A Ricardian Model of the Tragedy of the Commons[†]

by

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Abstract

This paper revisits the tragedy of the commons when agents have different capabilities in both production and encroachment activities, and can allocate their time between them. Under fairly general assumptions on production and encroachment technologies, an individual's expected income is convex with respect to his actions so that individuals specialize. Consequently, in equilibrium, the economy is divided into at most two homogenous groups: encroachers and producers. The partition obeys a relative advantage criterion. Several equilibria may exist. The 'tragedy of the commons' equilibrium without production always does; the Pareto optimal allocation of activities may not be an equilibrium. We show that minute changes in property right enforcement may lead to drastic improvements for society. We argue that, in convex games such as this paper's role choice game, bounded rationality is a natural assumption, and the concept of local Nash equilibrium is the natural analytical tool to handle it.

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1 Introduction

This paper revisits the 'tragedy of the commons' (Hardin, 1968; Brander and Taylor, 1998) when agents have different capabilities in both production activities and encroachment activities, and can allocate their time between these two activities. Although existing societies have equipped themselves with various institutions that help deal with appropriation failure, no institution suppresses the problem entirely so that, despite its caricatured simplicity, our model addresses a pervasive issue.

Role choices, i.e. individual decisions on the allocation of time between production and encroachment, are endogenous. In equilibrium, the waste associated with encroachment appears to vary enormously from one economy to the next one, more than differences in individual capabilities or differences in such institutions as property rights and enforcement explain at first sight.

The tragedy of the commons generally has been modelled as a problem arising from non exclusion. However non exclusion does not necessarily imply common production. In the model presented in this paper, the act of production is individual; but output is, to some extent, common because of an appropriation failure: other individuals encroach on private output. In fact, individual expected income depends both on individual production or, more precisely, on that part of production which is not robbed away from the individual by others, and on the booty derived by the individual from his encroachment activities. Productive abilities and abilities in encroachment differ between individuals in any given society. For identical individuals, encroachment abilities may vary from one society to the next because of differences in property rights and/or property rights enforcement. Since the proportion of his own production that an individual is able to appropriate himself depends on the level of encroachment activities by others, this defines a game similar to the games studied in the literature on private provision of public goods (Bergstrom *et al.*, 1986). Such *collective action games* (Olson, 1965) are characterized by the fact that the gain of each agent depends on his action and on the actions, usually the sum of the actions, of all agents.

Our game is described in Section 2; it is more complex than the standard collective action game because it exhibits increasing returns once the indirect effect of individual actions is taken into account. Although the direct effect of individual actions is linear, the payoff to each player is convex with respect to his actions whatever the actions of other players. This property gives our version of the tragedy of the commons a definite Ricardian flavor: individuals typically specialize into either encroachment activities, or production activities, according to their comparative advantages. As a result a society of many different individuals organizes itself in equilibrium into two homogeneous, specialized, groups. Unlike the Ricardian model, however, this property does not necessitate constant direct returns to individual actions.

Collective action games are pervasive in the literature. For example, contest games, reviewed by Nitzan (1994) and also in a recent special issue of the *European Journal of Political Economy* (1998), have been applied in a variety of contexts such as rent-seeking, labor economics, conflict theory, political economy, etc.. In market games, the emphasis is on externalities resulting from market power and strategic behavior (Shapley, 1976; Gabszewicz and Grazzini, 1998). Closer to our context are the various forms of tragedy of the commons games (Weitzman, 1974; Ito *et al.*, 1995; Sandler, 1992; Roemer and Silvestre, 1993), common property games (Lueck, 1994; de Meza and Gould, 1992), environmental or pollution games (Chander and Tulkens, 1997), to mention just a few.

The literature has privileged concave models almost exclusively. As Krugman (1991), we believe that this may have as much to do with mathematical convenience as with modeling realism. We argue that convexity is a robust and important property of many collective action games and deserves investigation. In a standard static tragedy of the commons game, producers, rather than focusing on marginal product, increase efforts as long as the net value of average product is non negative. This gives rise to a concave game under standard assumptions on the technology. In our variant of the tragedy of the commons game, the emphasis is on the appropriation issue. Production is not collective and each producer has complete control over marginal product. However an agent appropriates himself only a share of his own production; furthermore, his predatory activities encroach on other agents' outputs, which are entirely distinct from his own output. As we show in Section 3, this gives rise to a convex-payoffs game, with properties that are quite distinct from the properties of the standard tragedy of the commons game.

Standard collective action games admits a unique equilibrium; a game with convex payoffs may admit several equilibria. In our context, these equilibria arise according to the degree of heterogeneity of the agents in the economy. One possible equilibrium is the Pareto optimum, where all individuals are producers; another type of equilibrium is the Ricardian equilibrium just mentioned, which involves two specialized groups; finally there is the extreme version of the tragedy of the commons, an equilibrium where no-one produces. We discuss in Section 4 how these equilibria are affected by changes in property right enforcement.

Games with convex payoffs are characterized by 'all or nothing' decisions, i.e. by choices on the boundaries of the action spaces. An individual who is a full time producer must not only be sure that his choice is a local optimum; he must also verify that the alternative of being a full time encroacher is inferior. This alternative being remote from his current situation, this requirement imposes a heavy informational burden and thus raises the issue of bounded rationality. With a concave payoff, a local verification would suffice. It is easy to believe that an individual - normally a part-time encroacher and a part-time producer - evaluates without difficulty the consequences of small activity changes around his current position. It is more difficult to believe in the assumption, implied by rationality when payoffs are convex, that he fully apprehends the consequences of such drastic life changes as moving from one social group (e.g. full-time encroachers) to the group at the other end of the social spectrum (full-time producers). With this in mind, we define in Section 5 the concept of local Nash equilibrium of a game as a Nash equilibrium where, by assumption, agents do not consider deviations beyond an $\varepsilon-$neighborhood from their current situation.

While local Nash equilibria coincide with regular Nash equilibria in concave games (games where each payoff function has a unique maximum when other players play an equilibrium strategy), convex games normally admit more local Nash equilibria than regular Nash equilibria. Some of these equilibria Pareto dominate others. Few societies may claim to be completely read of any appropriability problem, so that the tragedy of the commons is always an issue. Multiple equilibria help rationalize the puzzling observation that societies with similar endowments may end up in widely different situations that seem to owe much to history. We conclude in Section 5 by raising some issues for further research such as the role of justice institutions, education, immigration, or income redistribution in alleviating tragedy of the commons problems.

2 Returns, payoffs, and individual differentiation

Let us consider an economy made up of I individuals who allocate their time, one unit each, between production and encroachment. Individual capabilities in both activities differ but are characterized by constant returns. Precisely, agent i is characterized by b_i, his 'productivity' in encroachment per unit of time, and by θ_i, his output per unit of time spent in productive activities. Someone who spends $e_i,\ e_i \in [0,1]$, units of time in productive activities and $1 - e_i$ encroaching, produces $q_i = \theta_i e_i$ and has a booty of $b_i [1 - e_i] = b_i - \beta_i q_i$ from encroachment activities, where $\beta_i \equiv \frac{b_i}{\theta_i}$ is the relative advantage of types i in encroachment.

