Nk}\left(\frac{x_A + x_B}{2}\right)$$

$$\hat{v}_{\underset{e}{-}}(x_A, x_B | L_{Nk}) = G_{Nk}\left(\frac{x_A + x_B}{2}\right) \tag{2.22}$$

$$\hat{y}(x_A, x_B | L_{Nk}) = \begin{cases} A \text{ if } \hat{v}_A > \hat{v}_B \\ B \text{ if } \hat{v}_B > v_A \\ \emptyset \text{ otherwise} \end{cases} \tag{2.23}$$

and

$$\hat{M}_k(x_A, x_B | L_{Nk}) = \begin{cases} 1 \text{ if } \hat{y}(x_A, x_B | L_{Nk}) = k \\ -1 \text{ if } \hat{y}(x_A, x_B | L_{Nk}) = \bar{k} \\ 0 \text{ otherwise} \end{cases} \tag{2.24}$$

Now, (2.21) becomes

$$\max_{x_k \in X} E[\hat{M}_k(x_A, x_B | L_{Nk})] \tag{2.25}$$

where the expectation is only over k's belief of $x_{\underset{k}{-}}$. But fixing L_{Nk}, the payoff function $\hat{M}_k(x_A, x_B | L_{Nk})$ has a dominant strategy for candidate k. Namely, it is easily shown that if x_k^* is any median of L_{Nk}, i.e., if $L_{Nk}(x_k^*) \leq \frac{\mu(N)}{2}$ and $G_{Nk}(x_k^*) \leq \frac{\mu(N)}{2}$, then x_k^* is a dominant strategy for the game with payoff function M_k. But then x_k^* will solve (2.25) regardless of candidate k's belief of $x_{\underset{k}{-}}$, so candidate k's behavior can be summarized by

$$x_k = x_k^* \text{ where } \begin{array}{l} L_{Nk}(x_k^*) \leq \mu(N)/2 \\ G_{Nk}(x_k^*) \leq \mu(N)/2 \end{array} \tag{2.26}$$

Clearly, a median always exists for any admissible L_{Nk}.

But now, in addition to (2.26), which describes the candidate's optimization behavior subject to his beliefs, the candidate's beliefs must be consistent with the information he has available. This means, in light of (2.9), that for L_{Nk} to be in equilibrium, we need

$$\begin{aligned} L_{Nk}\left(\frac{x_A + x_B}{2}\right) &= v_e(x_A, x_B, b) \\ G_{Nk}\left(\frac{x_A + x_B}{2}\right) &= v_{\underset{e}{-}}(x_A, x_B, b) \end{aligned} \tag{2.27}$$

We summarize the above developments in the following formal definition.

<u>Definition</u>: An <u>equilibrium</u> is a set $(x_A^*, x_B^*, \{b_\alpha^*\}_{\alpha\varepsilon N})$ of strategies together with a set $\{L_{NA}^*, L_{NB}^*, \{(\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*)\}_{\alpha\varepsilon N}\}$ of beliefs such that the strategies satisfy:

(a) For all $\alpha \varepsilon N$, b_α^* solves (2.10)

(a') For each $k \varepsilon K$, x_k^* satisfies (2.26)

and beliefs satisfy:

(b) For all $\alpha \varepsilon N$, $(\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*)$ satisfies (2.12) if $\alpha \varepsilon I$ and (2.17) if $\alpha \varepsilon U$. (For infinite N, replace (2.17) with (2.19).

(b') For each $k \in K$, L_{Nk} satisfies (2.27).

In short, our definition specifies that all agents are maximizing their expected payoffs subject to their beliefs, and that their beliefs are consistent with observed data.

We will also be concerned with a "partial equilibrium" in which the voters are in equilibrium, but the candidates are not. This is of interest because of possible exogenous constraints on candidate positions:

Definition: A voter equilibrium, conditional on $(x_A, x_B) \in X \times X$, is a set $\{b^*_\alpha\}_{\alpha \in N}$ of strategies together with a set $\{(\tilde{x}_{A\alpha}{}^*, \tilde{x}_{B\alpha}{}^*)\}_{\alpha \in N}$ of beliefs such that

(a) For each $\alpha \in N$, b^*_α solves (2.10)

(a') For all $\alpha \in N$, $(\tilde{x}_{A\alpha}{}^*, \tilde{x}_{B\alpha}{}^*)$ satisfies (2.12) if $\alpha \in I$ and (2.17) if $\alpha \in U$. (For infinite N, replace (2.17) with (2.19))

3. Results

We first prove some results about voter equilibria, given fixed, non equilibrium strategies by the candidates. When N is finite, we get:

Lemma 1 Assume N is finite. Given fixed strategies x_A, x_B by the candidates, with $x_A \neq x_B$, then if $b^* \in \underline{B}$ together with $\{(\tilde{x}_{A\alpha}{}^*, \tilde{x}_{B\alpha}{}^*)\}_{\alpha \in N}$ is a voter equilibrium, it must satisfy

(a) For all $\alpha \in I$

$$y_\alpha < \frac{x_A + x_B}{2} \Rightarrow b^*_\alpha = e(x_A, x_B)$$

$$y_\alpha > \frac{x_A + x_B}{2} \Rightarrow b^*_\alpha = \bar{e}(x_A, x_B)$$

(b) $\exists\ x_L, x_G \in X$, with $L_I(x_L) \geq L_I(\frac{x_A + x_B}{2}) - t + (\frac{1}{n})$ and $G_I(x_G) \geq G_I(\frac{x_A + x_B}{2}) - t + (\frac{1}{n})$ where $t = \sup_{x \in X} E(x)$, such that for all $\alpha \in U$,

$$y_\alpha < x_L \Rightarrow b_\alpha = e(x_A, x_B)$$
$$y_\alpha > x_L \Rightarrow b_\alpha = \bar{e}(x_A, x_B)$$

Proof: Part (a) follows directly from (2.14) so we must only show (b). We set

$$x_L = \sup\{x \mid L(x) \leq v_e\}$$

$$x_G = \inf\{x \mid G(x) \leq v_{\bar{e}}\} \qquad (3.1)$$

Since L(x) is lower semi continuous and monotone increasing, while G(x) is lower semi continuous and monotone decreasing, it follows

$$L_N(x_L) \leq v_e \qquad (3.2)$$

and

$$G_N(x_G) \le v_{\bar{e}}$$

So

$$y_\alpha < x_L \Rightarrow L_N(y_\alpha) + E_N(y_\alpha) \le L_N(x_L) \le v_e$$

and (3.3)

$$y_\alpha > x_G \Rightarrow G_N(y_\alpha) + E_N(y_\alpha) \le G_N(x_G) \le v_{\bar{e}}$$

But then, combining this with equation (2.18), it follows that for the uninformed voters to be in equilibrium, it must be that

$$y_\alpha < x_L \Rightarrow b_\alpha = e$$

(3.4)

$$y_\alpha > x_G \Rightarrow b_\alpha = \bar{e}$$

This proves the last inequalities of (b). We must only show the inequalities on $L_I(x_L)$ and $G_I(x_G)$. But from the above inequalities, if follows that the total vote among the uninformed voters for e must be at least $L_U(x_L)$, and the vote for $\bar{e}$ must be at least $G_U(x_G)$.

We can put together the results for the informed and the uninformed voters. Adding the votes from equations (2.14) and (3.4), we get that for all voters to be in equilibrium conditional on their information, we must have

$$v_e \ge L_I\left(\frac{x_A + x_B}{2}\right) + L_U(x_L)$$

(3.5)

$$v_{\bar{e}} \ge G_I\left(\frac{x_A + x_B}{2}\right) + G_U(x_G)$$

Then, adding and subtracting from the above equations, we get

$$v_e \ge L_I\left(\frac{x_A + x_B}{2}\right) - L_I(x_L) + (L_I(x_L) + L_U(x_L)) \qquad (3.6)$$

or

$$v_e - L_N(x_L) \ge L_I\left(\frac{x_A + x_B}{2}\right) - L_I(x_L) \qquad (3.7)$$

Similarly

$$v_{\bar{e}} - G_N(x_G) \ge G_I\left(\frac{x_A + x_B}{2}\right) - G_I(x_G) \qquad (3.8)$$

Now by definition of x_L, it follows that $L_N(x_L) \le v_e$. There are two cases, either $L_N(x_L) = v_e$ or $L_N(x_L) < v_e$. If $L_N(x_L) = v_e$, then $v_e - L_N(x_L) = 0 \le t - \frac{1}{n}$. If $L_N(x_L) < v_e$, then $L_N(x_L) + E_N(x_L) > v_e$. Otherwise, by finiteness of N, and semi continuity of L, there is an $x > x_L$ with $L_N(x) = L_N(x_L) + E_N(x_L) \le v_e$, a contradiction to the definition of x_L. But then, since all terms in the inequality $L_N(x_L) + E_N(x_L) > v_e$ are divisible by n, it follows that $L_N(x_L) + E_N(x_L) \ge v_e + (\frac{1}{n})$, so

$$v_e - L_N(x_L) \le E_N(x_L) - \frac{1}{n} \le t - \frac{1}{n}. \qquad (3.9)$$

So in either case, we get $v_e - L_N(x_L) \le t - \frac{1}{n}$. Plugging this into equation (3.7), we get

$$L_I(\frac{x_A + x_B}{2}) - L_I(x_L) \leq t - \frac{1}{n}$$

Rearranging terms, we get the desired result. A similar argument establishes the inequality for $G_I(x_G)$.

Q.E.D.

The above Lemma establishes that in equilibrium, all voters (informed and uninformed voters alike) with the exception of a few uninformed voters with ideal points in the neighborhood of $\frac{x_A + x_B}{2}$, vote as if they had correct information about candidate positions. In the special case when $t = \sup_{x \in X} E(x) = \frac{1}{n}$, we get

$$L_I(x_L) \geq L_I(\frac{x_A + x_B}{2})$$
$$G_I(x_G) \geq G_I(\frac{x_A + x_B}{2}) \qquad (3.10)$$

Figure 1 illustrates the interpretation of this result.

