Complex Systems Modeled by Multiple Interacting Agents

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(Dated: 10th July 2008)

In this work we investigate learning as performed by agents interacting as they infer models of a complex system representable by a complex network, under the presence of observation errors. The models correspond to estimations of the adjacency matrix of the complex system under investigation. We focus the specific case in which, at each time step, each agent takes into account its just performed observation as well as the average of the models of its neighbors. A series of interesting results are identified with respect for Barabási-Albert interaction networks. First, it is shown that the interaction among agents allows an overall improvement in the quality of the estimated models, a consequence of the averaging among neighbors. We then investigate situations in which one of the agents has different probability of observation error (twice as much higher or lower than the other agents). It is shown that the influence of this special agent over the quality of the models throughout the rest of the network is substantial and varies linearly with the respective degree of the agent with different estimation errors. In case the degree of each agent is taken as a fitness parameter, in the sense that the influence of the node over the other agents is proportional to its degree, the effect of the different estimation error is even more pronounced, becoming superlinear.

PACS numbers: 07.05.Mh, 64.60.aq, 01.40.Ha

'Knowledge is of two kinds: we know a subject ourselves, or we know where we can find information upon it.' (S. Johson)

I. INTRODUCTION

Several important systems in nature, from the brain to society, are characterized by intricate organization. Being intrinsically related to such systems, humans have been trying to understand them through the construction of models which can reasonably reproduce and predict the respectively observed properties. Indeed, model building is the key component in the scientific method.

The development of a model is an art which involves the observation of the phenomenon of interest (including several measurements), its representation in mathematical terms, followed by simulations and respective confrontation with further experimental evidences. With the increase in complexity of the systems still capable of challenging science (and many they are), model building became intrinsically related to collaborations between scientists or *agents*. The problem of multiple-agent knowledge acquisition and processing has been treated in the literature (e.g. [1, 2]), but often under assumption of simple schemes of interactions between the agents (e.g. lattice or pool).

Introduced recently, complex networks ([3–7]) have quickly become a key research area mainly because of the generality of this approach to represent virtually any discrete system, allied to the possibilities of relating network topology and dynamics. As such, complex networks stand out as being a fundamental resource for complementing and enhancing the scientific method.

The present work addresses the issue of modeling how one or more agents (e.g. scientists) progress while modeling a complex system. We start by considering a single agent and then proceed to more general situations involving several agents interacting through networks of relationships (see Figure 1). The complex system to be investigated is assumed to be representable by a complex network with respectively associated dynamics. The agents investigating the system (one or more) are allowed to make observations and take measurements of the system as they develop and complement their respective individual models. For instance, one agent may perform a random walk through the system while mapping the connections into a respective growing network representation (e.g. [8]). Errors, incompleteness, noise and forgetting are typically involved during such a model estimation. The main features of interest include the quality of the obtained models and the respective amount of time required for their estimation. The plural in 'models' stands for the fact that the models obtained by each agent are not necessarily identical and will often imply substantial diversity. Though corresponding to a simplified version of real scientific investigation, our approach captures some of the main elements characterizing the involvement of a large number of interacting scientists who continuously exchange information and modify their respective models and modeling approaches. As a matter of fact, in some cases the development of models may even affect the system being modeled (e.g. the perturbation implied by the measurements on the analysed systems or the change of paradigms).

Because interactions between scientists can be effectively represented in terms of complex networks (e.g. [9–15]), it is natural to resource to such an approach in our investigation. It is interesting to observe that the agents may not be limited to scientists, but can also include intelligent machines and even reference databases and libraries. Though most of the previous approaches to