Encroachment amounts to a tax on production. Let α represent the proportion of individual production that is taken away (stolen) from a producer; α is endogenous because it depends on the level of encroachment activities in the economy. We assume that all producers are affected in the same way by encroachment, so that, on average, individual i appropriates himself $(1 - \alpha)\, q_i$ from his own production. Consequently individual expected income, the sum of individual encroachment and expected appropriated individual production[1], is $y_i = b_i - \beta_i q_i + [1 - \alpha]\, q_i$.

The proportion α is determined at the aggregate level as $\alpha = \frac{B}{Q}$ where B is aggregate encroachment and Q is aggregate output, with B constrained not to exceed Q. Assuming

[1] We assume that encroachment activities are not affected by encroachment by others; this is not crucial an assumption.

this constraint met,

$$\alpha = \frac{\sum_{i \in I} (b_i - \beta_i q_i)}{\sum_{i \in I} q_i} \tag{1}$$

Substituting (1) into the expression for y_i gives

$$y_i(q_i, \mathbf{q}_{-i}) = b_i - \beta_i q_i + \left[1 - \frac{\sum_{j \in I} (b_j - \beta_j q_j)}{\sum_{j \in I} q_j}\right] q_i \tag{2}$$

Individuals are assumed to be risk neutral[2]; thus they choose q_i so as to maximize y_i, taking others' decisions $\mathbf{q}_{-i}$ as given and subject to the constraint that they have one unit of time to allocate between production and encroachment, so that $0 \leq q_i \leq \theta_i$. The first and second derivatives of y_i with respect to q_i are respectively

$$\begin{aligned} \frac{\partial y_i}{\partial q_i} &= 1 - \left[1 - \frac{q_i}{Q}\right][\alpha + \beta_i] \\ \frac{\partial^2 y_i}{\partial q_i^2} &= 2\left[1 - \frac{q_i}{Q}\right]\frac{\alpha + \beta_i}{Q} \end{aligned} \tag{3}$$

The aggregate constraints that $q_i \leq Q$ for all i and $B \leq Q$ are satisfied if $q_i \leq \theta_i$ and $0 \leq \alpha \leq 1$, which will be imposed throughout. Under these constraints, the second derivative is positive, implying that the optimal production choice is a corner solution.

Theorem 1 *At the individual optima, the economy is divided into at most two specialized groups: encroachers ($q_i^* = 0$) and producers ($q_i^* = \theta_i$).*

It should be emphasized that $\partial^2 y_i / \partial q_i^2$ is strictly positive so that our result is not a limiting case allowed by the linearity of the individual production and encroachment technologies. Constant return production and encroachment technologies produce the strictly convex (in q_i) revenue function $y_i(q_i, \mathbf{q}_{-i})$ for any $\mathbf{q}_{-i}$ (see Figure 1).

PLEASE INSERT FIGURE 1 HERE

[2]In Lasserre and Soubeyran (1998), we study a similar model under risk aversion.

In the Appendix, we show that the same result - a convex revenue function - may obtain when production and/or encroachment exhibit decreasing returns. Precisely, instead of using linear production and encroachment technologies, we assume that individual output is $q_i = \theta_i f(e_i)$ and individual encroachment is $b_i g(1 - e_i)$ where the functions f and g are both concave. Some straightforward substitutions yield that individual income is $y_i = b_i h\left(\frac{q_i}{\theta_i}\right) + [1 - \alpha] q_i$ where h is a non negative, decreasing, and concave function that reflects the production and encroachment technologies. We establish that

$$\frac{\partial^2 y_i}{\partial q_i^2} = \left(1 - \frac{q_i}{Q}\right)\left(\frac{2}{Q}(\alpha - \beta_i h') + \frac{1}{2}\frac{b_i}{\theta_i^2} h''\right) \tag{4}$$

which is positive provided the term involving $h'' < 0$ is not too large.

Thus convexity, implying the formation of strongly differentiated groups in an economy, is not an anecdotal possibility. Clearly it is also possible for the income function to be convex for some individuals, and concave for others, implying that some individuals specialize in production or encroachment, while others allocate part of their time to both roles. Allowing for such additional complexity would complicate the exposition without providing any additional insights.

3 Global equilibria, tragedy of the commons, and Pareto optimality

Theorem 1 implies that, in equilibrium, the set of all individuals $\mathcal{I}$ is divided into a group $\mathcal{P}$ of producers and a group $\mathcal{B}$ of encroachers, with $\mathcal{I} = \mathcal{P} \cup \mathcal{B}$ and $\mathcal{P} \cap \mathcal{B} = \Phi$. The convexity of the revenue functions implies that extrema may occur only at $q_i = \theta_i$ or at

$q_i = 0$. A Nash equilibrium is thus defined as a partition of $\mathcal{I}$, $\mathcal{N} = \{(\mathcal{P}, \mathcal{B})\}$ such that

$$\begin{aligned} y_i(\theta_i, \mathbf{q}_{-i}) &> y_i(0, \mathbf{q}_{-i}), \ i \in \mathcal{P} \\ y_i(\theta_i, \mathbf{q}_{-i}) &\leq y_i(0, \mathbf{q}_{-i}), \ i \in \mathcal{B} \end{aligned} \tag{5}$$

We call *global equilibrium* such a Nash equilibrium in order to distinguish it from the *local equilibrium* introduced in Section 5. Let (b^*, θ^*) be encroachment and productivity parameters ensuring that the second condition holds with equality for the partition $(\mathcal{P}, \mathcal{B})$; using (2), equality requires that

$$b^* = b^* - \beta^*\theta^* + \left[1 - \frac{\sum_{j\in\mathcal{B}} b_j}{\sum_{j\in\mathcal{P}} \theta_j}\right]\theta^*$$

where $\beta^* \equiv \frac{b^*}{\theta^*}$, or

$$\beta^* = 1 - \frac{\sum_{j\in\mathcal{B}} b_j}{\sum_{j\in\mathcal{P}} \theta_j} \tag{6}$$

Furthermore it follows that $\beta_i \geq \beta^*$ if and only if $i \in \mathcal{B}$, and $\beta_i < \beta^*$ if and only if $i \in \mathcal{P}$. Thus the partition must obey a criterion of relative advantage without any consideration for absolute abilities.

Let us arrange the β's by order of magnitude: $\beta_1 < \beta_2 < ... < \beta_J$ where J is the total number of different values of β. Let n_i be the number of individuals corresponding to β_i; let $\bar{\imath} = \max\{i,\ i \in \mathcal{P}\}$ and $\bar{j} = \min\{j,\ j \in \mathcal{B}\}$. Then

Lemma 1 *The game has the consecutive property:*

$$\begin{aligned} i &\in \mathcal{P} \Rightarrow i' \in \mathcal{P},\ i' \leq i \text{ and } \beta_i < \beta^e; \\ j &\in \mathcal{B} \Rightarrow j' \in \mathcal{B},\ j' \geq j \text{ and } \beta_j > \beta^e; \\ \bar{j} &= \bar{\imath} + 1; \end{aligned}$$

where $\beta^e = 1 - \alpha^e$ *and* $\alpha^e = \frac{\sum_{j \in \mathcal{B}} n_j b_j}{\sum_{i \in \mathcal{P}} n_i \theta_i}$.