Clearly, as the number of informed voters around $\frac{x_A + x_B}{2}$ becomes denser and denser, the uninformed voters' voting will also become more and more informed.

It is of considerable interest to determine the limiting behavior of the model as the number of voters becomes large. To do this, we consider the infinite voter model. Here, we can formalize the above notion of the denseness of the informed voters through the

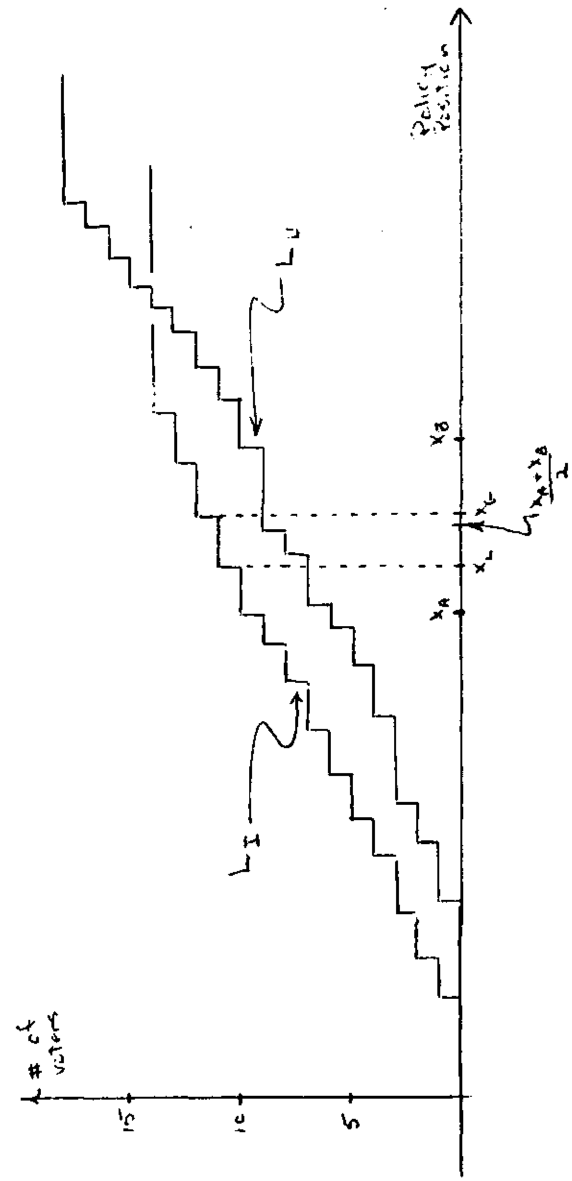


Figure 1 Illustration of Theorem 1 where t=1. All voters with $y_\alpha \leq x_L$ vote for A. Those with $y_\alpha \geq x_G$ vote for B.

invertability of the cumulative density function of ideal points. We get the following analogue of Lemma 1.

<u>Lemma 2</u> Assume N is infinite, and that L_I and L_U are invertible. Given fixed strategies, x_A, x_B by the candidates, with $x_A \neq x_B$, then if $b \in \underline{B}$ is in equilibrium for the voters, it must satisfy, for all $\alpha \in N$

$$y_\alpha < \frac{x_A + x_B}{2} \Rightarrow b_\alpha = e$$

$$y_\alpha > \frac{x_A + x_B}{2} \Rightarrow b_\alpha = \bar{e}$$

<u>Proof</u>: Invertability of L_I and L_U implies $E_I(x) = E_U(x) = 0$ for all x, which implies that for all x,

$$L_I(x) + G_I(x) = \mu(I)$$
$$L_U(x) + G_U(x) = \mu(U) \tag{3.11}$$

But now, using (2.19), and the fact L_N is invertible,

$$v_e > L_N(y_\alpha) \Rightarrow Pr(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \leq y_\alpha) = 0$$
$$v_{\bar{e}} > G_N(y_\alpha) \Rightarrow Pr(\frac{\tilde{x}_{A\alpha} + \tilde{x}_{B\alpha}}{2} \geq y_\alpha) = 0 \tag{3.12}$$

So, (2.20) becomes:

$$L_N(y_\alpha) < v_e \Rightarrow b_\alpha = e$$
$$G_N(y_\alpha) < v_{\bar{e}} \Rightarrow b_\alpha = \bar{e} \tag{3.13}$$

Now, define

$$x_L = L_N^{-1}(v_e)$$
$$x_G = G_N^{-1}(v_{\bar{e}}) \tag{3.14}$$

So

$$L_N(x_L) = v_e,$$
$$G_N(x_G) = v_{\bar{e}}, \tag{3.15}$$

and $x_G \geq x_L$, since

$$x_G = G_N^{-1}(v_{\bar{e}}) = L_N^{-1}(\mu(N) - v_{\bar{e}}) \geq L_N^{-1}(v_e) = x_L. \qquad (3.16)$$

So, by monotonicity of L_N and G_N,

$$y_\alpha < x_L \Rightarrow L_N(y_\alpha) < L_N(x_L) = v_e$$

$$(3.17)$$

and

$$y_\alpha > x_G \Rightarrow G_N(y_\alpha) < G_N(x_G) = v_{\bar{e}}$$

so

$$y_\alpha < x_L \Rightarrow b_\alpha = e$$

$$(3.18)$$

$$y_\alpha > x_G \Rightarrow b_\alpha = \bar{e}$$

Thus, the total vote among uninformed voters for e must be at least $L_U(x_L)$, and for $\bar{e}$ must be at least $G_U(x_G)$. We get an analogue of equation (3.5) for infinite voters. Namely

$$v_e \geq L_I(\frac{x_A + x_B}{2}) + L_U(x_L)$$

$$(3.19)$$

$$v_{\bar{e}} \geq G_I(\frac{x_A + x_B}{2}) + G_U(x_G)$$

Adding and subtracting from these equations, as in the proof of the previous Lemma, we get

$$L_I(\frac{x_A + x_B}{2}) - L_I(x_L) \leq v_e - L_N(x_L) = 0 \qquad (3.20)$$

or

$$L_I(x_L) \geq L_I(\frac{x_A + x_B}{2}) \qquad (3.21)$$

and similarly

$$G_I(x_G) \geq G_I(\frac{x_A + x_B}{2}) \qquad (3.22)$$

Using the monotonicity of L_I and G_I, we get

$$x_G \leq \frac{x_A + x_B}{2} \leq x_L \qquad (3.23)$$

But we have already shown $x_L \leq x_G$ So

$$x_L = x_G = (\frac{x_A + x_B}{2}). \qquad (3.24)$$

But together with (3.18) above, this proves the result.

Q.E.D.

Thus, the above lemmata show that equilibrium behavior by all voters implies that the aggregate voting behavior of the voters extracts all the revelant information about the candidate positions. As we see, for any choice of strategies by the candidates, in equilibrium, all voters vote as if they had perfect information, regardless of whether they are informed or uninformed. It should be noted that in equilibrium the uninformed voters still do not know or even have any common probability distribution on the positions of the candidates. However they each have enough probabilistic information about the location of the midpoint between x_A and x_B to allow them to make correct voting decisions. So the equilibrium extracts correct voting decisions without disseminating fully the information on candidate positions.

We next investigate the characteristics of full equilibria to the game. Here, for simplicity, we look only at the infinite voter case, where L_I, L_U and L_N (as well as candidate beliefs) are invertible. Similar theorems can be proven for finite N, but they are messier, because of the non uniqueness of admissible strategies for uninformed voters with ideal points near the candidate midpoint. We do not present results for finite voters here, since (via Lemma 1), in the limiting case, as n gets large, they are equivalent to the infinite voter results presented here.

<u>Theorem 1</u> There exists an equilibrium to the game defined by (2.5)-(2.7). Further, if N is infinite, with L_I and L_U invertible, any equilibrium $\langle(x_A^*, x_B^*, b^*), (L_{NA}^*, L_{NB}^*, [(\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*)]_{\alpha \in N})\rangle$ must satisfy $x_A^* = x_B^*$, with x_k^* a median for L_{Nk}^* for $k \in K$.

<u>Proof</u>: To show existence, set $L_{NA}^* = L_{NB}^* = L_N$, $(\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*) = (x_A^*, x_B^*)$ with probability 1 for all $\alpha \in N$, $b_\alpha = \emptyset$ for all α, and $x_A^* = x_B^* = x^*$, where x^* is a median for L_N. It is easily shown that this is an equilibrium.

Now assume $\langle(x_A^*, x_B^*, b^*), (L_{NA}^*, L_{NB}^*, [(\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*)]_{\alpha \in N})\rangle$ is an equilibrium. That x_k^* is a median for L_{Nk}^* follows directly from (2.26), so we must only show that $x_A^* = x_B^*$. Assume $x_A^* \neq x_B^*$. Say $x_A^* < x_B^*$). Then, by (2.27), $L_{Nk}^*(\frac{x_A^* + x_B^*}{2}) = v_A(x_A^*, x_B^*, b^*)$. By strict monotonicity of L_{Nk}^*, it follows that

$$L_{NA}^*(x_A^*) < L_{NA}^*(\frac{x_A^* + x_B^*}{2}) = v_A(x_A^*, x_B^*, b^*)$$
$$= L_{NB}^*(\frac{x_A^* + x_B^*}{2}) < L_{NB}^*(x_B^*)$$

But, since x_k^* is a median of L_{Nk}^*, which is invertible, we have $L_{NA}^*(x_A^*) = L_{NB}^*(x_B^*) = \mu(N)/2$, a contradiction. Hence we must have $x_A^* = x_B^*$.

Q.E.D.