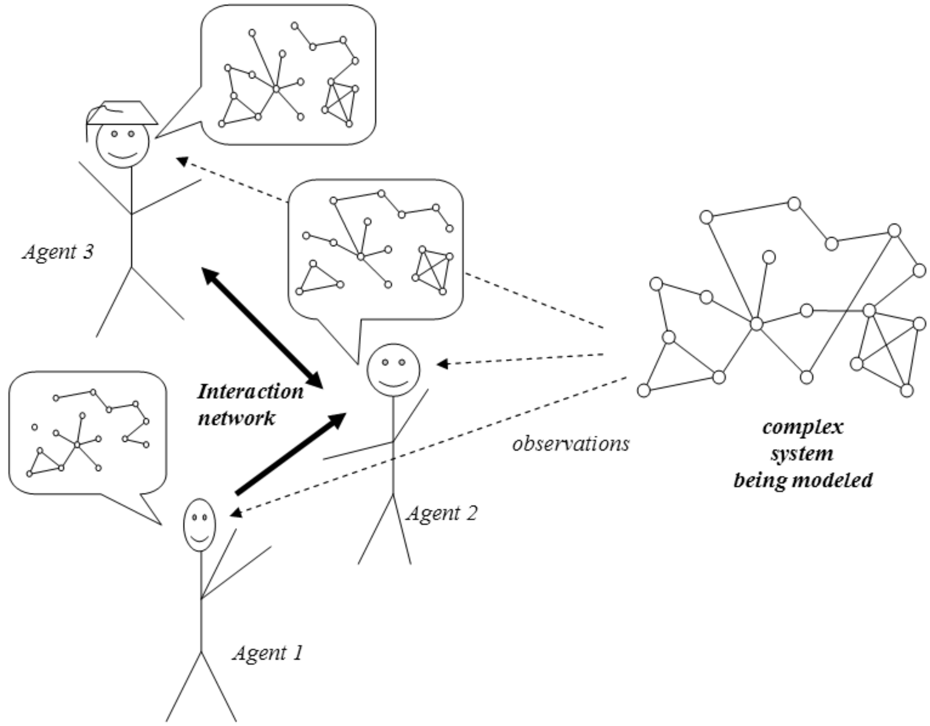


FIG. 1: Agents (scientists) develop their models of complex systems through observation and interactions.

modeling scientific interaction in terms of complex networks have focused on the topology of the collaborations, fewer approaches (e.g. [16]) have addressed the important issue of how the dynamics of learning/knowledge acquisition evolves in such systems. This is the main motivation of the current work.

This article starts by presenting the general assumptions and specifying and discussing successively more sophisticate levels of modeling. In order to illustrate the potential of the suggested approach, we include a simple illustration of the development of a model of the topology of a scale free network by a team of agents interacting through a network of that same type. During estimation, each agent takes into account not only its observation, but also the average of the models of its neighbors. The obtained results imply a series of interesting and important effects, such as the substantial role of hubs in affecting the estimation of models by all agents. While a linear relationship is identified between the overall estimation errors and the degree of single agents with different error rates, this relationship becomes superlinear when the degree of each node is considered as the respective fitness.

II. GENERAL ASSUMPTIONS

The first important assumption is that the complex systems under study be *representable as a discrete structure*, more specifically a graph or complex network. This hypothesis holds for a wide range of important natural and artificial systems, from protein-protein interaction to the Internet. Though the suggested methodology can be naturally used to investigate the learning about both structure and dynamics of complex systems, for simplicity's sake the attention in the present work is focused only on the structure/topology of the systems of interest. For instance, in the case of learning about the Internet, we would consider only its topological organization, involving servers represented as nodes and physical connections represented as links.

By *agent*, it is meant any entity capable of making observations/measurements of the system under analysis and storing this information for some period of time. Therefore, any scientist can be naturally represent as an agent. However, automated systems, from measurement stations to more sophisticated reasoning systems, can also be represented as agents. Actually, even less dynamic systems such as books or libraries can be thought of as a sort of passive agents, in the sense that they evolve (new editions and versions) as a consequence of incorporation of the evolution of knowledge.

Each agent is capable of making *observations/measurements* of the system under investigation. Though several types of observations and measurements are possible, here we focus attention on those more closely related to the topology of the complex network representing the system of interest, which may correspond to the identification of a node or link, as well as any of many possible topological measurements of the network organization (e.g. [6]). The process of measurement typically involves errors, which can be of several types such as getting an edge where there is none or missing an edge. Such errors can be a direct consequence of the limited accuracy of measurements devices as well as of the priorities and eventual biases (not to mention wishful thinking) of each agent. Several other possibilities, such as the existence of non-observable portions of the system, can also be considered and incorporated into the model. In addition, the model kept by each agent may undergo degradation as a consequence of noise and/or lost of memory.