Proof. $i \in \mathcal{P} \Leftrightarrow y_i(\theta_i, \mathbf{q}_{-i}) > y_i(0, \mathbf{q}_{-i}) \Leftrightarrow \beta_i < \beta^e$; $j \in \mathcal{B} \Leftrightarrow y_i(\theta_i, \mathbf{q}_{-i}) \leq y_i(0, \mathbf{q}_{-i}) \Leftrightarrow \beta_i \geq \beta^e$. It follows that $\beta_1 < \beta_2 < ... < \beta_{\bar{\imath}} < \beta^e < \beta_{\bar{\imath}+1} < ... < \beta_J$. ■

Theorem 2 below follows directly from the lemma and the definition of a Nash equilibrium.

Theorem 2 *Any global equilibrium may be described by some critical value* β^e*, such that* $q_i^* = \theta_i$ *if* $\beta_i < \beta^e$*,* $q_i^* = 0$ *if* $\beta_i \geq \beta^e$*.*

This theorem greatly simplifies the definition of an equilibrium: the criterion is unidimensional and all types are ranked on the β interval. The theorem is very reminiscent of the Ricardian theory of comparative advantage. Here too, individuals with two-dimensional characteristics end up being classified along a single characteristic. This result makes the study of existence which follows much easier. Before turning to that issue, one notes that the unique Pareto optimum allocation is an allocation where all agents are producers. Any equilibrium with $\mathcal{B} \neq \Phi$ is sub-optimum and illustrates more or less serious an instance of the tragedy of the commons.

Does a Pareto optimum equilibrium exist? Let β^{po} correspond to an allocation where every individual is a producer:

$$\beta^{po} = 1 - \frac{0}{\sum_{j \in \mathcal{J}} n_j \theta_j} = 1$$

Then the Pareto optimum allocation is an equilibrium if and only if, $\forall i,\ \beta_i < 1$: the Pareto optimum producers equilibrium exists if and only if, absent any encroachment, no-one is more productive in encroachment than in production. If it exists, the Pareto optimal equilibrium does not necessarily materialize; in fact, as we show now, it is never the sole equilibrium.

An extreme version of the tragedy of the commons is an economy where no-one produces. Let β^{ET} correspond to the allocation where every individual prefers encroachment to production:

$$\beta^{ET} = 1 - \frac{\sum_{j \mid \beta_j \geq \beta^{ET}} n_j b_j}{0} = -\infty$$

Then the encroachers equilibrium exists if and only if, $\forall i,\ \beta_i \geq \beta^{ET} = -\infty$: this condition is always satisfied.

Other possible global equilibria, which we call interior global equilibria, involve the simultaneous existence of a group of producers and a group of encroachers. In fact, since $\bar{\imath} = \max\{i,\ i \in \mathcal{P}\}$, Theorem 2 implies that $\bar{\imath}$ and $\bar{\jmath}$, as well as the corresponding number of individuals $n_{\bar{\imath}}$ and $n_{\bar{\jmath}}$, are functions of β^e. Using the definition of β^e and the lemma, it follows that

$$\beta^e = 1 - \frac{n_{\bar{\jmath}(\beta^e)} b_{\bar{\jmath}(\beta^e)} + \ldots + n_J b_J}{n_1 \theta_1 + \ldots + n_{\bar{\imath}(\beta^e)} \theta_{\bar{\imath}(\beta^e)}} \equiv F(\beta^e) \tag{7}$$

Thus β^e satisfies the fixed point property. Alternatively, this expression may be written as

$$\beta^e = 1 - \frac{B(\beta^e)}{Q(\beta^e)} = F(\beta^e) \tag{8}$$

where $B(\beta) = \sum_{\beta_j \geq \beta} n_j b_j$ is the aggregate bounty from encroachment when the group of encroachers satisfies $\beta_i \geq \beta$ and $Q(\beta) = \sum_{\beta_j < \beta} n_j \theta_j$ is the corresponding aggregate production. Thus the function $F(\beta)$ represents the proportion of output that remains to producers when the group of encroachers is such that $\beta_i \geq \beta$; we call it the appropriation rate function. Since $B(\beta)$ decreases from $\sum_{j \leq J} n_j b_j$ to zero, and $Q(\beta)$ rises from zero to $\sum_{j \leq J} n_j \theta_j$, as β rises from its minimum to its maximum, the function F has the general shape presented in Figure 2.[3] It is truncated below the horizontal axis because of the condition that $B(\beta)$ cannot exceed $Q(\beta)$ in equilibrium, so that F is non negative. Panels a) and b) give two typical alternative configurations; interior global equilibria occur when the $F(\beta)$ curve intersects the β curve.

[3] We have not represented F as a step function in order to make drawing easier; this does not affect the analysis in any significant way.

Let $\left[\beta^{\min}, \beta^{\max}\right]$ be the interval spanned by β. If, as in panel a), $\beta^{\min} < 1 < \beta^{\max}$, there is an even number of global equilibria. Since the shape of the F curve reflects the distribution of types, that number may be two, as drawn, as well as any even number including zero[4]. If there is a high proportion of low β's, individuals who have a comparative advantage in production, then the F curve is shifted to the left, so that at least two interior global equilibria exist. In the opposite case (too many encroachers), the F curve lies more to the right, so that it does not intersect the β line and there is no interior global equilibrium. In that case, it is easily shown that the sole global equilibrium is the extreme tragedy of the commons described earlier.

The two equilibria represented in panel a) have different stability properties. β_1^e is unstable: ask the individuals whose comparative advantage is the highest β_i such that $\beta_i \leq \beta_1^e$ to become encroachers rather than producers; then the proportion of output that producers appropriate themselves diminishes to $1 - \frac{B\left(\beta_1^e\right) + n_i b_i}{Q\left(\beta_1^e\right) - n_i \theta_i}$, a proportion at which the i types indeed find it preferable to encroach: $1 - \frac{B\left(\beta_1^e\right) + n_i b_i}{Q\left(\beta_1^e\right) - n_i \theta_i} < \beta_i < 1 - F\left(\beta_1^e\right)$.[5] Similarly, it is easy to see that β_2^e is stable, and that both the encroachers equilibrium $(q_i = 0 \ \forall i)$ and the producers equilibrium $(q_i = \theta_i \ \forall i)$ (if it exists), are stable.