Unfortunately it happens that the equilibria of the game (2.5)-(2.7) cannot be narrowed down any further than the set of candidate strategies defined by Theorem 1. In fact, it happens that all candidate strategies where $x_A^* = x_B^*$ and $L_{NA}^*(x_A^*) = L_{NB}^*(x_B^*) = \mu(N)/2$ are equilibria under the definition of equilibria we have given. However it seems apparent that if $x_A^* = x_B^* \neq x^*$, where x^* is the median of the true distribution of voter ideal points, then the equilibrium is somewhat unstable. Under our definition, this is formally an equilibrium by virtue of the fact that both candidates maintain the same incorrect beliefs about the median. Since they both agree on their incorrect beliefs, they both adopt the same position as their strategy. Voters vote randomly between them, yielding a tied election, which is consistent with their incorrect beliefs. However, this equilibrium is unstable in the sense that if either candidate makes a slight error in choice of strategy, then the beliefs of both candidates will be subjected to reality testing, and will be found to be inconsistent. This observation leads us to define a somewhat stronger notion of equilibrium. This stronger version requires that beliefs must be consistent not only with the information that is generated when candidates adopt their equilibrium strategies, but also with the information that is generated when they make small errors:

<u>Definition</u> An equilibrium, with corresponding strategies $(x_A^*, x_B^*, \{b_\alpha^*\}_{\alpha \in N})$ and beliefs $(L_{NA}^*, L_{NB}^*, \{\tilde{x}_{A\alpha}^*, \tilde{x}_{B\alpha}^*\}_{\alpha \in N})$ is <u>informationally stable</u> for the candidates if there is a neighborhood $N(x_A^*, x_B^*)$ of (x_A^*, x_B^*) such that whenever $(x_A, x_B) \in N(x_A^*, x_B^*)$, and voters adopt equilibrium strategies subject to the candidates being at (x_A, x_B), then L_{NA}^*, L_{NB}^* are consistent with the information so generated.

<u>Theorem 2</u> There exists an informationally stable equilibrium to the game defined by (2.5)-(2.7). Further, if N is infinite, and L_I and L_U are invertible, any informationally stable equilibrium must have $x_A^* = x_B^* = x^*$, where $L_N(x^*) = \mu(N)/2$.

<u>Proof</u> Existence follows with the same example as in the previous theorem. Assume the result of the second part of the theorem is false. Without loss of generality assume $x_A^* = x_B^* < x^*$. Then $L_N(x_A^*) = L_N(x_B^*) = L_N(\frac{x_A^* + x_B^*}{2}) < L_N(x^*) = \mu(N)/2$. But then pick $(x_A, x_B) \in N(x_A^*, x_B^*)$ such that $x_A < x_B$ and $L_N(\frac{x_A + x_B}{2}) = L_N(\frac{x_A^* + x_B^*}{2})$. By Lemma 2, if v_A' is the vote resulting from optimal voter behavior when candidates are at x_A, x_B, we must have $v_A' = L_N(\frac{x_A + x_B}{2})$, and by the definition of informational stability we must have $L_{NA}^*(\frac{x_A + x_B}{2}) = v_A'$. So it follows that $L_{NA}^*(x_A^*) = L_{NA}^*(\frac{x_A^* + x_B^*}{2}) = L_N(\frac{x_A + x_B}{2}) = L_N(\frac{x_A^* + x_B^*}{2}) < \mu(N)/2$. But then x_A^* is not equilibrium behavior for candidate A, given L_{NA}^*.

(Q.E.D.)

In summary, we have shown that the only informationally stable equilibrium involves both candidates converging to the true median of the <u>entire</u> electorate. If the candidates have converged exactly to the equilibrium, then, of course, there is no useful endorsement or poll information generated for the voters. They must vote randomly, and the outcome is a tie. If either of the candidates deviates at all from the equilibrium strategy, then the endorsement and poll information <u>will</u> be useful, and, in light of Lemma 1, the equilibrium behavior of the voters will extract all information. The outcome will be the same as the full information outcome, and all voters--informed and uninformed—will end up voting correctly.

4. Additional Results: Dynamics, Speed of Convergence, Bandwagons, and Manipulability

The previous section proves the existence of an equilibrium which extracts all relevant information. However, there is no guarantee that this equilibrium will ever be located. Here we concentrate on voter equilibria, when N is infinite, and we present a dynamic process by which such equilibria might be attained. The process corresponds to a series of successive polls. Candidate positions are fixed, and at each stage, all voters act rationally on the basis of information generated by the previous poll. We show that regardless of the initial starting behavior of the uninformed voters, this process converges to the full information voter equilibrium. The convergence properties of this process resemble in some respects a "Bandwagon effect." I.e., the vote share for one candidate increases monotonically, at the expense of the other candidate. Further, we obtain some results bearing on the speed of convergence. While any positive density of informed voters at the candidate midpoint guarantees eventual convergence, the speed of this convergence depends on the ratio of the density of informed and uninformed voters at that point. Finally, although we do not prove this formally, it appears that the above process, as well as the equilibrium associated with it is non manipulable. I.e., given our restrictions on preferences, no voter can gain by adopting strategies different from those prescribed

in the above dynamic. Similarly, and more obviously, in equilibrium no one can gain by misrepresenting his preferences.

We assume N is infinite, and that L_I and L_U are invertible. We fix $(x_A, x_B) \in X \times X$ and define, for $0 \leq t \leq \infty$,

$$\langle \{b_\alpha^t\}_{\alpha \in N}, \{(\tilde{x}_{A\alpha}{}^t, \tilde{x}_{B\alpha}{}^t)\}_{\alpha \in N} \rangle \tag{4.1}$$

to be a set of strategies and beliefs such that for all α, and all $t \geq 1$,

(a) b_α^t satisfies (2.10) with respect to beliefs $(\tilde{x}_{A\alpha}{}^{t-1}, \tilde{x}_{B\alpha}{}^{t-1})$

(b) $(\tilde{x}_{A\alpha}{}^t, \tilde{x}_{B\alpha}{}^t)$ satisfies (2.12) for $\alpha \in I$. $(\tilde{x}_{A\alpha}{}^t, \tilde{x}_{B\alpha}{}^t)$ satisfies (2.19) for $\alpha \in U$ with respect to information generated by b^{t-1}.

We also define

$$v_A^t = v_A(x_A, x_B, b^t)$$
$$v_B^t = v_B(x_A, x_B, b^t) \tag{4.2}$$

and

$$y^t = y(x_A, x_B, b^t) \tag{4.3}$$

Using the same argument as in Lemma 2, b^t can be characterized as follows for $t \geq 1$:

$$\text{For } \alpha \in I,\ b_\alpha^t = \begin{cases} e \text{ if } y_\alpha < \dfrac{x_A + x_B}{2} \\ \bar{e} \text{ if } y_\alpha > \dfrac{x_A + x_B}{2} \end{cases} \tag{4.4}$$

$$\text{For } \alpha \in U,\ b_\alpha^t = \begin{cases} e \text{ if } y_\alpha < L_N^{-1}(v_e^{t-1}) \\ \tilde{e} \text{ if } y_\alpha > L_N^{-1}(v_e^{t-1}) \end{cases} \tag{4.5}$$

We define, for $0 \leq t$

$$x^t = L_N^{-1}(v_e^t)$$

and (4.6)

$$x^* = \frac{x_A + x_B}{2}$$

It follows from (4.4) and (4.5) that the only voters voting incorrectly for $t \geq 1$ are the uninformed voters with ideal points in the interval between x^t and x^*. The measure of these voters is precisely $|v_e^t - v_e^*|$, where $v_e^* = L_N(\frac{x_A + x_B}{2})$. We can now prove:

<u>Theorem 3</u> If N is infinite, with L_I and L_U invertible, then the process defined by (4.1) converges to a voter equilibrium in the sense that $v_e^t \to v_e^*$, $v_{\bar{e}}^t \to v_{\bar{e}}^*$ and $x^t \to x^*$. The asympototic speed of convergence of v_e^t to v_e^* is $f_U(x^*)/f_N(x^*)$, where f_U and f_N are the p.d.f's of L_U and L_N, respectively.

<u>Proof</u> Since L_N is monotone increasing and invertible, it is continuous, so all three convergence results follow if we prove $v_e^t \to v_e^*$. Setting $v_e^* = L_N(\frac{x_A + x_B}{2})$, we have, from (4.4) and (4.5)

$$\begin{aligned} v_e^t &= L_I(L_N^{-1}(v_e^*)) + L_U(L_N^{-1}(v_e^{t-1})) \\ &= L_I(L_N^{-1}(v_e^*)) - L_I(L_N^{-1}(v_e^{t-1})) \\ &\quad + L_I(L_N^{-1}(v_e^{t-1})) + L_U(L_N^{-1}(v_e^{t-1})) \\ &= [L_I(L_N^{-1}(v_e^*)) - L_I(L_N^{-1}(v_e^{t-1}))] + L_N(L_N^{-1}(v_e^{t-1})) \end{aligned} \tag{4.7}$$

But since $v_e^{t-1} = L_N(L_N^{-1}(v_e^{t-1}))$,

$$v_e^t - v_e^{t-1} = L_I(L_N^{-1}(v_e^*)) - L_I(L_N^{-1}(v_e^{t-1})) \tag{4.8}$$

In a similar fashion, adding and subtracting $L_U(L_N^{-1}(v_e^*))$ to equation (4.7), we get

$$v_e^* - v_e^t = L_U(L_N^{-1}(v_e^*)) - L_U(L_N^{-1}(v_e^{t-1})) \tag{4.9}$$

Adding (4.8) and (4.9), we get the identity

$$v_e^* - v_e^{t-1} = L_N(L_N^{-1}(v_e^*)) - L_N(L_N^{-1}(v_e^{t-1})) \tag{4.10}$$

Further, by monotonicity of L_N, L_I, and L_U, it follows that $(v_e^* - v_e^{t-1})$ has the same sign as $L_N^{-1}(v_e^*) - L_N^{-1}(v_e^{t-1})$, which has the same sign as the right hand side of equations (4.8)-(4.10). But then, it follows that we can write v_e^t as the following convex combination of v_e^{t-1} and v_e^*

$$v_e^t = \lambda_1 v_e^{t-1} + \lambda_2 v_e^*$$

where

$$\lambda_1 = \frac{L_U(L_N^{-1}(v_e^{t-1})) - L_U(L_N^{-1}(v_e^*))}{v_e^{t-1} - v_e^*}$$

$$\lambda_2 = \frac{L_I(L_N^{-1}(v_e^{t-1})) - L_I(L_N^{-1}(v_e^*))}{v_e^{t-1} - v_e^*}$$