Another important element in our approach is the incorporation of interactions between the agents, which can be represented in terms of a graph or network (see Figure 1). In this case, each agent is represented as a node, while interactions among them are represented by links. The single-agent configuration can be immediately thought of as a special case where the graph involves only one node (agent). Several types of interactions are possible, including but being not limited to conversations, attendance to talks or courses (the speaker becomes a temporary hub), article/book reading, and internet exchanges (e.g. e-mailing and surfing). Such interactions may also involve errors (e.g. misunderstanding words during a talk), which can be incorporated into our model whenever necessary. It is interesting to observe that the network of interaction between the agents may present dynamic topology, varying with time as a consequence of new scientific partnerships, addition or removal of agents, and so on.

Therefore, the framework considered in our investigation includes the following basic components: (i) the complex system under analysis S; (ii) one or more agents capable of taking observations/measurements of the system S (subject to error) and interacting one another (also subjected to error and forgetting); (iii) a network of interaction between the agents.

In the following we address, in progressive levels of sophistication, the modeling of knowledge acquisition/model building in terms of complex networks concepts and tools.

III. SINGLE-AGENT MODELING

A very simple situation would be the case where a single agent is allowed to observe, in uniformly random fashion, the presence or not of connections between all pairs of nodes of the complex system being modeled. The observation of each edge involves a probability γ of error, i.e. observing an edge where there is none and vice-versa. A possible procedure of model building adopted by the agent involves taking the average of all individual observations up to the current time stet T, i.e.

$$\langle x \rangle_T = \frac{1}{T} \sum_{t=1}^{T} x_t, \tag{1}$$

where x is the value of a specific edge (0 if non-existent and 1 otherwise)and x_t is the observation of x at time step t. Observe that we are considering the observation error to be independent along the whole system under analysis. Let us quantify the error for estimation of any edge, after T time steps as follows

$$\epsilon(x)_T = \left| x_* - \langle x \rangle_T \right|, \tag{2}$$

where x_* is the original value of that edge. It can be easily shown that

$$\lim_{T \to \infty} \epsilon(x)_T = \gamma \tag{3}$$

Because the observation error is independent among the pairs of nodes, the average of the errors along the whole network is identical to the limit above.

Thus, in this configuration the best situation which can be aimed at by the agent is to reach a representation of the connectivity of the original complex system up to an overall limiting error γ, reached after a considerable period of time. Though the speed of convergence is an interesting additional aspect to be investigated while modeling multi-agent knowledge acquisition, we leave this development for a forthcoming investigation.

IV. MULTIPLE-AGENT MODELING

We now turn our attention to a more interesting configuration in which a total of A agents interact while making observations of the complex system under analysis, along a sequence of time steps. As before, each agent observes the whole system at each time step, with error probability γ. In case no communication is available between the agents, each one will evolve exactly as discussed in the previous section. However, our main interest in this work is to investigate how *interactions* and *influences* between the multiple agents can affect the quality and speed at which the system of interest is learned. The original system under study is henceforth represented in terms of its respective adjacency matrix K_*. We also assume that the agents exchange information through a complex network of contacts.

One of the simplest, and still interesting, modeling strategies to be adopted by the agents of such a system involves the following dynamics: at each time step t, each of the agents i observes the connectivity of the complex system with error γ, yielding the adjacency matrix K_i^t, and also receives the current models from each of its immediate neighbors j (i.e. the agents to which it is directly connected). The agent i then calculates the mean adjacency matrix $\langle K \rangle$ of the adjacency matrices K_{ji} received from all its neighbors j, i.e.

$$\langle K \rangle = \sum_j K_{ji}. \tag{4}$$

The agent i then makes a weighted average between its just obtained observation K_i^t and the immediate neighbors mean matrix, i.e.

$$v_i^{t+1} = (a \langle K \rangle + (1-a)K_i^t), \tag{5}$$

where $0 < a \leq 1$ is a relative weight. We henceforth assume $a = 0.5$, so that $v_i^{(t+1)}$ becomes equal to the average between the current observation and the mean of the observations received from the neighbors at that time step. Each agent i subsequently adds this value to its current state, i.e.