If, as in Panel b), $\beta^{\min} < \beta^{\max} \leq 1$, the existence of at least one interior global equilibrium is certain; however, if there is only one such equilibrium, it is unstable, as in the example drawn. In fact the economy represented in Panel b) may be less problematic than the economy represented in Panel a): when $\beta^{\max} \leq 1$, the Pareto optimum allocation is an equilibrium, which is not the case when $\beta^{\max} > 1$.

Finally an interesting special case arises when all individuals are characterized by

[4]In the continuous version of the model, $Q(\beta) = \int_{\beta}^{\beta^{\max}} \theta(l)\,\phi(l)\,dl$ and $B(\beta) = \int_{\beta^{\min}}^{\beta} b(l)\,\phi(l)\,dl$, where $\beta^{\max}$ ($\beta^{\min}$) is the upper (lower) bound of the β interval and ϕ is the density of types. Then it may be shown that $F'(\beta) = \frac{\phi(\beta)}{Q(\beta)}\theta(\beta)\left[\frac{b(\beta)}{\theta(\beta)} + \frac{B(\beta)}{Q(\beta)}\right]$ which reduces to $\frac{\phi(\beta^e)}{Q(\beta^e)}\theta(\beta^e)$ at the equilibrium. Within the class of rising functions satisfying the specified bounds, F may assume just about any shape according to the density. If that density gives more weight to low β's, then F' is larger at low values of β so that the F curve is shifted to the left, implying that the existence of an equilibrium is more likely.

[5]Dealing with steps in the F function complicates the analysis without bringing up any new light; therefore we do not provide a formal proof. See the example below for an explicit discrete treatment of the fixed point argument.

the same relative efficiency in encroachment, $\bar{\beta}$. Then the $\beta = \beta$ curve reduces to the point $(\bar{\beta}, \bar{\beta})$ so that no interior equilibrium exists. In such an economy, Ricardian specialization into a single group normally obtains (i.e. all agents are either producers or encroachers).

The results about equilibrium existence and stability are summarized in the next theorem. For reasonable parameter configurations, several global equilibria exist, which may help explain the variety of outcomes found in societies whose individuals do not differ widely.

Theorem 3 *Three types of global equilibria are possible.*

1. *The encroachers equilibrium, where everyone is an encroacher and aggregate production is null, always exists; it is stable. If $\beta_i > 1$ $\forall i$, then the encroachers equilibrium is the unique global equilibrium.*

2. *The producers equilibrium, where everyone produces and aggregate encroachment is null, is the sole Pareto optimal equilibrium; it exists if and only if $\beta_i \leq 1$ $\forall i$; It is stable.*

3. *Interior global equilibria, involving both producers and encroachers, exist if the proportion of low β's in the economy ($\beta_i < 1$) is high enough. Let $m \geq 0$ be the number of interior global equilibria. If $\beta^{\min} < 1 < \beta^{\max}$, then m is even (possibly null) and there are $\frac{m}{2}$ stable global equilibria and $\frac{m}{2}$ unstable ones. If $\beta^{\min} < \beta^{\max} < 1$, then m is uneven; there are $1 + \max\left(0, \frac{m-1}{2}\right)$ unstable global equilibria and $\max\left(0, \frac{m-1}{2}\right)$ stable ones.*

An example: the two group case:

Suppose that there are only two groups of agents with different β_i : $\beta_1 < \beta_2$. The fixed point argument works as follows. Let us locate β^e with respect to β_1 and β_2. If $\beta_1 < \beta_2 < \beta^e$, $\mathcal{B} = \Phi$ and $\mathcal{P} = \mathcal{I}$. All agents are producers, so that $\alpha^e = 0$. This is the Pareto optimum equilibrium. If $\beta^e < \beta_1 < \beta_2$, $\mathcal{B} = \mathcal{I}$ and $\mathcal{P} = \Phi$. All agents are encroachers and the economy fails to materialize. If $\beta_1 < \beta^e < \beta_2$, there exists an equilibrium where agents of type β_1 are producers while type β_2 agents are encroachers: $\mathcal{B} = \{1\}$, $\mathcal{P} = \{2\}$, and

$$\frac{b_1}{\theta_1} = \beta_1 < \beta^e = 1 - \frac{n_2 b_2}{n_1 b_1} \leq \beta_2 = \frac{b_2}{\theta_2}$$

This case is possible if and only if $\theta_1 \left[\frac{1}{b_2} - \frac{1}{\theta_2}\right] < \frac{n_2}{n_1} < \frac{1}{b_2}[\theta_1 - b_1]$ with $\theta_1 > b_1$. Thus the relative size of the two groups matters for the existence of the interior equilibrium. Irrespective of group sizes, the encroachers equilibrium always exists and the producers equilibrium exists only if $b_2 < \theta_2$.

4 Effects of changes in property rights

It is natural to interpret the ability to encroach as reflecting the existence of property rights and the quality of their enforcement. It is not our purpose to study how these characteristics are determined; see, e.g. Fender (1999) for a treatment under complete information, or Lasserre and Soubeyran (2000) for an agency setting. However, we are interested in how property rights affect role choices and equilibrium production.

Thus let λ be an index of property right quality such that the individual productivity in encroachment per unit of time is now λb_i. The analysis presented sofar remains valid if we substitute λb_i, $\lambda > 0$, for b_i everywhere; in particular, β_i is redefined as $\frac{\lambda b_i}{\theta_i}$. An improvement in property right enforcement reduces β for every agent, but does not affect individual ranking according to comparative advantage. The analysis sofar has taken λ to be unity. Setting $\lambda < 1$ means an improvement in property right enforcement and $\lambda > 1$ reflects a degradation in property rights.

The implications are quite obvious. An improvement in property rights ($\lambda < 1$) does not affect the range of F but shifts its domains to the left: $\lambda\beta^{\min} < \beta^{\min}$ and $\lambda\beta^{\max} < \beta^{\max}$. In Figure 2 a), this means that the curve F is shifted to the left while the $\beta = \beta$ line (now $\lambda\beta = \lambda\beta$) is unchanged, implying that the new equilibrium cutoffs

$\lambda\beta_1^e$ and $\lambda\beta_2^e$ (not represented) involve a higher proportion of individuals choosing to produce rather than encroach. The reduction in λ may furthermore involve a shift from a situation such as panel a) where Pareto optimality is impossible in equilibrium to a situation described by panel b): the economy moves from the inefficient equilibrium corresponding to β_2^e in Panel a) to the Pareto optimal equilibrium corresponding to $\beta^{\max}$ (now $\lambda\beta^{\max}$) in Panel b).

Changes in property rights may have more dramatic effects in case of multiple equilibria. In Figure 3, we represent an initial situation that may be called an institutional trap equilibrium: although a good equilibrium A exists, the economy is stuck at the stable equilibrium B. Here an increase in property right enforcement, by shifting the $F(\lambda\beta)$ curve and both the $\lambda\beta^{min}$ and the $\lambda\beta^{max}$ limits to the left, may have the miraculous effect of putting the institutional trap equilibrium out of existence, causing the economy to move to the good equilibrium A'. However, similarly, a slight degradation in institutions may have a catastrophic effect of sending the economy to the extreme tragedy of the commons equilibrium C'.