It is easily verified that $1 > \lambda_i > 0$ for $v_e^{t-1} \neq v_e^*$ and $\lambda_1 + \lambda_2 = 1$

Thus, it follows that the sequence $\{v_e^t\}_{t=0}^{\infty}$ is either a monotone increasing or monotone decreasing sequence, converging to v_e^*. To address the speed of convergence, we note that if $\rho_t = |v_e^t - v_e^*|$, then

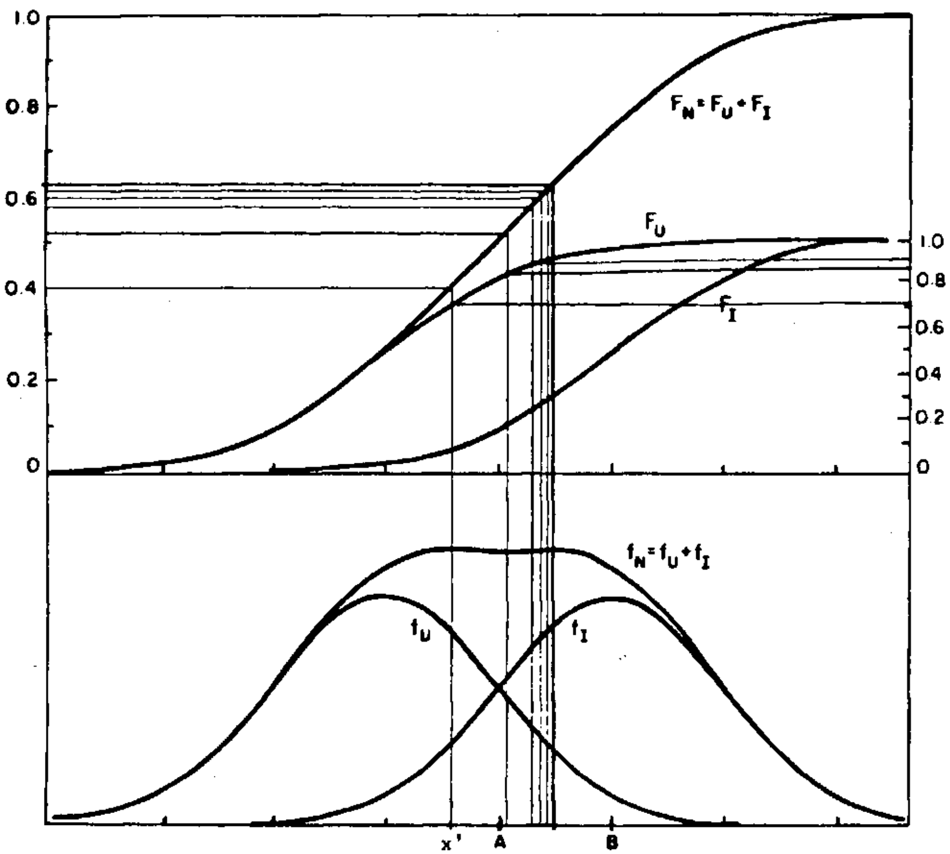
$$\frac{\rho_t}{\rho_{t-1}} = \frac{L_U(L_N^{-1}(v_e^*)) - L_U(L_N^{-1}(v_e^{t-1}))}{L_N(L_N^{-1}(v_e^*)) - L_N(L_N^{-1}(v_e^{t-1}))}$$

$$= \frac{\dfrac{L_U(L_N^{-1}(v_e^*)) - L_U(L_N^{-1}(v_e^{t-1}))}{L_N^{-1}(v_e^*) - L_N^{-1}(v_e^{t-1})}}{\dfrac{L_N(L_N^{-1}(v_e^*)) - L_N(L_N^{-1}(v_e^{t-1}))}{L_N^{-1}(v_e^*) - L_N^{-1}(v_e^{t-1})}}$$

$$\simeq \frac{f_U(L_N^{-1}(v_e^*))}{f_N(L_N^{-1}(v_e^*))} = \frac{f_U(x^*)}{f_N(x^*)} \text{ for large t.}$$

Q.E.D.

To illustrate the above dynamic model, we give an example. Figure 2 portrays cumulative distribution of ideal points for uninformed, informed, and all voters (denoted F_U, F_I and F_N respectively), and supposes that candidates A and B are at the points 4 and 5 respectively. We assume all voters have symmetric single peaked preferences, so with full information they would vote for the candidate closest to their ideal point. The corresponding density functions for the uninformed, informed, and all voters are illustrated in the lower half of the figure and are denoted f_U, f_I and f_N. An initial poll of voters, now, might reveal a random response by uninformed voters (hence they split 50-50 between A and B) while the informed voters vote correctly, and split 30-70 to an overall straw vote of .40 for A, .60 for B. If uninformed voters know where they are on the issue relative to the entire electorate, then if they assume they are the only uninformed voter, such voters can infer where the midpoint between the candidates is and hence how they ought to vote. Specifically, from the endorsement information that A is to the left of B, voters can infer from the straw vote that everyone to the left of the point x' ought to vote for A and everyone to the right ought to vote for B. That is, uninformed voters who are below the 40th percentile on the overall distribution vote for A, the remaining vote for B. This produces a second straw vote of 51% for A, 49% for



First Poll

	A	B
I	0.3	0.7
U	0.5	0.5
	0.4	0.6

Second Poll

	A	B
I	0.3	0.7
U	0.72	0.28
	0.51	0.49

Third Poll

	A	B
I	0.3	0.7
U	0.85	0.15
	0.575	0.425

Fourth Poll

	A	B
I	0.3	0.7
U	0.9	0.11
	0.6	0.4

. . . . etc.

FIGURE 2

Example of Successive Polls:

B. Repeating this for a third poll yields a 57.5% - 42.5% division and a final and correct vote of 60% for A, 40% for B. Hence, as we show formally in Section 3, in equilibrium, all voters vote as if they had perfect information.

Finally, we say a few words about manipulability of the above dynamic process. Although we do not formally prove it here, it should be evident that given his state of information at time t, it will never be to any voter's advantage to vote differently than it is assumed he votes in the above dynamic process. The reason for this is because of the "multiplier effect" which drives the above process. Namely, uninformed voters cue off of the total vote, and the larger the vote for a given candidate, the greater is the number of uninformed voters who will infer it is their interest to vote for that candidate (since all uninformed voters to the left of $L_N^{-1}(v_e^{t-1})$ vote for e and those to the right vote for $\bar{e}$). Thus, given his state of information at time t, in order to encourage other voters to vote for the candidate he believes he prefers, a voter should always vote his truthful preferences, as we assume he does.

5. Test of The Model

This section describes two experiments that are designed to test the model developed above. We wish to test both the hypothesis that uninformed voters use poll information to inform their vote and the hypothesis that candidates converge to positions that reflect the preferences of the uninformed as well as of the informed voters.

Thus, it is necessary to design an experiment that allows candidates to adjust their policy positions, but at the same time keeps candidate positions stationary enough to allow voters to collect useful information on candidate positions through the poll results.

The experiments we conducted each had between forty and fifty subjects. Two of the subjects played the part of candidates, while the rest were voters. Each experiment consists of a sequence of periods, or elections. (See Figure 3 for a schematic diagram of the sequence of events.) In each period, the two candidates first adopt policy positions in a one dimensional policy space. Candidate positions are fixed for the duration of the period, after which the candidates are able to adopt new positions. Once the candidates have selected positions, a sequence of two polls is taken, followed by a final election. Each poll is like a Gallup poll, in that voters are asked how they would vote if the election were held now. There are two classes of voters: informed and uninformed, who are selectively provided with information about the candidate positions. The informed voters are told the positions of both candidates at the beginning of each period. The uninformed voters, on the other hand are never told the position of the candidates. The uninformed voters are only told which candidate position is furthest to the left. All voters, however, are informed of the poll results, and hence, if they wish, can attempt to infer candidate positions on the basis of these results.

Voters are paid for their participation in each period on the

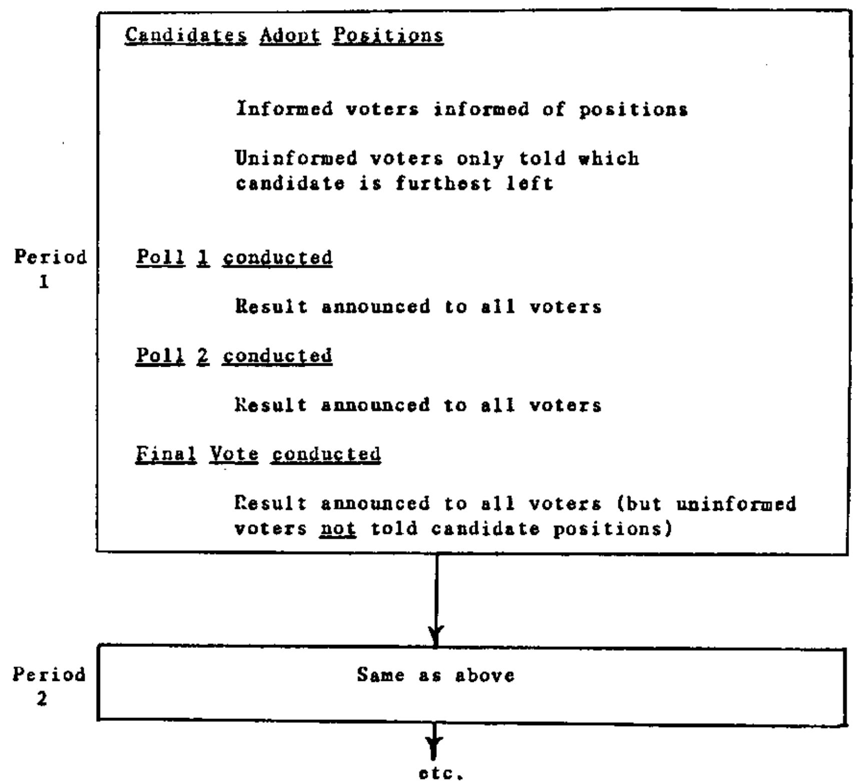


Figure 3

Sequence of Events for Experiments 1 and 2

Note: All polls and votes are by secret ballot.

basis of the position of the winning candidate and their individual payoff functions. A sample payoff function for a typical voter is given in Figure 4. The payoff function determines the amount the voter will be paid if the winning candidate adopts a given position. For example, in this sample, if the winning candidate adopted the position 70, the voter would earn $1.28 in that period. All voters have single peaked, symmetric payoff functions, but the location of the individual ideal points will differ for different voters. Although voters do not know the distribution of voter ideal points, they do know where their own ideal point is in relation to those of the rest of the electorate. As seen in the sample of Figure 4, each voter is informed about how many voters have ideal points to the left and to the right of his.