$$S_i^{(t+1)} = S_i^{(t)} + v_i^{(t+1)}, \tag{6}$$

so that agent i estimation of the adjacency matrix at any time step t can be given as

$$K_i^{(t+1)} = S_i^{(t)}/t. \tag{7}$$

This simple type of dynamics has some immediate important consequences. First, because each node receive estimations from its neighbors at each time step, a better quality of estimation can be obtained faster than it would be in case the agent operated in isolation. Indeed, by averaging between its current observation and the mean estimation from the neighbors, even the limit error can be actually decreased with respect to the single agent situation. This situation can be clarified by considering the case in which a specific agent i is connected to p neighbors with unit degree (i.e. a star configuration, with agent i at the center). The quantity p may refer to the average node degree of the agents network or to individual agents. For simplicity's sake, in our current calculation these neighbors are considered to operate isolated, i.e. they do not consider the observations performed by agent i. Let us now focus attention on a specific edge known to exist in the original network. At a very large time instant, i.e. $t \to \infty$, the average value of this edge at any of the neighbors will necessarily be equal to γ as seen in the previous section. If the agent observes correctly this edge, its estimation after considering the observations from the p neighbors can be easily calculated as $(1 + (1-\gamma))/2 = 1 - \gamma/2$. However, in case the agent i observed the wrong value of the edge, i.e. 0, the estimation considering the mean value from the neighbors becomes $(0 + (1-\gamma))/2 = 0.5 - \gamma/2$. This implies in an absolute error estimation value necessarily smaller or equal than γ, meaning a reduction of the estimation error for $0 < \gamma < 1$. A similar situation arises regarding the observation of a non-existing edge, with identical respective estimation errors. This simple situation illustrates why the consideration of the mean of the neighbors observation at each step leads to a reduction of the overall mean error in the estimation. This is clearly a case where we could apply the old saying that 'The whole is greater than the sum of its parts'.

Several variations and elaborations of this model are possible, including the consideration of noise while transmitting the observations between adjacent agents, forgetting, the adoption of other values of a, as well as other averaging and noise schemes. In this article, however, we focus attention o the multi-agent model described in this section with error being present only during the observation by each agent. In the rest of our work, we report results of numerical simulation considering three particularly important situations: (i) multiple-agents with equal observation errors; (ii) as in (i) but with one of the agents having different observation error (half or twice as large as those of the other agents); and (iii) as in the previous configuration, but now with the degrees of each agent being considered as weights (fitness) during the neighborhood averaging step. The latter configuration is aimed at representing the situation in which the importance (henceforth called /emphfitness) of the opinion of each agent is proportional to its degree, so that hubs would be the most influent agents in the system. Interesting results are obtained for all these three configurations.

V. CASE EXAMPLES: SCALE-FREE NETWORKS OF AGENTS

In this section we investigate further the dynamics of multi-agent learning by considering simulations performed with a fixed collaboration network. More specifically, we assume that the agents collaborate through a scale free network, more specifically a Barabási-Albert network containing 20 nodes and average degree equal to 6. For simplicity's sake, we consider only a single realization of such a model in our subsequent investigation. The performance of each agent is quantified in terms of the error between the original network and the models obtained by that agent after a large number of time steps (henceforth assumed to be equal to $T = 300$ time steps). This error is calculated as

$$\varepsilon_p^{(T)} = \frac{1}{\gamma N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} |K_p(i,j)^{(T)} - K_*(i,j)| \qquad (8)$$

where K_p is the adjacency matrix representation of the model of agent p at time step T. The overall performance of the network, including all its agents, is henceforth expressed in terms of the average of the above error considering all agents, which is here called the *overall error* of the system.

The original network to be learnt by the agents, is a Barabási-Albert network containing 50 nodes and average degree equal to 6, represented by the respective adjacency matrix K_*.