5 Local equilibria and limited rationality

An important characteristic of equilibrium behavior in our model is the fact that optimal individual activities are corner solutions: individuals choose to be full-time encroachers or full-time producers. A person who considers a deviation from the current equilibrium position not only must examine local changes - doing some production for an encroacher, stealing occasionally for a producer - but also must consider the drastic life change involved in a full switch from production to encroachment or vice-versa.

It is easy to believe that individuals evaluate the effects of small changes around their current position accurately; it is more difficult to imagine that they fully apprehend the consequences of decisions that would take them all the way to the opposite end of the social spectrum. Thus the presence of non convexities in the model quite naturally raises the issue of bounded rationality.

A natural way to introduce bounded rationality in the model consists in assuming that changes by more than ε in the allocation of time are prohibitively costly, while any feasible deviation from the current allocation of the time unit in the neighborhood of that allocation is possible at no cost. Consider Figure 1; such an assumption would imply that the local maxima at both ends of the income curve might be admissible while not necessarily being global maxima.

The concept of local equilibrium introduced now is based on these ideas. It could be used in many games, where players choose corner solutions in equilibrium as in the present time allocation game. Let $Y_i = Y_i(a_i, \mathbf{a}_{-i})$ be the payoff to player i when i plays action a_i and other players choose $\mathbf{a}_{-i}$. The set of feasible actions for agent i is $\mathcal{A}_i$, a closed, convex subset of $\mathbb{R}^m$.

Definition 1 *(Convex payoff game) A game has convex payoffs if and only if all payoff functions* $Y_i(a_i, \mathbf{a}_{-i})$, $i = 1, ..., n$ *are convex in* a_i $\forall$ $a_{-i} \in \Pi_{j \neq i} \mathcal{A}_j$.

Definition 2 *(Local Nash equilibrium of a game) The action vector* $a^* = (a_1^*, ...a_n)$ *is a local Nash equilibrium if and only if*

$$Y_i\left(a_i^*, \mathbf{a}_{-i}^*\right) \geq Y_i\left(a_i, \mathbf{a}_{-i}^*\right) \ \forall\ a_i \in \mathcal{A}_i \cap \mathcal{N}(\varepsilon)$$

where $\mathcal{N}(\varepsilon) = \{a_i \in \mathbb{R}^m \mid d(a_i, a_i^*) \leq \varepsilon_i\}$ *is the ball of radius* ε_i *around* a_i^* *in* $\mathbb{R}^m$.

Note that the concept of local Nash Equilibrium does not differ from the standard Nash equilibrium in concave games, where local optima are also global. In the current context, $a_i = q_i$, $\mathcal{A}_i = [0, \theta_i]$, and $Y_i(a_i, \mathbf{a}_{-i}) = y_i(q_i, \mathbf{q}_{-i})$. Consider a partition of $\mathcal{I}$ into $\mathcal{P}$ and $\mathcal{B}$ such that $q_i^* = 0$, $i \in \mathcal{B}$ and $q_i^* = \theta_i$, $i \in \mathcal{P}$. Then by the above definition, given the convexity of y_i in q_i, $(q_i^*, \mathbf{q}_{-i})$ is a local Nash equilibrium if and only if $\frac{\partial y_i\left(\theta_i, \mathbf{q}^*_{-i}\right)}{\partial q_i} > 0$, $\forall\ i \in \mathcal{P}$ and $\frac{\partial y_i\left(0, \mathbf{q}^*_{-i}\right)}{\partial q_i} < 0$, $\forall\ i \in \mathcal{B}$. This is equivalent to

$$\beta_i < \frac{Q}{Q - \theta_i} - \alpha,\ i \in \mathcal{P} \tag{9}$$

and

$$\beta_i > 1 - \alpha,\ i \in \mathcal{B} \tag{10}$$

with $\alpha = \frac{\sum_{i \in \mathcal{B}} n_i b_i}{\sum_{i \in \mathcal{P}} n_i \theta_i} \leq 1$ in equilibrium and $Q = \sum_{i \in \mathcal{P}} \theta_i$. The local maximum at θ_i represented in Figure 1 is now admissible given α. Since the concept of local equilibrium is less demanding than the global concept, one expects an increase in the number of equilibria relative to the previous section.

Perhaps more surprisingly, comparative advantage does not necessarily govern role choices in equilibrium; absolute advantage also plays a role. Indeed, for a given level of comparative advantage, measured by $\beta_i = \frac{b_i}{\theta_i}$, condition (9) may be satisfied for individual i, whose absolute advantage is measured by θ_i, while that condition may be violated for another individual with the same comparative advantage but different absolute capabilities: $\beta_j = \beta_i$ but $\theta_j \neq \theta_i$.

Also, the two conditions (9) and (10) that define the local equilibrium are not mutually exclusive. Thus, unlike the standard (global) equilibrium, two identical individuals may be choosing opposite positions in equilibrium, one being an encroacher and the other a producer. Local equilibrium allows unequal behavior of identical individuals. If θ_i is small relative to aggregate equilibrium output, only individuals in a narrow range of β will meet conditions (9) and (10) simultaneously; on the contrary, if θ_i is a significant contribution to aggregate output,

whether because there are few different types or because few types produce in equilibrium, then the range of β_i compatible with both occupations is broader. In either case, for a level of relative efficacy compatible with encroachment, i.e. compatible with (10), a person with a high level of absolute productivity is more likely also to meet condition (9) for being a producer.

The same types of local equilibria are possible as in the case of global equilibria. The encroacher equilibrium (everybody is an encroacher) always exists. The producer equilibrium (everybody is a producer) exists if and only if, $\forall i,\ \beta_i < \frac{\sum_{j \in \mathcal{I}} n_j \theta_j}{\left(\sum_{j \in \mathcal{I}} n_j \theta_j\right) - n_i \theta_i}$.

(see (9)). This condition is less restrictive than the corresponding condition for a global equilibrium, $\beta_i \leq 1$ (see Theorem 3).

The conditions given in Theorem 3 for the existence of global interior equilibria are sufficient for local interior equilibria. They are not necessary, however, for two reasons. First Theorem 2 no longer applies. The partitioning of individuals into producers or encroachers may be done in more complex fashions than along the $\left[\beta^{\min}, \beta^{\max}\right]$ interval; the fixed point condition illustrated in Figure 2 is no longer necessary. Second, suppose we look for the subset of local interior equilibria obtained by partitioning individuals according to their β_i. Then some of these equilibria will be found by studying the fixed points of the relation obtained by transforming (10) into an equality: this gives (8) whose fixed points have been studied in the previous section. Some other local interior equilibria will be found by studying the fixed points of the relation obtained by transforming (9) into an equality:

$$\beta = \frac{Q(\beta)}{Q(\beta) - \theta_i} - \frac{B(\beta)}{Q(\beta)} \equiv G(\beta, \theta_i) \tag{11}$$

It may be shown that $G(\beta, \theta_i) \geq F(\beta)$, with equality if $\theta_i = 0$. Thus the existence of one or several fixed points of the $\beta = G(\beta, \theta_i)$ relation requires less stringent conditions than in the case of the $\beta = F(\beta)$ relation studied in Figure 2.

