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Evolutionary Stability in Common Pool Resources

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Abstract

The Tragedy of the Commons refers to the dissipation of a common-pool ressource when any appropriator has free access to it. Under the behavior of absolute payoff maximisation, the common-pool resource game leads to a Nash equilibrium in which the resource is overexploited. However, some empirical studies show that the overutilization is even larger than the Nash equilibrium predicts. We account for these results in an evolutionary framework. Under an imitation-experimentation dynamics, the long run stable behavior implies a larger exploitation of the resource than in the classical Nash equilibrium.

keywords: common-pool resource, imitation behavior, evolutionary stable strategy, evolutionary games.
JEL Classification: C73 , D41 , Q20

1 introduction

In 'The Tragedy of the Commons', Hardin (1968) raises the problem of exploitation of the common resources, like water, forests, oil fields, pastures and many others. If several appropriators have free access to a common-pool resource (CPR), then the resource is overexploited because agents do not consider the degradation of the resource when they appropriate it. The classical income maximizing Nash equilibrium is consistent with this phenomena. When the number of players is unlimited, the common resource is dissipated to the level where the average value of extraction equals the cost of the individual effort. As a result, the exploitation of the resource is higher than in a pareto-optimal equilibrium. Thus, every situations corresponding to a Tragedy of the Commons seem to be explained by the absolute maximization behavior adopted by fully rational agents.

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However, some laboratory experiments show that the common-pool resource is dissipated more than the Nash equilibrium predicts[1] . And empirical evidences like the dramatic deforestation of the amazonian forest suggest that the degradation sometimes exeeds the Nash equilibrium, as pointed out by Ito, Saijo and Une (1995). We demonstrate here that these facts are consistent with an evolutionary approach in which a limited number of agents adopt an imitation and experimentation behavior. Although most of the recent evolutionary analysis attempt to prove the existence of cooperation and altruism in games[2], our aim is conversly to study the competition resulting from the imitation behavior. This kind of competition is interesting because it can explain the situations where the dissipation exeeds the Nash equilibrium.

Why should we study the imitation behavior? Do the economic agents really imitate each other? Several empirical and theoretical studies proved that imitation is widely used by economic agents. It is often the best way to learn and optimize under imperfect information and uncertainty (Pingle and Day 1996). That's why there is a place for the imitation behavior and it is not absurd to assume that imitation could exist in a CPR game.

Vega-Redondo (1997) analysed a finite n-number Cournot Oligopoly where firms imitate those who earn the higher profit in the earlier period. The result is a convergence toward the walrasian outcome, instead of the Cournot-Nash equilibrium. The imitation dynamics describes a logic of *relative* profit maximization, leading to the perfect competition equilibrium. This competitive behavior has been extended to the class of submodular and quasisubmodular games (Schipper 2004, Alós-Ferrer and Ania 2005, Leininger 2006), which include the CPR game. We use a framework close to Hehenkamp et al. (2004) to show that the dissipation of a common resource is higher under the imitation-experimentation dynamics than in the classic Nash equilibrium (proposition 4).

2 The common-pool resource game

A common-pool resource game is a tuple $\Gamma = (n, S^n, \pi)$ where $n \geq 2$ is the number of players, $S \in \mathbb{R}_+$ is the strategy set common to all players, and π the payoff function. Each player chooses a level of effort (input) $s_i \in S^n$ in order to extract the resource. $S = \sum_{i=1}^n s_i$ is the sum of all the individual efforts. The extraction process is represented by a production function facing decreasing returns to scale $Y = f(S) = S^\theta$ with $\theta \in]0, 1[$. Each player recieves a part of the total output y in proportion of his individual effort.

[1]See Ostrom, Gardner and Walker (1994), chapters 5 and 6.

[2]Cooperation in the CPR game has been studied by Sethi and Somanathan (1996), Noailly et al. (2007).

We assume the game to be symmetric, then the payoff of agent i is given for all $s = (s_1, ..., s_n) \in S^n, i = (1, ..., n)$, by

$$\pi_i(s_i, S) = \frac{s_i}{S} f(S) - cs_i$$

where $c > 0$ is the cost of the individual effort, for example the wage rate. The individual payoff depends both on the player's strategy and the aggregate of all strategies chosen. Assume that all players adopt the following imitation and experimentation behavioral principles. For each discrete time period t, they imitate with probability $1 - \epsilon$ the strategy that gave the highest payoff in the earlier period among all the participants. With a small probability $\epsilon > 0$, they try a new strategy randomly choosen in the strategy set S^n. Note that this behavior does not require any knowledge about the payoff function. An agent only needs to know the individual strategy s_i of all the participants and their associated payoffs for the previous period.