A. Fixed Observation Errors Without Fitness

In this first configuration, all nodes have the same estimation error $\gamma = 0.2$ and fitness (equal to 1), and therefore the same influence, over the averages performed by each agent. However, nodes with higher degree, especially hubs, are still expected to influence more strongly the overall dynamics as a consequence of the fact that their estimated models are taken into account by a larger number of neighbors. Figure 2 shows the distribution of the average of the errors $\varepsilon_p^{(T)}$ obtained for this case in terms of the respective degrees. Two important results can be immediately identified: (i) the errors obtained among the agents are very similar one another; and (ii) the errors are smaller than γ, as a consequence of the averaging among neighbors. As could be expected, there is no major difference in the learning quality among the agents. A rather different situation arises in the next sections, where we consider different error rates.

B. Varying Observation Errors Withouth Fitness

We now repeat the previous configuration, but with one of the agents having half or twice the error rate of the others. Simulations are performed independently while placing the higher error rate at all of the possible nodes, while the respective overall errors are calculated. Figure 3(a) shows the results obtained while considering error probability $\gamma = 0.2$ for one of the agents and 0.4 for all other agents. Differently from the previous configuration, we now have a substantial difference of the quality of the models which depends linearly on the degree of each respective node. More specifically, it is clear that the errors are much smaller for nodes with higher degrees. In other words, the best models will be obtained when the hubs have smaller observation errors, influencing strongly the rest of the agents through the diffusion, along time, of their respectively estimated models.

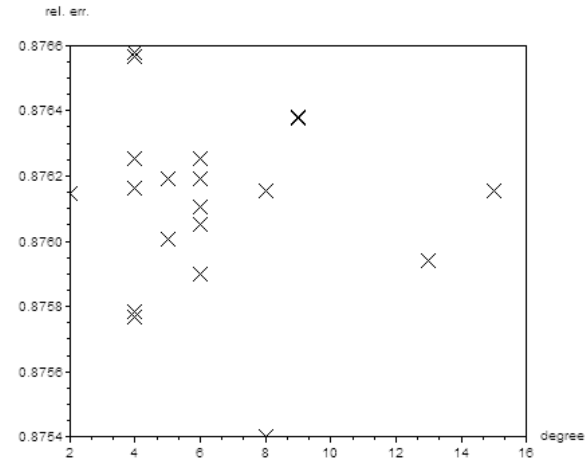


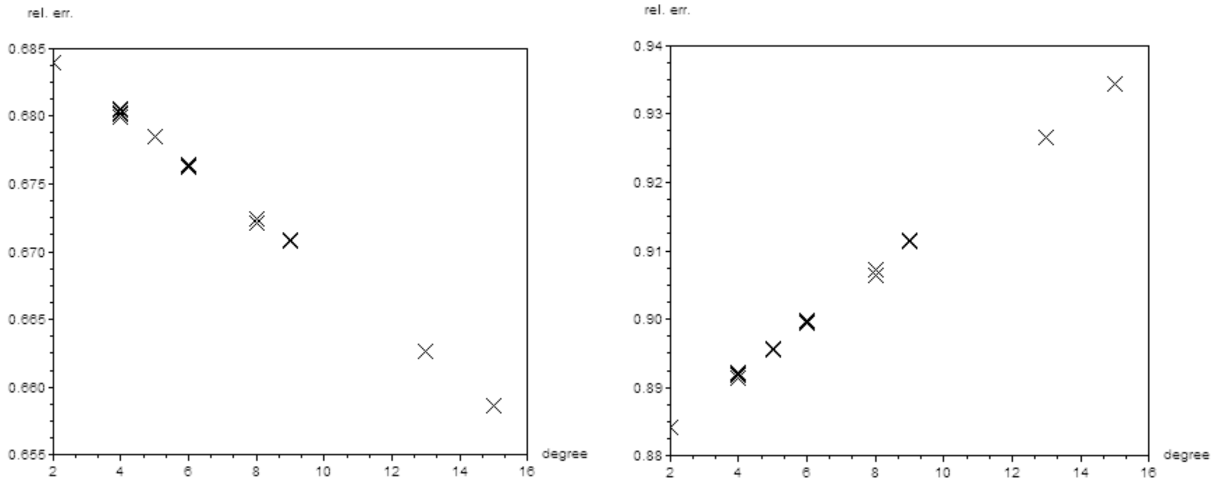
FIG. 2: The overall estimation errors in terms of the degree of the agents.