More generally, some partitions may be drawn according to other criteria than comparative advantage, measured by β_i. Consider the special case where comparative advantages are identical across individuals. There exists no interior global equilibrium in that case since the admissible portion of the $\beta = \beta$ line reduces to a point (see Figure 2). However an interior local equilibrium may exist which separates producers from encroachers on the basis of absolute productivity. With $\beta_i = \bar{\beta}\ \forall i$, such an equilibrium may be constructed as follows. Define a partition of $\mathcal{I}$ based on absolute advantage in production: $q_i = \theta_i$ if $\theta_i > \theta$ and $q_i = 0$ if $\theta_i \leq \theta$. Define $Q(\theta)$, $B(\theta)$, and $\alpha(\theta)$ accordingly. We look for a value θ^e such that, when $\theta = \theta^e$, the partition is a local Nash equilibrium. Let θ^e be defined by writing (10) as an equality: $\bar{\beta} = 1 - \alpha(\theta)$. Considering

(9) and (10), if θ^e exists, then the partition is an equilibrium. Since $1-\alpha(\theta)$ decreases from 1 to zero as θ increases from its minimum to its maximum, an equilibrium exists if $\bar{\beta} \leq 1$, as illustrated in Figure 4. Since the $1-\alpha(\theta)$ curve reaches the horizontal axis for $\theta < \theta^{max}$, the groups of encroachers and producers defined by the partition are both non empty if $\bar{\beta} < 1$. Unlike the Ricardian trade model, specialization occurs despite the fact that no agent has any comparative advantage over others; it is based on absolute advantage.

PLEASE INSERT FIGURE 4 AROUND HERE

To illustrate further, consider the case of two agents. There are four candidate equilibria: $(q_1 = q_2 = 0)$ which is always a global (and local) equilibrium; $(q_1 = \theta_1;\ q_2 = 0)$; $(q_1 = 0;\ q_2 = \theta_2)$; and $(q_1 = \theta_1;\ q_2 = \theta_2)$. The corresponding values of Q and α to be inserted into (9) and (10) are respectively $(Q = 0;\ \alpha = 1)$; $\left(Q = \theta_1;\ \alpha = \frac{b_2}{\theta_1}\right)$; $\left(Q = \theta_2;\ \alpha = \frac{b_1}{\theta_2}\right)$; $(Q = \theta_1 + \theta_2;\ \alpha = 0)$. Take $b_1 = 1,\ b_2 = 1.26,\ \theta_1 = 2,\ \theta_2 = 3$. Since $\beta_1 = .5 > \beta_2 = .42$, Theorem 2 implies that $(q_1 = \theta_1;\ q_2 = 0)$ is not a global equilibrium. However, for that allocation, (9) and (10) respectively give $\beta_1 = .5 < \frac{2}{2-2} - \frac{1.26}{2}$ and $\beta_2 = .42 > 1 - .63$, so that $(q_1 = \theta_1;\ q_2 = 0)$ is a local equilibrium, where the agent with a comparative advantage in production is an encroacher, and vice-versa. All other candidate allocations are equilibria as well.

6 Discussion

Like its ancestor the trade model of Ricardo, our model with constant returns in both production and encroachment implies that individuals specialize in either activity in equilibrium. Thus societies composed of individuals that differ only slightly from each other end up divided into strongly differentiated groups of producers and encroachers. Unlike the Ricardian trade model, this result does not rely on constant returns.

As is normal in games involving externalities, our model admits a multiplicity of equilibria in general. The extreme case of tragedy of the commons - no-one produces -

always exists. The Pareto optimal allocation is an equilibrium when all individuals are relatively productive; if some individual have high encroachment abilities, then there exists no Pareto optimal equilibrium. Despite the fact that individuals differ both in their abilities to encroach and in their productivities, the global Nash equilibria rank individuals by order of comparative advantage so that the equilibria are relatively easy to characterize.

The convexity of payoff functions leads one naturally to question the assumption of rationality. Indeed, in our model rational individuals must consider deviations that take them all the way from one end to the other end of the activity spectrum. The computational and informational requirements may be considered excessive. The concept of local Nash equilibrium introduced in the paper may be more appropriate in such circumstances. When local equilibria are allowed, individuals still make all or nothing decisions, but the division of society into encroachers and producers obeys more complex criteria than ranking by comparative advantage. In the special case where comparative advantages are identical there exists an interior equilibrium where society is divided according to absolute productivity.

We have represented individual abilities by the production and encroachment parameters θ_i and b_i; we have represented institutions as mitigating the ability to encroach by a factor λ. The multiplicity of equilibria and the important differences between them may explain why we observe large differences between societies whose individuals and even institutions are not very different. Our model is also compatible with the observation that individuals who migrate often change their behavior drastically: they adjust to a different equilibrium. Similarly, exporting institutions is rarely successful: similar institutions may give rise to very different equilibria. This also emphasizes the importance of history and coordination in explaining observed outcomes.

Encroachment redistributes income. Does that suggest that justice institutions should tolerate some level of encroachment or is a zero tolerance policy preferable? Our static analysis reveals a multiplicity of equilibria. In a dynamic setup, productivities may be affected by learning, and learning may not follow the same path in repetitions of one static equilibrium as in repetitions of another equilibrium. If tolerance of some encroachment induces faster growth in encroachment abilities and erosion in productive abilities, then this may lead to situations where the earlier equilibrium no longer exists and the economy may suddenly experience a dramatic shock as it moves to less productive an equilibrium or even disappears. From a more optimistic angle, a slight improvement in institutions may precipitate an economy toward a Pareto superior equilibrium (Figure 3).

Similar remarks apply with respect to immigration, or education. These activities change the proportion of individuals in each type category β_i. As we have illustrated such changes (for example an increase in n_2 in our two type example) may cause the disappearance of the interior equilibrium, leaving the extreme version of the tragedy of the commons as sole equilibrium.

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Appendix: decreasing returns in production and encroachment

Let individual output be $q_i = \theta_i f(e_i)$ and let individual encroachment be $b_i g(1-e_i)$ where the functions f and g are both positive, strictly increasing, nul at zero, and concave. Since f is invertible, the effort e_i necessary to produce q_i is $e_i = f^{-1}\left(\frac{q_i}{\theta_i}\right)$ and the reward from encroachment is $t_i = b_i g\left(1 - f^{-1}\left(\frac{q_i}{\theta_i}\right)\right)$. Let $h\left(\frac{q_i}{\theta_i}\right) \equiv g\left(1 - f^{-1}\left(\frac{q_i}{\theta_i}\right)\right)$. The expected income from producing q_i is thus $y_i = b_i h\left(\frac{q_i}{\theta_i}\right) + [1-\alpha]\, q_i$ where $\alpha = \frac{\sum_{j \in I} b_j h\left(\frac{q_j}{\theta_j}\right)}{\sum_{j \in I} q_j}$, $0 \leq q_i \leq \theta_i f(1)$.