Although the experiment consists of a number of periods, voter preferences remain fixed across periods, as does the partition of informed and uninformed voters. Voters know only their own payoff functions, not those of any other voters, and candidates do not have any information about voter payoff functions. Further, the uninformed voters never learn anything about the policy position adopted by either candidate in a given period until the termination of the entire experiment. Thus, there is no possibility for uninformed voters to make inferences about candidate positions from the historical record of candidate positions in previous periods, as they could in our previous experiments. Uninformed voters are truly uninformed. For a complete listing of the instructions, see Appendix A.

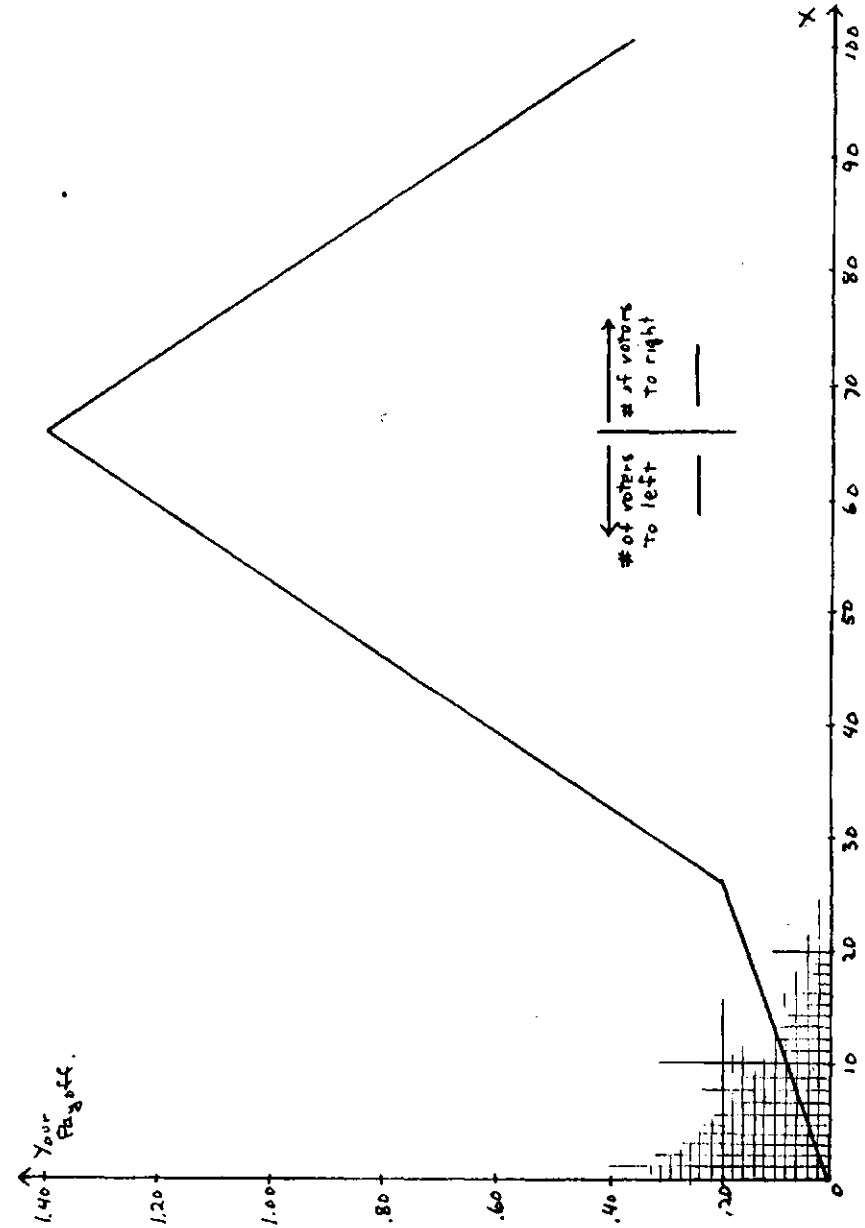


Figure 4

Sample Payoff Functions

The distribution of ideal points of the informed and uninformed voters for our two experiments is given in Figure 5. Notice that in each experiment there are approximately equal numbers of informed and uninformed voters, with the distribution of the uninformed voters being stochastically dominated by that of the informed voters. The median informed voter is at 75, the median uninformed voter is at 4S-48, and the overall median is at 60.

Our model makes predictions about the candidate behavior as well as about voter choice. First, with regard to voters, we would expect, in each period, the poll results to converge to the perfect information poll result. In the first poll, the informed voters should vote correctly, with the uninformed being indifferent or voting arbitrarily. In the second poll and in the final vote, uninformed voters should sort themselves into the appropriate category, by using information available from the previous polls. If no voters err, then as proven in the previous section, this process should converge to the situation where uninformed voters vote as if they had complete information.

Tables 1 and 2 report the results of the polls and final vote in all periods for Experiment 1 and 2 respectively. The raw data on which these tables are based is given in Appendix B. The right hand side of these tables compares the predicted with the actual results of the final vote in each period, and displays the number of voters making errors. An "error" in this table is simply a difference from the behavior the subject would exhibit if he had full information.

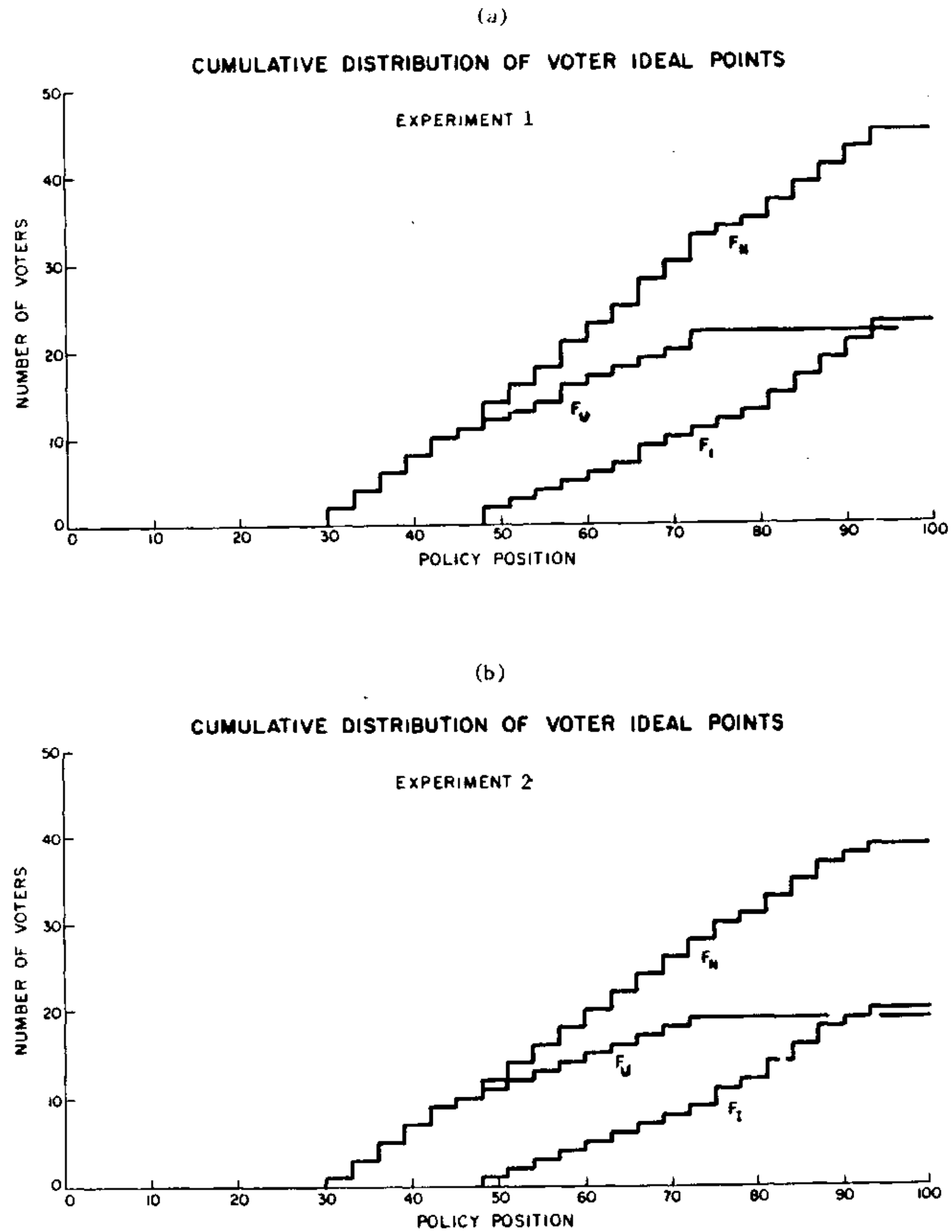


Figure 5

TABLE 1

POLL RESULTS
FOR EXPERIMENT 1

Period #		Poll #1 A	Poll #1 B	Poll #1 –	Poll #2 A	Poll #2 B	Poll #2 –	Final Vote A	Final Vote B	Predicted Vote A	Predicted Vote B	# of Errors in Final Vote Total	# of Errors in Final Vote Net
	I	2	20	1	3	19	1	2	21	2	21	0	0
1)	U	8	5	9	11	8	3	13	9	12	10	5	1
	T	10	25	10	14	27	4	15	30	14	31	5	1
	I	17	6	0	18	5	0	18	5	18	5	0	0
2)	U	4	11	7	8	12	2	8	14	6	16	6	2
	T	21	17	7	26	17	2	26	19	24	21	6	2
	I	5	16	2	4	17	2	6	17	6	17	0	0
3)	U	9	6	7	10	10	2	11	11	17	5	8	6
	T	14	22	9	14	27	4	17	28	23	22	8	6
	I	15	6	2	14	7	2	14	9	14	9	0	0
4)	U	4	9	9	6	12	4	7	15	3	19	6	4
	T	19	15	11	20	19	6	21	24	17	28	6	4
	I	15	5	3	16	5	2	16	7	16	7	0	0
5)	U	3	11	8	3	14	5	5	17	4	18	3	1
	T	18	16	11	19	19	7	21	24	20	25	3	1
	I	4	16	3	6	15	2	7	16	7	16	0	0
6)	U	11	5	6	11	8	3	13	9	18	4	7	5
	T	15	21	9	17	23	5	20	25	25	20	7	5
	I	5	16	3	5	17	1	8	15	7	16	1	1
7)	U	11	4	6	12	7	3	15	7	18	4	5	3
	T	16	20	9	17	24	4	23	22	25	20	6	2