Figure 3(b) shows the results obtained when the estimation error of one of the agents is 0.4 while the rest of the agents have error rate 0.2. The opposite effect is verified, with the overall error increasing linearly with the degree of the node with higher estimation error.

A better picture of the influence of the degree over the model development by other agents can be obtained by considering the errors $\varepsilon_p^{(T)}$ of individual agents in terms of their respective degree. This is shown in Figure 4 respectively to the situations of having twice the error probability ($\gamma = 0.4$) for the agent with the smallest degree (a) and the network hub (b). These two agents have degrees 2 and 15 respectively. It is clear from these results that the degree of the differentiated agent affects the estimation of the whole set of agents, shifting substantially the average individual error. Observe also that, in both cases, the largest individual error results precisely at the less accurate agent.

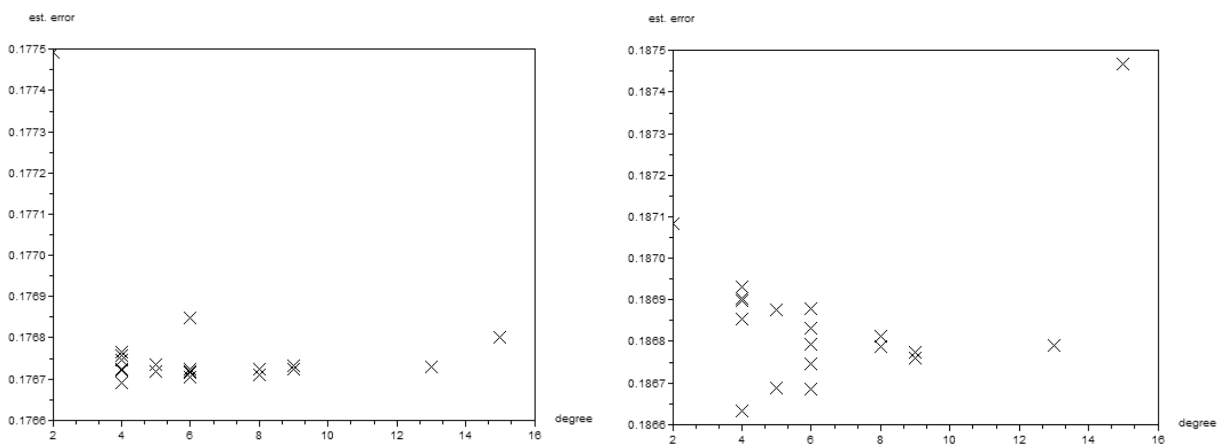
C. Varying Observation Errors With Fitness Proportional to the Degree

We now address the same situation as in the previous section, but with the difference that the influence (fitness) of the each node over the others (during the averaging) is assumed to be proportional to its degree. More specifically, the degree of each node is used to weight the corresponding adjacency matrix during the estimation of the averages by its neighbors in Equation 4. Figures 5(a) and (b) show the overall errors obtained by each agent in terms of the respective degree in case one of the agents has half (a) or twice (b)the estimation error (i.e. 0.2 against 0.4 for all other agents). Though similar to the



(a) (b)

FIG. 3: The overall estimation errors in terms of the degree of for agents with different error rates: (a) one agent with $\gamma = 0.2$ and all other agents with 0.4; and (b) one node with $\gamma = 0.4$ and all other nodes with 0.2.