The function $h(z_i)$ is non negative, decreasing, and concave:

$$
\begin{aligned}
h'(z_i) &= -\frac{g'\left(1 - f^{-1}\left(\frac{q_i}{\theta_i}\right)\right)}{f'(e_i)} < 0 \\
h''(z_i) &= \frac{g''\left(1 - f^{-1}\left(\frac{q_i}{\theta_i}\right)\right) + g'\left(1 - f^{-1}\left(\frac{q_i}{\theta_i}\right)\right)\frac{f''(e_i)}{f'(e_i)}}{f'(e_i)^2} < 0
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\frac{\partial y_i}{\partial q_i} &= -\frac{\partial \alpha}{\partial q_i} q_i + (1-\alpha) + \beta_i h'\left(\frac{q_i}{\theta_i}\right) \\
\frac{\partial^2 y_i}{\partial q_i^2} &= -\frac{\partial^2 \alpha}{\partial q_i^2} q_i - 2\frac{\partial \alpha}{\partial q_i} + \frac{b_i}{\theta_i^2} h''\left(\frac{q_i}{\theta_i}\right)
\end{aligned}
$$

with

$$\begin{aligned}\frac{\partial\alpha}{\partial q_i} &= \frac{1}{Q^2}\left[\beta_i h'\left(\frac{q_i}{\theta_i}\right)Q - \sum_{j\in I} b_j h\left(\frac{q_j}{\theta_j}\right)\right] \\ &= \frac{1}{Q}\left[\beta_i h'\left(\frac{q_i}{\theta_i}\right) - \alpha\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2\alpha}{\partial q_i^2} &= \frac{1}{Q^2}\left[\frac{\beta_i}{\theta_i}h''\left(\frac{q_i}{\theta_i}\right)Q - \frac{\partial\alpha}{\partial q_i}Q - \left[\beta_i h'\left(\frac{q_i}{\theta_i}\right) - \alpha\right]\right] \\ &= \frac{1}{Q^2}\left[\frac{\beta_i}{\theta_i}h''\left(\frac{q_i}{\theta_i}\right)Q - 2\beta_i h'\left(\frac{q_i}{\theta_i}\right) + 2\alpha\right]\end{aligned}$$

Some straightforward substitutions yield

$$\frac{\partial^2 y_i}{\partial q_i^2} = \left(1 - \frac{q_i}{Q}\right)\frac{2}{Q}\left(\alpha - \beta_i h'\right) + \left(1 - \frac{q_i}{Q}\right)\frac{1}{2}\frac{b_i}{\theta_i^2}h''$$

which is expression (4), where the first term is strictly positive, and the second term is non positive. Under constant returns, $h(z) = 1 - z$ and $h''(z) = 0$ so that $\frac{\partial^2 y_i}{\partial q_i^2} > 0$.

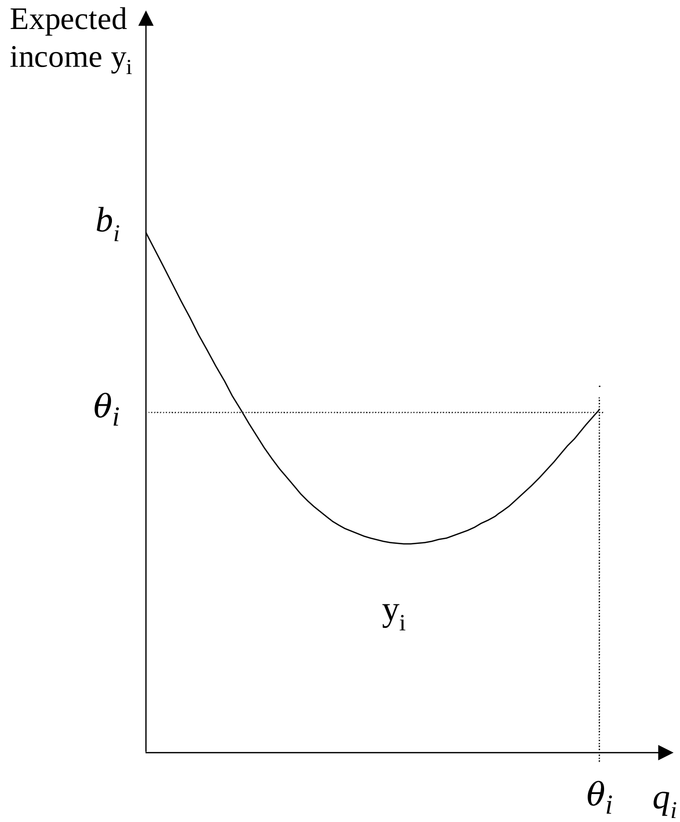
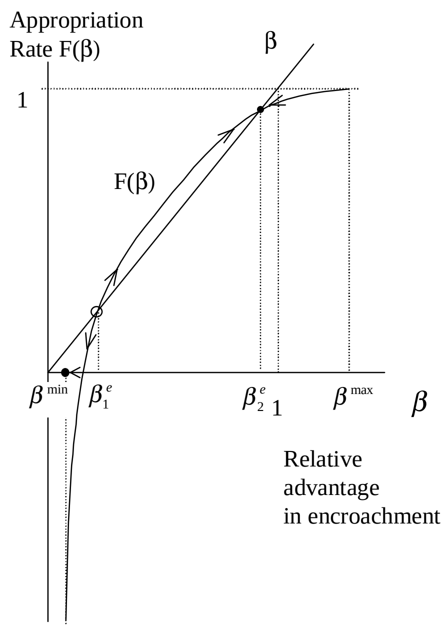
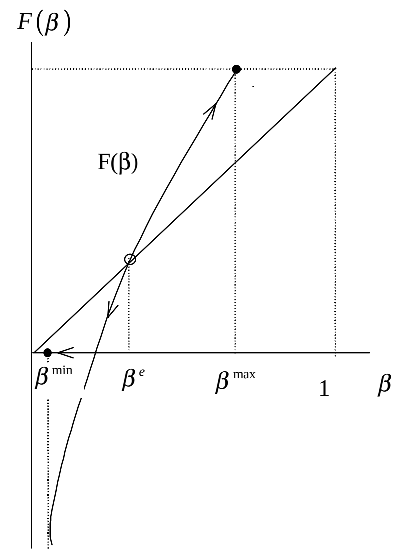