TABLE 2

POLL RESULTS FOR
EXPERIMENT 2

Period #		Poll #1 A	Poll #1 B	Poll #1 –	Poll #2 A	Poll #2 B	Poll #2 –	Final Vote A	Final Vote B	Predicted Vote A	Predicted Vote B	# of Errors in Final Vote Total	# of Errors in Final Vote Net
	I	6	14		7	13		6	14	5	15	1	1
1)	U	10	5	4	7	9	3	11	8	15	4	4	4
	T	16	19	4	14	22	3	17	22	20	19	5	3
	I	7	13		9	11		8	12	7	13	1	1
2)	U	8	5	6	10	5	4	13	6	16	3	3	3
	T	15	18	6	19	16	4	21	18	23	16	4	2
	I	7	13		5	15		5	15	5	15	0	0
3)	U	6	6	7	8	5	6	11	8	15	4	4	4
	T	13	19	7	13	20	6	16	23	20	19	4	4
	I	12	8		12	8		13	7	14	6	3	1
4)	U	6	7	6	6	9	4	5	14	3	16	2	0
	T	18	15	6	18	17	4	18	21	17	22	6	1
	I	10	8	2	11	7	2	13	7	15	5	2	2
5)	U	6	6	7	5	7	7	8	11	4	15	3	3
	T	16	14	9	16	14	9	21	18	19	20	5	2
	I	10	10		10	9	1	13	7	14	6	1	1
6)	U	4	7	8	3	10	6	6	13	3	16	3	3
	T	14	17	8	13	19	7	19	20	17	22	4	2
	I	6	13	1	6	14		5	15	5	15	0	0
7)	U	7	4	8	9	5	5	11	8	15	4	4	4
	T	13	17	9	15	19	5	16	23	20	19	4	4
	I	6	14		6	14		5	15	5	15	0	0
8)	U	8	3	8	9	5	5	12	7	15	4	3	3
	T	14	17	8	15	19	5	17	22	20	19	3	3

This data is summarized in Table 3. We see that the informed voters virtually always vote correctly. In Experiment 1 there is only one error in the entire experiment (for the final votes), while for Experiment 2, there are 8 errors, or an average of one per period. Of all the votes cast by the informed voters in the final period, over 97 percent are cast correctly. The error rate for the uninformed voters is substantially higher than that for the informed voters, but still, across both experiments, approximately 80 percent of the votes cast by the uninformed voters are cast correctly.

It is important to note that the above computation of the error rate is actually an over estimate of the individual level errors. Since uninformed voters can only make inferences about candidate positions on the basis of poll data, it follows that errors made by one voter can affect the decisions made by other voters. Under the above computation, a voter may be making a completely rational vote based on the information he observes, but if this information is itself incorrect, he will not necessarily vote as if he had complete information. We wish, therefore, to determine the proportion of voters who make correct voting decisions based on the information available to them. We assume, then, that voters use the decision rule in equation (2.18) in a dynamic setting—as described in section 4. Thus, we assume that for $\alpha \in U$, $t > 0$,

$$L_N(y_\alpha) + E_N(y_\alpha) \leq v_e^t \Rightarrow b_\alpha^{t+1} = e$$

$$L_N(y_\alpha) + E_N(y_\alpha) \leq v_{\bar{e}}^t \Rightarrow b_\alpha^{t+1} = \bar{e} \qquad (5.1)$$

TABLE 3

SUMMARY OF ERRORS IN FINAL VOTE
(Errors Based on Assumption of Full Information)

Informed Voters

	Expt 1	Expt 2	All
Correct Choice	160 (99.4)	149 (94.9)	309 (97.2)
Error	1 (0.6)	8 (5.1)	9 (2.8)
Total	161	157	318

Uninformed Voters

	Expt 1	Expt 2	All
Correct Choice	121 (75.2)	131 (83.4)	252 (79.2)
Error	40 (24.8)	26 (16.6)	66 (20.8)
Total	161	157	318

where $v_k^t = v_k(x_A, x_B, b^t)$. We look only at those voters who have a unique choice given the information available to them. Table 4 compiles this data for the second poll and final vote in all periods of Experiments 1 and 2. We see that the error rate for uninformed voters averages around fifteen percent across both experiments for both the final vote and the second poll. Finally, a glance at Figures 6 and 7 illustrates that most of the errors which do occur can be attributed to two or three voters in each experiment. Voters 33 and 11 together account for twenty of the errors in Experiment 1.

Our second hypothesis concerns candidate behavior. Figure 8 shows the sequence of candidate positions in each experiment. We see that in both experiments, the candidates converge quickly to a point between 63 and 65. The point to which they converge lies between the median of the informed voters and that of the total electorate, but they are closer to the total median. This is consistent with what we should anticipate given the individual voting behavior of the uninformed voters. The fact that some proportion (about 1/3) of the uninformed voters are not utilizing the poll information causes the effective equilibrium for the candidates to slide up by several voters from the total median.

Overall, this experiment provides qualified support for model II. We do not have full support for either hypothesis. Rather about 2/3 of the uninformed voters appear to end up voting as if they had perfect information, and the positions to which the candidates converge is correspondingly to a point about 2/3 of the distance from the informed median to the total median.

TABLE 4

SUMMARY OF INDIVIDUAL LEVEL ERRORS
(Errors Based on Failure of Equation (2.1B))

Second Poll

	Expt 1	Expt 2	All
Correct Choice	88 (71.5)	88 (68.8)	176 (70.1)
Abstain	14 (11.4)	29 (22.7)	43 (17.1)
Errors	21 (17.1)	11 (8.6)	32 (12.8)
Total	123	128	251

Final Vote

	Expt 1	Expt 2	All
Correct Choice	119 (83.2)	118 (86.8)	237 (84.9)
Error	24 (16.8)	18 (13.2)	42 (15.1)
Total	143	136	279

INDIVIDUAL VOTES OF UNINFORMED VOTERS (final vote)

EXPERIMENT 1

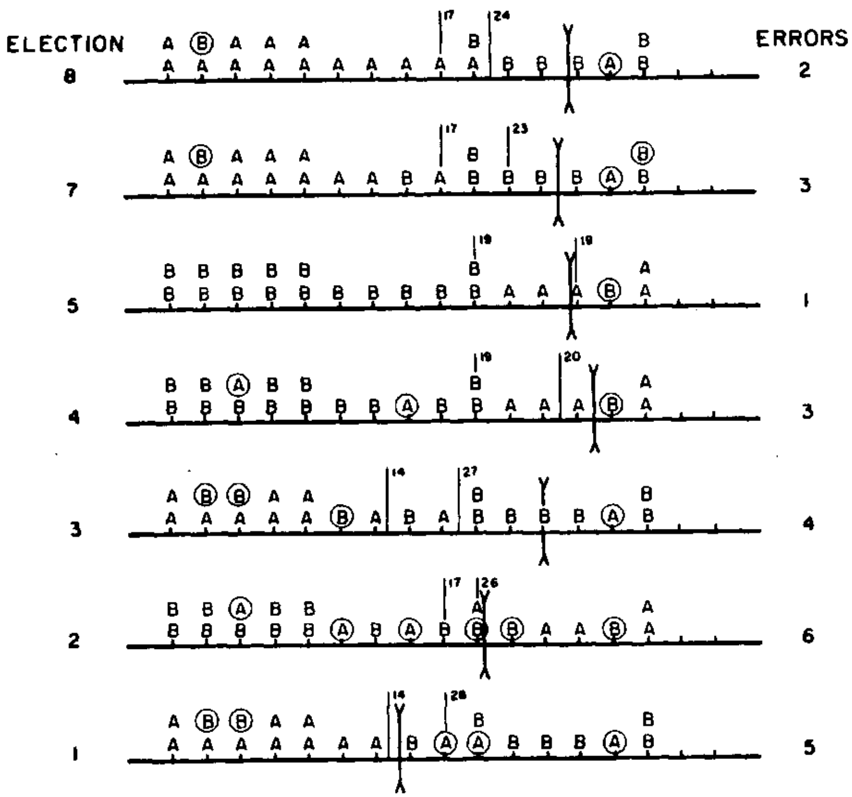


Figure 6

INDIVIDUAL VOTES OF UNINFORMED VOTERS (final vote)