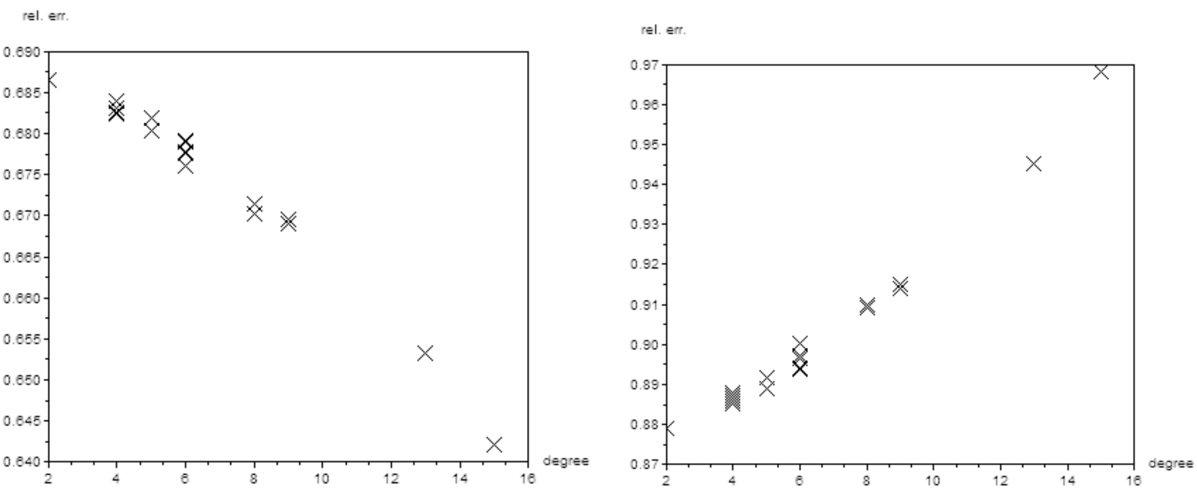
(a) (b)

FIG. 4: The errors of individual agents with respect to their degree obtained while having twice as large errors for the agent with the smallest degree (a) and the hub (b). These two agents have degrees 2 and 15, respectively.

results obtained in Section V B, the dependency of the overall errors with the degree is now superlinear, indicating that the nodes with higher degree have an even stronger influence on the estimations by the other agents along time.

VI. CONCLUDING REMARKS

The important problem of scientific interaction has been effectively investigated in terms of concepts and tools of complex networks research (e.g.[9–14]). However, most of the interest has been so far concentrated on characterizing the connectivity between scientists and institutions. The present work reported what is possibly the first approach to model the dynamics of knowledge



(a) (b)

FIG. 5: The overall estimation errors obtained by considering the fitness of each node (i.e. its degree) in terms of the degree with different error rates: (a) one node with $\gamma = 0.2$ and all other nodes with 0.4; and (b) one node with $\gamma = 0.4$ and all other nodes with 0.2.

acquisition (building a model of a complex system) by a system of multiple agents interacting through a specific complex network. Several configurations were considered at increasing levels of sophistication. Each agent was assumed to make an observation of the system of interest at each time step with error probability γ. A series of interesting and important results have been identified analytically and through simulations. First, we have that the overall estimation error tends to γ when the agents do not interact one another. However, smaller overall errors were observed when the agents were allowed to consider the average of models at each of their immediate neighbors. We also investigated the case in which one of the agents has a different observation error, yielding the important result that the overall error in such a configuration tends to vary linearly with the degree of the agent with different observation error. In other words, hubs will exert substantial influence over the models developed by the other agents, for better or for worse. In case the hubs have smaller observation errors, all agents will obtain improved models. Otherwise, the negative influence of the hubs is strongly felt throughout the system of agents. The situation in which the current models of each agent were taken with weight (fitness) proportional to their degree led to an intensification of the influence of degree and hubs, with a superlinear dependency between the overall estimation error and the degree of the node with different observation error. However, it is interesting to observe that agents with many connections will imply strong influences over the whole network even in case those agents have no special fitness. Such an effect is a direct consequence of the fact that those agents are heard by more people. In this work, such interesting phenomena were identified for a Barabási-Albert scale-free network, which is believed to be a good model of scientific collaborations.

This investigation has paved the way to a number of subsequent works, including but not being limited to: consideration of model degradation along time, other learning strategies, other types of networks, observation errors conditional to specific local features (e.g. degree or clustering coefficient) of the network being modeled, as well as other distribution of observation errors among the agents.

Acknowledgments

Luciano da F. Costa thanks CNPq (308231/03-1) and FAPESP (05/00587-5) for sponsorship. Part of this work was developed during the author's Visiting Scholarship at St. Catharine's College, University of Cambridge, UK.

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