Figure 1: Individual income as a function of individual production



a) No Pareto optimal equilibrium



b) Pareto optimal equilibrium exists

Figure 2: Existence and stability of equilibria

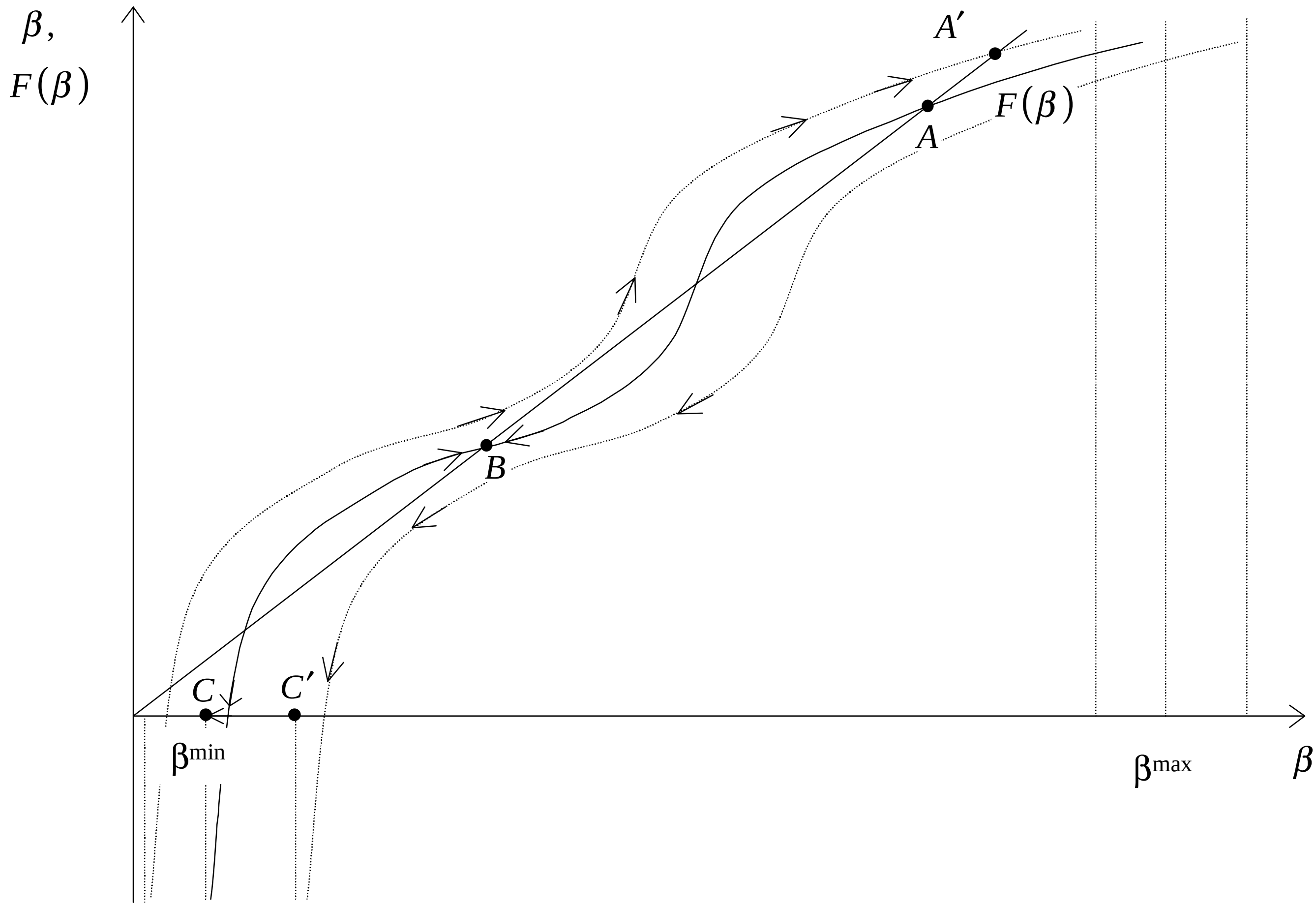


Figure 3: Institutional trap, miracle, and catastrophy

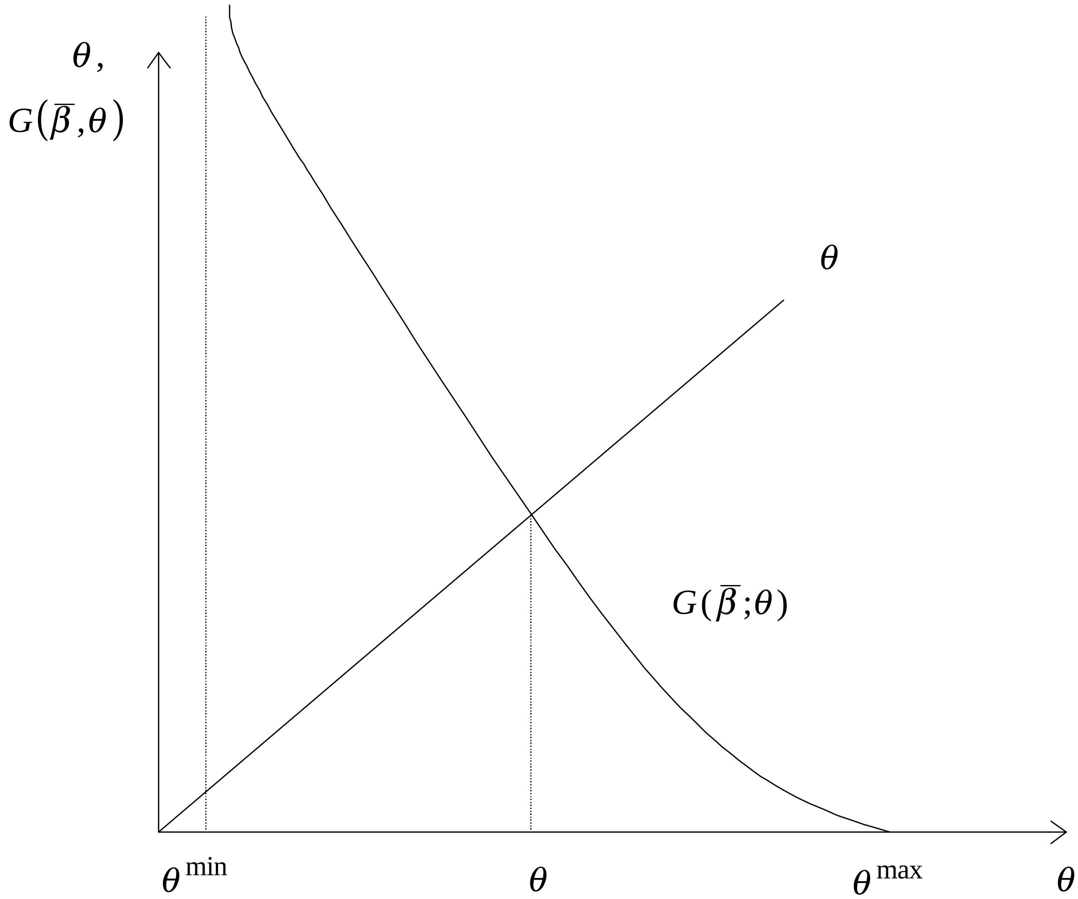


Figure 4: Local equilibrium with identical comparative advantages