3 Evolutionary stability

The main purpose in evolutionary games is to describe the long run behavior adopted by players. This is generally done by using the concept of *evolutionary stable strategy* (ESS). Here we define the ESS in the case of a finite population, and we show that it corresponds to the unique globally stable ESS, which is the long run behavior under the imitation-experimentation dynamics.

3.1 Evolutionary stable behavior

Maynard Smith (1982) defined the concept of an evolutionary stable strategy to characterize the long run equilibrium in evolutionary games. A strategy is said to be evolutionary stable if a population using this strategy cannot be invaded by a small group of mutants using another strategy. In other words, there exist an invasion barrier such that the ESS yields higher payoffs than the other strategies.

The Maynard Smith formal definition of an ESS holds for pairwise contests where two players are repeatedly chosen at random in an infinite population. But in many economic situations, including the common-pool extraction game, it is more realistic to assume that players take part simultaneously to the game and that the number of players, even large, is limited. Schaffer (1988) precisely adapted the notion of ESS to a finite population of agents who compete simultaneously.

Definition 1. (1-stable ESS) *Let $\Gamma = (n, S^n, \pi)$ be a symmetric game. The strategy s^* is a (strictly) 1-stable*[3] *ESS if $\forall\, s \in S^n$ and $s \neq s^*$,*

[3] Who resists to the appearance of one mutant at a time

$$\pi(s^*|s, s^*, \ldots, s^*) \geq (>) \, \pi(s|s^*, \ldots, s^*).$$

This definition means that in a 1-stable ESS, a single mutant always performs badly compared with the other players. We now determine the ESS of the CPR game in the next proposition.

proposition 1. *In the CPR game, the 1-stable ESS is the strategy* $s^* = c^{\frac{1}{\theta-1}}/n$.

Proof. The definition of an ESS means that

$$s^* \in arg \max_s [\pi_1(s|s^*, \ldots, s^*) - \pi_j(s^*|s, s^*, \ldots, s^*)], \; j = 2, \ldots, n.$$

If s^* is the candidate strategy for an ESS and s is the mutant strategy,

$$\begin{aligned}
\pi_1(s|s^*, \ldots, s^*) - \pi_j(s^*|s, s^*, \ldots, s^*) &= \frac{s - s^*}{s + (n-1)s^*}(s + (n-1)s^*)^\theta \\
&\quad -c(s - s^*) \\
&= (s - s^*)(s + (n-1)s^*)^{\theta-1} \\
&\quad -c(s - s^*)
\end{aligned}$$

the first order condition is given by

$$(s + (n-1)s^*)^{\theta-1} + \theta(s - s^*)(s + (n-1)s^*)^{\theta-2} = c$$

Let set $s = s^*$ because we look for a monomorphic state [4]. Then,

$$(ns^*)^{\theta-1} = c$$

$$ns^* = c^{\frac{1}{\theta-1}}$$

$$s^* = \frac{c^{\frac{1}{\theta-1}}}{n}$$

□

The 1-stable ESS s^* resists to the appearance of one mutant at a time. If we allow the mutants to appear in groups of any size $m \in [1, n-1]$, we must define an ESS as a globally stable ESS, like in the following definition.

[4] A state is said to be monomorphic when all agents play the same strategy.

Definition 2. **(Globally Stable ESS)** *Let* $\Gamma = (n, S^n, \pi)$ *be a symmetric game. The strategy* s^* *is a (strictly) globally stable ESS if* $\forall\, s \in S^n$ *and* $s \neq s^*$*,*

$$\pi(s^* | \underbrace{s, \ldots, s}_{m}, s^*, \ldots, s^*) \geq (>)\ \pi(s | \underbrace{s, \ldots, s}_{m-1}, s^*, \ldots, s^*) \quad \forall\, 1 \leq m \leq n-1.$$

We show now that the 1-stable ESS $s^* = c^{\frac{1}{\theta-1}}/n$ is globally stable. Resistance to one mutant implies resistance to m mutants in the CPR game.

Proposition 2. *Stability against invadibility by one mutant implies stability against any number of mutants. Then the ESS* $s^* = c^{\frac{1}{\theta-1}}/n$ *is globally stable.*

Proof. We use the same method than Hehenkamp et al. (2004). Write π_s^m the relative payoff of a mutant in respect to the ESS when the population contains m mutants.

$$\pi_s^m = (s - s^*)[ms + (n-m)s^*]^{\theta-1} - c(s - s^*) \qquad \forall s \neq s^*$$

Note that since s^* is an ESS, the relative profit of a single mutant is negative, $\pi_s^1 < 0$.