EXPERIMENT 2

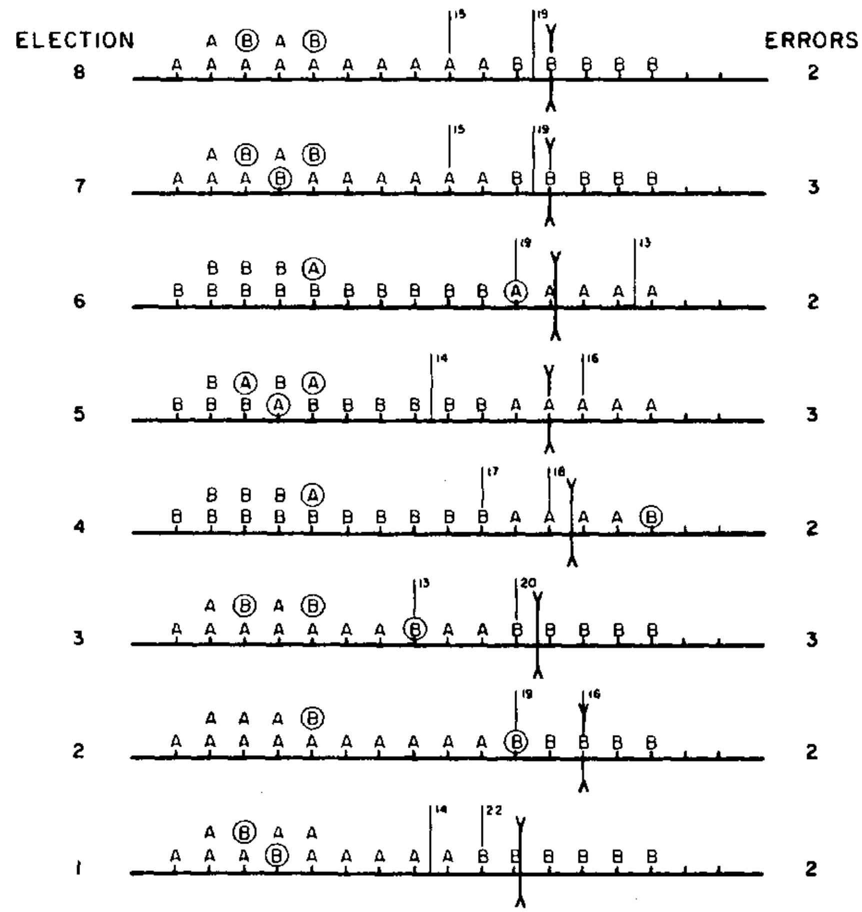


Figure 7

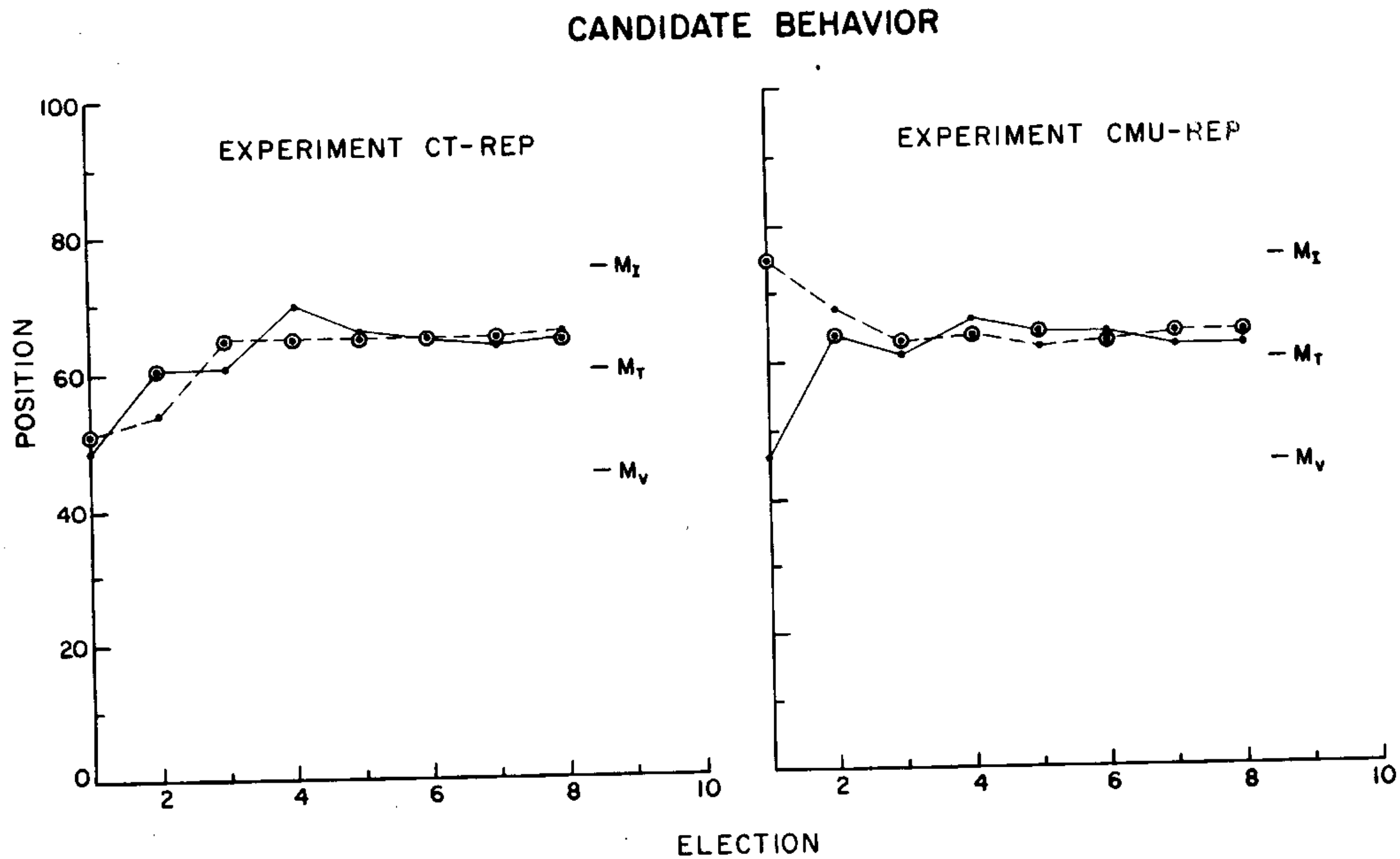

CANDIDATE BEHAVIOR
EXPERIMENT CT-REP
EXPERIMENT CMU-REP
POSITION
ELECTION
100
80
60
40
20
0
2
4
6
8
10
— M_I
— M_T
— M_V

Figure 8

APPENDIX A

INSTRUCTIONS

1-ISSUE RATIONAL EXPECTATIONS WITH POLLS

This experiment is a study of voting in two candidate elections. As subjects in the experiment, you will each be paid for your participation in the experiment on the basis of the decisions you make. If you are careful, and make good decisions, you can make a substantial amount of money.

In this experiment, there are two candidates, labeled A and B, and the rest of you are voters. The purpose of this experiment is to test certain ideas about how voters and candidates make decisions in elections and, in particular, how they make decisions when their information is imperfect or incomplete. The experiment will consist of a number of elections. In each election the candidates will adopt positions in a one dimensional policy space. Two successive polls will then be taken in which voters can indicate which candidate they prefer, and the outcome of each poll will be announced. After the second poll, voters will vote for the candidate of their choice, and the outcome of this vote will determine the winning candidate for that election. Candidates will then be permitted to adopt new positions, and the process will repeat itself. Voters are paid for their participation on the basis of their payoff function-to be described in more detail shortly, and candidates are paid for their participation on the basis of how many elections they win.

Before describing the experiment in detail, let me describe the policy space and the payoff function of the voters.

At the beginning of the experiment, voters will be given a payoff chart similar to the sample chart in front of you. This chart depicts the policy space and a sample payoff function for a voter. Candidates will be given a similar chart. However, the candidate chart will only contain the policy space, and will not have <u>any</u> voter payoff functions. The policy space is simply the set of all numbers between 0 and 100, and is represented on the horizontal axis in the diagram. During the experiment, in each period, candidates will select positions in the policy space. At the end of each period, each voter will be paid for his or her participation in that period on the basis of his payoff function and the position of the winning candidate. Thus, with the sample payoff chart, if the winning candidate were to adopt the position 48, then the voter would earn 90 cents for that period.

In the actual experiment, the payoff charts for each of the voters will be different from the sample chart. Further, the payoff functions for different voters may also be different. Each voter will have a payoff function which has a peak, or ideal point at some point in the policy space, and decreases symmetrically as we move away in either direction, as in the example in the sample chart. However, different voters' ideal points, or peaks, may be at different points in the space, and their payoff functions may decrease at different rates. One important rule in the experiment is that the information on your payoff chart is private information. None of the other voters or candidates should know the information on your chart. At no time should you show, talk about, or in any other way reveal any information about your payoff chart to the other subjects. Further, at no time during the experiment are you to have any communications with any of the other subjects except those explicitly provided for in the rules.

Axe there any questions about the payoff chart? If not, I will proceed to a description of the experiment itself.

The experiment itself is divided into a number of periods or elections. Each period will consist of a sequence of two polls followed by an election. At the beginning of each period, the two candidates, A and B, will each adopt policy positions. These positions will hold throughout that period, and the candidates will not be permitted to modify or change these positions until the next period. The positions adopted by the two candidates will not be made public. Rather, you, as voters will selectively be provided with information about the candidate positions. Before each poll, some of the voters will be informed as to the actual positions of the candidates. The remaining voters will only be given limited information about the candidate positions. They will only be told which candidate is farthest to the left and which is furthest to the right.

After the candidates have adopted their positions, and the voters have been given their information, we will then take a poll of

all voters. You may think of this as a Gallup Poll. Voters will be asked to indicate the candidate they would vote for if the election were held now. All voters will fill in their ballot cards and hand them in. This will be done as a secret ballot. The vote will be tallied and announced. At this point, additional information will be provided, and a second poll will be taken. After two such polls, the final information will be provided, and we will proceed to the final vote. These votes will also be cast in secret, and the experimenter will then tally the vote. We will then announce the winning candidate and proceed to the next period.