We can now determine the effect of m on the relative profit by derivating,

$$\frac{\partial \pi_s^m}{\partial m} \;=\; (s - s^*)^2(\theta - 1)[ms + (n-m)s^*]^{\theta-2} < 0$$

This expression is negative because $(\theta - 1)$ is negative and the two others terms in brackets are positive. Hence the influence of m is always negative. A group of m mutants is penalised by an additional mutant for all $m \in [1, n-1]$. Then stability against one mutant involves stability against any invasion by m mutants [5]. □

3.2 Stochastic stability

A globally stable ESS is the long run equilibrium of an evolutionary game if it is the unique stochastically stable state. Alós-Ferrer and Ania (2005) use results of Ellison (2000) to show that a globally stable ESS is effectively the unique stochastically stable state of a learning process based on imitation and experimentation behavior.

[5] This property applies more generally to symmetric aggregative games (Alós-Ferrer and Ania 2005, Leininger 2006)

Proposition 3. *Let s^* be a globally stable ESS. The profile $w^* = (s^*, \ldots, s^*)$ is the unique stochastically stable state of the imitation-experimentation dynamics. Then, in the CPR game all players adopt the behavior $s^* = c^{\frac{1}{\theta-1}}/n$ in the long run.*

Proof. cf Alós-Ferrer and Ania (proposition 4) ☐

4 Evolutionary stability and Nash equilibrium

In the classical form of the game where players maximize their absolute profit, the individual optimal strategy corresponds to a Nash equilibrium. It is well known that this behavior leads to an overutilization of the common resource. We show in the next proposition that the situation is worse in our evolutionary framework.

Proposition 4. *The optimal input effort is higher in the globally stable ESS than in the Nash equilibrium, which is itself higher than the pareto optimum. $S^* > S^N > S^P$ for all $n \geq 2$ players and for any cost level $c > 0$. Then, the resource is overexploited in a larger mesure under the logic of relative optimization than in the absolute maximization standard game, $Y^* > Y^N > Y^P$.*

Proof. First, we show that the aggregate effort level is higher in the Nash equilibrium than under the pareto-optimal exploitation, $S^N > S^P$. The pareto optimal condition is given by

$$\max S^P (S^P)^\theta / S^P - cS^P = (S^P)^\theta - cS^P$$

leads to

$$\theta (S^P)^{\theta-1} = c$$

$$S^P = \left(\frac{c}{\theta}\right)^{\frac{1}{\theta-1}}$$

S^P is the aggregate effort which maximises the sum of the individual profits. The condition to have a maximizing Nash equilibrium is

$$\frac{\partial \pi_i(s_1^N, \ldots, s_n^N)}{\partial s^N} = \frac{(S^N - s^N)}{(S^N)^2} f(S^N) + \frac{s^N}{S^N} f'(S^N) - c = 0$$

The sum of theses equations gives

$$\sum_{i=1}^{n} \frac{\partial \pi_i(s_1^N, \ldots, s_n^N)}{\partial s^N} = (n-1)\frac{f(S^N)}{S^N} + f'(S^N) - nc = 0$$

$$(n-1)\frac{f(S^N)}{S^N} + f'(S^N) = nc$$

We substitute $f(S)$ by S^θ

$$(n-1)(S^N)^{\theta-1} + \theta(S^N)^{\theta-1} = cn$$

$$[(n-1)+\theta](S^N)^{\theta-1} = cn$$

$$S^N = \left(\frac{cn}{n-1+\theta}\right)^{\frac{1}{\theta-1}}$$

is the aggregate effort level in the symmetric Nash equilibrium. $S^N > S^P$ if

$$\left(\frac{cn}{n-1+\theta}\right)^{\frac{1}{\theta-1}} > \left(\frac{c}{\theta}\right)^{\frac{1}{\theta-1}}$$

$$\frac{cn}{n-1+\theta} < \frac{c}{\theta}$$

$$\theta n < n-1+\theta$$

$$\theta < 1$$

which is true because of the decreasing returns to scale of the production function. So $S^N > S^P$ is the classical result that Nash equilibrium leads to the overexploitation of the resource.

Secondly. Recall that the ESS is $S^* = c^{1/(\theta-1)}$.
$S^* > S^N$ if

$$c^{\frac{1}{\theta-1}} > \left(\frac{cn}{n-1+\theta}\right)^{\frac{1}{\theta-1}}$$

$$c^{\frac{1}{1-\theta}} < \left(\frac{cn}{n-1+\theta}\right)^{\frac{1}{1-\theta}}$$

$$c < \frac{cn}{n-1+\theta}$$

$$\theta < 1$$

which, as before, is true because of the decreasing returns to scale. Then $S^* > S^N$, the aggregate effort is higher in the evolutionary stable equilibrium than in the Nash equilibrium. Finally, we have $S^* > S^N > S^P$. And since $Y = S^\theta$, $Y^* > Y^N > Y^P$. □

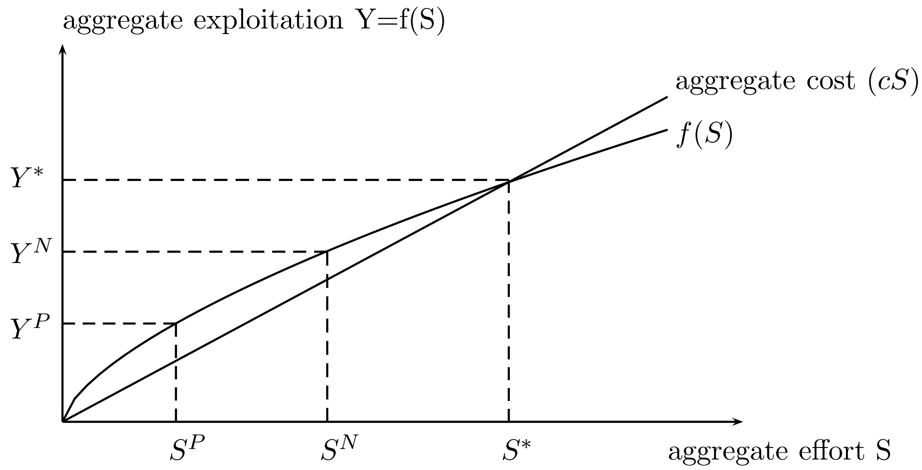

Figure 1 : Pareto-efficient, Nash equilibrium and ESS exploitation

In the standard Nash equilibrium, the resource can be dissipated at most to the level where the average value of extraction equals the individual cost of effort. This situation generates a lower profit than the benefit obtained under pareto optimality, but the profit is still positive. Under the logic of relative maximization, players increase their efforts until the average product equals the cost (figure 1), that is until the profit equals zero.

$$\begin{aligned}
\pi_i(s^*, S^*) &= \frac{s^*}{S^*} S^{*\theta} - cs^* \\
&= s^*(S^{*\theta-1} - c) \\
&= s^*(c - c) \\
&= 0
\end{aligned}$$

The agents increase their efforts until all opportunities of profit disappear. This result is similar to Vega-Redondo's Cournot oligopoly (1997). Under imitation and experimentation, the long run outcome of the Cournot oligopoly is the competitive equilibrium where profits equal zero.

Another interesting feature is that the Nash equilibrium approaches the finite population ESS when the number of players becomes large. If the population size grows up, $\lim_{n\to\infty} \frac{n}{n-1+\theta} = 1$ and then $\lim_{n\to\infty} S^N = S^*$. So when many rational players have free access to the ressource, the exploitation level approaches the finite population ESS.

5 Resource stock

What happens when we take into consideration the stock level of the resource? Noally et al. (2007) introduce the stock level K in the production function, which gives

$$Y_k = f(S, K) = S^\theta K^{1-\theta}$$

We can compute the new Nash equilibrium

$$(n-1)S^{\theta-1}K^{1-\theta} + \theta S^{\theta-1}K^{1-\theta} = nc$$

$$S^{\theta-1}K^{1-\theta} = \frac{nc}{n-1+\theta}$$

$$S_K^N = K\left(\frac{cn}{n-1+\theta}\right)^{\frac{1}{\theta-1}} = KS^N$$

Similarly the new ESS is

$$S_K^* = Kc^{\frac{1}{\theta-1}} = KS^*$$

and the Pareto optimal effort

$$S_K^P = K\left(\frac{c}{\theta}\right)^{\frac{1}{\theta-1}} = KS^P$$

We see that the effort is proportional to K in the three equilibrium concepts. Hence the result of the previous section holds when we introduce the stock level K.

6 conclusion

We have shown that the dissipation of a common resource is higher in an imitation-experimentation dynamics than in the classical Nash equilibrium. The Tragedy of the Commons is then attested in our evolutionary framework. The degradation of the resource is worse with imitators than with fully rational agents. The competition for capturing a common resource is tough because of the imitation behavior implying a logic of relative optimization. Agents are willing to grab more of the resource in order to maximize the difference between their own payoffs and the payoffs of the other players, until their profits equal zero.

Although many of evolutionary games treats about the emergence and the stability of cooperation and altruism, we should not forget that competition is at least important and is not the privilege of the non-evolutionary theory. A concrete example of fierce competition in common-pool resource is given by the rapid deforestation in South America. This 'Tragedy' could be explained by the evolutionary process based on the imitation behavior.

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