In order to selectively give information only to some voters, the following procedure will be used. Before each poll, the experimenter will write, on the blackboard, coded information about the position of each of the candidates. You will note, on the record sheet, (which is the second sheet in the sample packet you have been given), that for each period, and each poll, there is an entry for a code for each candidate. If this entry is filled in, you are an informed voter in that period and that poll. If it is not, you are uninformed. If you are an informed voter, you may obtain the correct position for the candidate from the coded information on the board. To do so, you just add the code to the coded information on the blackboard. Thus, in the example, if the coded positions on the board were A= 106, B= 157, then the correct position of the candidates in the first period are A= 48, B= 34. Thus, if you are an informed voter, you'll be able to obtain the exact positions adopted by the candidates. If you are uninformed, the only information you will be given is information on which candidate is furthest to the left. This will be posted on the blackboard at the beginning of each period.

It should be emphasized that all voters will get the correct information if they are informed at all. Further, if you are informed for two successive polls in the same period, then using your code, you will get the same information in each period. So it is really only necessary for you to compute the candidate positions once. It is important to emphasize that there is no attempt in this experiment to mislead voters as to the position of the candidates. To emphasize this point, we invite any interested voter, after the experiment, to compare his decoded position with the actual position of the candidates. If there is any discrepancy, you will be awarded a $10 bonus. Note that there are three possibilities in terms of the information yon might receive. You may be an informed voter throughout an entire election (i.e. for both polls and the final vote), you may be uninformed for the entire election, or you may become informed part way through. On the sample record sheet, these possibilities are represented in periods 1, 2, and 3 respectively.

At this point the task of the uninformed voters might seem an impossible one. However, you will be given one additional piece of information to assist you in your decisions. Specifically, while neither you nor the candidates will know the exact distribution of voter ideal points, each of you, as indicated on the sample, will be told the total number of the voters who have ideal points to the left

and to the right of your most preferred position. You can use this information, in conjunction with the information you do have, as to which candidate is to the left and which candidate is on the right, and the fact that it is in the interest of the informed voters to vote sincerely, to formulate your guess about the locations of the candidates and hence to determine your preference for one candidate over the other.

To recapitulate, then, the sequence of events will be as follows. Candidates adopt positions, the experimenter will write the coded positions on the board for the first poll, and the first poll will be taken and announced. The experimenter will write the coded for the second poll, and the second poll will be taken and announced, then the coded for the final election will be written, and the final election will take place. We then proceed to the next period.

After a predetermined number of periods, the experiment will end. At this point, the position of the winning candidate will be announced, and voters will be paid the sum of their payoffs for the position of the winning candidate in each election. (Note that in each election all voters are paid for the position of the winning candidate, regardless of whether or not they voted for that candidate.) The candidate payoffs are as follows: A candidate will receive $2 for each election won and nothing for each election lost.

APPENDIX B

EXPERIMENT 1

Raw Vote Data
Sorted by Ideal Point

	Ideal Point	Voter No.	Period 1 (1 2 3)	Period 2 (1 2 3)	Period 3 (1 2 3)	Period 4 (1 2 3)	Period 5 (1 2 3)	Period 7 (1 2 3)	Period 8 (1 2 3)
INFORMED VOTERS	48	13	A A A	B B B	A A A	- - B	- - B	- - A	- A A
	48	43	A A A	B B B	A A A	B B B	B B B	A A A	A A A
	51	23	B (A) B	B B B	A (B) A	(A)(A) B	B B B	(B) A A	(B)(B) A
	54	35	- - B	B B B	- - A	- - B	- - B	- - A	- - A
	57	09	B B B	B B B	A A A	B B B	B B B	A A A	A A A
	60	17	B B B	A A A	A A A	B B B	B B B	A A A	A A A
	63	31	B B B	A A A	- - B	B B B	B B B	- A A	A (B) A
	66	03	B B B	A A A	B B B	B B B	A A A	B B B	B B B
	66	39	B B B	A A A	B B B	B B B	A A A	B B B	B B B
	69	19	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	72	27	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	75	01	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	78	07	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	81	11	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	81	29	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	84	05	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	84	33	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	87	15	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	87	37	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	90	25	B B B	(B) A A	B B B	A (B) A	A A A	B B B	(A)(A) B
	90	41	B B B	A A A	B B B	A A A	A A A	(A)(A) B	B B B
	93	21	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	93	45	B B B	A A A	B B B	A A A	- A A	B B B	B B (A)
UNINFORMED VOTERS	30	30	A A A	B B B	B (B) A	A (A) B	B B B	A A A	A A A
	30	42	A A A	B B B	A A A	B B B	B B B	A A A	A A A
	33	22	A A A	B B B	A A A	B B B	B B B	A A A	A A A
	33	34	B (B)(B)	B (A) B	A (B)(B)	B B B	B B B	B (B)(B)	B (B)(B)
	36	12	- A A	- B B	- - A	- B B	- B B	- A A	A A A
	36	26	- (B)(B)	- B (A)	- A (B)	- (A)(A)	- (A) B	- - A	- A A
	39	06	A (B) A	B B B	A A A	B B B	B - B	A - A	A A A
	39	16	- - A	B B B	A A A	B B B	B B B	A A A	A A A
	42	10	A A A	B B B	A A A	B B B	B B B	A A A	A A A
	42	38	A A A	B B B	A A A	B B B	B B B	A A A	A A A
	45	02	B A A	A B (A)	B (B)(B)	- - B	- B B	- A A	- - A
	48	36	A A A	B - B	A A A	B B B	A B B	A A A	A (B) A
	51	28	B - B	A (A)(A)	B B B	A (A)(A)	A B B	A (B)(B)	A A A
	54	04	- A (A)	B B B	A A A	B B B	B B B	A A A	A A A
	57	24	- (A)(A)	- A (B)	- A B	- B B	B B B	B B B	- A A
	57	40	- B B	- A A	B B B	- A B	- - B	- B B	- - B
	60	08	- B B	- - (B)	- B B	- - A	- - A	- - B	- B B
	63	18	- - B	- A A	- B B	- - A	- A A	B B B	B B B
	66	20	- B B	- A A	- B B	- - A	- - A	- B B	- B B
	69	32	A (A)(A)	B (B)(B)	- - (A)	- (B)(B)	- - (B)	A (A)(A)	- - (A)
	72	14	B B B	A A A	B B B	A A A	A A A	B B B	B B B
	72	44	B B B	A A A	B B B	A A A	B (B) A	B B B	B B B

Informed voters, upper group (ideal points 48–63): predicted vote = e

Informed voters: Candidate midpoint

Informed voters, lower group (ideal points 66–93): predicted vote = $\bar{e}$

Uninformed voters: predicted vote = e

Uninformed voters: Candidate midpoint

Uninformed voters: predicted vote = $\bar{e}$

note: Circled votes denote errors

EXPERIMENT 2

Raw Vote Data
Sorted by Ideal Point

	Ideal Point	Voter No.	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	
			1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	
INFORMED VOTERS	48	14	(B) A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	51	23	A A A	A A A	A A A	B B (A)	B B B	B B B	A A A	A A A	predicted vote = e
	54	35	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	57	11	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	60	18	A A A	A A A	A A A	(A) B B	B B B	B B B	A A A	A A A	Candidate midpoint
	63	31	B B B	A A A	B B B	(A) B B	– A A	B B B	B B B	B B B	
	66	05	B B B	B A A	B B B	A A A	A A A	A A A	B B B	B B B	
	69	19	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	
	72	27	B B B	B B B	B B B	A A A	(B)(B) A	(B)(B) A	B B B	B B B	
	75	01	(A)(A) B	(A)(A) B	(A) B B	(B)(B)(B)	A – A	(B)(B) A	(A)(A) B	(A)(A) B	
	75	03	B B B	B B B	B B B	A A A	A A A	(B) A A	B B B	B B B	
	78	09	B B B	B B B	(A) B B	(B) A A	A A A	A A A	B B B	B B B	
	81	13	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	predicted vote = $\bar{e}$
	81	29	B B B	B B B	B B B	(B)(B)(B)	(B)(B)(B)	A A A	B B B	B B B	
	84	07	B B B	B B B	B B B	A A A	– – A	(B) – A	(A) B B	B B B	
	84	33	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	
	87	16	B B B	B B B	B B B	(B) A A	A A A	A A A	B B B	B B B	
	87	37	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	
	90	25	(A)(A)(A)	B (A)(A)	B B B	A A A	(B) A (B)	A (B)(B)	B B B	B B B	
	93	21	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	
UNINFORMED VOTERS	30	32	A A A	A A A	A A A	B B B	B – B	B B B	A A A	A A A	
	33	24	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	33	36	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	36	12	– (B) A	– A A	– – A	– B B	– B B	– B B	– – A	– – A	
	36	28	A (B)(B)	A (B) A	B (B)(B)	A (A) B	A (A)(A)	A B B	– (B)(B)	– (B)(B)	
	39	06	A A (B)	A A A	B A A	B B B	A B A	B B B	– – (B)	A A A	
	39	17	– – A	– – A	– A A	– – B	– – B	– – B	– – A	– – A	predicted vote = e
	42	10	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	42	39	A (B) A	– – (B)	– – (B)	– – (A)	– – (A)	– – (A)	– – (B)	– – (B)	
	45	02	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	48	38	– – A	– – A	– – A	– – B	– – B	– – B	– A A	A A A	
	51	30	– (B) A	– A A	– (B)(B)	– B B	– – B	– B B	– A A	– A A	
	54	04	A A A	A A A	A A A	B B B	B B B	B B B	A A A	A A A	
	57	26	B B B	B A A	– – A	– – B	– – B	– – B	– A A	– B A	
	60	08	A – B	– – (B)	B B B	A A A	– – A	– – (A)	– – B	– – B	Candidate midpoint
	63	20	B B B	B B B	B – B	A A A	A A A	A – A	B B B	B B B	
	66	22	B B B	B B B	B B B	A A A	A A A	A A A	B B B	B B B	
	69	34	B B B	B B B	– – B	A A A	A A A	– A A	B B B	– B B	predicted vote = $\bar{e}$
	72	15	B B B	B B B	B B B	A A (B)	A A A	A A A	B B B	B – B	

Note: Circled votes denote errors

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